Assignment 10

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Outline

Example 34

Find the mean of the binomial distribution B $\left(4, \dfrac{1}{3}\right)$

Theory

A binomial distribution with n- Bernoulli trials and probability of success in each trial as p , is denoted by B (n, p)

The probability of k successes $\Pr(X = k)$ is also denoted by $P(k)$ and is given by

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \tag{1}$$

for $x = 0, 1, 2, ..., n - 1, n$

Solution

Let X be the random variable whose probability distribution is $B\left(4, \dfrac{1}{3}\right)$.

So , we can write that ,

$$n = 4 \tag{2}$$

$$p = \frac{1}{3} \tag{3}$$

$$q = 1 - p = \frac{2}{3} \tag{4}$$

From (1) we can say,

$$\Pr\left(X = k\right) = {}^{4}C_{k}\left(\frac{1}{3}\right)^{k}\left(\frac{2}{3}\right)^{4-k} \tag{5}$$
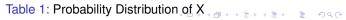
for $k = 0, 1, 2, 3, 4$

Distribution of X

x_i	$P(x_i)$	$x_i P(x_i)$
0	${}^4C_0 \left(\dfrac{1}{3}\right)^0 \left(\dfrac{2}{3}\right)^4$	0
1	${}^4C_1 \left(\dfrac{1}{3}\right)^1 \left(\dfrac{2}{3}\right)^3$	${}^4C_1 \left(\dfrac{1}{3}\right)^1 \left(\dfrac{2}{3}\right)^3$
2	${}^4C_2 \left(\dfrac{1}{3}\right)^2 \left(\dfrac{2}{3}\right)^2$	$2\left({}^4C_2 \left(\dfrac{1}{3}\right)^2 \left(\dfrac{2}{3}\right)^2\right)$
3	${}^4C_3 \left(\dfrac{1}{3}\right)^3 \left(\dfrac{2}{3}\right)^1$	$3\left({}^4C_3 \left(\dfrac{1}{3}\right)^3 \left(\dfrac{2}{3}\right)^1\right)$
4	${}^4C_4 \left(\dfrac{1}{3}\right)^4 \left(\dfrac{2}{3}\right)^0$	$4\left({}^4C_4 \left(\dfrac{1}{3}\right)^4 \left(\dfrac{2}{3}\right)^0\right)$

Table 1: Probability Distribution of X

Mean (μ)

$$\mu = \sum_{i=1}^{4} x_i P(x_i) \tag{6}$$

$$= 0 + 4 \times \frac{2^3}{3^4} + 2 \times 6 \times \frac{2^2}{3^4} + 3 \times 4 \times \frac{2}{3^4} + 4 \times 1 \times \frac{1}{3^4} \tag{7}$$

$$= \frac{32 + 48 + 24 + 4}{3^4} \tag{8}$$

$$= \frac{108}{81} = \frac{4}{3} \tag{9}$$

Result

The mean of the binomial distribution B $\left(4, \dfrac{1}{3}\right) = \dfrac{4}{3}$