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Problem 6:

Calculate the centroids

$$\vec{x}_c = \frac{1}{N} \vec{x}_{sum}$$

$x_{sum} \rightarrow$ Sum of all points in X

$N \rightarrow$ Number of points

$$\vec{y}_c = \frac{1}{N} \vec{y}_{sum}$$

$y_{sum} \rightarrow$ Sum of all points in Y

Considering 2 points in images

$$\vec{y}_2 - \vec{y}_c = R(\vec{x}_2 - \vec{x}_c)$$

$$\begin{pmatrix} \vec{y}_{21} - \vec{y}_{c1} \\ \vec{y}_{22} - \vec{y}_{c2} \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \begin{pmatrix} \vec{x}_{21} - \vec{x}_{c1} \\ \vec{x}_{22} - \vec{x}_{c2} \end{pmatrix}$$

y_{21}, x_{21}, y_{22} and x_{22} are 2 points in images x and y

$$\vec{y}_{21} - \vec{y}_{c1} = r_{11}(\vec{x}_{21} - \vec{x}_{c1}) + r_{12}(\vec{x}_{22} - \vec{x}_{c2}) \rightarrow ①$$

$$\vec{y}_{22} - \vec{y}_{c2} = r_{21}(\vec{x}_{21} - \vec{x}_{c1}) + r_{22}(\vec{x}_{22} - \vec{x}_{c2}) \rightarrow ②$$

$$\vec{y}_1 - \vec{y}_c = R(\vec{x}_1 - \vec{x}_c)$$

$$\begin{pmatrix} y_{11} - y_{c1} \\ y_{12} - y_{c2} \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \begin{pmatrix} x_{11} - x_{c1} \\ x_{12} - x_{c2} \end{pmatrix}$$

$$y_{11} - y_{c1} = r_{11}(x_{11} - x_{c1}) + r_{12}(x_{12} - x_{c2}) \rightarrow (3)$$
$$y_{12} - y_{c2} = r_{21}(x_{11} - x_{c1}) + r_{22}(x_{12} - x_{c2}) \rightarrow (4)$$

$$\vec{y} = R\vec{x} + \vec{t}$$

$$\begin{pmatrix} y_{c1} \\ y_{c2} \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \begin{pmatrix} x_{c1} \\ x_{c2} \end{pmatrix} + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$$

$$\cancel{y_1 = r_{11}x_1 + r_{12}x_2 + t_1 \rightarrow (5)}$$
$$\cancel{y_2 = r_{21}x_1 + r_{22}x_2 + t_2 \rightarrow (6)}$$

$$y_{c1} = r_{11}x_{c1} + r_{12}x_{c2} + t_1 \rightarrow (5)$$
$$y_{c2} = r_{21}x_{c1} + r_{22}x_{c2} + t_2 \rightarrow (6)$$

From the above equations;

$$y_{21} = r_{11}x_{21} + r_{12}x_{22} \rightarrow (7)$$

$$y_{22} = r_{21}x_{21} + r_{22}x_{22} \rightarrow (8)$$

$$y_{11} = r_{11}x_{11} + r_{12}x_{12} \rightarrow (9)$$

$$y_{12} = r_{21}x_{11} + r_{22}x_{12} \rightarrow (10)$$

writing eq^{n}s 5, 6, 7, 8, 9, 10 in matrix form

$$
\begin{pmatrix} y_{21} \\ y_{22} \\ y_{11} \\ y_{12} \\ y_{c1} \\ y_{c2} \end{pmatrix}
=
\begin{pmatrix}
x_{21} & x_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & x_{21} & x_{22} & 0 & 0 \\
x_{11} & x_{12} & 0 & 0 & 0 & 0 \\
0 & 0 & x_{11} & x_{12} & 0 & 0 \\
x_{c1} & x_{c2} & 0 & 0 & 1 & 0 \\
0 & 0 & x_{c1} & x_{c2} & 0 & 1
\end{pmatrix}
\begin{pmatrix} r_{11} \\ r_{12} \\ r_{21} \\ r_{22} \\ t_1 \\ t_2 \end{pmatrix}
$$

$$\vec{d} = M \; \vec{u}$$