

Srinidhi-HW5

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1 ECE 271A - Assignment 5

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3 A59010584

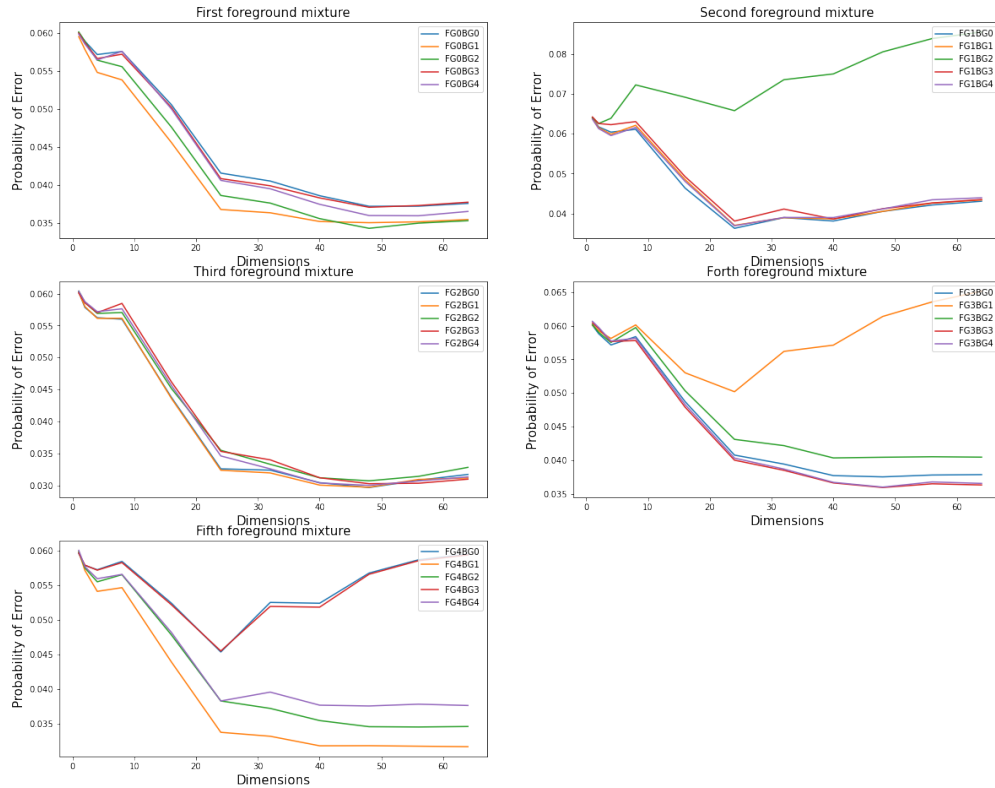
- a) For each class, learn 5 mixtures of $C = 8$ components, using a random initialization (recall that the mixture weights must add up to one). Plot the probability of error vs. dimension for each of the 25 classifiers obtained with all possible mixture pairs. Comment the dependence of the probability of error on the initialization.

Solution:

Below is the plot for probability of error for 25 classifiers. The curves varies slightly every-time the code is run because of random initialization.

As the number of dimensions are increased, the probability of error starts to decrease. It can be observed from the plot that the probability of error is at its minimum when the dimension is between 40 and 50. This is corroborated with what was observed in homework 2 where selecting best 8 features yielded better results than when 64 features were used.

The effect of random initialization is that the probabilities of error are different for different combinations of foreground and background mixtures



Probability of Error versus Dimensions

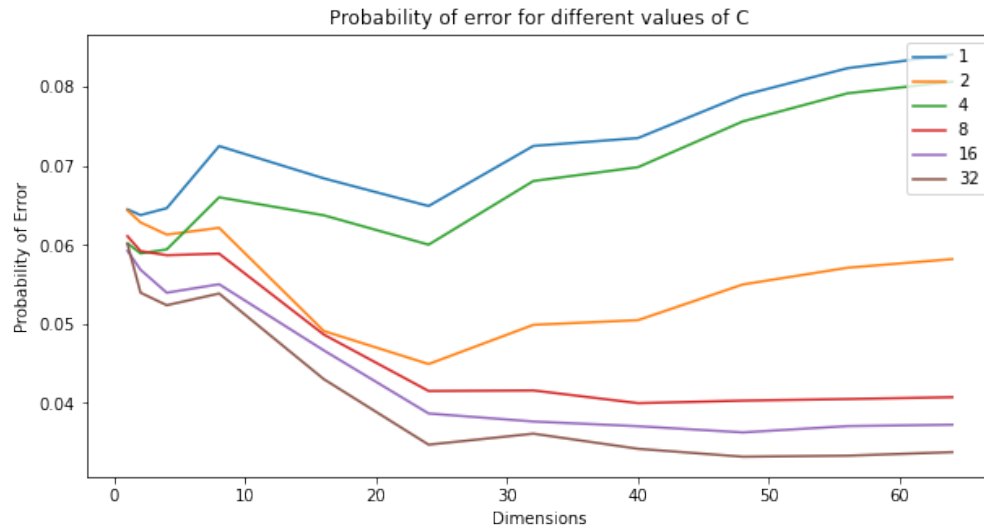
- b) For each class, learn mixtures with $C = 2, 4, 8, 16, 32$. Plot the probability of error vs. dimension for each number of mixture components. What is the effect of the number of mixture components on the probability of error?

