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Problem 3:

(a) $A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ 5 & 5 & 5 \end{bmatrix}$ $b = \begin{bmatrix} 10 \\ -10 \\ 0 \end{bmatrix}$

$Ax = b$

$x = A^{-1}b$

Augmented matrix:

$$\begin{pmatrix} 2 & 1 & 3 & 10 \\ 2 & 1 & 2 & -10 \\ 5 & 5 & 5 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1/2 & 3/2 & 5 \\ 2 & 1 & 2 & -10 \\ 5 & 5 & 5 & 0 \end{pmatrix}$$

$R_1 = R_1/2$

$R_2 = R_2 - 2R_1$

$$\begin{pmatrix} 1 & 1/2 & 3/2 & 5 \\ 0 & 0 & -1 & -20 \\ 5 & 5 & 5 & 0 \end{pmatrix} \Leftarrow \begin{pmatrix} 1 & 1/2 & 3/2 & 5 \\ 0 & 0 & -1 & -20 \\ 0 & 5/2 & -5/2 & -25 \end{pmatrix}$$

$R_3 = R_3 - 5R_1$

$R_2 \Leftrightarrow R_3$

$$\begin{pmatrix} 1 & 1/2 & 3/2 & 5 \\ 0 & 5/2 & -5/2 & -25 \\ 0 & 0 & -1 & -20 \end{pmatrix}$$

$x_1 + x_2/2 + 3x_3/2 = 5$

$5x_2/2 - 5x_3/2 = -25$

$\Rightarrow x_3 = -20$

Contd..

Problem 3 contd..

$$x_1 + x_2/2 + 3x_3/2 = 5 \quad \rightarrow (1)$$

$$5x_2/2 - 5x_3/2 = -25 \quad \rightarrow (2)$$

$$\boxed{x_3 = 20} \quad \rightarrow (3)$$

(3) in (2)

$$5x_2/2 - \frac{5 \times 20^{10}}{2} = -25$$

$$\frac{5x_2}{2} = -25 + 50$$

$$\frac{5x_2}{2} = 25$$

$$\boxed{x_2 = 10} \quad \rightarrow (4)$$

(4) and (3) in (1)

$$x_1 + 10/2 + 3 \times 20^{10}/2 = 5$$

$$x_1 + 5 + 30 = 25$$

$$\boxed{x_1 = -30}$$

Soln for the given set of linear equations

$$x = \begin{bmatrix} -30 \\ 10 \\ 20 \end{bmatrix}$$

Problem 3b:

$$A = \begin{pmatrix} 8 & 14 & 0 \\ 2 & 2 & -6 \\ 1 & 2 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 6 \\ 5 \\ 1 \end{pmatrix}$$

Reducing to echelon form:

$$\begin{pmatrix} 8 & 14 & 0 & 6 \\ 2 & 2 & -6 & 5 \\ 1 & 2 & 1 & 1 \end{pmatrix} \Longrightarrow_{R_1 = R_1/8} \begin{pmatrix} 1 & 7/4 & 0 & 3/4 \\ 2 & 2 & -6 & 5 \\ 1 & 2 & 1 & 1 \end{pmatrix}$$

$\Downarrow R_2 = R_2 - 2R_1$

$$\begin{pmatrix} 1 & 7/4 & 0 & 3/4 \\ 0 & -3/2 & -6 & 7/2 \\ 0 & 1/4 & 1 & 1/4 \end{pmatrix} \Longleftarrow_{R_3 = R_3 - R_1} \begin{pmatrix} 1 & 7/4 & 0 & 3/4 \\ 0 & -3/2 & -6 & 7/2 \\ 1 & 2 & 1 & 1 \end{pmatrix}$$

$R_2 = \frac{-2R_2}{3}$

$$\begin{pmatrix} 1 & 7/4 & 0 & 3/4 \\ 0 & 1 & 4 & -7/3 \\ 0 & 1/4 & 1 & 1/4 \end{pmatrix} \Longrightarrow_{R_3 = R_3 - \frac{R_2}{4}} \begin{pmatrix} 1 & 7/4 & 0 & 3/4 \\ 0 & 1 & 4 & -7/3 \\ 0 & 0 & 0 & 115/12 \end{pmatrix}$$

System has no solutions

Problem 3c:

$$A = \begin{pmatrix} 4 & 7 & 0 \\ 2 & 2 & -6 \\ 1 & 2 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 18 \\ -12 \\ 8 \end{pmatrix}$$

$|A| = 4(2+12) - 7(2+6) = 56 - 56 = 0$

To determine whether the system has zero or infinitely many solutions,

$$\begin{pmatrix} 4 & 7 & 0 & 18 \\ 2 & 2 & -6 & -12 \\ 1 & 2 & 1 & 8 \end{pmatrix} \xrightarrow{R_1 = R_1/4} \begin{pmatrix} 1 & 7/4 & 0 & 9/2 \\ 2 & 2 & -6 & -12 \\ 1 & 2 & 1 & 8 \end{pmatrix} \xrightarrow{R_2 = R_2 - 2R_1} \begin{pmatrix} 1 & 7/4 & 0 & 9/2 \\ 0 & -3/2 & -6 & -21 \\ 1 & 2 & 1 & 8 \end{pmatrix}$$

$$\downarrow\downarrow R_3 = R_3 - R_1$$

$$\begin{pmatrix} 1 & 7/4 & 0 & 9/2 \\ 0 & -3/2 & -6 & -21 \\ 0 & 1/4 & 1 & 7/2 \end{pmatrix} \xrightarrow{R_2 = \frac{-2}{3}R_2} \begin{pmatrix} 1 & 7/4 & 0 & 9/2 \\ 0 & 1 & 4 & 14 \\ 0 & 1/4 & 1 & 7/2 \end{pmatrix}$$

$$\downarrow\downarrow R_3 = R_3 - R_2/4$$

$$\begin{pmatrix} 1 & 7/4 & 0 & 9/2 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Rank of the given matrix is 2, hence the system has infinitely many solutions