

AI1110 ASSIGNMENT-7

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Abstract—This document contains the solution for Assignment 7 (NCERT GRADE 11 CHAPTER 16 Exercise 16.2 Question 5)

QUESTION 5 :

Three coins are tossed. Describe

- Two events which are mutually exclusive.
- Three events which are mutually exclusive and exhaustive.
- Two events , which are not mutually exclusive.
- Two events which are mutually exclusive but not exhaustive.
- Three events which are mutually exclusive but not exhaustive.

SOLUTION :

- Let us consider the trial of tossing a coin once. And let us label the outcome of the trial with the Bernoulli random variable Y .
- Let us label the outcomes 1(Head) and 0(Tail) respectively for success and failure , let

$$\Pr(Y = 1) = p \quad (1)$$

$$\Pr(Y = 0) = 1 - p \quad (2)$$

- Now let us consider three Bernoulli trials for tossing a coin. Let X be a binomial random variable for the trials, with parameters n and p , where

$$a) \ n = \text{No.of trials} = 3$$

$$b) \ p = \text{Probability with which we get a favourable outcome (here let us consider getting Head as a favourable outcome)} = 0.5$$

- So ,

- The possible outcomes when 3 coins are tossed are (or) the sample space contains ,

$$S = \{000, 001, 010, 100, 011, 101, 110, 111\} \quad (3)$$

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$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \quad (4)$$

where $k = 0, \dots, n$ is number of heads according to this question.

We can tabulate the probabilities of each event into a binomial probability table as shown in Table I

Event	Description of event	Probability of event
$X = 0$	Zero heads in the trials	$\frac{1}{8}$
$X = 1$	Exactly one head in the trials	$\frac{3}{8}$
$X = 2$	Exactly two heads in the trials	$\frac{3}{8}$
$X = 3$	All three heads in the trials	$\frac{1}{8}$

TABLE I
BINOMIAL PROBABILITY DISTRIBUTION

- Two events which are mutually exclusive**

Let us take the events A,B as shown in Table II. So ,

Event	Description of event
A	Getting all 3 Heads
B	Getting all 3 Tails

TABLE II
EVENTS FOR QUESTION 1

$$A = \{(X = 3)\} \quad (5)$$

$$B = \{(X = 0)\} \quad (6)$$

and Since ,

$$\Pr(A \cap B) = \Pr((X = 3) \cap (X = 0)) = 0 \quad (7)$$

So ,events A and B are mutually exclusive.

- Three events which are mutually exclusive and exhaustive**

Let us take events C,D,E as shown in Table III. So ,

$$C = \{(X = 0)\} \quad (8)$$

$$D = \{(X = 1)\} \quad (9)$$

$$E = \{(X = 2) \cup (X = 3)\} \quad (10)$$

Event	Description of event
C	Getting all 3 Tails
D	Getting exactly 2 Tails
E	Getting at least 2 Heads

TABLE III
EVENTS FOR QUESTION 2

and Since ,

$$\Pr(C \cap D) = 0 \quad (11)$$

$$\Pr(D \cap E) = 0 \quad (12)$$

$$\Pr(C \cap E) = 0 \quad (13)$$

We can say that , events C,D,E are mutually exclusive. And from Table I,

$$\Pr(C \cup D \cup E) = \Pr((X \geq 0)) \quad (14)$$

$$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1 \quad (15)$$

So, events C,D,E are mutually exclusive and exhaustive.

(iii) **Two events ,which are not mutually exclusive**

Let us take the events F,G as shown in Table IV. So ,

Event	Description of event
F	Getting all 3 Heads
G	Getting at least 2 Heads

TABLE IV
EVENTS FOR QUESTION 3

$$F = \{(X = 3)\} \quad (16)$$

$$G = \{(X = 2) \cup (X = 3)\} \quad (17)$$

and Since from Table I,

$$\Pr(F \cap G) = \Pr((X = 3)) = \frac{1}{8} \neq 0 \quad (18)$$

So ,events F and G are not mutually exclusive.

(iv) **Two events ,which are mutually exclusive but not exhaustive**

Let us take the events H,I as shown in Table V. So ,

$$H = \{(X = 3)\} \quad (19)$$

$$I = \{(X = 0)\} \quad (20)$$

Event	Description of event
H	Getting all 3 Heads
I	Getting all 3 Tails

TABLE V
EVENTS FOR QUESTION 4

and Since ,

$$\Pr(H \cap I) = 0 \quad (21)$$

So ,events H and I are mutually exclusive. And from Table I,

$$\Pr(H \cup I) = \Pr((X = 0)) + \Pr((X = 3)) \quad (22)$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \neq 1 \quad (23)$$

So , events H and I are mutually exclusive but not exhaustive.

(v) **Three events which are mutually exclusive but not exhaustive**

Let us take events J,K,L as shown in Table VI. So ,

Event	Description of event
J	Getting all 3 Tails
K	Getting exactly 2 Tails
L	Getting exactly 1 Tail

TABLE VI
EVENTS FOR QUESTION 5

$$J = \{(X = 0)\} \quad (24)$$

$$K = \{(X = 1)\} \quad (25)$$

$$L = \{(X = 2)\} \quad (26)$$

and Since ,

$$\Pr(J \cap K) = 0 \quad (27)$$

$$\Pr(K \cap L) = 0 \quad (28)$$

$$\Pr(J \cap L) = 0 \quad (29)$$

We can say that , events J,K,L are mutually exclusive. And from Table I,

$$\Pr(J \cup K \cup L) = \Pr((X = 0)) + \Pr((X = 1)) + \Pr((X = 2)) \quad (30)$$

$$\Pr(J \cup K \cup L) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8} \quad (31)$$

$$\neq 1 \quad (32)$$

So, events J,K,L are mutually exclusive but not exhaustive.