Assignment 9

Dasari Srinith

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Outline

Exercise 13.3.5

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive ?

Bayes' Theorem

Bayes theorem is used to determine conditional probability.

$$\Pr\left(\left(E_i|A\right)\right) = \frac{\Pr\left(\left(AE_i\right)\right)}{\Pr\left(A\right)} \tag{1}$$

$$= \frac{\Pr\left(\left(AE_i\right)\right)}{\sum_{i=0}^{n} \Pr\left(AE_i\right)} \tag{2}$$

Solution

Let X, Y be two random variables which maps to following set of real numbers , $X \in \{0, 1\}$, $Y \in \{0, 1\}$ as defined below in Table 1 , Table 2 respectively.

Event	**Description of event**
$X = 0$	Person doesn't have the disease
$X = 1$	Person has the disease

Table 1: Defining the events for X

Event	Description of event
$Y = 0$	Test result isn't positive
$Y = 1$	Test result is positive

Table 2: Defining the events for Y

We have to find $\Pr\left((X = 1)|(Y = 1)\right)$.

Input data

From the data given in the question we can write the following ,

Expression	**Value**
$\Pr\left((Y = 1)\vert(X = 1)\right)$	0.99
$\Pr\left((Y = 1)\vert(X = 0)\right)$	0.005
$\Pr\left((X = 1)\right)$	0.001

Table 3: Input data

Computation

Since the events $X = 0$ and $X = 1$ are complimentary to each other ,

$$\Pr\left((X = 0)\right) + \Pr\left((X = 1)\right) = 1 \tag{3}$$

$$\Pr\left((X = 0)\right) = 0.999 \tag{4}$$

By using Bayes' theorem i.e, (2) we have

$$\Pr\left((X=1)|(Y=1)\right) = \frac{\Pr\left((X=1)\right)\Pr\left((Y=1)|(X=1)\right)}{\Pr\left((X=1)\right)\Pr\left((Y=1)|(X=1)\right)+\Pr\left((X=0)\right)\Pr\left((Y=1)|(X=0)\right)} \tag{5}$$

$$= \frac{(0.001)(0.99)}{(0.001)(0.99)+(0.999)(0.005)} \tag{6}$$

$$= \frac{0.00099}{0.005985} \tag{7}$$

$$= \frac{990}{5985} = \frac{22}{133} \tag{8}$$

Result

If 0.1 percent of the population actually has the disease , the probability that a person has the disease given that his test result is positive is $\dfrac{22}{133}$