Solution:

Below is the plot for Probabilities of Error versus different mixture components $C = \{1, 2, 4, 8, 16, 32\}$

It can be observed from the plot that using just 1 components to fit the data will result in high probability of error which is expected as not all data points can be fit perfectly. For the given randomly initialized parameters, 32 components yielded the best result. As the number of components increases, PoE decreases and a similar trend is followed for larger components. When the dimension is low, regardless of the number of components used, the value of PoE is high.

For a given dimension, PoE is dependent on the value of the number of components and the random initialization.



Probability of Error versus Dimensions for different components

```
[1]: import numpy as np
from scipy.io import loadmat
import matplotlib.pyplot as plt
import scipy
import scipy.fftpack
import cv2
from scipy import stats
%matplotlib inline

[2]: # Helper functions
def dct2d(feature):
    return scipy.fftpack.dct(scipy.fftpack.dct(feature, axis=0, norm='ortho'),
    ↪axis=1, norm='ortho')

#ZigZag transform
zigzag = np.array([[0,1,5,6,14,15,27,28],
                  [2,4,7,13,16,26,29,42],
                  [3,8,12,17,25,30,41,43],
                  [9,11,18,24,31,40,44,53],
                  [10,19,23,32,39,45,52,54],
                  [20,22,33,38,46,51,55,60],
                  [21,34,37,47,50,56,59,61],
                  [35,36,48,49,57,58,62,63]])

zigzagFlat = zigzag.flatten()
def zig_zag_transform(a):
    result = np.zeros(64)
    for i in range(64):
        result[zigzagFlat[i]] = a[i]
    return result
```

```

# Helper function that takes in the input of components and outputs random
→probability, mu and covariance values
def rand_init(components):
    pi = np.ones(components) * (1 / components) # Normalized values
    mu = np.random.randn(components, 64)
    cov = []
    for i in range(components):
        cov_temp = np.random.normal(5, 0.3, size=64)
        cov.append(np.diag(cov_temp))
    cov = np.array(cov)
    return pi,mu,cov

#Helper function to perform BDR
def gaussian_mixture_decision(pi_FG, mu_FG, cov_FG, pi_BG, mu_BG, cov_BG
→,image,dim):
    c = mu_FG.shape[0]
    im_blocks = calculate_dct(image)[:,:dim]

    #Vectorizing BDR
    p_y_x_cheetah, p_y_x_grass, A = np.zeros(247*262),np.zeros(247*262),np.
→zeros(247*262)

    #Calculate foreground probability
    for k in range(c):
        p_y_x_cheetah += stats.multivariate_normal.pdf(im_blocks,mean =
→mu_FG[k],cov = cov_FG[k]) * pi_FG[k]

    #Calculate background probability
    for k in range(c):
        p_y_x_grass += stats.multivariate_normal.pdf(im_blocks,mean =
→mu_BG[k],cov = cov_BG[k]) * pi_BG[k]
    A = p_y_x_cheetah - p_y_x_grass
    #print(A)
    A = np.where(A > 0,1,0)
    decision = np.reshape(A,(247,262))
    decision = np.lib.pad(decision,(4,4),'constant',constant_values = 0)
    return A, decision

#Helper function to calculate dct of input image
def calculate_dct(image):
    result = []
    for i in range(image.shape[0]-8):
        for j in range(image.shape[1]-8):
            row_start,row_end = i,i+8
            col_start,col_end = j,j+8
            block = image[row_start:row_end,col_start:col_end]
            block_dct = dct2d(block).flatten()
            block_dct= zig_zag_transform(block_dct)

```

```

        result.append(block_dct)
    result = np.array(result)
    return result

def expectation_maximization(c,sample,max_iter):
    pi,mu,cov = rand_init(c)
    for i in range(max_iter):
        # E-step
        H = []
        for j in range(c):
            H_temp = stats.multivariate_normal.pdf(sample,mean=mu[j,:
→],cov=cov[j,:,:]) * pi[j]
            H.append(H_temp)
        H = np.array(H).T
        H = H / np.sum(H,axis = 1)[:,np.newaxis]
        H_sum = np.sum(H,axis = 0)
        # M-step
        pi = 1 / sample.shape[0] * H_sum
        mu_update = []
        for j in range(c):
            mu_temp = np.sum(H[:,j][:,np.newaxis] * sample,axis = 0) / H_sum[j]
            mu_update.append(mu_temp)
        # update covariance
        cov_update = []
        for j in range(c):
            x_temp = sample - mu[j,:]
            cov_temp = np.sum((x_temp ** 2) * H[:,j][:,np.newaxis],axis = 0) /
→H_sum[j]
            cov_temp[cov_temp < 1e-6] = 1e-6 # Ensuring the covariance matrix
→is not singular
            cov_temp = np.diag(cov_temp)
            cov_update.append(cov_temp)
        cov = np.array(cov_update)
        mu = np.array(mu_update)
    return pi,mu,cov

def probability_of_error(predicted, image_mask, rows, cols):
    return np.sum(np.absolute(image_mask - predicted)) / (rows*cols)

```

```

[3]: TrainingSet = loadmat('TrainingSamplesDCT_8_new.mat')
    foreground,background =
→TrainingSet['TrainsampleDCT_FG'],TrainingSet['TrainsampleDCT_BG']

```

```

[4]: [fg_rows, fg_cols] = foreground.shape
    [bg_rows, bg_cols] = background.shape

    total_samples = fg_rows + bg_rows;

```

```

#Prior calculation, used later
prior_foreground = round(fg_rows / total_samples, 4);
prior_background = round(bg_rows / total_samples, 4);
print("Foreground prior value is {0} and Background prior value is {1}".
      →format(prior_foreground, prior_background))

