

## 1. Logistic Regression:

### 1.2.1 Behavior of the algorithm when there is no regularization:

The behavior of the logistic regression with no regularization tries to fit the training set as much as possible, given the constraints and thus will have big values for  $w$  and small classification errors.

When there is no regularization, the weights and error counts on training and validation sets are:

Linearly Separable:

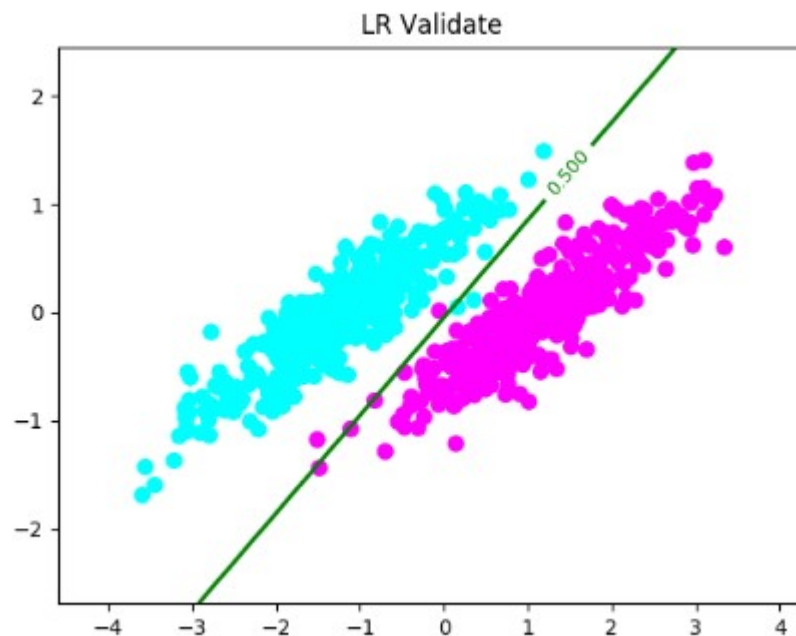
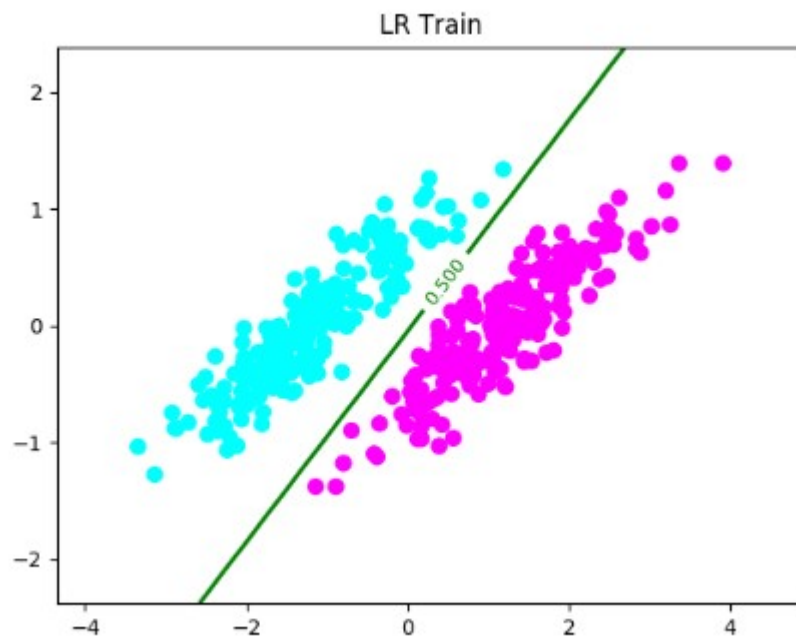
Weights :  $w_0 = 2.59665133$

$w_1 = -47.68478956$

$w_2 = 52.84677869$

Training set error count : 0

Validation set error count : 4



Non Linearly Separable:

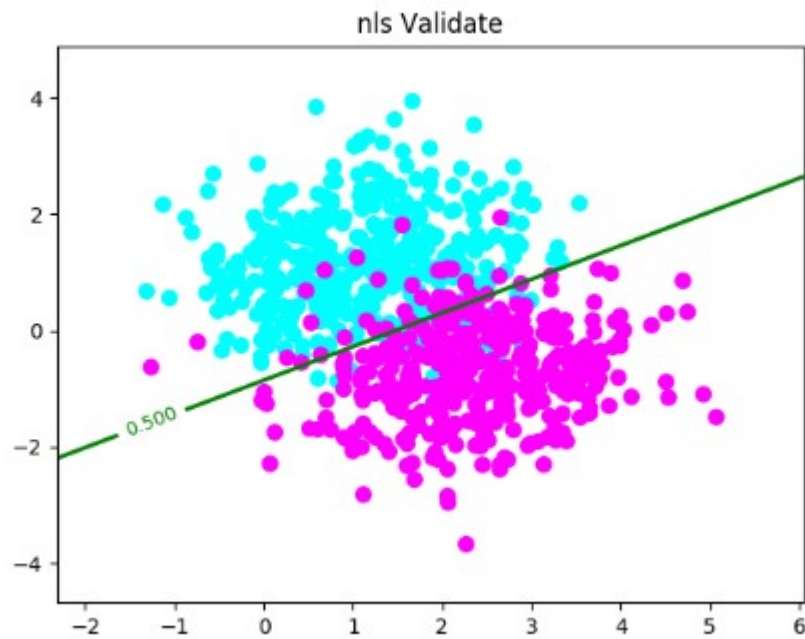
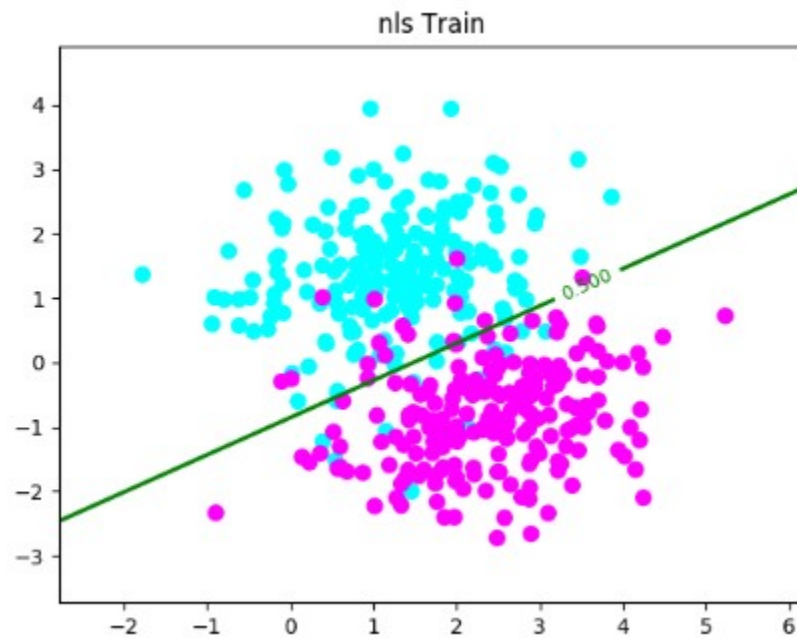
Weights :  $w_0 = 2.09378157$

$w_1 = -1.41668237$

$w_2 = 2.44940736$

Training set error count : 30

Validation set error count : 66



Non Linearly Separable(2):

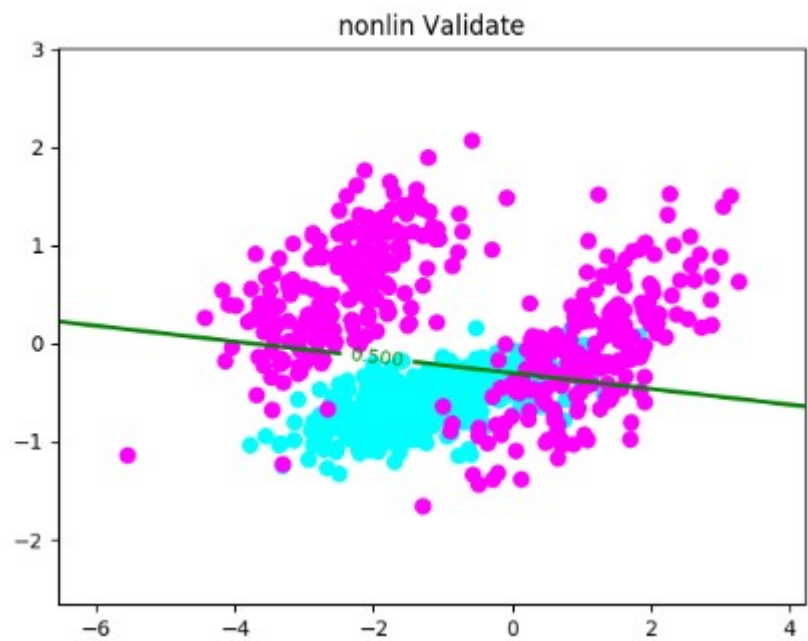
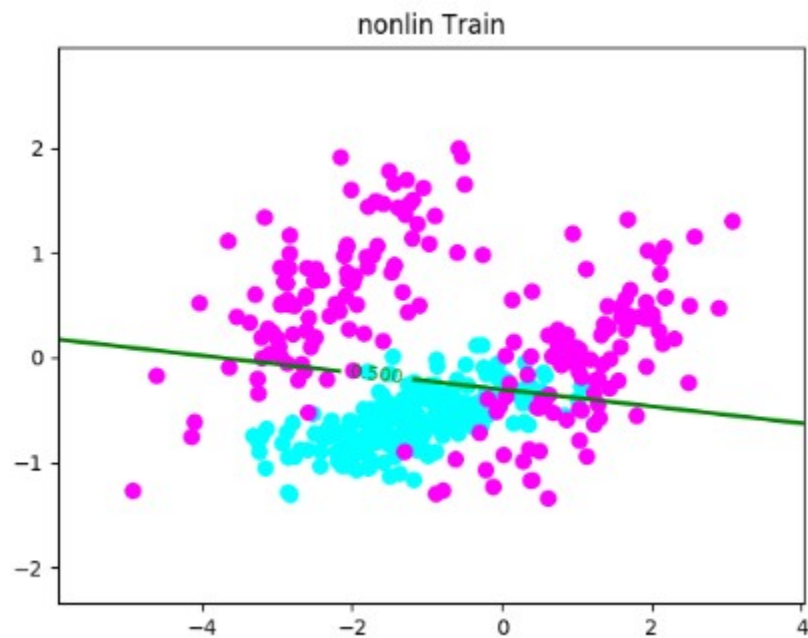
Weights :  $w_0 = -1.01674506$

$w_1 = -0.26996381$

$w_2 = -3.35432192$

Training set error count : 69

Validation set error count : 130



### 1.2.2 Behavior of the algorithm when lambda increases:

When lambda increases, the model will be regularized and hence the number of errors increases and the weight vector will be smaller. As lambda increases, the slope of the decision boundary increases until a point when it becomes parallel to y axis. ( $w_1$ ,  $w_2$  will be heavily regularized that they become equal to 0). When  $L = 10$ , the graphs and the corresponding weights are:

Linearly Separable:

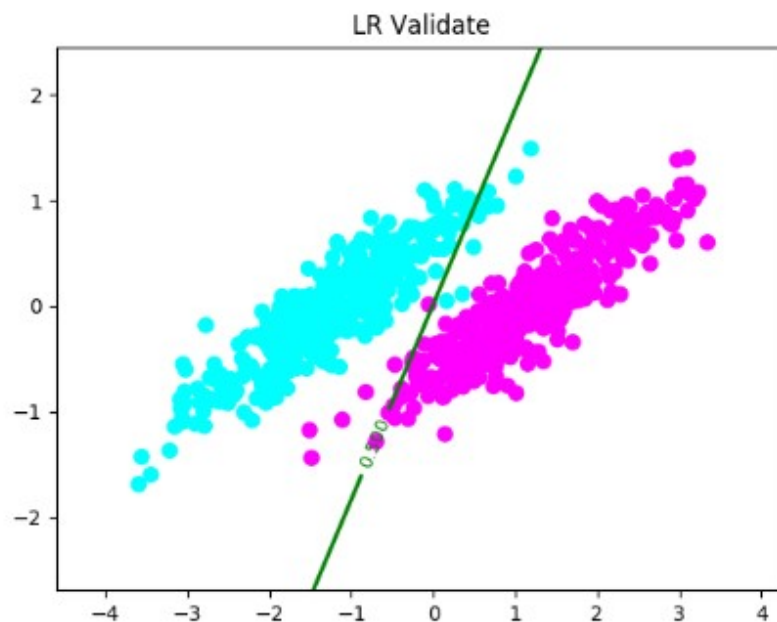
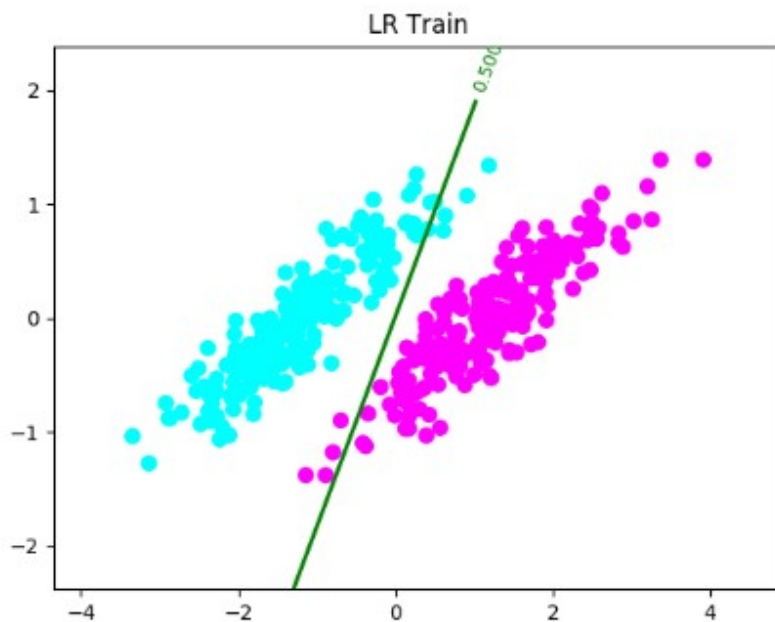
Weights :  $w_0 = -0.01506527$

