Axioms of $\mathbb{R}$

Readings

- Part 3 of Introduction, Calculus Volume 1 — Apostol
- Sections 1-5 of Elementary Analysis — Kenneth Ross

Textbook Exercises

From Apostol Calculus vol 1 — Sections from the introduction — 3.3, 3.5, 3.12. Also prove all the properties listed in the entire part 3 of Introduction.

Exercises at the end of sections 3 and 4 of Analysis by Ross

Problems

Find the supremum and infimum (if they exist) of the following sets. Also decide which sets have maximum and minimum elements respectively.

(a) $\{x \mid x = 0 \text{ or } x = \frac{1}{n},\ n \in \mathbb{N}\}$
(b) $\{x \in \mathbb{R} \mid 0 \leq x \leq \sqrt{2},\ \text{and } x \text{ is a rational}\}$
(c) $\{x \in \mathbb{R} | x^2 + 2x + 2 \geq 0\}$
(d) $\{x \in \mathbb{R} | x < 0 \text{ and } x^2 + x - 1 < 0\}$
(e) $\{\frac{1}{n} + (-1)^n \mid\ n \in \mathbb{N}\}$
(f) $\left\{\left(1 - \frac{1}{\sqrt{n}}\right) \mid n \in \mathbb{N}\right\}$

2. Prove that $\inf(A + B) = \inf A + \inf B$ for nonempty sets A and B