

Logistic regression:

where $g(z) = \frac{1}{1 + e^{-z}}$

→ Sigmoid or logistic function

Want $h_{\theta}(x) \in [0, 1]$

So we take ;

$$h_{\theta}(x) = g(\theta^T x) = \frac{1}{1 + e^{-\theta^T x}}$$

→ $P(y=1 | x; \theta) = h_{\theta}(x)$

So, $P(y=0 | x; \theta) = 1 - h_{\theta}(x)$

and y can take only two values $\{0, 1\}$

→ So, $P(y | x; \theta) = (h_{\theta}(x))^y (1 - h_{\theta}(x))^{1-y}$

$$\mathcal{L}(\theta) = P(\bar{y} | x; \theta) = \prod_{i=1}^m P(y^{(i)} | x^{(i)}; \theta)$$

$$= \prod_{i=1}^m (h_{\theta}(x^{(i)}))^{y^{(i)}} (1 - h_{\theta}(x^{(i)}))^{1-y^{(i)}}$$

14 Thursday

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287-078

Reason to take logistic function as $h_{\theta}(x)$

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"log likelihood"

$$l(\theta) = \log \mathcal{L}(\theta)$$

$$= \sum_{i=1}^m y^{(i)} \log h_{\theta}(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_{\theta}(x^{(i)}))$$

$\mathcal{L}(\theta) \rightarrow$ Should be maximised

$\Rightarrow l(\theta) \rightarrow$ Should be maximised

Can be used because $l(\theta)$ won't have local maximum, it will be concave and has single Maximum

To do that we shall use "Batch gradient

$$\theta_j := \theta_j + \alpha \frac{\partial}{\partial \theta_j} l(\theta)$$

i.e

ascent

On substituting $l(\theta)$ & doing the derivatives we get

$$\theta_j := \theta_j + \alpha \sum_{i=1}^m (y^{(i)} - h_{\theta}(x^{(i)})) \cdot x_j^{(i)}$$

"Happiness, like unhappiness, is a proactive choice."

For logistic regression:

$$\theta_j := \theta_j - \alpha \sum_{i=1}^m \left(h_{\theta}(x^{(i)}) - y^{(i)} \right) \cdot x_j^{(i)}$$

where

$$h_{\theta}(x^{(i)}) = \frac{1}{1 + e^{-\vec{x}^{(i)} \vec{\theta}}}$$