$w_1 = -1.72425101$

$w_2 = 0.93339326$

Training set error count : 8

Validation set error count : 17



Non Linearly Separable:

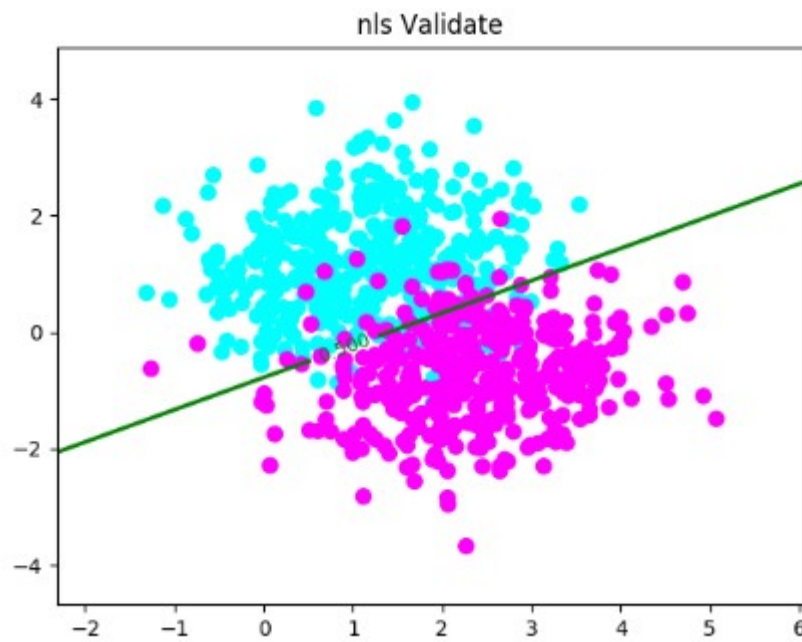
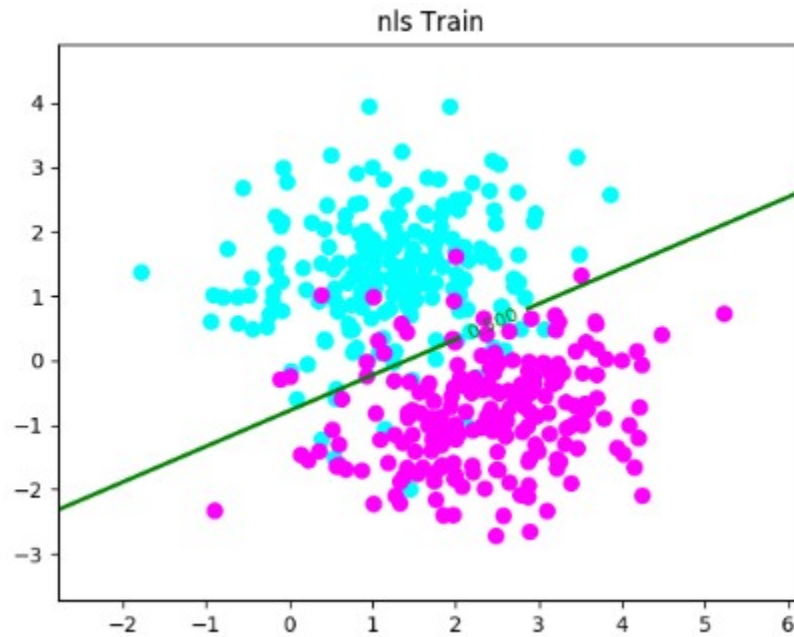
Weights :  $w_0 = 1.08993567$

$w_1 = -0.77421673$

$w_2 = 1.39863589$

Training set error count : 30

Validation set error count : 68



Non Linearly Separable(2):

Weights :  $w_0 = -0.45733524$

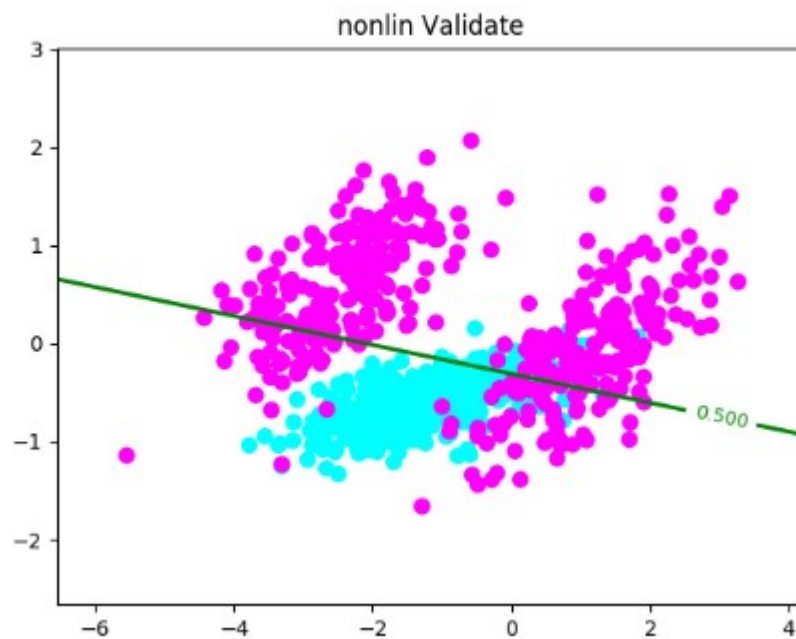
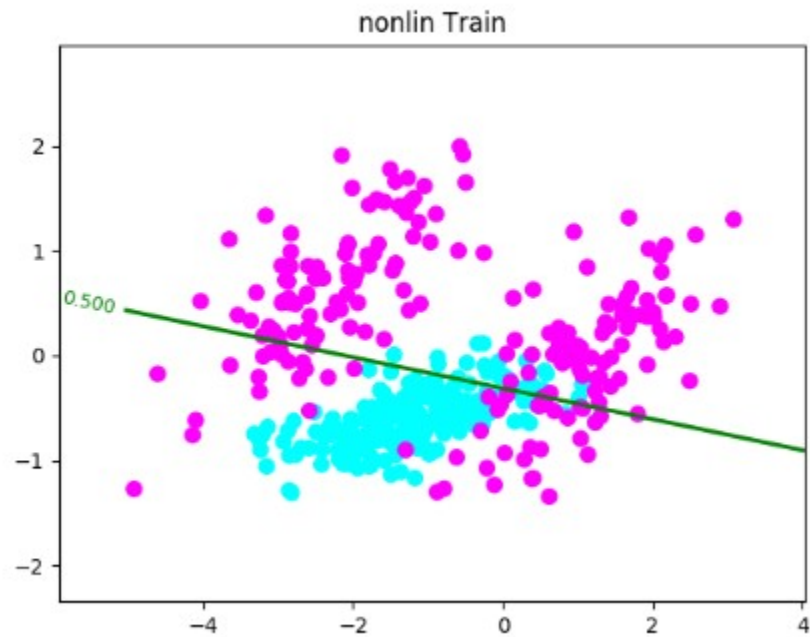
$w_1 = -0.2172903$

$w_2 = -1.47284982$

Training set error count : 73

Validation set error count : 143

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**When  $l = 100$ ,**

Linearly Separable:

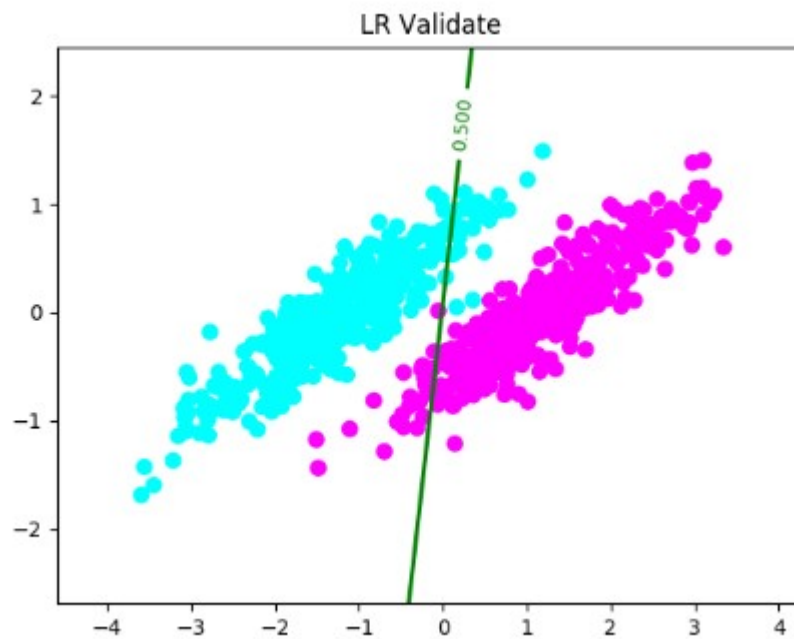
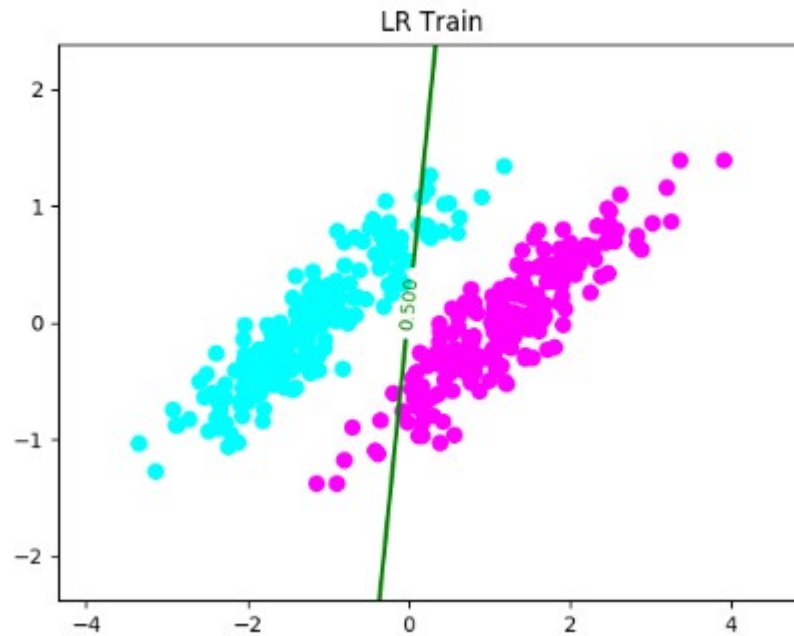
Weights :  $w_0 = -0.00817829$

$w_1 = -0.62546143$

$w_2 = 0.09179153$

Training set error count : 23

Validation set error count : 50





Non-Linearly Separable:

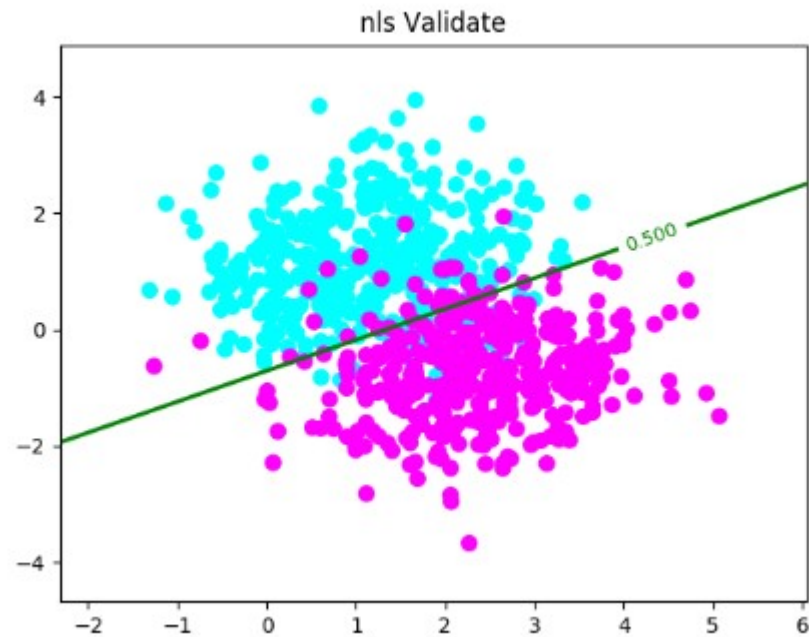
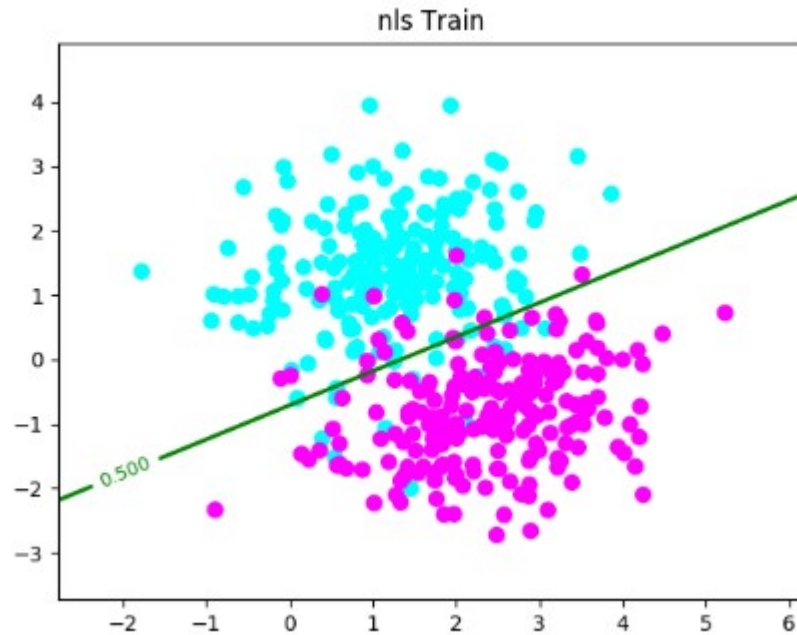
Weights :  $w_0 = 0.38704908$