```

Foreground prior value is 0.1919 and Background prior value is 0.8081

```

[5]: import imageio
mask_image = cv2.imread('../cheetah_mask.bmp', 0)
mask_image = np.array(mask_image)
mask_image = mask_image / 255

input_image = cv2.imread('../cheetah.bmp', 0)
input_image = input_image / 255 - 0.23529412
image_size_row = input_image.shape[0]
image_size_col = input_image.shape[1]

```

```

[6]: num_mixtures = 5
      num_components = 8

```

4 a)

```

[17]: error_dic = {}
dimensions = np.array([1,2,4,8,16,24,32,40,48,56,64])
for i in range(num_mixtures):
    pi_FG,mu_FG,cov_FG = expectation_maximization(8,foreground,200)
    print("EM for 5 different BG values for FG" + str(i) + " started.")
    for j in range(num_mixtures):
        pi_BG,mu_BG,cov_BG = expectation_maximization(8,background,200)
        error_list = []
        for dim in dimensions:
            mu_FG_cur,cov_FG_cur = mu_FG[:, :dim], cov_FG[:, :dim, :dim]
            mu_BG_cur,cov_BG_cur = mu_BG[:, :dim], cov_BG[:, :dim, :dim]

            A, bdr_out =
            →gaussian_mixture_decision(pi_FG,mu_FG_cur,cov_FG_cur,pi_BG,mu_BG_cur,
                                     cov_BG_cur, input_image, dim)

            #plt.imshow(bdr_out, cmap='gray')
            #plt.show()
            error = probability_of_error(bdr_out.flatten(),mask_image.
            →flatten(), image_size_row, image_size_col)
            error_list.append(error)
            print("Processing dimension {0} completed and the error for current
            →dim is {1}".format(dim, (error)))

```

```
error_list = np.array(error_list)
label = 'FG' + str(i) + 'BG' + str(j)
error_dic[label] = error_list
```

EM for 5 different BG values for FGO started.

Processing dimension 1 completed and the error for current dim is
0.056223674655047205
Processing dimension 2 completed and the error for current dim is
0.05519244734931009
Processing dimension 4 completed and the error for current dim is
0.054974582425562815
Processing dimension 8 completed and the error for current dim is
0.057342047930283226
Processing dimension 16 completed and the error for current dim is
0.05445170660856936
Processing dimension 24 completed and the error for current dim is
0.04946986201888163
Processing dimension 32 completed and the error for current dim is
0.056296296296296296
Processing dimension 40 completed and the error for current dim is
0.05803921568627451
Processing dimension 48 completed and the error for current dim is
0.06286129266521423
Processing dimension 56 completed and the error for current dim is
0.0643282498184459
Processing dimension 64 completed and the error for current dim is
0.06521423384168482
Processing dimension 1 completed and the error for current dim is
0.05648511256354394
Processing dimension 2 completed and the error for current dim is
0.05564270152505447
Processing dimension 4 completed and the error for current dim is
0.05449527959331881
Processing dimension 8 completed and the error for current dim is
0.05600580973129993
Processing dimension 16 completed and the error for current dim is
0.048932461873638346
Processing dimension 24 completed and the error for current dim is
0.0405519244734931
Processing dimension 32 completed and the error for current dim is
0.03957879448075526
Processing dimension 40 completed and the error for current dim is
0.03748729121278141
Processing dimension 48 completed and the error for current dim is
0.03652868554829339
Processing dimension 56 completed and the error for current dim is
0.036891793754538854

Processing dimension 64 completed and the error for current dim is
0.03726942628903413
Processing dimension 1 completed and the error for current dim is
0.05618010167029775
Processing dimension 2 completed and the error for current dim is
0.05507625272331155
Processing dimension 4 completed and the error for current dim is
0.054625998547567174
Processing dimension 8 completed and the error for current dim is
0.05632534495279593
Processing dimension 16 completed and the error for current dim is
0.05095134350036311
Processing dimension 24 completed and the error for current dim is
0.04336964415395788
Processing dimension 32 completed and the error for current dim is
0.04371822803195352
Processing dimension 40 completed and the error for current dim is
0.04395061728395062
Processing dimension 48 completed and the error for current dim is
0.04402323892519971
Processing dimension 56 completed and the error for current dim is
0.045214233841684826
Processing dimension 64 completed and the error for current dim is
0.04533042846768337
Processing dimension 1 completed and the error for current dim is
0.056659404502541755
Processing dimension 2 completed and the error for current dim is
0.05596223674655047
Processing dimension 4 completed and the error for current dim is
0.05488743645606391
Processing dimension 8 completed and the error for current dim is
0.056092955700798836
Processing dimension 16 completed and the error for current dim is
0.049920116194625996
Processing dimension 24 completed and the error for current dim is
0.04226579520697168
Processing dimension 32 completed and the error for current dim is
0.04246913580246914
Processing dimension 40 completed and the error for current dim is
0.04201888162672476
Processing dimension 48 completed and the error for current dim is
0.04184458968772694
Processing dimension 56 completed and the error for current dim is
0.0420479302832244
Processing dimension 64 completed and the error for current dim is
0.0418881626724764
Processing dimension 1 completed and the error for current dim is
0.056427015250544665

