AI1110 ASSIGNMENT-7

DASARI SRINITH (CS21BTECH11015)

***Abstract*—This document contains the solution for Assignment 7 (NCERT GRADE 11 CHAPTER 16 Exercise 16.2 Question 5)**

QUESTION 5 :

Three coins are tossed. Describe

(i) Two events which are mutually exclusive.
(ii) Three events which are mutually exclusive and exhaustive.
(iii) Two events , which are not mutually exclusive.
(iv) Two events which are mutually exclusive but not exhaustive.
(v) Three events which are mutually exclusive but not exhaustive.

SOLUTION :

1) Let us consider the trial of tossing a coin once. And let us label the outcome of the trial with the Bernoulli random variable Y.
2) Let us label the outcomes 1(Head) and 0(Tail) respectively for success and failure , let

$$\Pr(Y = 1) = p \tag{1}$$
$$\Pr(Y = 0) = 1 - p \tag{2}$$

3) Now let us consider three Bernoulli trials for tossing a coin. Let X be a binomial random variable for the trials, with parameters n and p , where
   a) n = No.of trials = 3
   b) p = Probability with which we get a favourable outcome (here let us consider getting Head as a favourable outcome) = 0.5
4) So ,
   a) The possible outcomes when 3 coins are tossed are (or) the sample space contains ,

$$\text{S} = \{000, 001, 010, 100, 011, 101, 110, 111\} \tag{3}$$

   b)

$$\Pr(X = k) = {}^{n}C_k p^k (1 - p)^{n-k} \tag{4}$$

   where $k = 0, \ldots, n$ is number of heads according to this question.

We can tabulate the probabilities of each event into a binomial probability table as shown in Table I

Event	Description of event	Probability of event
$X = 0$	Zero heads in the trials	$\frac{1}{8}$
$X = 1$	Exactly one head in the trials	$\frac{3}{8}$
$X = 2$	Exactly two heads in the trials	$\frac{3}{8}$
$X = 3$	All three heads in the trials	$\frac{1}{8}$

TABLE I
BINOMIAL PROBABILITY DISTRIBUTION

(i) **Two events which are mutually exclusive**
Let us take the events A,B as shown in Table II. So ,

Event	Description of event
A	$X = 3$
B	$X = 0$

TABLE II
EVENTS FOR QUESTION 1

$$A = \{(X = 3)\} \tag{5}$$
$$B = \{(X = 0)\} \tag{6}$$

and Since ,

$$\Pr(A \cap B) = \Pr((X = 3) \cap (X = 0)) = 0 \tag{7}$$

So ,events A and B are mutually exclusive.

(ii) **Three events which are mutually exclusive and exhaustive**
Let us take events C,D,E as shown in Table III. So ,

$$C = \{(X = 0)\} \tag{8}$$
$$D = \{(X = 1)\} \tag{9}$$
$$E = \{(X = 2) \cup (X = 3)\} \tag{10}$$

Event	Description of event
C	$X=0$
D	$X=1$
E	$X\geq 2$

TABLE III
EVENTS FOR QUESTION 2

and Since ,

$$\Pr\left(C\cap D\right)=0 \tag{11}$$
$$\Pr\left(D\cap E\right)=0 \tag{12}$$
$$\Pr\left(C\cap E\right)=0 \tag{13}$$

We can say that , events C,D,E are mutually exclusive. And from Table I,

$$\Pr\left(C\cup D\cup E\right)=\Pr\left(\left(X\geq 0\right)\right) \tag{14}$$

$$=\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=1 \tag{15}$$

So, events C,D,E are mutually exclusive and exhaustive.

(iii) **Two events ,which are not mutually exclusive**

Let us take the events F,G as shown in Table IV. So ,

Event	Description of event
F	$X=3$
G	$X\geq 2$

TABLE IV
EVENTS FOR QUESTION 3

$$F=\{(X=3)\} \tag{16}$$
$$G=\{(X=2)\cup(X=3)\} \tag{17}$$

and Since from Table I,

$$\Pr\left(F\cap G\right)=\Pr\left(\left(X=3\right)\right)=\frac{1}{8}\neq 0 \tag{18}$$

So ,events F and G are not mutually exclusive.

(iv) **Two events ,which are mutually exclusive but not exhaustive**

Let us take the events H,I as shown in Table V. So ,

$$H=\{(X=3)\} \tag{19}$$
$$I=\{(X=0)\} \tag{20}$$

Event	Description of event
H	$X=3$
I	$X=0$

TABLE V
EVENTS FOR QUESTION 4

and Since ,

$$\Pr\left(H\cap I\right)=0 \tag{21}$$

So ,events H and I are mutually exclusive. And from Table I,

$$\Pr\left(H\cup I\right)=\Pr\left(\left(X=0\right)\right)+\Pr\left(\left(X=3\right)\right) \tag{22}$$

$$=\frac{1}{8}+\frac{1}{8}=\frac{1}{4}\neq 1 \tag{23}$$

So , events H and I are mutually exclusive but not exhaustive.

(v) **Three events which are mutually exclusive but not exhaustive**

Let us take events J,K,L as shown in Table VI. So ,

Event	Description of event
J	$X=0$
K	$X=1$
L	$X=2$

TABLE VI
EVENTS FOR QUESTION 5

$$J=\{(X=0)\} \tag{24}$$
$$K=\{(X=1)\} \tag{25}$$
$$L=\{(X=2)\} \tag{26}$$

and Since ,

$$\Pr\left(J\cap K\right)=0 \tag{27}$$
$$\Pr\left(K\cap L\right)=0 \tag{28}$$
$$\Pr\left(J\cap L\right)=0 \tag{29}$$

We can say that , events J,K,L are mutually exclusive. And from Table I,

$$\Pr\left(J\cup K\cup L\right)=\Pr\left(\left(X=0\right)\right)+\Pr\left(\left(X=1\right)\right)+\Pr\left(\left(X=2\right)\right) \tag{30}$$

$$\Pr\left(J \cup K \cup L\right) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8} \tag{31}$$

$$\neq 1 \tag{32}$$

So, events J,K,L are mutually exclusive but not exhaustive.