Solutions to Homework Set Four
ECE 271A
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1.

a) The main difference with respect to what we have seen so far is that, in the regression problem everything is conditioned on the knowledge of x. That is, we have

$$P_{\mathbf{z}|\mathbf{x},\theta}(\mathbf{z}|\mathbf{x},\theta) = \mathcal{G}(\mathbf{\Phi}(\mathbf{x})\theta, \mathbf{\Sigma}).$$

We break down $\mathbf{T}$ into the $\mathbf{x}$ and $\mathbf{z}$ components, i.e. $\mathbf{T} = (\mathbf{T_z}, \mathbf{T_x})$, $\mathcal{D}_x$ ($\mathcal{D}_y$) being a sample of the random variable $\mathbf{T_x}$ ($\mathbf{T_y}$). Hence, for the posterior we have

$$\begin{aligned}
P_{\theta|\mathbf{T}}(\theta|\mathcal{D}) &= P_{\theta|\mathbf{T_x},\mathbf{T_z}}(\theta|\mathcal{D}_x, \mathcal{D}_z) \\
&= \frac{P_{\mathbf{T_z}|\theta,\mathbf{T_x}}(\mathcal{D}_z|\theta, \mathcal{D}_x)P_{\theta|\mathbf{T_x}}(\theta|\mathcal{D}_x)}{\int P_{\mathbf{T_z}|\theta,\mathbf{T_x}}(\mathcal{D}_z|\theta, \mathcal{D}_x)P_{\theta|\mathbf{T_x}}(\theta|\mathcal{D}_x)} d\theta \\
&= \frac{P_{\mathbf{T_z}|\theta,\mathbf{T_x}}(\mathcal{D}_z|\theta, \mathcal{D}_x)P_{\theta}(\theta)}{\int P_{\mathbf{T_z}|\theta,\mathbf{T_x}}(\mathcal{D}_z|\theta, \mathcal{D}_x)P_{\theta}(\theta)} d\theta,
\end{aligned}$$

and, therefore,

$$\begin{aligned}
P_{\theta|\mathbf{T}}(\theta|\mathcal{D}) &\propto \exp\left\{-\frac{1}{2}\left[(\mathbf{z}-\mathbf{\Phi}\theta)^T\mathbf{\Sigma}^{-1}(\mathbf{z}-\mathbf{\Phi}\theta) + \theta^T\mathbf{\Gamma}^{-1}\theta\right]\right\} \\
&\propto \exp\left\{-\frac{1}{2}\left[\theta^T(\mathbf{\Phi}^T\mathbf{\Sigma}^{-1}\mathbf{\Phi} + \mathbf{\Gamma}^{-1})\theta - 2\theta^T\mathbf{\Phi}^T\mathbf{\Sigma}^{-1}\mathbf{z}\right]\right\}.
\end{aligned}$$

This is the same as

$$\begin{aligned}
P_{\theta|\mathbf{T}}(\theta|\mathcal{D}) &\propto \exp\left\{-\frac{1}{2}\left[(\theta-\mu_\theta)^T\mathbf{\Sigma}_\theta^{-1}(\theta-\mu_\theta)\right]\right\} \\
&\propto \exp\left\{-\frac{1}{2}\left[\theta^T\mathbf{\Sigma}_\theta^{-1}\theta - 2\theta^T\mathbf{\Sigma}_\theta^{-1}\mu_\theta\right]\right\}
\end{aligned}$$

when

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It follows that