Processing dimension 2 completed and the error for current dim is
0.05562817719680465
Processing dimension 4 completed and the error for current dim is
0.05504720406681191
Processing dimension 8 completed and the error for current dim is
0.05760348583877996
Processing dimension 16 completed and the error for current dim is
0.05163398692810457
Processing dimension 24 completed and the error for current dim is
0.04329702251270879
Processing dimension 32 completed and the error for current dim is
0.042367465504720404
Processing dimension 40 completed and the error for current dim is
0.04021786492374728
Processing dimension 48 completed and the error for current dim is
0.03931735657225853
Processing dimension 56 completed and the error for current dim is
0.038707334785766156
Processing dimension 64 completed and the error for current dim is
0.039201161946259985
EM for 5 different BG values for FG1 started.
Processing dimension 1 completed and the error for current dim is
0.05808278867102396
Processing dimension 2 completed and the error for current dim is
0.05729847494553377
Processing dimension 4 completed and the error for current dim is
0.05571532316630356
Processing dimension 8 completed and the error for current dim is
0.05753086419753087
Processing dimension 16 completed and the error for current dim is
0.05112563543936093
Processing dimension 24 completed and the error for current dim is
0.04268700072621641
Processing dimension 32 completed and the error for current dim is
0.041089324618736385
Processing dimension 40 completed and the error for current dim is
0.039201161946259985
Processing dimension 48 completed and the error for current dim is
0.03832970225127088
Processing dimension 56 completed and the error for current dim is
0.03783587509077705
Processing dimension 64 completed and the error for current dim is
0.038082788671023965
Processing dimension 1 completed and the error for current dim is
0.05822803195352215
Processing dimension 2 completed and the error for current dim is
0.05803921568627451
Processing dimension 4 completed and the error for current dim is

0.05516339869281046
Processing dimension 8 completed and the error for current dim is
0.056223674655047205
Processing dimension 16 completed and the error for current dim is
0.04896151053013798
Processing dimension 24 completed and the error for current dim is
0.039694989106753814
Processing dimension 32 completed and the error for current dim is
0.038445896877269424
Processing dimension 40 completed and the error for current dim is
0.036485112563543934
Processing dimension 48 completed and the error for current dim is
0.03545388525780683
Processing dimension 56 completed and the error for current dim is
0.03525054466230937
Processing dimension 64 completed and the error for current dim is
0.03545388525780683
Processing dimension 1 completed and the error for current dim is
0.05837327523602034
Processing dimension 2 completed and the error for current dim is
0.05718228031953522
Processing dimension 4 completed and the error for current dim is
0.05655773420479303
Processing dimension 8 completed and the error for current dim is
0.05760348583877996
Processing dimension 16 completed and the error for current dim is
0.05177923021060276
Processing dimension 24 completed and the error for current dim is
0.043979665940450254
Processing dimension 32 completed and the error for current dim is
0.0458242556281772
Processing dimension 40 completed and the error for current dim is
0.04485112563543936
Processing dimension 48 completed and the error for current dim is
0.045170660856935364
Processing dimension 56 completed and the error for current dim is
0.04554829339143065
Processing dimension 64 completed and the error for current dim is
0.045649963689179375
Processing dimension 1 completed and the error for current dim is
0.05799564270152505
Processing dimension 2 completed and the error for current dim is
0.05776325344952796
Processing dimension 4 completed and the error for current dim is
0.05577342047930283
Processing dimension 8 completed and the error for current dim is
0.05774872912127814
Processing dimension 16 completed and the error for current dim is

0.051169208424110386
 Processing dimension 24 completed and the error for current dim is
 0.043050108932461875
 Processing dimension 32 completed and the error for current dim is
 0.04196078431372549
 Processing dimension 40 completed and the error for current dim is
 0.03972403776325345
 Processing dimension 48 completed and the error for current dim is
 0.03827160493827161
 Processing dimension 56 completed and the error for current dim is
 0.037937545388525784
 Processing dimension 64 completed and the error for current dim is
 0.03859114015976761
 Processing dimension 1 completed and the error for current dim is
 0.05828612926652142
 Processing dimension 2 completed and the error for current dim is
 0.058111837327523605
 Processing dimension 4 completed and the error for current dim is
 0.05507625272331155
 Processing dimension 8 completed and the error for current dim is
 0.056427015250544665
 Processing dimension 16 completed and the error for current dim is
 0.04903413217138707
 Processing dimension 24 completed and the error for current dim is
 0.039767610748002905
 Processing dimension 32 completed and the error for current dim is
 0.038707334785766156
 Processing dimension 40 completed and the error for current dim is
 0.0367755991285403
 Processing dimension 48 completed and the error for current dim is
 0.03546840958605665
 Processing dimension 56 completed and the error for current dim is
 0.03572984749455338
 Processing dimension 64 completed and the error for current dim is
 0.0357443718228032
 EM for 5 different BG values for FG2 started.
 Processing dimension 1 completed and the error for current dim is
 0.058053740014524326
 Processing dimension 2 completed and the error for current dim is
 0.05541031227305737
 Processing dimension 4 completed and the error for current dim is
 0.052578068264342775
 Processing dimension 8 completed and the error for current dim is
 0.05459694989106754
 Processing dimension 16 completed and the error for current dim is
 0.04549019607843137
 Processing dimension 24 completed and the error for current dim is
 0.03497458242556282

