

Assignment 10

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Example 34

Find the mean of the binomial distribution $B\left(4, \frac{1}{3}\right)$

Theory

A binomial distribution with n - Bernoulli trials and probability of success in each trial as p , is denoted by $B(n, p)$

The probability of k successes $\Pr(X = k)$ is also denoted by $P(k)$ and is given by

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \quad (1)$$

for $x = 0, 1, 2, \dots, n - 1, n$

Mean

$$\mu = \sum_{i=1}^n x_i P(x_i) \quad (2)$$

$$\mu = \sum_{r=1}^n r \times {}^n C_r p^r (1-p)^{n-r} \quad (3)$$

$$\mu = (1-p)^n \sum_{r=1}^n r \times {}^n C_r \left(\frac{p}{1-p} \right)^r \quad (4)$$

$$\mu = (1-p)^n \frac{np}{(1-p)^n} \quad (5)$$

$$\mu = np \quad (6)$$

Solution

Let X be the random variable whose probability distribution is $B\left(4, \frac{1}{3}\right)$.

So , we can write that ,

$$n = 4 \quad (7)$$

$$p = \frac{1}{3} \quad (8)$$

$$q = 1 - p = \frac{2}{3} \quad (9)$$

From (1) we can say,

$$\Pr(X = k) = {}^4C_k \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{4-k} \quad (10)$$

for $k = 0, 1, 2, 3, 4$

Distribution of X

x_i	$P(x_i)$	$x_i P(x_i)$
0	${}^4C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4$	0
1	${}^4C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3$	${}^4C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3$
2	${}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2$	$2 \left({}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 \right)$
3	${}^4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1$	$3 \left({}^4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1 \right)$
4	${}^4C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^0$	$4 \left({}^4C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^0 \right)$

Table 1: Probability Distribution of X

Mean (μ)

We know that, from (6)

$$\mu = np \quad (11)$$

$$\mu = \frac{4}{3} \quad (12)$$

Result

The mean of the binomial distribution $B\left(4, \frac{1}{3}\right) = \frac{4}{3}$