Solutions to Homework Set Five
ECE 271A
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1.

a) In this case the BDR is to say

$$\begin{aligned}
g(\mathcal{X}) &= \arg\max_i \log P_{\mathbf{X}|Y}(\mathcal{X}|i) + \log \frac{1}{c} \\
&= \arg\max_i \sum_k \log P_{\mathbf{X}|Y}(\mathbf{x}_k|i) \\
&= \arg\max_i \frac{1}{n} \sum_k \log P_{\mathbf{X}|Y}(\mathbf{x}_k|i) \\
&\rightarrow \arg\max_i E_{\mathbf{X}}[\log P_{\mathbf{X}|Y}(\mathbf{x}|i)] \quad (\text{as } n \rightarrow \infty)
\end{aligned}$$

where the convergence is in probability and follows from the law of large numbers. This is equivalent to

$$\begin{aligned}
g(\mathcal{X}) &= \arg\min_i -E_{\mathbf{X}}[\log P_{\mathbf{X}|Y}(\mathbf{x}|i)] \\
&= \arg\min_i E_{\mathbf{X}}[\log Q_{\mathbf{X}}(\mathbf{x})] - E_{\mathbf{X}}[\log P_{\mathbf{X}|Y}(\mathbf{x}|i)] \\
&= \arg\min_i E_{\mathbf{X}}\left[\log \frac{Q_{\mathbf{X}}(\mathbf{x})}{P_{\mathbf{X}|Y}(\mathbf{x}|i)}\right] \\
&= \arg\min_i \int Q_{\mathbf{X}}(\mathbf{x}) \log \frac{Q_{\mathbf{X}}(\mathbf{x})}{P_{\mathbf{X}|Y}(\mathbf{x}|i)} d\mathbf{x} \\
&= \arg\min_i \mathcal{D}[Q_{\mathbf{X}}(\mathbf{x})||P_{\mathbf{X}|Y}(\mathbf{x}|i)]
\end{aligned}$$

where $Q_{\mathbf{X}}(\mathbf{x})$ is the density from which $\mathcal{X}$ was sampled. Hence, the BDR is equivalent to search for the class-conditional pdf $P_{\mathbf{X}|Y}(\mathbf{x}|i)$ that is closest, in the Kullback-Leibler sense, to $Q_{\mathbf{X}}(\mathbf{x})$.

b) Denoting

$$Q_{\mathbf{X}}(\mathbf{x}) = \mathcal{G}(\mathbf{x}, \mu_x, \mathbf{\Sigma}) = \frac{1}{\sqrt{(2\pi)^d |\mathbf{\Sigma}|}} e^{-\frac{1}{2}(\mathbf{x}-\mu_x)^T \mathbf{\Sigma}^{-1} (\mathbf{x}-\mu_x)^T}$$

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we have

$$\mathcal{D}[Q_{\mathbf{X}}(\mathbf{x})||P_{\mathbf{X}|Y}(\mathbf{x}|i)] = E_{\mathbf{X}}[\log \frac{Q_{\mathbf{X}}}{\ldots}]$$