

AI1110 ASSIGNMENT-7

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Abstract—This document contains the solution for Assignment 7 (NCERT GRADE 11 CHAPTER 16 Exercise 16.2 Question 5)

QUESTION 5 :

Three coins are tossed. Describe

- Two events which are mutually exclusive.
- Three events which are mutually exclusive and exhaustive.
- Two events , which are not mutually exclusive.
- Two events which are mutually exclusive but not exhaustive.
- Three events which are mutually exclusive but not exhaustive.

SOLUTION :

- Let us consider the trial of tossing a coin once. And let us label the outcome of the trial with the Bernoulli random variable Y .
- Let us label the outcomes 1(Head) and 0(Tail) respectively for success and failure , let

$$\Pr(Y = 1) = p \quad (1)$$

$$\Pr(Y = 0) = 1 - p \quad (2)$$

- Now let us consider three Bernoulli trials for tossing a coin. Let X be a binomial random variable for the trials, with parameters n and p , where

a) n = No.of trials = 3

b) p = Probability with which we get a favourable outcome (here let us consider getting Head as a favourable outcome) = 0.5

- So ,

a) The possible outcomes when 3 coins are tossed are (or) the sample space contains ,

$$S = \{000, 001, 010, 100, 011, 101, 110, 111\} \quad (3)$$

b)

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \quad (4)$$

where $k = 0, \dots, n$ is number of heads according to this question.

We can tabulate the probabilities of each event into a binomial probability table as shown in Table I

Event	Description of event	Probability of event
$X = 0$	Zero heads in the trials	$\frac{1}{8}$
$X = 1$	Exactly one head in the trials	$\frac{3}{8}$
$X = 2$	Exactly two heads in the trials	$\frac{3}{8}$
$X = 3$	All three heads in the trials	$\frac{1}{8}$

TABLE I
BINOMIAL PROBABILITY DISTRIBUTION

- Two events which are mutually exclusive**

Let us take the two events as shown in Table II.

Event	Probability of event
$X = 3$	$\frac{1}{8}$
$X = 0$	$\frac{1}{8}$

TABLE II
EVENTS FOR QUESTION 1

Since ,

$$\Pr((X = 3) \cap (X = 0)) = 0 \quad (5)$$

So ,the two events are mutually exclusive.

- Three events which are mutually exclusive and exhaustive**

Let us take the three events as shown in Table III.

Since ,

$$\Pr((X = 0) \cap (X = 1)) = 0 \quad (6)$$

$$\Pr((X = 1) \cap (X \geq 2)) = 0 \quad (7)$$

$$\Pr((X = 0) \cap (X \geq 2)) = 0 \quad (8)$$

Event	Probability of event
$X = 0$	$\frac{1}{8}$
$X = 1$	$\frac{3}{8}$
$X \geq 2$	$\frac{1}{2}$

TABLE III
EVENTS FOR QUESTION 2

We can say that , the three events are mutually exclusive.

$$\begin{aligned} \Pr((X = 0) \cup (X = 1) \cup (X \geq 2)) \\ = \Pr((X \geq 0)) \end{aligned} \quad (9)$$

And from Table III,

$$\Pr((X \geq 0)) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1 \quad (10)$$

So, the three events are mutually exclusive and exhaustive.

(iii) **Two events ,which are not mutually exclusive**

Let us take the two events as shown in Table IV.

Event	Probability of event
$X = 3$	$\frac{1}{8}$
$X \geq 2$	$\frac{1}{2}$

TABLE IV
EVENTS FOR QUESTION 3

Since from Table IV,

$$\Pr((X = 3) \cap (X \geq 2)) = \Pr((X = 3)) \quad (11)$$

$$\Pr((X = 3)) = \frac{1}{8} \neq 0 \quad (12)$$

So ,the two events are not mutually exclusive.

(iv) **Two events ,which are mutually exclusive but not exhaustive**

Let us take the two events as shown in Table V. Since ,

Event	Probability of event
$X = 3$	$\frac{1}{8}$
$X = 0$	$\frac{1}{8}$

TABLE V
EVENTS FOR QUESTION 4

$$\Pr((X = 3) \cap (X = 0)) = 0 \quad (13)$$

So ,the two events are mutually exclusive. And from Table V,

$$\begin{aligned} \Pr((X = 3) \cup (X = 0)) &= \frac{1}{8} + \frac{1}{8} \quad (14) \\ &= \frac{1}{4} \neq 1 \quad (15) \end{aligned}$$

So , the two events are mutually exclusive but not exhaustive.

(v) **Three events which are mutually exclusive but not exhaustive**

Let us take the three events as shown in Table VI. Since ,

Event	Probability of event
$X = 0$	$\frac{1}{8}$
$X = 1$	$\frac{3}{8}$
$X = 2$	$\frac{3}{8}$

TABLE VI
EVENTS FOR QUESTION 5

$$\Pr((X = 0) \cap (X = 1)) = 0 \quad (16)$$

$$\Pr((X = 1) \cap (X = 2)) = 0 \quad (17)$$

$$\Pr((X = 0) \cap (X = 2)) = 0 \quad (18)$$

We can say that , the three events are mutually exclusive.

And from Table VI,

$$\begin{aligned} \Pr((X = 0) \cup (X = 1) \cup (X = 2)) \\ = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1 \end{aligned} \quad (19)$$

So, the three events are mutually exclusive but not exhaustive.