$w_1 = -0.28980222$

$w_2 = 0.5459234$

Training set error count : 29

Validation set error count : 69





Non-Linearly Separable(2):

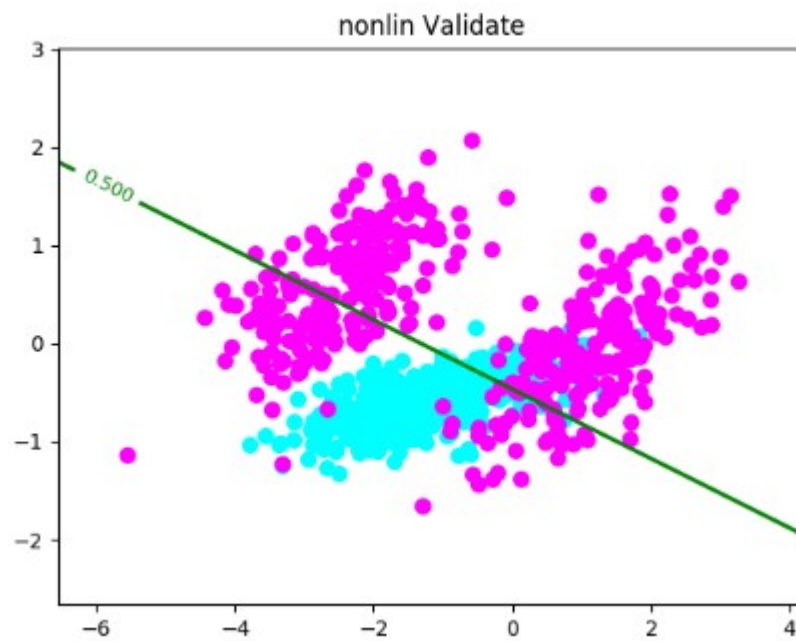
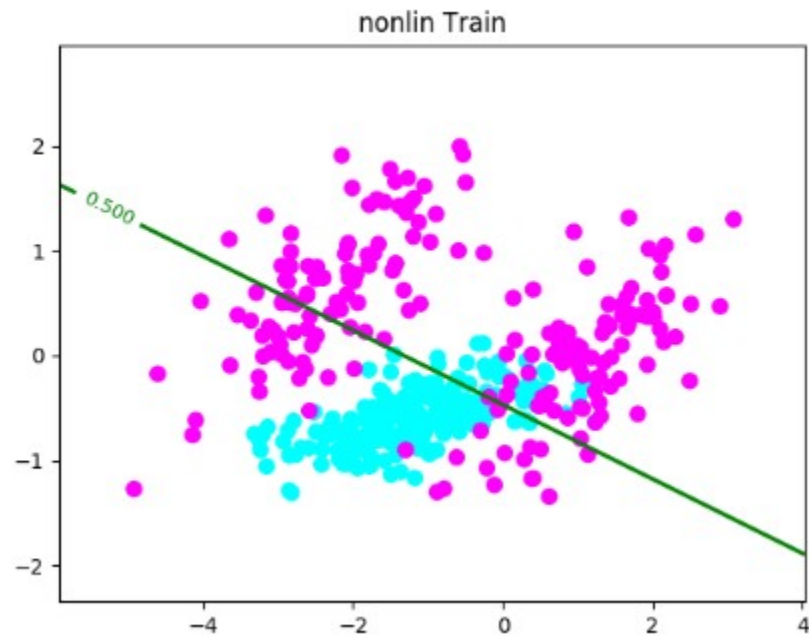
Weights :  $w_0 = -0.16595198$

$w_1 = -0.12572148$

$w_2 = -0.3552611$

Training set error count : 86

Validation set error count : 193



### 1.2.3 General trends:

Since the regularized logistic regression is a for a linearly separable input data, it is not able to accurately classify the data points for non separable data. Hence, the error on non separable data set is very high.

### 1.3: Regularization on polynomial basis functions:

Since we are mapping the input to higher order, it is possible for the logistic regression to handle second order polynomial basis. Hence without regularization, the model over-fits. Regularization causes the weights to be small and the weights will be smaller as we increase  $\lambda$ . For non-linear and non separable cases, the error is less since the features are mapped in higher dimensions. For linearly separable data, mapping to higher order feature space provided worse results but for nls and nonlin data sets, there was a better performance because mapping into higher space allowed the decision boundary to linearly separate the data in the new feature space.

As we can see from the above graphs and data, as we increase the regularizing term  $\lambda$ , over-fitting reduces and the number of errors decreases. The rate of change of error rate in validation set is less and this shows us that when using regularization, over-fitting is reduced and hence the model is a little flexible but the number of errors in the training data increases since the model is not tightly fit to it anymore.

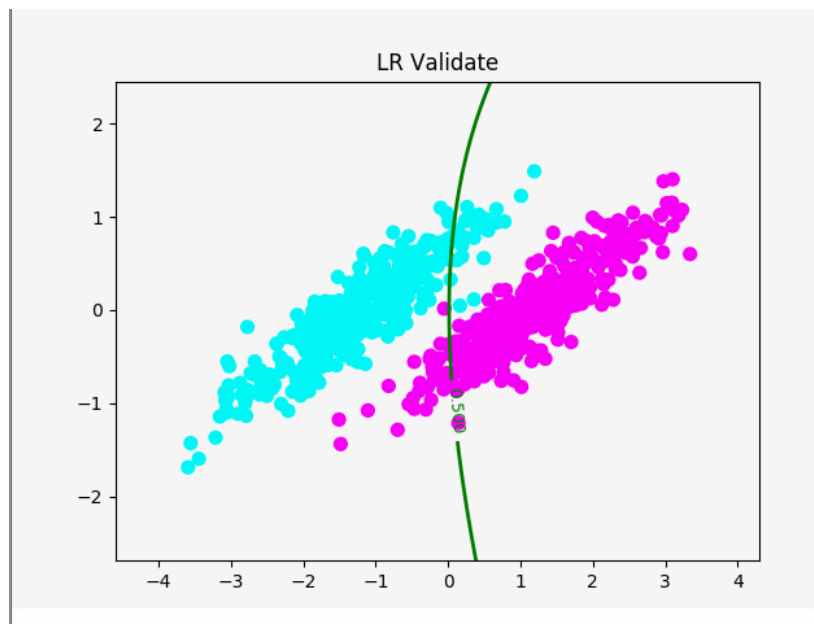
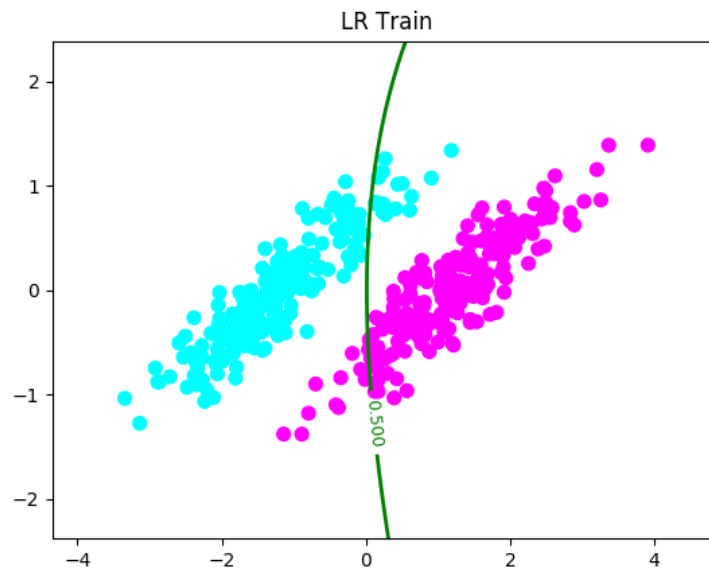
The result of using polynomial basis functions setting  $\lambda = 0$  (**No regularization**) is given below. The weights and the number of errors are also specified.

Linearly Separable:

Weights :  $w_0 = 0.04729134$   
 $w_1 = -2.42724697$   
 $w_3 = 0.1450316$   
 $w_4 = 0.14503517$   
 $w_5 = -0.14854224$   
 $w_6 = 0.23866279$

Training set error count : 27

Validation set error count : 59

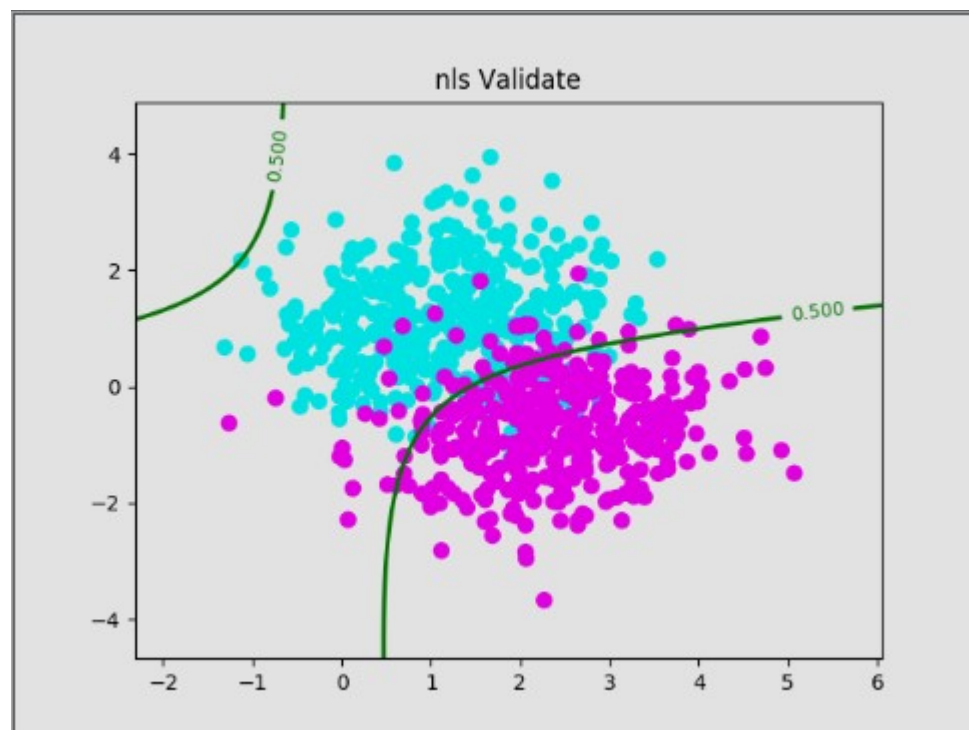
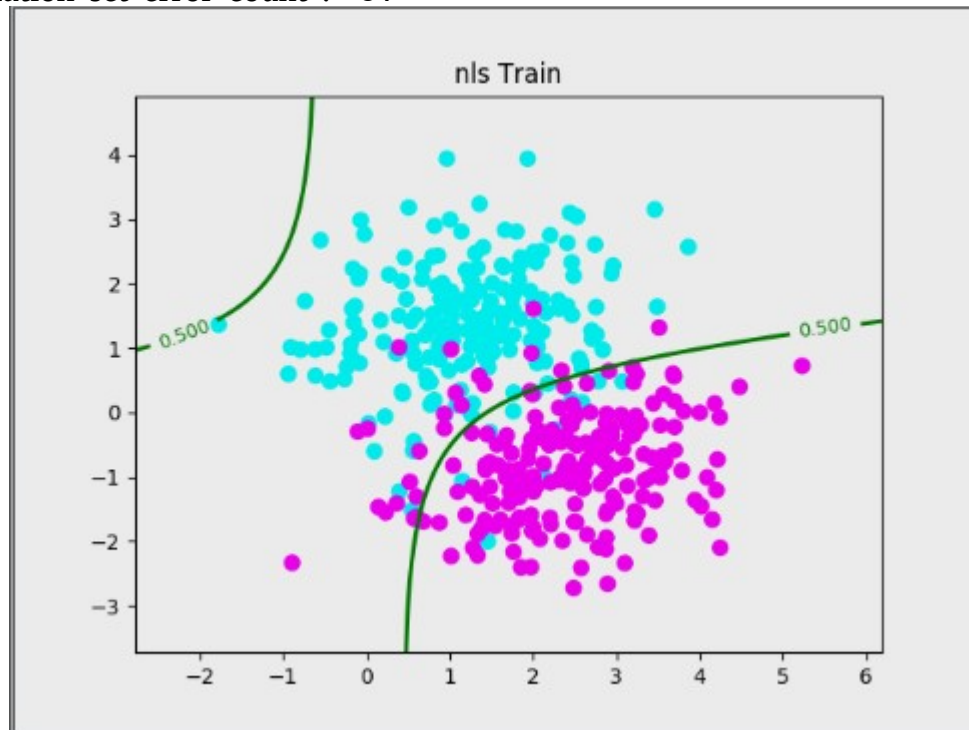


