Assignment 10

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Outline

- Question
- 2 Theory
- Solution
- Result



Example 34

Find the mean of the binomial distribution B $\left(4, \frac{1}{3}\right)$



Theory

A binomial distribution with n- Bernoulli trials and probability of success in each trial as p , is denoted by B (n,p)

The probability of k successes Pr(X = k) is also denoted by P(k) and is given by

$$\Pr(X = k) = {}^{n}C_{k}p^{k}(1-p)^{n-k}$$
(1)

for x = 0, 1, 2, ..., n - 1, n



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Mean

The mean of X is defined as

$$E(X) = \left(\frac{\partial M_X(z^{-1})}{\partial z}\right)_{z=1} \tag{2}$$

We know that

$$M_{x}(z^{-1}) = \sum_{k=-\infty}^{\infty} z^{k} P_{x}(k)$$
(3)

$$M_{X}(z^{-1}) = \sum_{k=-\infty}^{\infty} z^{k} \times {}^{n}C_{k}p^{k} (1-p)^{n-k}$$
 (4)

$$M_{X}(z^{-1}) = \sum_{k=-\infty}^{\infty} {}^{n}C_{k}(zp)^{k} (1-p)^{n-k}$$
 (5)

5/10

$$M_{x}(z^{-1}) = \sum_{k=0}^{n} {^{n}C_{k}(zp)^{k} (1-p)^{n-k}}$$
 (6)

$$M_X(z^{-1}) = (zp + 1 - p)^n$$
 (7)

SO, Mean is,

$$E(X) = \left(\frac{\partial (zp + 1 - p)^n}{\partial z}\right)_{z=1}$$
 (8)

$$E(X) = np \times (zp + 1 - p)^{n-1}$$
(9)

$$E(X) = np \times 1 \tag{10}$$

$$E(X) = np (11)$$



Solution

Let X be the random variable whose probability distribution is B $\left(4,\frac{1}{3}\right)$. So, we can write that,

$$n=4 (12)$$

$$\rho = \frac{1}{3}
q = 1 - p = \frac{2}{3}$$
(13)

$$q = 1 - p = \frac{2}{3} \tag{14}$$

From (1) we can say,

$$\Pr(X = k) = {}^{4}C_{k} \left(\frac{1}{3}\right)^{k} \left(\frac{2}{3}\right)^{4-k}$$
 (15)

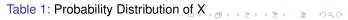
for k = 0, 1, 2, 3, 4



7/10

Distribution of X

Xi	$P(x_i)$	$x_i P(x_i)$
0	$^4C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^4$	0
1	${}^4C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3$	${}^4C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3$
2	${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$	$2\left(^{4}C_{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^{2}\right)$
3	$^4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1$	$3\left(^4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1\right)$
4	${}^{4}C_{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{0}$	$4\left({}^4C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0\right)$



Dasari Srinith Assignment 10 May 24, 2022 8/10

Mean (μ)

We know that, from (11)

$$\mu = np \tag{16}$$

$$\mu = \frac{4}{3} \tag{17}$$



Result

The mean of the binomial distribution B $\left(4, \frac{1}{3}\right) = \frac{4}{3}$