Processing dimension 32 completed and the error for current dim is
0.03420479302832244
Processing dimension 40 completed and the error for current dim is
0.0338562091503268
Processing dimension 48 completed and the error for current dim is
0.03263616557734205
Processing dimension 56 completed and the error for current dim is
0.03241830065359477
Processing dimension 64 completed and the error for current dim is
0.03286855482933914
Processing dimension 1 completed and the error for current dim is
0.058198983297022513
Processing dimension 2 completed and the error for current dim is
0.055875090777051564
Processing dimension 4 completed and the error for current dim is
0.053071895424836604
Processing dimension 8 completed and the error for current dim is
0.0540159767610748
Processing dimension 16 completed and the error for current dim is
0.04502541757443718
Processing dimension 24 completed and the error for current dim is
0.03488743645606391
Processing dimension 32 completed and the error for current dim is
0.03326071169208424
Processing dimension 40 completed and the error for current dim is
0.03270878721859114
Processing dimension 48 completed and the error for current dim is
0.031270878721859116
Processing dimension 56 completed and the error for current dim is
0.031067538126361656
Processing dimension 64 completed and the error for current dim is
0.03129992737835875
Processing dimension 1 completed and the error for current dim is
0.058053740014524326
Processing dimension 2 completed and the error for current dim is
0.05536673928830792
Processing dimension 4 completed and the error for current dim is
0.05260711692084241
Processing dimension 8 completed and the error for current dim is
0.054538852578068266
Processing dimension 16 completed and the error for current dim is
0.04560639070442992
Processing dimension 24 completed and the error for current dim is
0.03510530137981118
Processing dimension 32 completed and the error for current dim is
0.03448075526506899
Processing dimension 40 completed and the error for current dim is
0.03397240377632534

Processing dimension 48 completed and the error for current dim is
0.03294117647058824
Processing dimension 56 completed and the error for current dim is
0.0330718954248366
Processing dimension 64 completed and the error for current dim is
0.03349310094408134
Processing dimension 1 completed and the error for current dim is
0.057835875090777054
Processing dimension 2 completed and the error for current dim is
0.05516339869281046
Processing dimension 4 completed and the error for current dim is
0.052171387073347855
Processing dimension 8 completed and the error for current dim is
0.054422657952069714
Processing dimension 16 completed and the error for current dim is
0.04644880174291939
Processing dimension 24 completed and the error for current dim is
0.036891793754538854
Processing dimension 32 completed and the error for current dim is
0.036194625998547565
Processing dimension 40 completed and the error for current dim is
0.03522149600580973
Processing dimension 48 completed and the error for current dim is
0.034074074074074076
Processing dimension 56 completed and the error for current dim is
0.034088598402323894
Processing dimension 64 completed and the error for current dim is
0.03446623093681917
Processing dimension 1 completed and the error for current dim is
0.057777777777777775
Processing dimension 2 completed and the error for current dim is
0.055279593318809
Processing dimension 4 completed and the error for current dim is
0.05231663035584604
Processing dimension 8 completed and the error for current dim is
0.0542483660130719
Processing dimension 16 completed and the error for current dim is
0.04534495279593319
Processing dimension 24 completed and the error for current dim is
0.035003631082062454
Processing dimension 32 completed and the error for current dim is
0.03424836601307189
Processing dimension 40 completed and the error for current dim is
0.03342047930283224
Processing dimension 48 completed and the error for current dim is
0.032476397966594045
Processing dimension 56 completed and the error for current dim is
0.032766884531590414

Processing dimension 64 completed and the error for current dim is
0.033028322440087146
EM for 5 different BG values for FG3 started.
Processing dimension 1 completed and the error for current dim is
0.06342774146695715
Processing dimension 2 completed and the error for current dim is
0.06196078431372549
Processing dimension 4 completed and the error for current dim is
0.060159767610748004
Processing dimension 8 completed and the error for current dim is
0.06264342774146696
Processing dimension 16 completed and the error for current dim is
0.04820624546114742
Processing dimension 24 completed and the error for current dim is
0.037095134350036314
Processing dimension 32 completed and the error for current dim is
0.039694989106753814
Processing dimension 40 completed and the error for current dim is
0.03934640522875817
Processing dimension 48 completed and the error for current dim is
0.04225127087872186
Processing dimension 56 completed and the error for current dim is
0.04447349310094408
Processing dimension 64 completed and the error for current dim is
0.045083514887436456
Processing dimension 1 completed and the error for current dim is
0.06341321713870733
Processing dimension 2 completed and the error for current dim is
0.06209150326797386
Processing dimension 4 completed and the error for current dim is
0.06136528685548293
Processing dimension 8 completed and the error for current dim is
0.06261437908496732
Processing dimension 16 completed and the error for current dim is
0.04916485112563544
Processing dimension 24 completed and the error for current dim is
0.04495279593318809
Processing dimension 32 completed and the error for current dim is
0.05060275962236747
Processing dimension 40 completed and the error for current dim is
0.0514161220043573
Processing dimension 48 completed and the error for current dim is
0.05607843137254902
Processing dimension 56 completed and the error for current dim is
0.05844589687726943
Processing dimension 64 completed and the error for current dim is
0.05933188090050835
Processing dimension 1 completed and the error for current dim is

