Solutions to Homework Set One
ECE 271A
Electrical and Computer Engineering
University of California San Diego

1.

a) For this problem, the Bayesian decision rule is to guess *heads* when

$$\begin{aligned} P_{S|R}(heads|heads) &> P_{S|R}(tails|heads) && (1)\\ P_{R|S}(heads|heads)P_S(heads) &> P_{R|S}(heads|tails)P_S(tails) && (2)\\ (1-\theta_1)\alpha &> \theta_2(1-\alpha) && (3)\\ \alpha &> \frac{\theta_2}{1-\theta_1+\theta_2} && (4) \end{aligned}$$

and *tails* when

$$\alpha < \frac{\theta_2}{1-\theta_1+\theta_2}. \qquad (5)$$

When

$$\alpha = \frac{\theta_2}{1-\theta_1+\theta_2} \qquad (6)$$

any guess is equally good.

b)When $\theta_1 = \theta_2 = \theta$ the minimum probability of error decision is to declare *heads* if

$$\alpha > \theta \qquad (7)$$

and *tails* otherwise. This means that you should only believe your friend's report if your prior for *heads* is greater than the probability that he lies. To see that this makes a lot of sense let's look at a few different scenarios.

- If your friend is a pathological lier ($\theta = 1$), then you know for sure that the answer is not *heads* and you should always say tails. This is the decision that (7) advises you to take.
- If he never lies ($\theta = 0$) you know that the answer is *heads*. Once again this is the decision that (7) advises you to take.
- If both $\alpha = 0$ and $\theta = 0$ we have a contradiction, i.e. you know for sure that the result of the toss is always *tails* but this person that never lies is telling you that it is *heads*. In this case Bayes just gives up ... thing wrong with the mo...

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- If your frien... l ignore him. If you believe that that the coin is more likely to land on *heads* say *heads* otherwise say *tails*. As we have seen in class, Bayes has no problem with ignoring the observations, whenever these are