Homework 2 - CSE 276C - Math for Robotics
Due: 21 October 2021

1. Prove that the first derivative $p_2'(x)$ of the parabola interpolating $f(x)$ at $x_0 < x_1 < x_2$ is equal to the straight line which takes on the value $f[x_{i-1}, x_i]$ at the point $(x_{i-1} + x_i)/2$, for $i = 1, 2$.

2. We decided to build a cheap robot with no floating point unit. Suppose you need to build an interpolation table with entries of the form $(x, f(x))$ for the function $f(x) = \cos x$ over the interval $[0, \pi]$. Please use uniform spacing between points.

   - What table spacing is required to ensure 6 decimal digit accuracy, assuming that you will use linear interpolation between adjacent points in the table?

   - How fine must it be if you use quadratic interpolation?

   - In each case, how many entries do you need in the table?

3. Implement Newton's Method. Consider the following equation:

$$x = \tan x.$$

   There are an infinite number of solutions x to this equation. Use Newton's method (and any techniques you need to start Newton in regions of convergence) to find the two solutions that are closest to 5.

4. We want to find the period of a trip around the sun as a function of distance to the sun, i.e., a year in that planets calendar. In the table shown below you can find 5 planets and their distance from the sun (in million kilometer) the number of days it takes to travel around the sun.

planets	Mercury	Venus	Earth	Mars	Jupiter
distance from the sun	58	108	149.5	227	778
days in a (planet) year	88	224.7	365.3	687	4331.5

   Find the function with the Lagrange method.

5. Alice's internet speed follows function f, and we know:

$$f'(x) = e^x - 4$$
$$f(0) = 2$$

   Help Alice figure out when her internet will stop working (by finding the root of f). Use the bisection method for this question.