AI1110 ASSIGNMENT-8

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***Abstract*—This document contains the solution for Assignment 8 (NCERT GRADE 12 CHAPTER 13 Example 9)**

EXAMPLE 9 :

Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is an ace ?

SOLUTION :

In a deck of 52 cards there are 4 King cards and 4 Ace cards in total. Let us take three random variables X,Y,Z for three trials of drawing a card each of which takes values from the set of real numbers 0,1,2 . Let us define those events in Table I, Table II, Table III.

Event	Description of event
$X = 0$	The drawn card is a King
$X = 1$	The drawn card is an ace
$X = 2$	The drawn card is neither an ace nor a King

TABLE I
DEFINING THE EVENTS FOR FIRST DRAW

Event	Description of event
$Y = 0$	The drawn card is a King
$Y = 1$	The drawn card is an ace
$Y = 2$	The drawn card is neither an ace nor a King

TABLE II
DEFINING THE EVENTS FOR SECOND DRAW

Event	Description of event
$Z = 0$	The drawn card is a King
$Z = 1$	The drawn card is an ace
$Z = 2$	The drawn card is neither an ace nor a King

TABLE III
DEFINING THE EVENTS FOR THIRD DRAW

We have to find $\Pr\left((X=0),(Y=0),(Z=1)\right)$.

We know that ,

$$\Pr\left((X=0)\right) = \frac{\text{Number of Kings}}{\text{Total number of cards}} \tag{1}$$

$$\Pr\left((X=0)\right) = \frac{4}{52} \tag{2}$$

and , also $\Pr\left((Y=0)|(X=0)\right)$ is the probability of second king with the condition that one king has already been drawn . As now ,there are 3 Kings in 51 cards .

$$\Pr\left((Y=0)|(X=0)\right) = \frac{3}{51} \tag{3}$$

Lastly , $\Pr\left((Z=1)|(Y=0),(X=0)\right)$ is the probability of third drawn card to be an ace ,with the condition that two kings have already been drawn . As now , there are 4 aces in 50 cards .

$$\Pr\left((Z=1)|(Y=0),(X=0)\right) = \frac{4}{50} \tag{4}$$

By multiplication law of probability ,we have

$$\begin{aligned}\Pr\left((X=0),(Y=0),(Z=1)\right) = \\ \Pr\left((X=0)\right)\Pr\left((Y=0)|(X=0)\right) \\ \Pr\left((Z=1)|(X=0),(Y=0)\right)\end{aligned} \tag{5}$$

$$\Pr\left((X=0),(Y=0),(Z=1)\right) = \frac{4}{52}\frac{3}{51}\frac{4}{50} \tag{6}$$

$$= \frac{2}{5525} \tag{7}$$