Assignment 11

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Outline

Ex 6.6

The joint p.d.f. of x and y is given by

$$f_{xy}(x,y) = \begin{cases} 2(1-x) & 0 < x \leq 1, 0 < y \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Determine the probability density function of z = xy.

Theory

Suppose X and Y are jointly continuous random variables and we define a new random variable by Z = g(X,Y) then ,

The cumulative distributive function of Z is

$$F_Z(z) = P(Z \leq z) = \iint_A f_{XY}(x,y)\, dy\, dx \tag{2}$$

On differentiating the c.d.f wrt to Z , we get the probability density function

$$f_Z(z) = \frac{d}{dz} F_Z(z) \tag{3}$$

Solution

It is given that a new random variable Z = XY where each of X,Y are uniformly distributed over [0,1].

We can also note that the range of Z is [0,1].

So ,we can write that ,

$$F_Z(z) = P(Z \leq z) = \iint_A (2 - 2x)\, dy\, dx \tag{4}$$

where ,

$$A = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1, xy \leq z\} \tag{5}$$

So ,

$$F_Z(z) = \int_0^z \left[\int_0^1 (2-2x)\,dy\right] dx + \int_z^1 \left[\int_0^{\frac{z}{x}} (2-2x)\,dy\right] dx \tag{6}$$

$$F_Z(z) = \int_0^z \left[(2-2x)\right] dx + \int_z^1 \left[(2-2x)\frac{z}{x}\right] dx \tag{7}$$

$$F_Z(z) = (2z - z^2) + (-2z\ln z - 2z(1-z)) \tag{8}$$

$$F_Z(z) = z^2 - 2z\ln z \tag{9}$$

So ,

$$f_Z(z) = \frac{d}{dz} F_Z(z) \tag{10}$$

$$f_Z(z) = \frac{d}{dz}\left(z^2 - 2z \ln z\right) \tag{11}$$

$$f_Z(z) = 2z - 2(1 + \ln z) \tag{12}$$

$$f_Z(z) = 2z - 2 - 2 \ln z \tag{13}$$

Result

The probability density of z is

$$f_Z(z) = 2z - 2 - 2\ln z$$