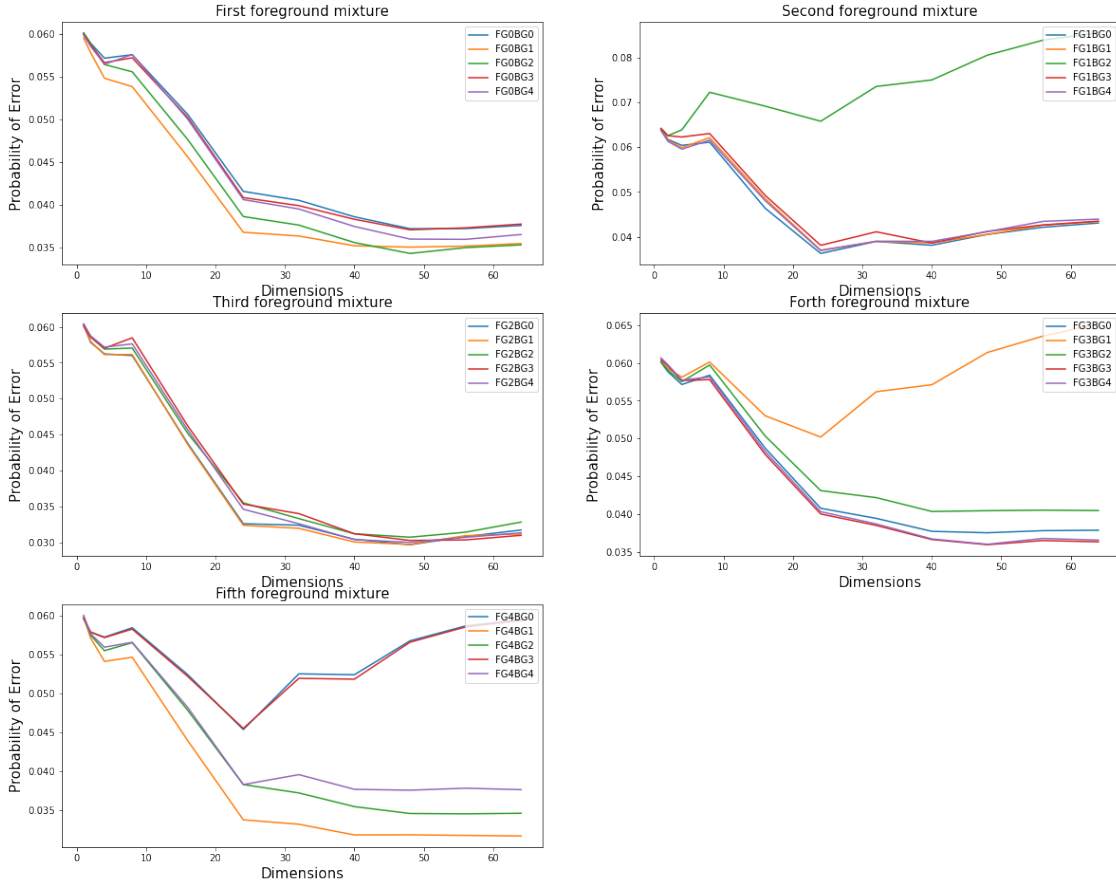
0.0633841684822077
Processing dimension 2 completed and the error for current dim is
0.06185911401597676
Processing dimension 4 completed and the error for current dim is
0.06061002178649237
Processing dimension 8 completed and the error for current dim is
0.06233841684822077
Processing dimension 16 completed and the error for current dim is
0.049368191721132895
Processing dimension 24 completed and the error for current dim is
0.045156136528685546
Processing dimension 32 completed and the error for current dim is
0.050210602759622365
Processing dimension 40 completed and the error for current dim is
0.05122730573710966
Processing dimension 48 completed and the error for current dim is
0.055381263616557735
Processing dimension 56 completed and the error for current dim is
0.0579520697167756
Processing dimension 64 completed and the error for current dim is
0.05864923747276688
Processing dimension 1 completed and the error for current dim is
0.06342774146695715
Processing dimension 2 completed and the error for current dim is
0.06194625998547567
Processing dimension 4 completed and the error for current dim is
0.060871459694989104
Processing dimension 8 completed and the error for current dim is
0.062193173565722584
Processing dimension 16 completed and the error for current dim is
0.04919389978213508
Processing dimension 24 completed and the error for current dim is
0.04505446623093682
Processing dimension 32 completed and the error for current dim is
0.050268700072621644
Processing dimension 40 completed and the error for current dim is
0.05115468409586057
Processing dimension 48 completed and the error for current dim is
0.05568627450980392
Processing dimension 56 completed and the error for current dim is
0.05779230210602759
Processing dimension 64 completed and the error for current dim is
0.059302832244008716
Processing dimension 1 completed and the error for current dim is
0.06342774146695715
Processing dimension 2 completed and the error for current dim is
0.061917211328976034
Processing dimension 4 completed and the error for current dim is

0.06011619462599855
Processing dimension 8 completed and the error for current dim is
0.06161220043572985
Processing dimension 16 completed and the error for current dim is
0.045737109658678284
Processing dimension 24 completed and the error for current dim is
0.03523602033405955
Processing dimension 32 completed and the error for current dim is
0.03798111837327524
Processing dimension 40 completed and the error for current dim is
0.037443718228031955
Processing dimension 48 completed and the error for current dim is
0.038997821350762525
Processing dimension 56 completed and the error for current dim is
0.040740740740740744
Processing dimension 64 completed and the error for current dim is
0.04148148148148148
EM for 5 different BG values for FG4 started.
Processing dimension 1 completed and the error for current dim is
0.057545388525780686
Processing dimension 2 completed and the error for current dim is
0.05493100944081336
Processing dimension 4 completed and the error for current dim is
0.05243282498184459
Processing dimension 8 completed and the error for current dim is
0.05281045751633987
Processing dimension 16 completed and the error for current dim is
0.042556281771968046
Processing dimension 24 completed and the error for current dim is
0.034800290486564994
Processing dimension 32 completed and the error for current dim is
0.03360929557007988
Processing dimension 40 completed and the error for current dim is
0.03195352214960058
Processing dimension 48 completed and the error for current dim is
0.031154684095860568
Processing dimension 56 completed and the error for current dim is
0.031430646332607115
Processing dimension 64 completed and the error for current dim is
0.031343500363108207
Processing dimension 1 completed and the error for current dim is
0.05753086419753087
Processing dimension 2 completed and the error for current dim is
0.054466230936819175
Processing dimension 4 completed and the error for current dim is
0.0523602033405955
Processing dimension 8 completed and the error for current dim is
0.05400145243282498

