Problem 5:

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 3 & x \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 3 \\ y \end{bmatrix}$$

(a) Values of x for the system to have unique solution.

$$\left(\begin{array}{cccc} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & x & y \end{array}\right) \xRightarrow[R_2 = R_2 - R_1]{} \left(\begin{array}{cccc} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 3 & x & y \end{array}\right)$$

$$\Downarrow R_3 = R_3 - R_1$$

$$(1) \Leftarrow \left(\begin{array}{cccc} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & (x-5) & (y-4) \end{array}\right) \xLeftarrow[R_3 = R_3 - 2R_2]{} \left(\begin{array}{cccc} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & (x-3) & (y-2) \end{array}\right)$$

$$x - 5 \neq 0$$

$$x \neq 5$$

System will have a unique solution for all $x \neq 5$

(b) and (c) contd...

(b) Values of x and y for the system to have no solution:

$$x - 5 = 0 \qquad y - 4 \neq 0$$
$$x = 5 \qquad y \neq 4$$

System will have no solⁿ when $x = 5$ and $y \neq 4$

(c) Values of x and y for the system to have infinitely many solutions

$$x - 5 = 0 \qquad y - 4 = 0$$
$$x = 5 \qquad y = 4$$

System will have infinitely many solⁿs when $x = 5$ and $y = 4$

Reason (b): System will have no solⁿ when $x = 5$ and $y \neq 4$ because from row echelon form (1),

$\cancel{0 \cdot x_3 = k}$ $\quad 0 = $ (A non zero value) which is inconsistent

Reason (c): System will have infinite solⁿs when $x = 5$ and $y = 4$ as one of the rows (in turn a variable) is redundant