Non Linearly Separable:

Weights :     $w_0 = 1.93069782$   
               $w_1 = -0.76020259$   
               $w_3 = 0.46761887$   
               $w_4 = 0.46760018$   
               $w_5 = -0.18408145$   
               $w_6 = 0.06980724$

Training set error count : 35

Validation set error count : 81

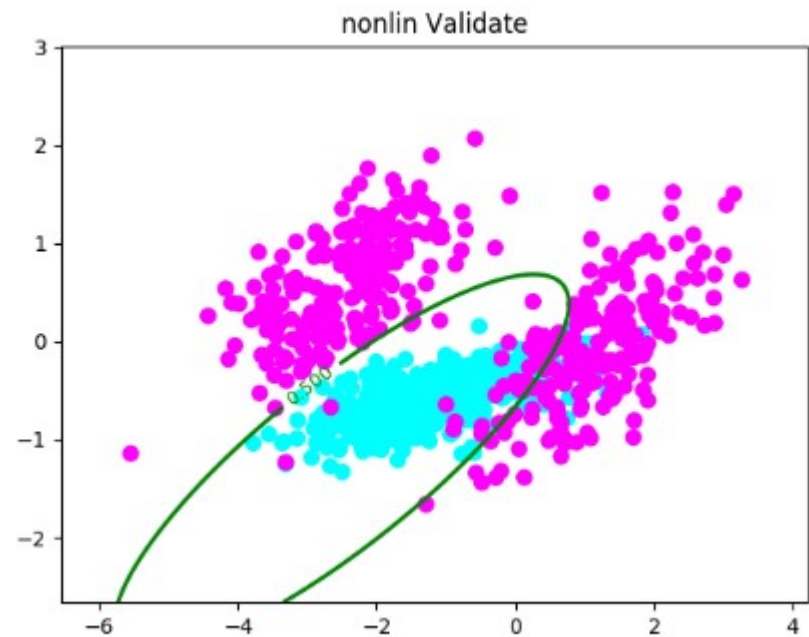
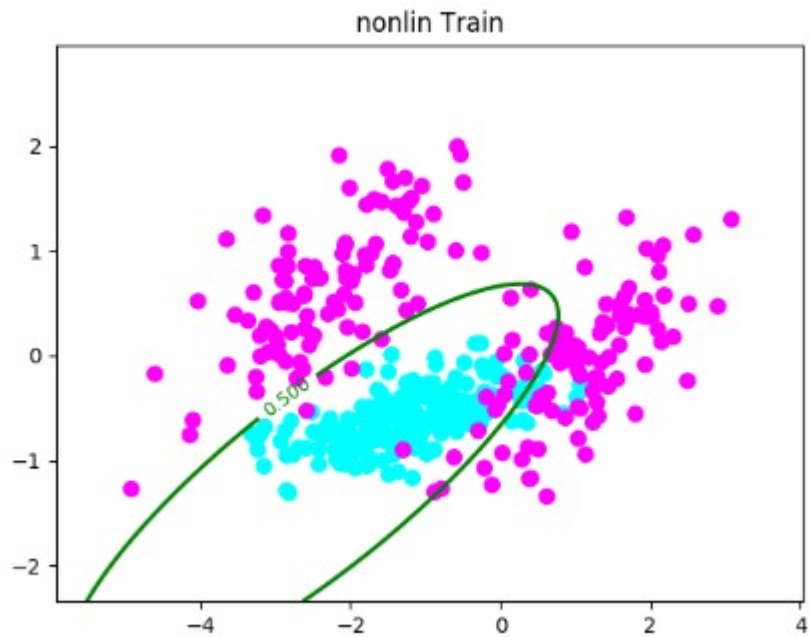


Non Linearly Separable(2):

Weights :     $w_0 = 1.73782384$   
               $w_1 = -1.37659095$   
               $w_3 = 1.3825534$   
               $w_4 = 1.38255344$   
               $w_5 = -1.36161498$   
               $w_6 = -3.92934765$

Training set error count : 35

Validation set error count : 72

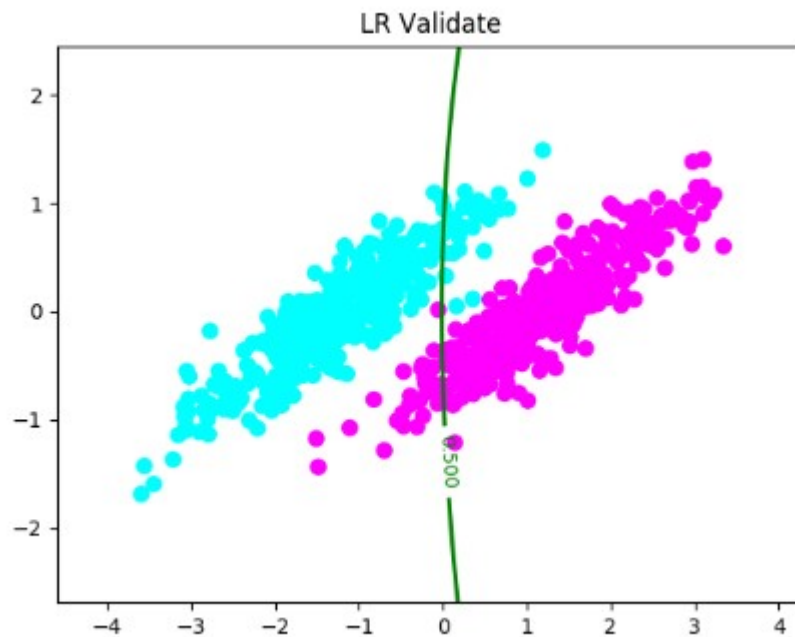
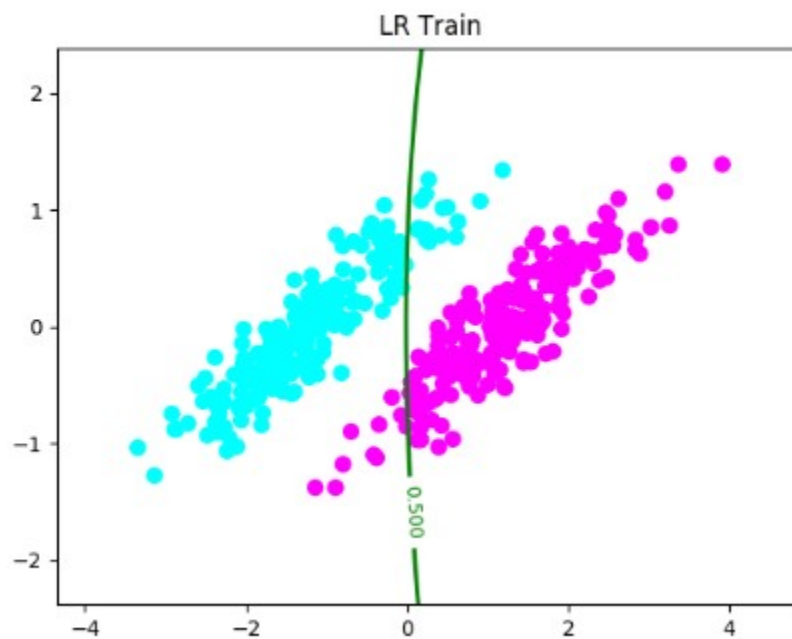


When  $l = 10$ ,  
Linearly Separable:

Weights :  $w_0 = -0.02418571$   
 $w_1 = -1.4304912$   
 $w_3 = 0.03984169$   
 $w_4 = 0.03984173$   
 $w_5 = -0.00655184$   
 $w_6 = 0.06099804$

Training set error count : 25

Validation set error count : 60

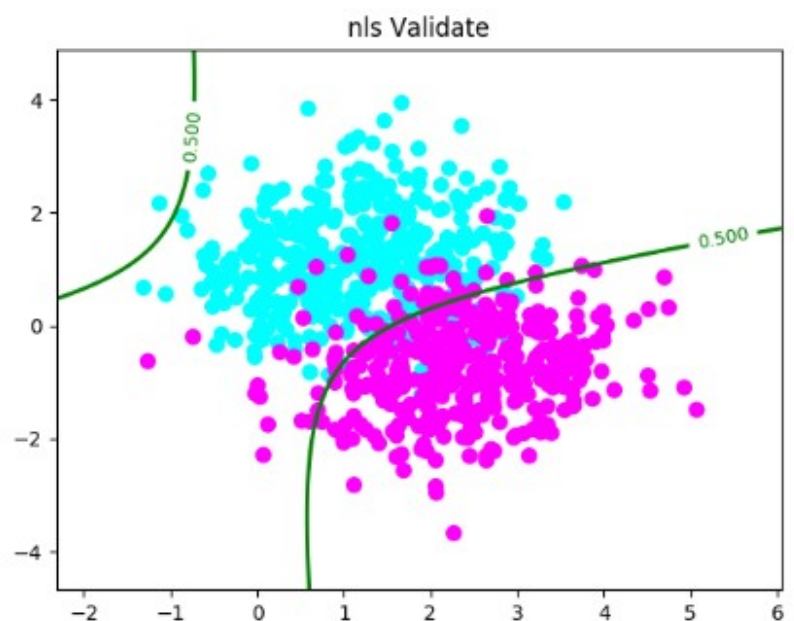
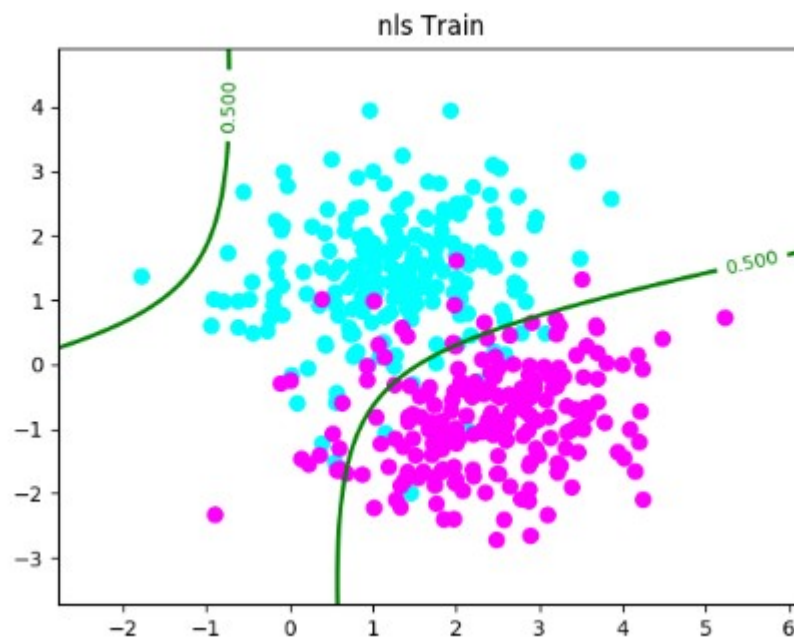


Non Linearly Separable:

Weights :     $w_0 = 1.53472266$   
               $w_1 = -0.38436523$   
               $w_3 = 0.4076053$   
               $w_4 = 0.40760526$   
               $w_5 = -0.28855404$   
               $w_6 = 0.0977048$

Training set error count : 37

Validation set error count : 81



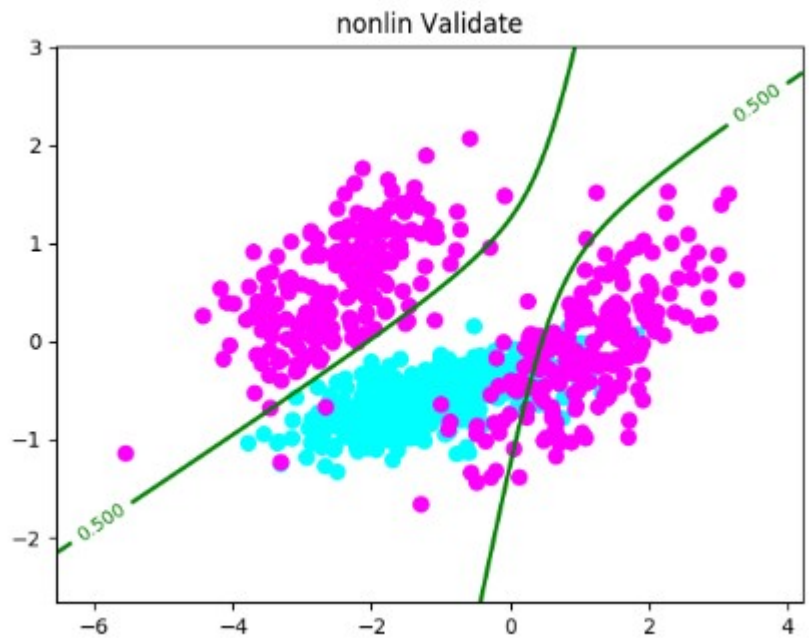
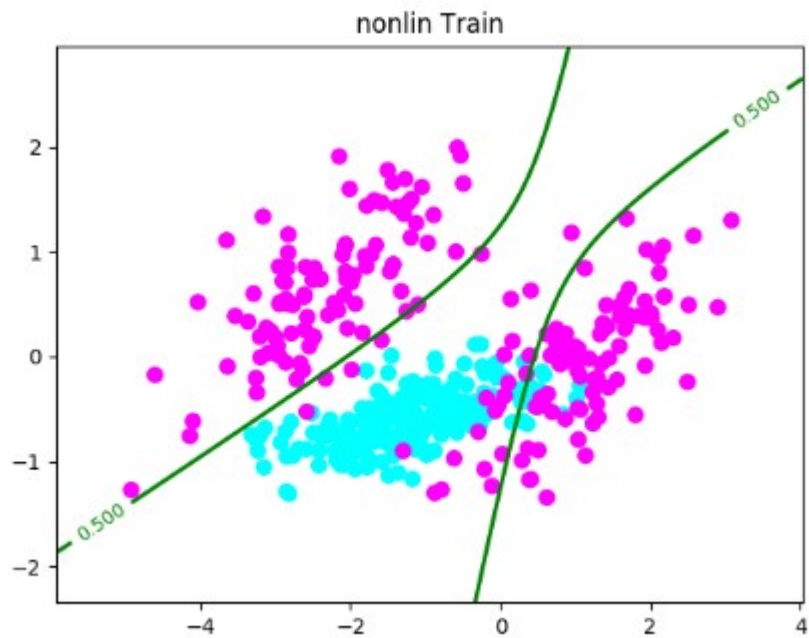