Processing dimension 16 completed and the error for current dim is
0.04411038489469862
Processing dimension 24 completed and the error for current dim is
0.03594771241830065
Processing dimension 32 completed and the error for current dim is
0.03536673928830791
Processing dimension 40 completed and the error for current dim is
0.03404502541757444
Processing dimension 48 completed and the error for current dim is
0.03328976034858388
Processing dimension 56 completed and the error for current dim is
0.033667392883079156
Processing dimension 64 completed and the error for current dim is
0.03403050108932462
Processing dimension 1 completed and the error for current dim is
0.057545388525780686
Processing dimension 2 completed and the error for current dim is
0.054742193173565726
Processing dimension 4 completed and the error for current dim is
0.05211328976034858
Processing dimension 8 completed and the error for current dim is
0.05324618736383442
Processing dimension 16 completed and the error for current dim is
0.04258533042846768
Processing dimension 24 completed and the error for current dim is
0.03490196078431373
Processing dimension 32 completed and the error for current dim is
0.03410312273057371
Processing dimension 40 completed and the error for current dim is
0.03209876543209877
Processing dimension 48 completed and the error for current dim is
0.03129992737835875
Processing dimension 56 completed and the error for current dim is
0.03144517066085693
Processing dimension 64 completed and the error for current dim is
0.03145969498910675
Processing dimension 1 completed and the error for current dim is
0.05767610748002905
Processing dimension 2 completed and the error for current dim is
0.05458242556281772
Processing dimension 4 completed and the error for current dim is
0.052650689905591866
Processing dimension 8 completed and the error for current dim is
0.053986928104575164
Processing dimension 16 completed and the error for current dim is
0.04437182280319535
Processing dimension 24 completed and the error for current dim is
0.03599128540305011

Processing dimension 32 completed and the error for current dim is
0.03514887436456064
Processing dimension 40 completed and the error for current dim is
0.03378358750907771
Processing dimension 48 completed and the error for current dim is
0.03327523602033406
Processing dimension 56 completed and the error for current dim is
0.03314451706608569
Processing dimension 64 completed and the error for current dim is
0.03337690631808279
Processing dimension 1 completed and the error for current dim is
0.057705156136528685
Processing dimension 2 completed and the error for current dim is
0.055250544662309366
Processing dimension 4 completed and the error for current dim is
0.05249092229484386
Processing dimension 8 completed and the error for current dim is
0.05336238198983297
Processing dimension 16 completed and the error for current dim is
0.04366013071895425
Processing dimension 24 completed and the error for current dim is
0.03572984749455338
Processing dimension 32 completed and the error for current dim is
0.03488743645606391
Processing dimension 40 completed and the error for current dim is
0.033231663035584606
Processing dimension 48 completed and the error for current dim is
0.03237472766884532
Processing dimension 56 completed and the error for current dim is
0.03292665214233842
Processing dimension 64 completed and the error for current dim is
0.03288307915758896

```
[16]: fig=plt.figure(figsize=(20,16))
      voc = ['First', 'Second', 'Third', 'Forth', 'Fifth']
      for i in range(num_mixtures):
          fig.add_subplot(3,2,i+1)
          plt.plot(dimensions,error_dic['FG'+str(i)+'BG0'],label = 'FG'+str(i)+'BG0')
          plt.plot(dimensions,error_dic['FG'+str(i)+'BG1'],label = 'FG'+str(i)+'BG1')
          plt.plot(dimensions,error_dic['FG'+str(i)+'BG2'],label = 'FG'+str(i)+'BG2')
          plt.plot(dimensions,error_dic['FG'+str(i)+'BG3'],label = 'FG'+str(i)+'BG3')
          plt.plot(dimensions,error_dic['FG'+str(i)+'BG4'],label = 'FG'+str(i)+'BG4')
          plt.title(voc[i] + ' foreground mixture', fontdict={ 'size' : 15})
          plt.legend(loc='upper right')
          plt.xlabel('Dimensions', fontdict={ 'size' : 15})
          plt.ylabel('Probability of Error', fontdict={'size': 15})
```



5 b)

```
[12]: error_mixtures = {}
dimensions = np.array([1,2,4,8,16,24,32,40,48,56,64])
mixtures = np.array([1,2,4,8,16,32])
for c in mixtures:
    error_list = []
    pi_FG, mu_FG, cov_FG = expectation_maximization(c,foreground,200)
    pi_BG, mu_BG, cov_BG = expectation_maximization(c,background,200)
    for dim in dimensions:
        mu_FG_cur,cov_FG_cur = mu_FG[:, :dim],cov_FG[:, :dim,:dim]
        mu_BG_cur,cov_BG_cur = mu_BG[:, :dim],cov_BG[:, :dim,:dim]
        A, bdr_out =  $\square$ 
         $\rightarrow$ gaussian_mixture_decision(pi_FG,mu_FG_cur,cov_FG_cur,pi_BG,mu_BG_cur, $\square$ 
         $\rightarrow$ cov_BG_cur, input_image, dim)
        error = probability_of_error(bdr_out.flatten(),mask_image.
         $\rightarrow$ flatten(),image_size_row, image_size_col)
        error_list.append(error)
```

```

    print("Probability of error for C = {0} and dim = {1} is {2}".format(c,
→dim, error))
    error_list = np.array(error_list)
    label = str(c)
    error_mixtures[label] = error_list

