Assignment 12

Dasari Srinith

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Outline

Ex 6.6

Show that

$$E\{y|x \leq 0\} = \frac{1}{F_x(0)} \int_{-\infty}^{0} E\{y|x\} f_x(x)\, dx \tag{1}$$

Theory

$$F_y\{y|X < x\} = \frac{P(X \leq x, Y \leq y)}{X \leq x} = \frac{F(x,y)}{F_x(x)} \tag{2}$$

$$f_y\{y|X < x\} = \frac{1}{F_x(x)} \frac{\partial F(x,y)}{\partial y} \tag{3}$$

Solution

$$E\{y|x \leq 0\} = \int_{-\infty}^{\infty} y.f_y(y|x \leq 0)\,dy \tag{4}$$

From (3) we can write,

$$E\{y|x \leq 0\} = \int_{-\infty}^{\infty} y.\left\{\frac{1}{F_x(0)}\frac{\partial F(0,y)}{\partial y}\right\} dy \tag{5}$$

$$E\{y|x \leq 0\} = \frac{1}{F_x(0)}\int_{-\infty}^{\infty} y.\left\{\frac{\partial F(0,y)}{\partial y}\right\} dy \tag{6}$$

And ,

$$\int_{-\infty}^{0} E\{y|x\} f_x(x)\,dx = \int_{-\infty}^{0} \left\{ \int_{-\infty}^{\infty} y f(y|x)\,dy \right\} f_x(x)\,dx \tag{7}$$

$$\int_{-\infty}^{0} E\{y|x\} f_x(x)\,dx = \int_{-\infty}^{\infty} y \left\{ \int_{-\infty}^{0} f(y|x) f_x(x)\,dy \right\} dx \tag{8}$$

$$\int_{-\infty}^{0} E\{y|x\} f_x(x)\,dx = \int_{-\infty}^{\infty} y \left\{ \int_{-\infty}^{0} f(x,y)\,dy \right\} dx \tag{9}$$

$$\int_{-\infty}^{0} E\{y|x\} f_x(x)\,dx = \int_{-\infty}^{\infty} y \left\{ \frac{\partial F(0,y)}{\partial y} \right\} dx \tag{10}$$

So , from (6) , (10)

$$E\{y|x \le 0\} = \frac{1}{F_x(0)} \int_{-\infty}^{0} E\{y|x\} f_x(x)\, dx \tag{11}$$