Homework 3 - CSE 276 - Math for Robotics

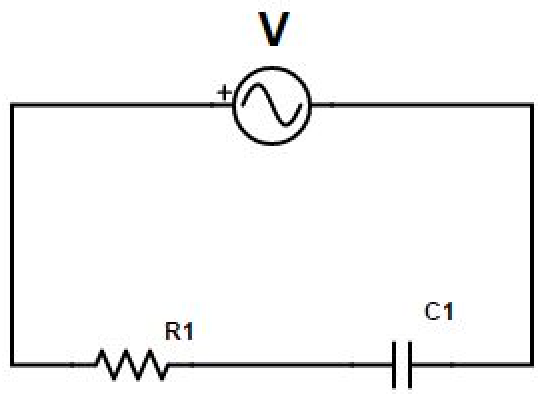
Due: 4 November 2021

1. In the following circuit, $V = 10^3 sin\sqrt{\pi t}$, $R_1 = 1k\Omega$ and $C_1 = 2mF$. The parameter q is considered as electrical charge for capacitor c_1. If $q(0) = 4C$ and $h = 0.1$, determine $q(t)$ at $t = 0.1$ using the following methods:

   (a) Euler's Method

   (b) 4^{th} Order Runge-Kutta's Method

   (hint: Write KVL and simplify it: $V - IR - \frac{q}{C} = 0 \rightarrow I = \frac{V}{R} - \frac{q}{RC} \rightarrow \frac{dq(t)}{dt} = \frac{V}{R} - \frac{q}{RC}$)



2. Let X be a random variable with probability density function (PDF) $f(x)$, as defined below:

$$f_X(x) = \begin{cases} \frac{1}{e}\left[e^x(x+1)\right] & \text{if } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

   Find the expected value of X, $\mathbf{E}[x]$, by ($h = 0.1$):

   (a) Rectangular method

   (b) Midpoint method

   (c) Trapezoidal method

3. Consider the following differential equation over the interval $(0, 1]$:

$$\frac{dy}{dx} = \frac{1}{x^2(1-y)}$$

   with y(1) = -1.

- Obtain an exact analytical solution to the equation

  In the following solve for y(0) even though in theory the equation is not defined for x=0.

- Implement and use Euler's method to solve the differential equation numerically. Use a step size of 0.05. How accurate is your numerical solution?

- Implement and use a fourth-order Runge-Kutta method to solve the differential equation numerically. Again, use a step size of 0.05. Again, how accurate is your numerical solution?

- Implement and use a Richardson extrapolation to solve the equation, again with a step size of 0.05. How accuracy is your solution compared to the analytical solution?

4. We have multiple robots that can generate point clouds such as those coming from a RealSense camera. In many cases we want to use the robots to detect objects in its enviroment. We provide three data files:

   (a) Empty2.asc which containts a data for an empty table

   (b) TableWithObjects2.asc contains point cloud for a cluttered table

   (c) hallway1b.asc contains data from a hallway

   Each file has the point cloud file in a format with each line contains x_i y_i z_i. You can use np.loadtxt to load a pointcloud into a numpy array.

   - Provide a method to estimate the plane parameter for the table. Test it both with the empty and cluttered table. Describe how you filter out the data from the objects. You have to be able to estimate the table parameters in the presence of clutter.

   - Describe and show how the method can be generalized to extract all the dominant planes in a relatively empty hallway.