```

```

Probability of error for C = 1 and dim = 1 is 0.06450254175744372
Probability of error for C = 1 and dim = 2 is 0.0637763253449528
Probability of error for C = 1 and dim = 4 is 0.0646477850399419
Probability of error for C = 1 and dim = 8 is 0.07251997095134351
Probability of error for C = 1 and dim = 16 is 0.0684241103848947
Probability of error for C = 1 and dim = 24 is 0.06492374727668845
Probability of error for C = 1 and dim = 32 is 0.07253449527959333
Probability of error for C = 1 and dim = 40 is 0.07353667392883079
Probability of error for C = 1 and dim = 48 is 0.07895424836601307
Probability of error for C = 1 and dim = 56 is 0.08238198983297022
Probability of error for C = 1 and dim = 64 is 0.08411038489469862
Probability of error for C = 2 and dim = 1 is 0.06437182280319535
Probability of error for C = 2 and dim = 2 is 0.06284676833696441
Probability of error for C = 2 and dim = 4 is 0.06130718954248366
Probability of error for C = 2 and dim = 8 is 0.06216412490922295
Probability of error for C = 2 and dim = 16 is 0.049092229484386345
Probability of error for C = 2 and dim = 24 is 0.04490922294843863
Probability of error for C = 2 and dim = 32 is 0.04987654320987654
Probability of error for C = 2 and dim = 40 is 0.0504720406681191
Probability of error for C = 2 and dim = 48 is 0.05498910675381263
Probability of error for C = 2 and dim = 56 is 0.05710965867828613
Probability of error for C = 2 and dim = 64 is 0.058198983297022513
Probability of error for C = 4 and dim = 1 is 0.06020334059549746
Probability of error for C = 4 and dim = 2 is 0.05892519970951343
Probability of error for C = 4 and dim = 4 is 0.05943355119825708
Probability of error for C = 4 and dim = 8 is 0.06602759622367466
Probability of error for C = 4 and dim = 16 is 0.06374727668845316
Probability of error for C = 4 and dim = 24 is 0.060029048656499634
Probability of error for C = 4 and dim = 32 is 0.06809005083514888
Probability of error for C = 4 and dim = 40 is 0.06984749455337691
Probability of error for C = 4 and dim = 48 is 0.07564270152505446
Probability of error for C = 4 and dim = 56 is 0.07920116194625998
Probability of error for C = 4 and dim = 64 is 0.08066811909949165
Probability of error for C = 8 and dim = 1 is 0.0611038489469862
Probability of error for C = 8 and dim = 2 is 0.05921568627450981
Probability of error for C = 8 and dim = 4 is 0.05869281045751634
Probability of error for C = 8 and dim = 8 is 0.058896151053013795
Probability of error for C = 8 and dim = 16 is 0.04864197530864198
Probability of error for C = 8 and dim = 24 is 0.0414960058097313
Probability of error for C = 8 and dim = 32 is 0.04156862745098039
Probability of error for C = 8 and dim = 40 is 0.03995642701525055

```

Probability of error for C = 8 and dim = 48 is 0.04027596223674655
 Probability of error for C = 8 and dim = 56 is 0.04047930283224401
 Probability of error for C = 8 and dim = 64 is 0.0407116920842411
 Probability of error for C = 16 and dim = 1 is 0.05924473493100944
 Probability of error for C = 16 and dim = 2 is 0.0568482207697894
 Probability of error for C = 16 and dim = 4 is 0.05394335511982571
 Probability of error for C = 16 and dim = 8 is 0.055018155410312276
 Probability of error for C = 16 and dim = 16 is 0.04663761801016703
 Probability of error for C = 16 and dim = 24 is 0.038649237472766884
 Probability of error for C = 16 and dim = 32 is 0.03763253449527959
 Probability of error for C = 16 and dim = 40 is 0.037037037037037035
 Probability of error for C = 16 and dim = 48 is 0.03625272331154684
 Probability of error for C = 16 and dim = 56 is 0.03705156136528685
 Probability of error for C = 16 and dim = 64 is 0.03722585330428468
 Probability of error for C = 32 and dim = 1 is 0.06011619462599855
 Probability of error for C = 32 and dim = 2 is 0.05394335511982571
 Probability of error for C = 32 and dim = 4 is 0.0523602033405955
 Probability of error for C = 32 and dim = 8 is 0.05384168482207698
 Probability of error for C = 32 and dim = 16 is 0.0429920116194626
 Probability of error for C = 32 and dim = 24 is 0.03469862018881627
 Probability of error for C = 32 and dim = 32 is 0.03609295570079884
 Probability of error for C = 32 and dim = 40 is 0.0341757443718228
 Probability of error for C = 32 and dim = 48 is 0.03317356572258533
 Probability of error for C = 32 and dim = 56 is 0.0333042846768337
 Probability of error for C = 32 and dim = 64 is 0.03375453885257807

```

[15]: plt.figure(figsize=(10, 5))
      for i in range(mixtures.shape[0]):
          plt.plot(dimensions, error_mixtures[str(2**(i))],label = str(2**(i)))
      plt.title('Probability of error for different values of C')
      plt.legend(loc='upper right')
      plt.xlabel('Dimensions', )
      plt.ylabel('Probability of Error')
      plt.show()
  
```

