Mid-term review solutions
ECE 271A
Electrical and Computer Engineering
University of California San Diego

Nuno Vasconcelos Fall 2014

1. a) The posterior is given by

$$\begin{aligned} P_{Y|\mathbf{X}}(1|\mathbf{x}) &= \frac{P_{\mathbf{X}|Y}(\mathbf{x}|1)P_Y(1)}{P_{\mathbf{X}|Y}(\mathbf{x}|1)P_Y(1)+P_{\mathbf{X}|Y}(\mathbf{x}|0)P_Y(0)} \\ &= \frac{P_{\mathbf{X}|Y}(\mathbf{x}|1)}{P_{\mathbf{X}|Y}(\mathbf{x}|1)+P_{\mathbf{X}|Y}(\mathbf{x}|0)} \\ &= \frac{1}{1+\frac{P_{\mathbf{X}|Y}(\mathbf{x}|0)}{P_{\mathbf{X}|Y}(\mathbf{x}|1)}} \\ &= \frac{1}{1+\frac{e^{-\frac{1}{2}(\mathbf{x}-\mu_0)^T\Sigma^{-1}(\mathbf{x}-\mu_0)}}{e^{-\frac{1}{2}(\mathbf{x}-\mu_1)^T\Sigma^{-1}(\mathbf{x}-\mu_1)}}} \\ &= \frac{1}{1+\frac{e^{\mu_0^T\Sigma^{-1}\mathbf{x}-\frac{1}{2}\mu_0^T\Sigma^{-1}\mu_0}}{e^{\mu_1^T\Sigma^{-1}\mathbf{x}-\frac{1}{2}\mu_1^T\Sigma^{-1}\mu_1}}} \\ &= \frac{1}{1+e^{(\mu_0-\mu_1)^T\Sigma^{-1}\mathbf{x}-\frac{1}{2}(\mu_0^T\Sigma^{-1}\mu_0-\mu_1^T\Sigma^{-1}\mu_1)}} \\ &= \frac{1}{1+e^{-\mathbf{w}^T\mathbf{t}}} \end{aligned}$$

with

$$\mathbf{w} = \begin{bmatrix} \Sigma^{-1}(\mu_1-\mu_0) \\ \frac{\mu_0^T\Sigma^{-1}\mu_0-\mu_1^T\Sigma^{-1}\mu_1}{2} \end{bmatrix}. \quad (1)$$

b) We start by noting that

$$P_{Y|\mathbf{X}}(y_i|\mathbf{x}_i) = \begin{cases} \frac{1}{1+e^{-\mathbf{w}^T\mathbf{t}_i}}, & y_i = 1 \\ 1-\frac{1}{1+e^{-\mathbf{w}^T\mathbf{t}_i}}, & y_i = 0 \end{cases}$$

which can be writ