Non Linearly Separable(2):

Weights :     $w_0 = 0.67617748$   
               $w_1 = -0.81251499$   
               $w_3 = 0.6192013$   
               $w_4 = 0.61920118$   
               $w_5 = -0.72197768$   
               $w_6 = -0.43022213$

Training set error count : 40

Validation set error count : 74



## 2. Support Vector Machine:

### 2.2 SVM Implementation report:

The results of implementing SVM is depicted in the graphs below. Since nls and nonlin are not linearly separable, the upper bound of  $c$  on alpha was required to produce a classification. The primal form of SVM is also implemented and the graphs corresponding to  $c = 50$  is given below for both primal and dual on all the data sets. The weights and the misclassification errors on the training and validation set is also listed. As we can see from the graphs below, as we increase  $c$ , the model over-fits and the number of misclassification error reduces. The effect of  $c$  in primal is more noticeable than that of dual.

#### Primal:

Linearly Separable:

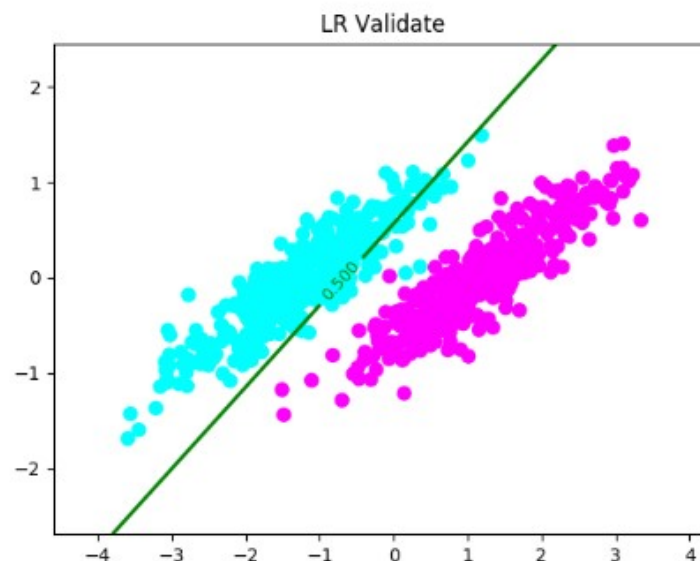
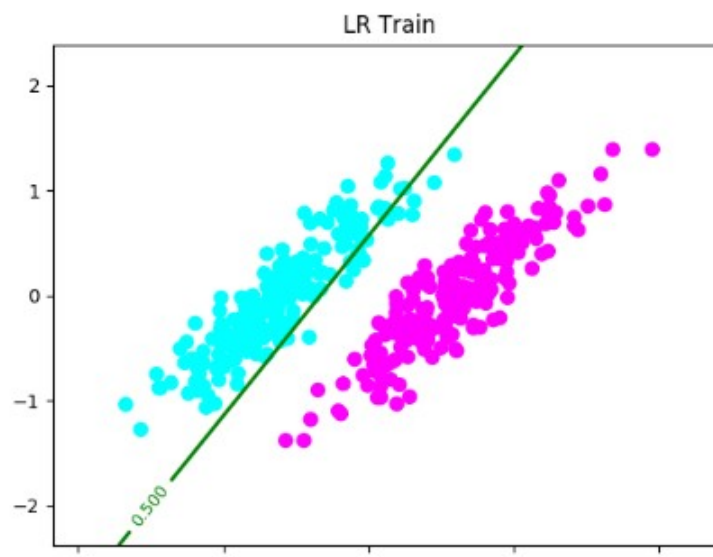
Weights :  $w_0 = -0.24505618$

$w_1 = -1.12852829$

$w_2 = 1.31886421$

Training set error count : 0

Validation set error count : 3



Non Linearly Separable:

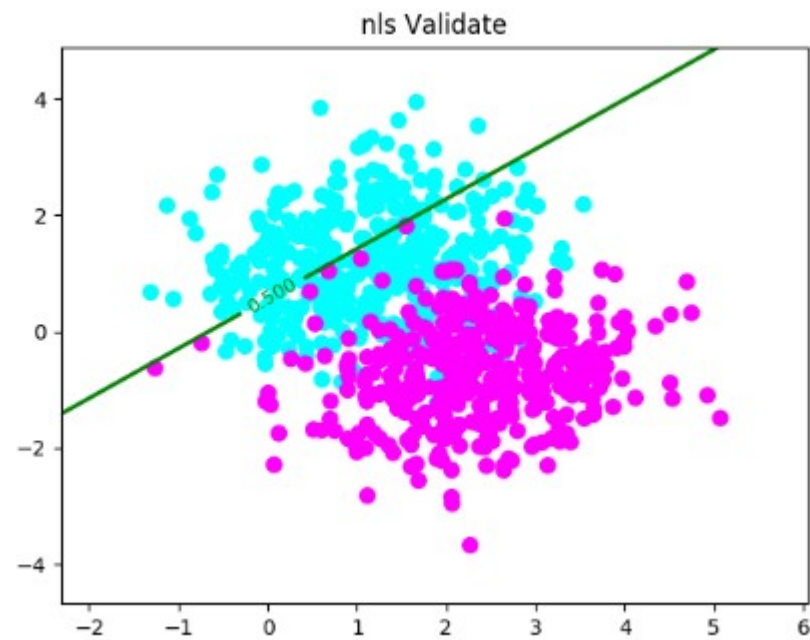
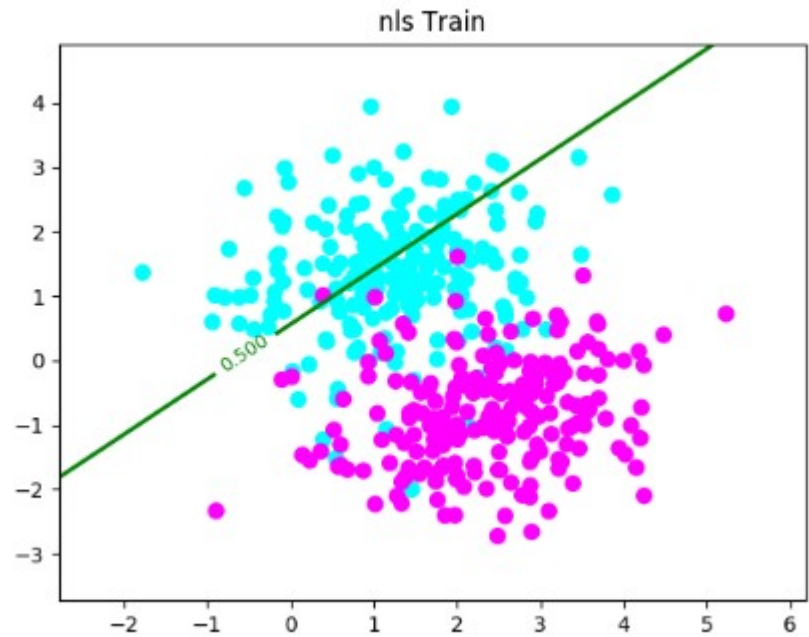
Weights :  $w_0 = -0.48839257$

$w_1 = 0.19219624$

$w_2 = 1.95725041$

Training set error count : 91

Validation set error count : 214



Non Linearly Separable(2):

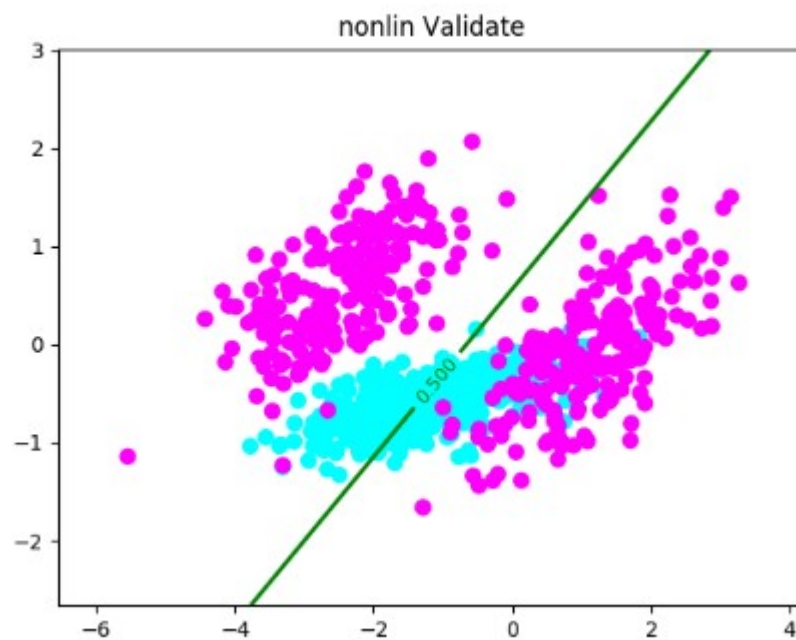
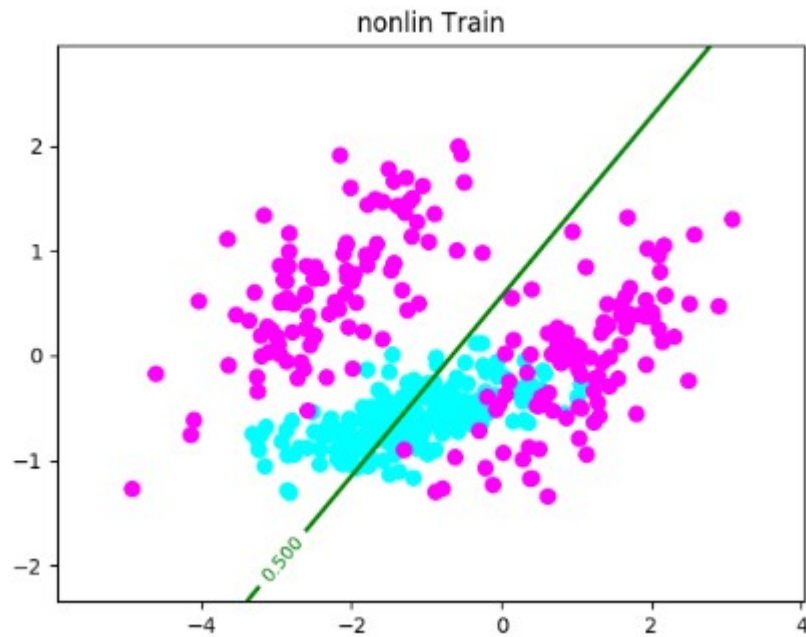
Weights :  $w_0 = -0.50677567$

