Assignment 10

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Outline

Example 34

Find the mean of the binomial distribution $\mathrm{B}\left(4, \frac{1}{3}\right)$

Theory

A binomial distribution with n- Bernoulli trials and probability of success in each trial as p , is denoted by B (n, p)

The probability of k successes $\Pr(X = k)$ is also denoted by $P(k)$ and is given by

$$\Pr(X = k) = {}^{n}C_{k} p^{k} (1 - p)^{n-k} \tag{1}$$

for $x = 0, 1, 2, ..., n - 1, n$

Mean

The mean of X is defined as

$$E(X) = \left(\frac{\partial M_x(z^{-1})}{\partial z}\right)_{z=1} \tag{2}$$

We know that

$$M_x(z^{-1}) = \sum_{k=-\infty}^{\infty} z^k P_x(k) \tag{3}$$

$$M_x(z^{-1}) = \sum_{k=-\infty}^{\infty} z^k \times {}^nC_k p^k (1-p)^{n-k} \tag{4}$$

$$M_x(z^{-1}) = \sum_{k=-\infty}^{\infty} {}^nC_k (zp)^k (1-p)^{n-k} \tag{5}$$

$$M_x(z^{-1}) = \sum_{k=0}^{n} {}^{n}C_k (zp)^k (1-p)^{n-k} \tag{6}$$

$$M_x(z^{-1}) = (zp + 1 - p)^n \tag{7}$$

SO, Mean is ,

$$E(X) = \left(\frac{\partial (zp + 1 - p)^n}{\partial z} \right)_{z=1} \tag{8}$$

$$E(X) = np \times (zp + 1 - p)^{n-1} \tag{9}$$

$$E(X) = np \times 1 \tag{10}$$

$$E(X) = np \tag{11}$$

Solution

Let X be the random variable whose probability distribution is B $\left(4, \frac{1}{3}\right)$.

So , we can write that ,

$$n = 4 \tag{12}$$

$$p = \frac{1}{3} \tag{13}$$

$$q = 1 - p = \frac{2}{3} \tag{14}$$

From (1) we can say,

$$\Pr(X = k) = {}^{4}C_{k} \left(\frac{1}{3}\right)^{k} \left(\frac{2}{3}\right)^{4-k} \tag{15}$$

for $k = 0, 1, 2, 3, 4$

Distribution of X

x_i	$P(x_i)$	$x_i P(x_i)$
0	$^4C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4$	0
1	$^4C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3$	$^4C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3$
2	$^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2$	$2\left(^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2\right)$
3	$^4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1$	$3\left(^4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1\right)$
4	$^4C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^0$	$4\left(^4C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^0\right)$

Table 1: Probability Distribution of X

Mean (μ)

We know that, from (11)

$$\mu = np \tag{16}$$

$$\mu = \frac{4}{3} \tag{17}$$

Result

The mean of the binomial distribution $\mathrm{B}\left(4, \frac{1}{3}\right) = \frac{4}{3}$