# Assignment 10

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# Outline

# Example 34

Find the mean of the binomial distribution $B\left(4, \frac{1}{3}\right)$

# Theory

A binomial distribution with n- Bernoulli trials and probability of success in each trial as p , is denoted by B $(n, p)$

The probability of k successes $\Pr(X = k)$ is also denoted by $P(k)$ and is given by

$$\Pr(X = k) = {}^{n}C_{k} p^{k} (1 - p)^{n-k} \tag{1}$$

for $x = 0, 1, 2, ..., n - 1, n$

# Mean

$$\mu = \sum_{i=1}^{n} x_i P(x_i) \tag{2}$$

$$\mu = \sum_{r=1}^{n} r \times {}^{n}C_r p^r \left(1 - p\right)^{n-r} \tag{3}$$

$$\mu = (1 - p)^n \sum_{r=1}^{n} r \times {}^{n}C_r \left(\frac{p}{1 - p}\right)^r \tag{4}$$

$$\mu = (1 - p)^n \frac{np}{(1 - p)^n} \tag{5}$$

$$\mu = np \tag{6}$$

# Solution

Let $X$ be the random variable whose probability distribution is $\mathrm{B}\left(4, \frac{1}{3}\right)$.

So , we can write that ,

$$n = 4 \tag{7}$$

$$p = \frac{1}{3} \tag{8}$$

$$q = 1 - p = \frac{2}{3} \tag{9}$$

From (1) we can say,

$$\Pr\left(X = k\right) = {}^{4}C_{k} \left(\frac{1}{3}\right)^{k} \left(\frac{2}{3}\right)^{4-k} \tag{10}$$

for $k = 0, 1, 2, 3, 4$

# Distribution of X

| $x_i$ | $P(x_i)$ | $x_iP(x_i)$ |
|---|---|---|
| 0 | ${}^4C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^4$ | 0 |
| 1 | ${}^4C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3$ | ${}^4C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3$ |
| 2 | ${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$ | $2\left({}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2\right)$ |
| 3 | ${}^4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1$ | $3\left({}^4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1\right)$ |
| 4 | ${}^4C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0$ | $4\left({}^4C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0\right)$ |

Table 1: Probability Distribution of X

# Mean ($\mu$)

We know that, from (6)

$$\mu = np \tag{11}$$

$$\mu = \frac{4}{3} \tag{12}$$

# Result

The mean of the binomial distribution $\mathrm{B}\left(4, \dfrac{1}{3}\right) = \dfrac{4}{3}$