AI1110 ASSIGNMENT-7

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***Abstract*—This document contains the solution for Assignment 7 (NCERT GRADE 11 CHAPTER 16 Exercise 16.2 Question 5)**

QUESTION 5 :

Three coins are tossed. Describe

(i) Two events which are mutually exclusive.
(ii) Three events which are mutually exclusive and exhaustive.
(iii) Two events , which are not mutually exclusive.
(iv) Two events which are mutually exclusive but not exhaustive.
(v) Three events which are mutually exclusive but not exhaustive.

SOLUTION :

Since a coin can either turn up Tail(0) or Head(1) ,each with equal probability of 0.5 and the possible outcomes when 3 coins are tossed are (or) the sample space contains ,

$$\text{S} = \{000, 001, 010, 100, 011, 101, 110, 111\} \quad (1)$$

Let X be a binomial random variable, with parameters n and p , where

1) n = No.of trials = 3
2) p = Probability with which we get a favourable outcome (here let us consider getting Head as a favourable outcome) = $\frac{1}{2}$

then ,

$$\Pr(X = k) = {}^{n}C_k p^k (1-p)^{n-k}, \quad k = 0, \ldots, n \quad (2)$$

where k is number of heads according to this question.

We can tabulate the probabilities of each event into a binomial probability table as shown in Table I

Event	Description of event	Probability of event
$X = 0$	Zero heads in the trial	$\frac{1}{8}$
$X = 1$	Exactly one head in the trial	$\frac{3}{8}$
$X = 2$	Exactly two heads in the trial	$\frac{3}{8}$
$X = 3$	All three heads in the trial	$\frac{1}{8}$

TABLE I
BINOMIAL PROBABILITY DISTRIBUTION

(i) **Two events which are mutually exclusive**
Let us take the events A,B as shown in Table II. So ,

Event	Description of event
A	Getting all 3 Heads
B	Getting all 3 Tails

TABLE II
EVENTS FOR QUESTION 1

$$A = \{(X = 3)\} \quad (3)$$
$$B = \{(X = 0)\} \quad (4)$$

and Since ,

$$\Pr(A \cap B) = \Pr((X = 3) \cap (X = 0)) = 0 \quad (5)$$

So ,events A and B are mutually exclusive.

(ii) **Three events which are mutually exclusive and exhaustive**
Let us take events C,D,E as shown in Table III. So ,

Event	Description of event
C	Getting all 3 Tails
D	Getting exactly 2 Tails
E	Getting at least 2 Heads

TABLE III
EVENTS FOR QUESTION 2

$$C = \{(X = 0)\} \quad (6)$$
$$D = \{(X = 1)\} \quad (7)$$
$$E = \{(X = 2) \cup (X = 3)\} \quad (8)$$

and Since ,

$$\Pr\left(C \cap D\right) = 0 \tag{9}$$
$$\Pr\left(D \cap E\right) = 0 \tag{10}$$
$$\Pr\left(C \cap E\right) = 0 \tag{11}$$

We can say that , events C,D,E are mutually exclusive. And from Table I,

$$\Pr\left(C \cup D \cup E\right) = \Pr\left((X \geq 0)\right) \tag{12}$$
$$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1 \tag{13}$$

So, events C,D,E are mutually exclusive and exhaustive.

(iii) **Two events ,which are not mutually exclusive**
Let us take the events F,G as shown in Table IV. So ,

Event	Description of event
F	Getting all 3 Heads
G	Getting at least 2 Heads

TABLE IV
EVENTS FOR QUESTION 3

$$F = \{(X = 3)\} \tag{14}$$
$$G = \{(X = 2) \cup (X = 3)\} \tag{15}$$

and Since from Table I,

$$\Pr\left(F \cap G\right) = \Pr\left((X = 3)\right) = \frac{1}{8} \neq 0 \tag{16}$$

So ,events F and G are not mutually exclusive.

(iv) **Two events ,which are mutually exclusive but not exhaustive**
Let us take the events H,I as shown in Table V. So ,

Event	Description of event
H	Getting all 3 Heads
I	Getting all 3 Tails

TABLE V
EVENTS FOR QUESTION 4

$$H = \{(X = 3)\} \tag{17}$$
$$I = \{(X = 0)\} \tag{18}$$

and Since ,

$$\Pr\left(H \cap I\right) = 0 \tag{19}$$

So ,events H and I are mutually exclusive. And from Table I,

$$\Pr\left(H \cup I\right) = \Pr\left((X = 0)\right) + \Pr\left((X = 3)\right) \tag{20}$$
$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \neq 1 \tag{21}$$

So , events H and I are mutually exclusive but not exhaustive.

(v) **Three events which are mutually exclusive but not exhaustive**
Let us take events J,K,L as shown in Table VI. So ,

Event	Description of event
J	Getting all 3 Tails
K	Getting exactly 2 Tails
L	Getting exactly 1 Tail

TABLE VI
EVENTS FOR QUESTION 5

$$J = \{(X = 0)\} \tag{22}$$
$$K = \{(X = 1)\} \tag{23}$$
$$L = \{(X = 2)\} \tag{24}$$

and Since ,

$$\Pr\left(J \cap K\right) = 0 \tag{25}$$
$$\Pr\left(K \cap L\right) = 0 \tag{26}$$
$$\Pr\left(J \cap L\right) = 0 \tag{27}$$

We can say that , events J,K,L are mutually exclusive. And from Table I,

$$\Pr\left(J \cup K \cup L\right) = \Pr\left((X = 0)\right) + \Pr\left((X = 1)\right) + \Pr\left((X = 2)\right) \tag{28}$$
$$\Pr\left(J \cup K \cup L\right) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8} \tag{29}$$
$$\neq 1 \tag{30}$$

So, events J,K,L are mutually exclusive but not exhaustive.