$w_1 = -0.21785751$

$w_2 = -1.90695535$

Training set error count : 180

Validation set error count : 352



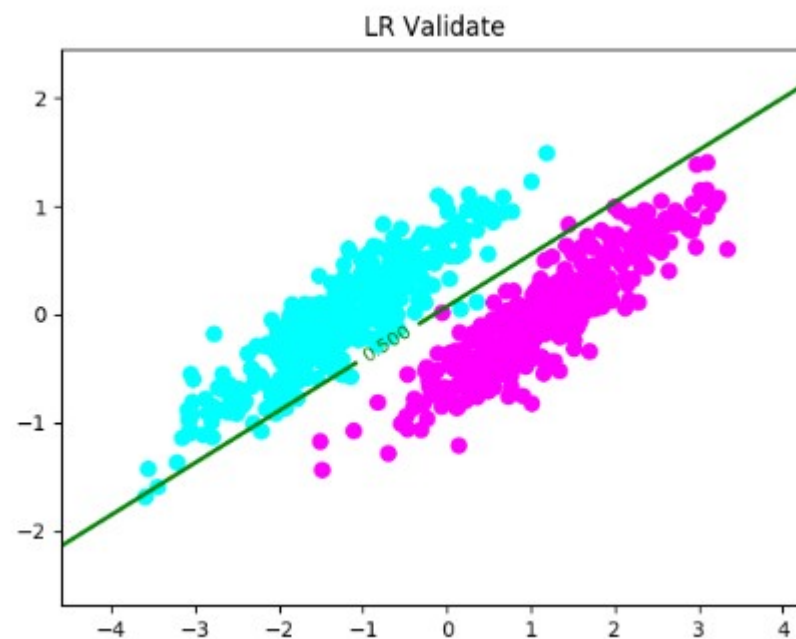
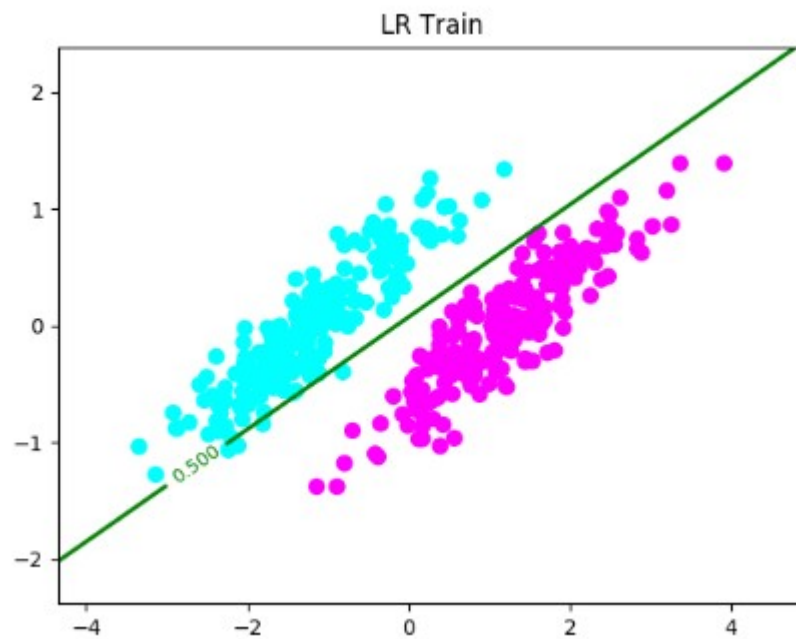
**When  $c = 10$ , for primal we get the following output**

Linearly Separable:

Weights :  $w_0 = 0.30644623$   
 $w_1 = -1.27487651$   
 $w_2 = 2.65057583$

Training set error count : 4

Validation set error count : 11



Non Linearly Separable:

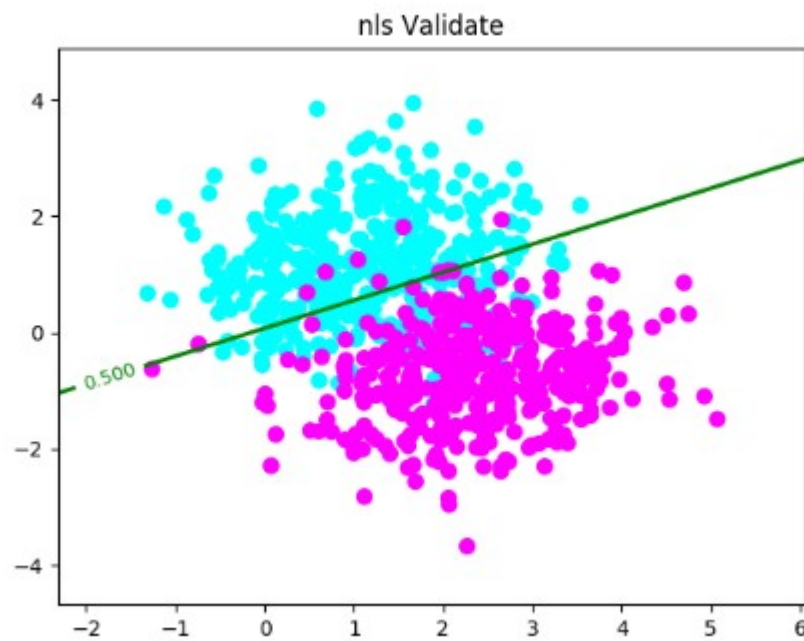
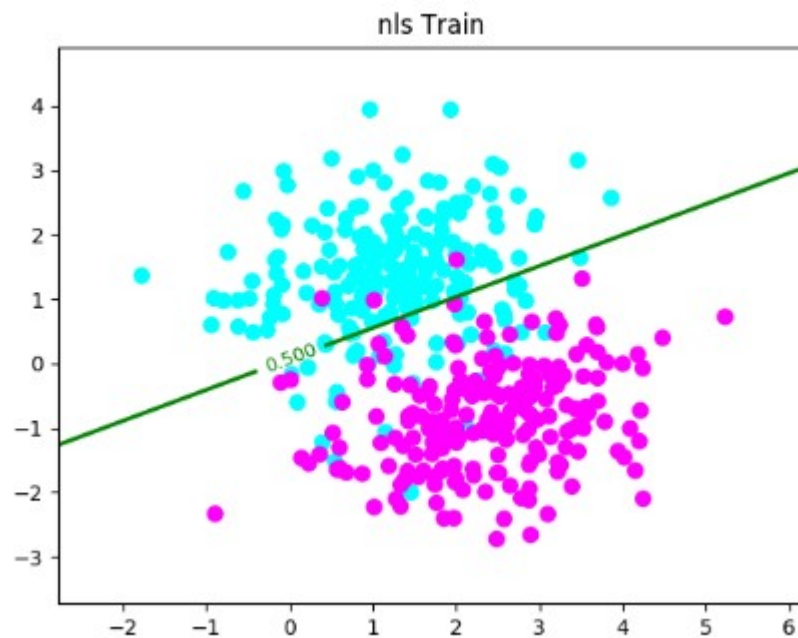
Weights :  $w_0 = -0.48839257$

$w_1 = 0.19219624$

$w_2 = 1.95725041$

Training set error count : 42

Validation set error count : 103



Non Linearly Separable(2):

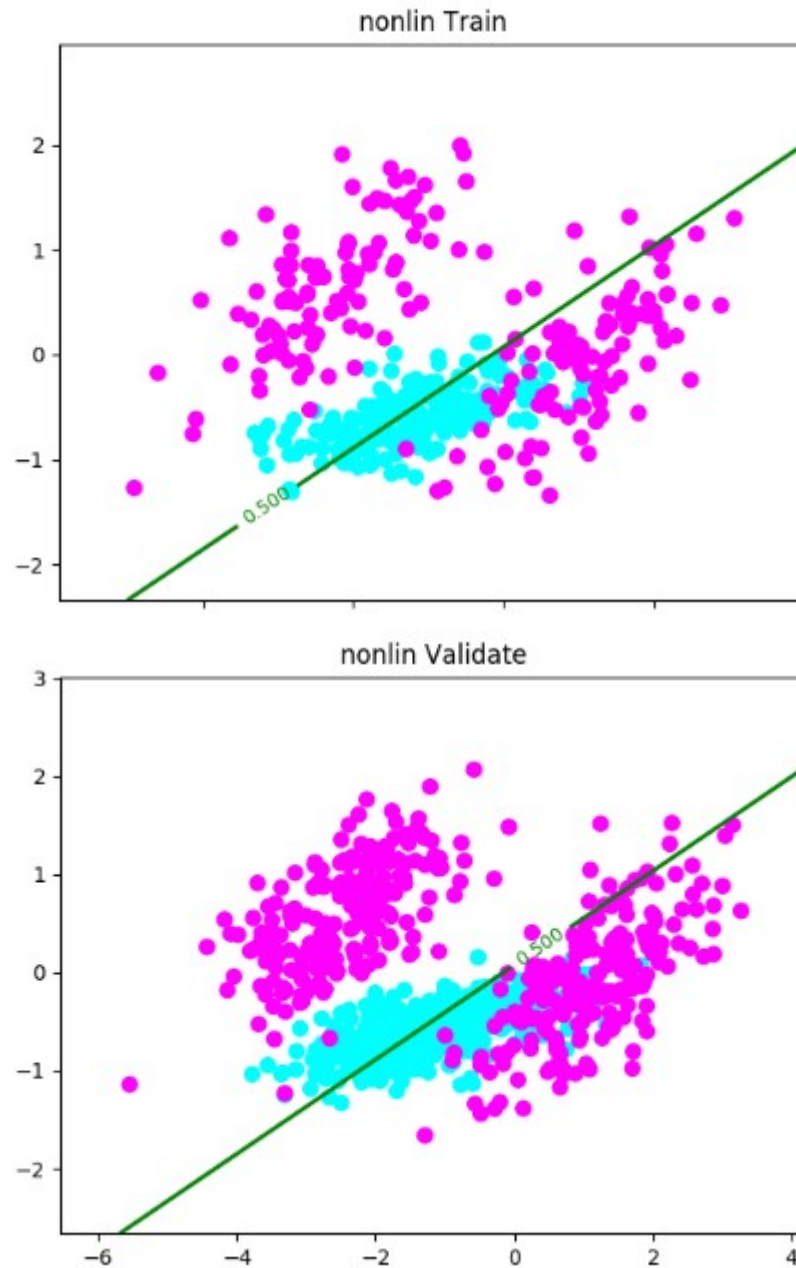
Weights :  $w_0 = -0.50677567$

$w_1 = -0.21785751$

$w_2 = -1.90695535$

Training set error count : 194

Validation set error count : 397





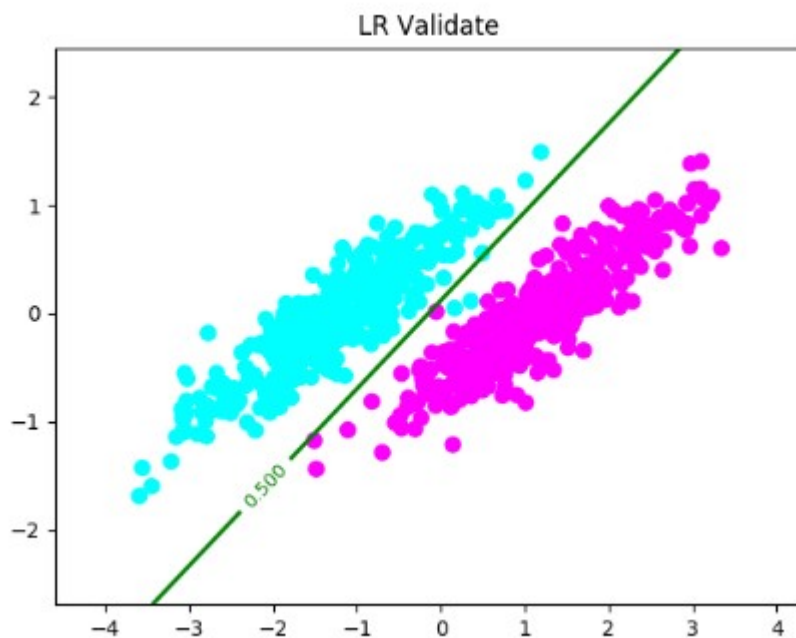
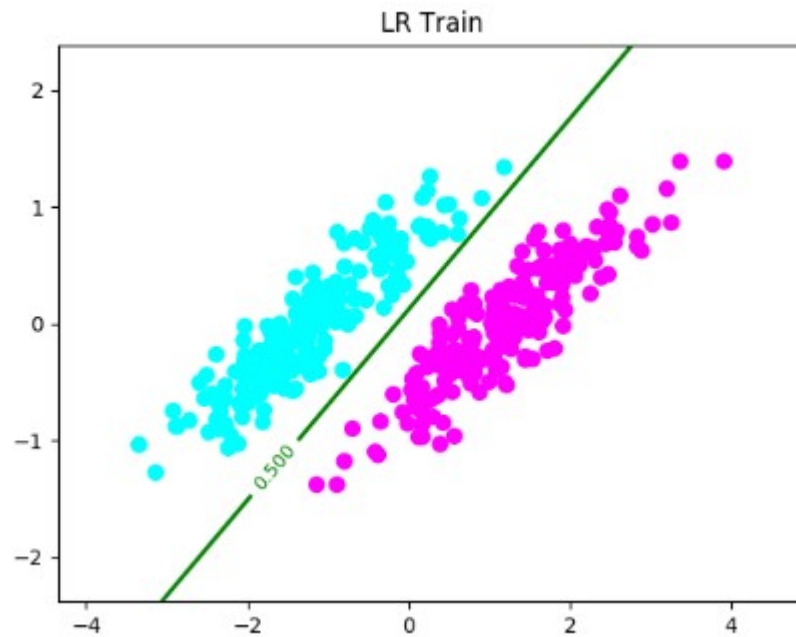
**For dual, when  $C = 50$**

Linearly Separable:

Weights :  $w_0 = 0.09222021$   
 $w_1 = -2.76438112$   
 $w_2 = 3.38479058$

Training set error count : 0

Validation set error count : 4



Non Linearly Separable:

