SRINIDHI BHARADWAJ KANGUNDI SRINIVAS

Problem 4: Null spaces

(a)

$$A_1 = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 6 \end{bmatrix}$$

Row reduction:

$R_2 = R_1 + R_2$

$$A_1 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 6 \end{bmatrix}$$

$R_2 = R_2/3$

$$A_1 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$R_1 = R_1 + -2R_2$

$$A_1 = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$x_1 + 0 \times x_2 - 4x_3 = 0$$

$$\Rightarrow x_1 - 4x_3 = 0$$

$$x_2 + 2x_3 = 0$$

Let $x_3 = t$

$x_1 = 4t$

$x_2 = -2t$

Let $t = 1$

$$\therefore x = \begin{bmatrix} 4t \\ -2t \\ t \end{bmatrix}$$

Nullspace(A) = Span $\left\{ \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} \right\}$

Problem 4:

(b) $A_1 = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

Row echelon conversion:

$1 - 2/3 \times 2$
$= -4/3$
$3 - 4/3 = -1/3$

$R_2 = R_1 - 2/3 R_2$

$= \begin{bmatrix} 2 & 1 \\ 0 & -1/3 \end{bmatrix}$

$R_1 \to R_1/2$

$= \begin{bmatrix} 1 & 1/2 \\ 0 & -1/3 \end{bmatrix}$

$R_1 \to R_1 - 3/2 R_2$

$= \begin{bmatrix} 1 & 0 \\ 0 & -1/3 \end{bmatrix}$

Converting to equations:

$x_1 = 0$

$-x_2/3 = 0 \Rightarrow x_2 = 0$

Given system has only one solution

Nullspace $(A) = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$