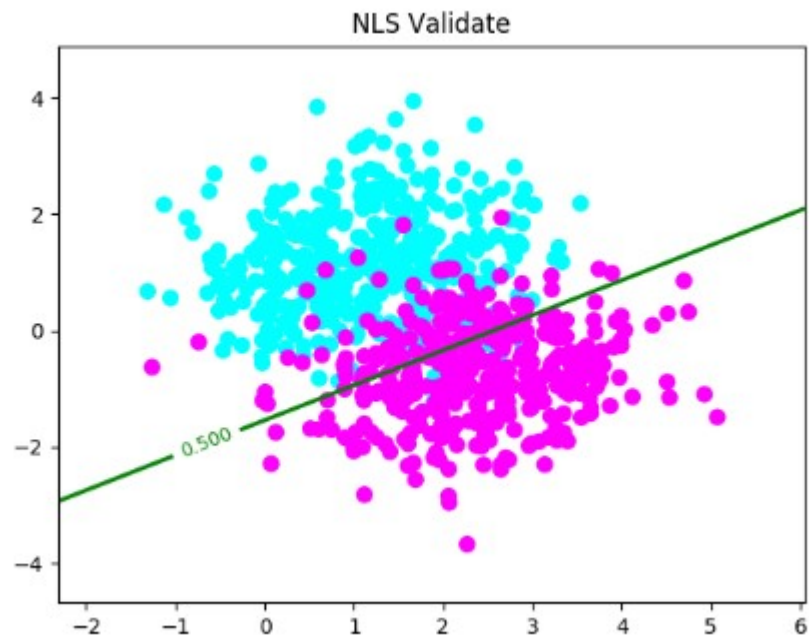
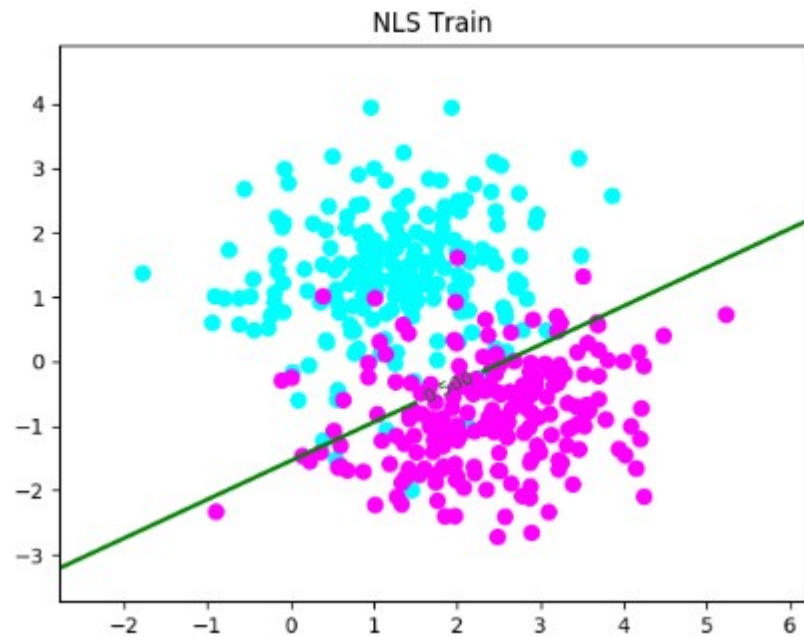
Weights :  $w_0 = 2.68168858$

$w_1 = -0.8509694$

$w_2 = 1.41568194$

Training set error count : 67

Validation set error count : 156

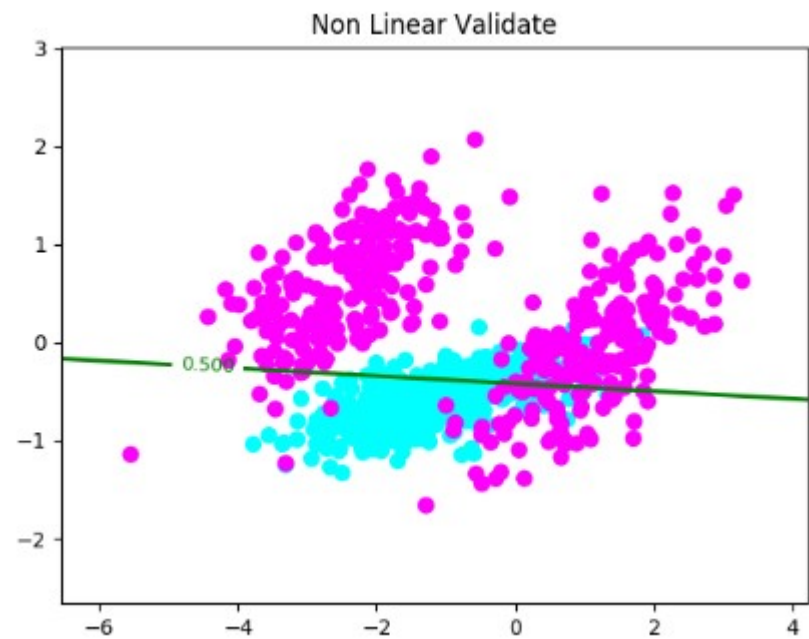
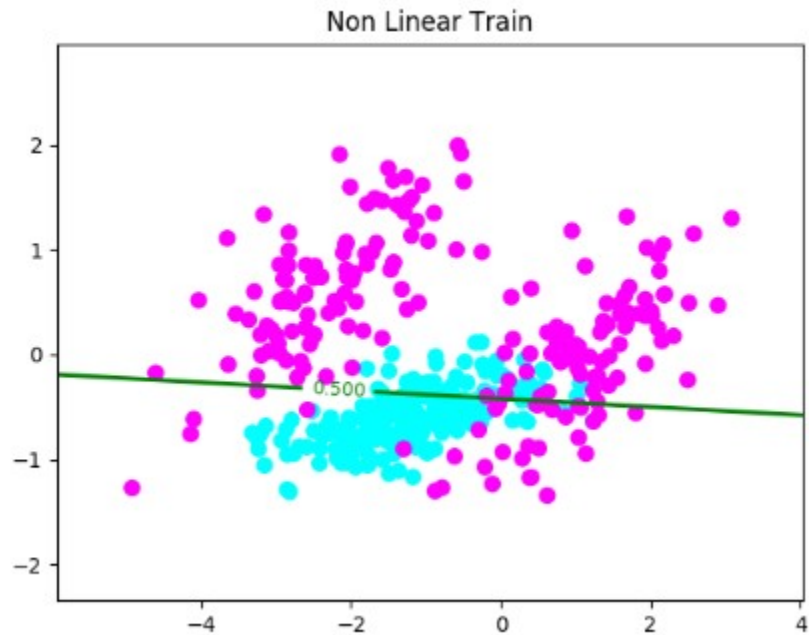


Non Linearly Separable(2):

Weights :  $w_0 = -0.6156045$   
 $w_1 = -0.10330625$   
 $w_2 = -2.6558326$

Training set error count : 64

Validation set error count : 126



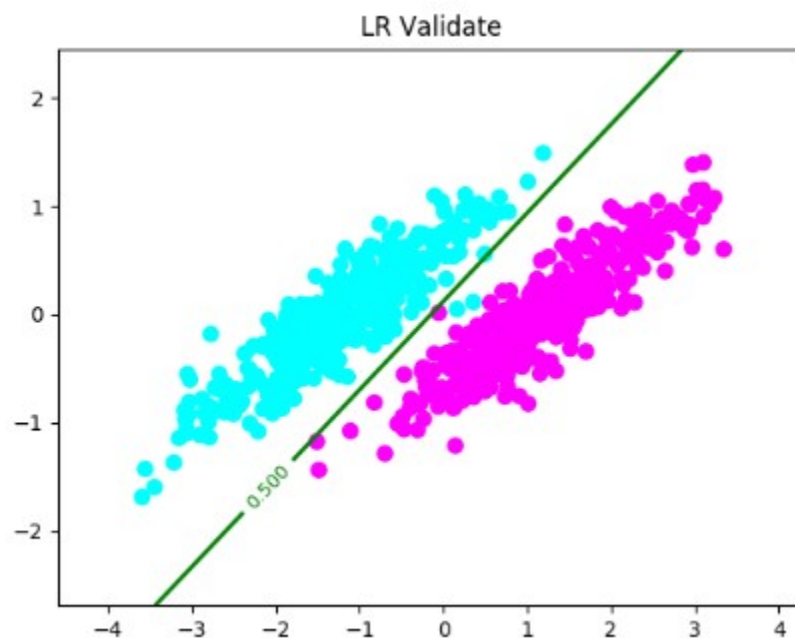
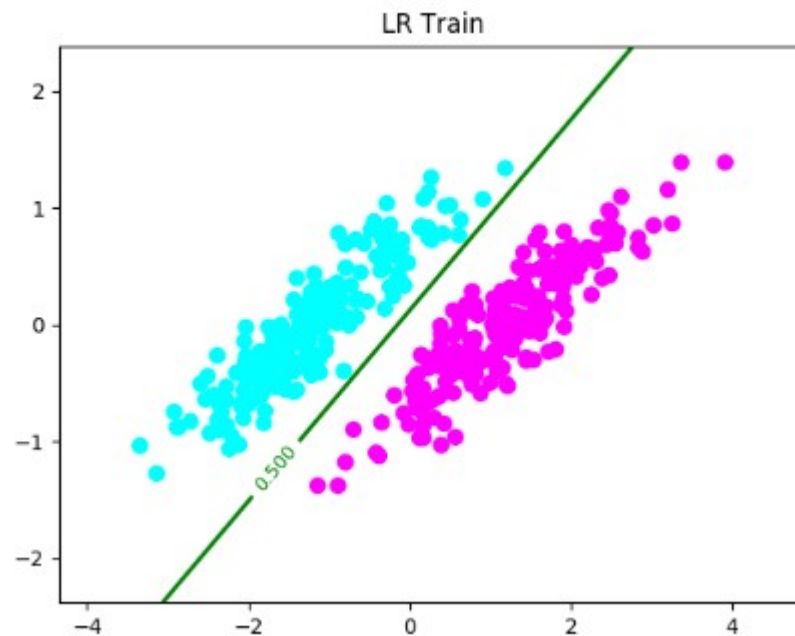
**When  $C = 10$ ,**

Linearly Separable:

Weights :  $w_0 = 0.09222021$   
 $w_1 = -2.76438112$   
 $w_2 = 3.38479058$

Training set error count : 0

Validation set error count : 4

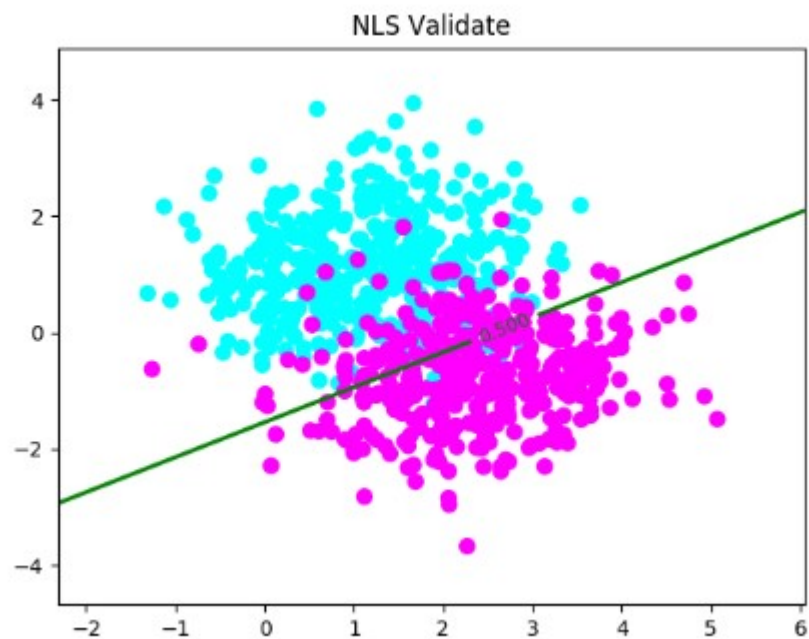
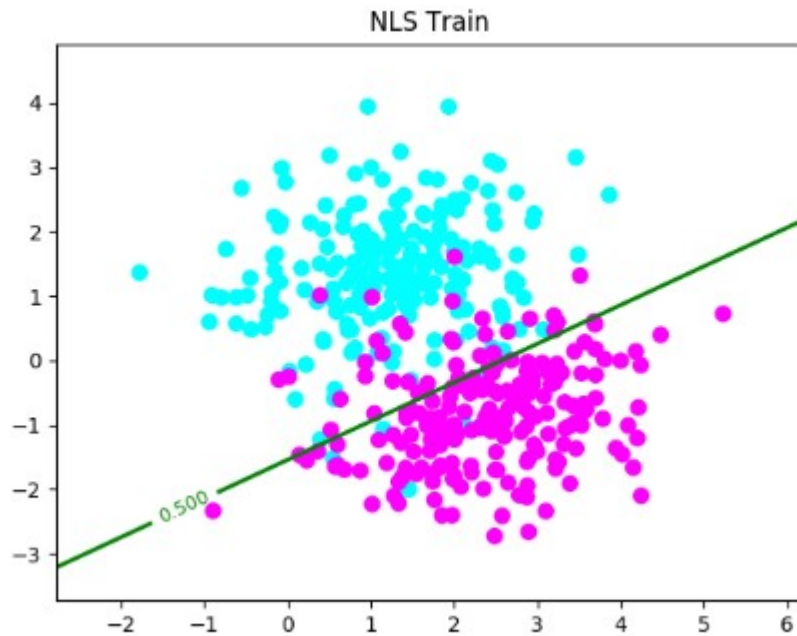


Non Linearly Separable:

Weights :  $w_0 = 2.68168855$   
 $w_1 = -0.85096936$   
 $w_2 = 1.41568182$

Training set error count : 67

Validation set error count : 156



Non Linearly Separable(2):

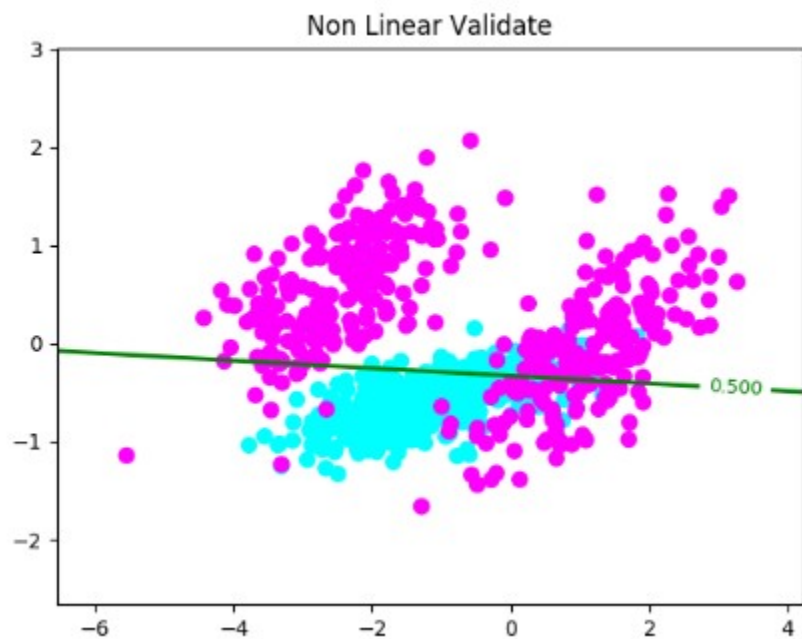
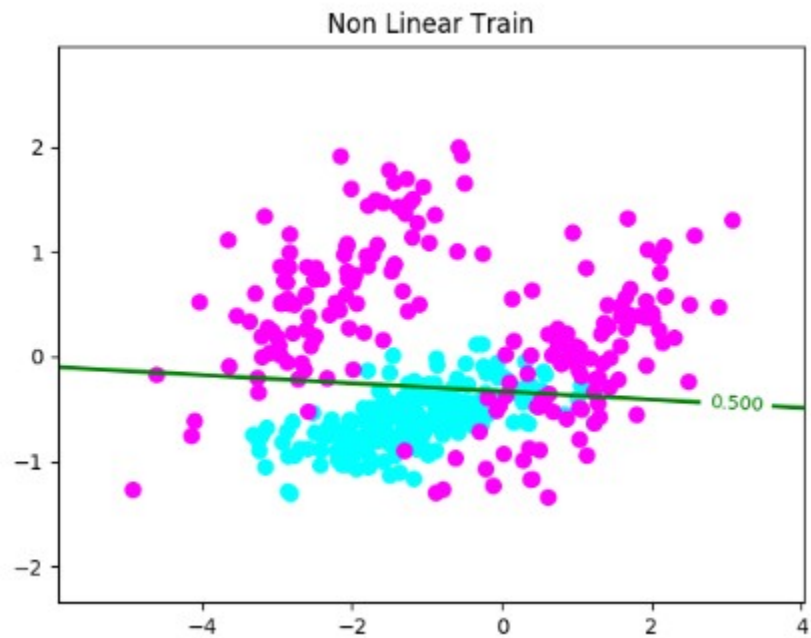
Weights :  $w_0 = -0.37877077$

$w_1 = -0.10330611$

$w_2 = -2.65583341$

Training set error count : 67

Validation set error count : 127



### 3. Kernel Support Vector Machine:

Some of the graphs obtained by applying a polynomial kernel are:

**When  $c = 50$ ,**

Linearly Separable:

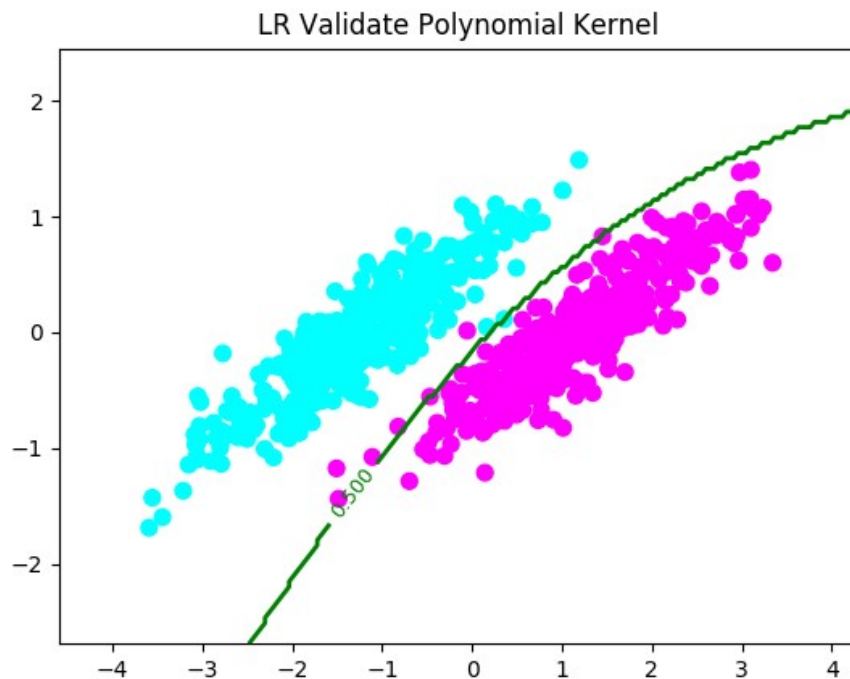
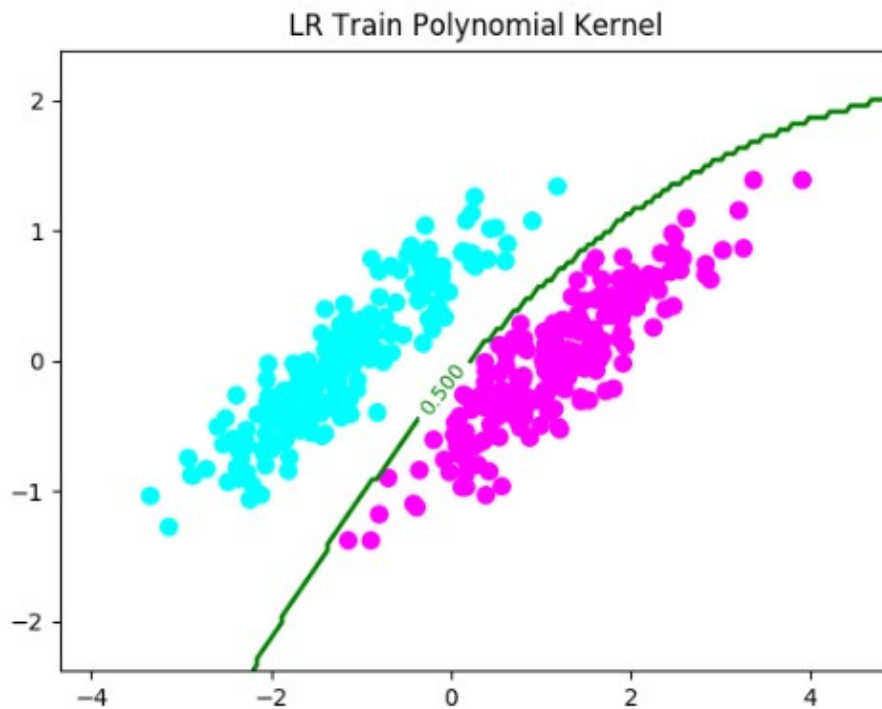
Weights :  $w_0 = 0.57428486$

$w_1 = -1.32673392$

$w_2 = 1.64958729$

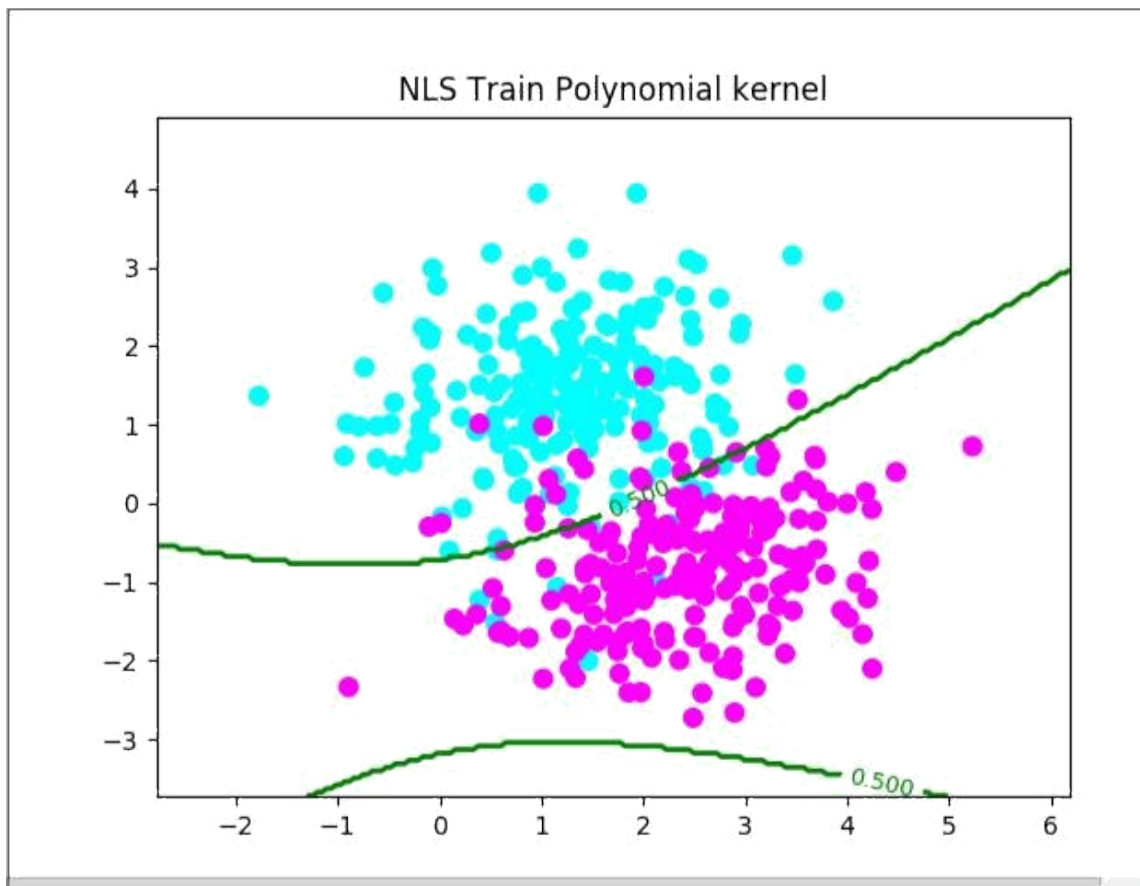
Training set error count : 0

Validation set error count : 7

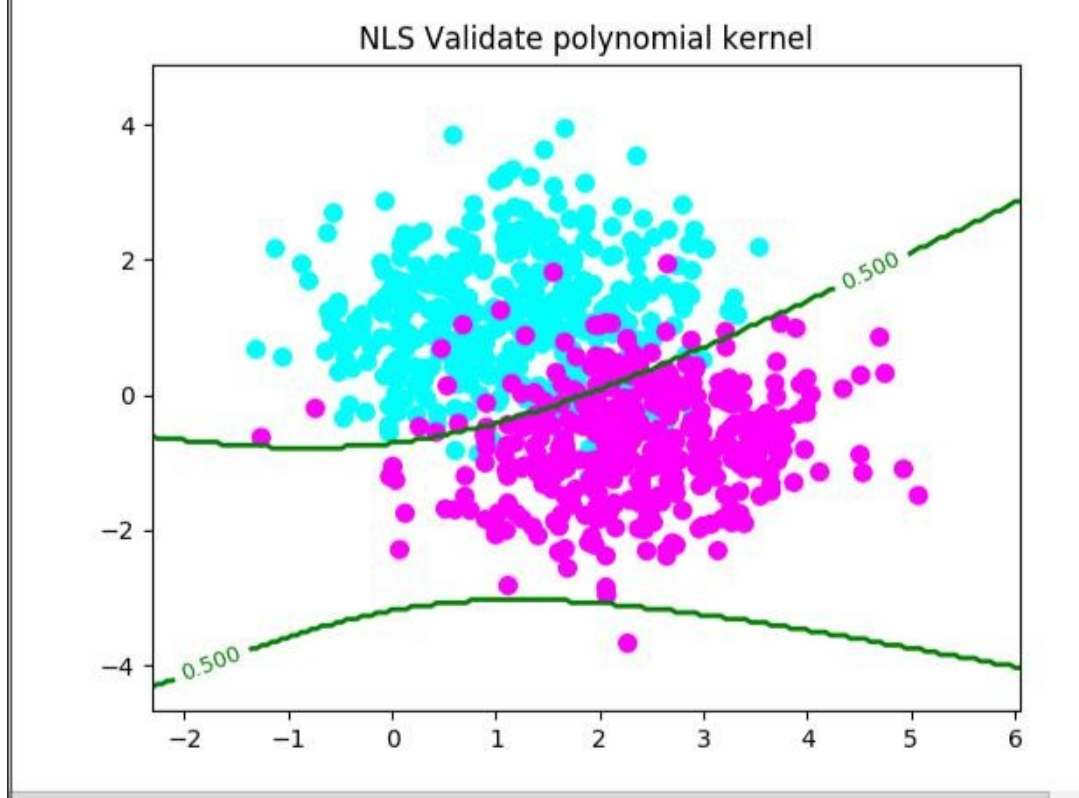




When  $C = 0$ ,



Number of classification errors: 31

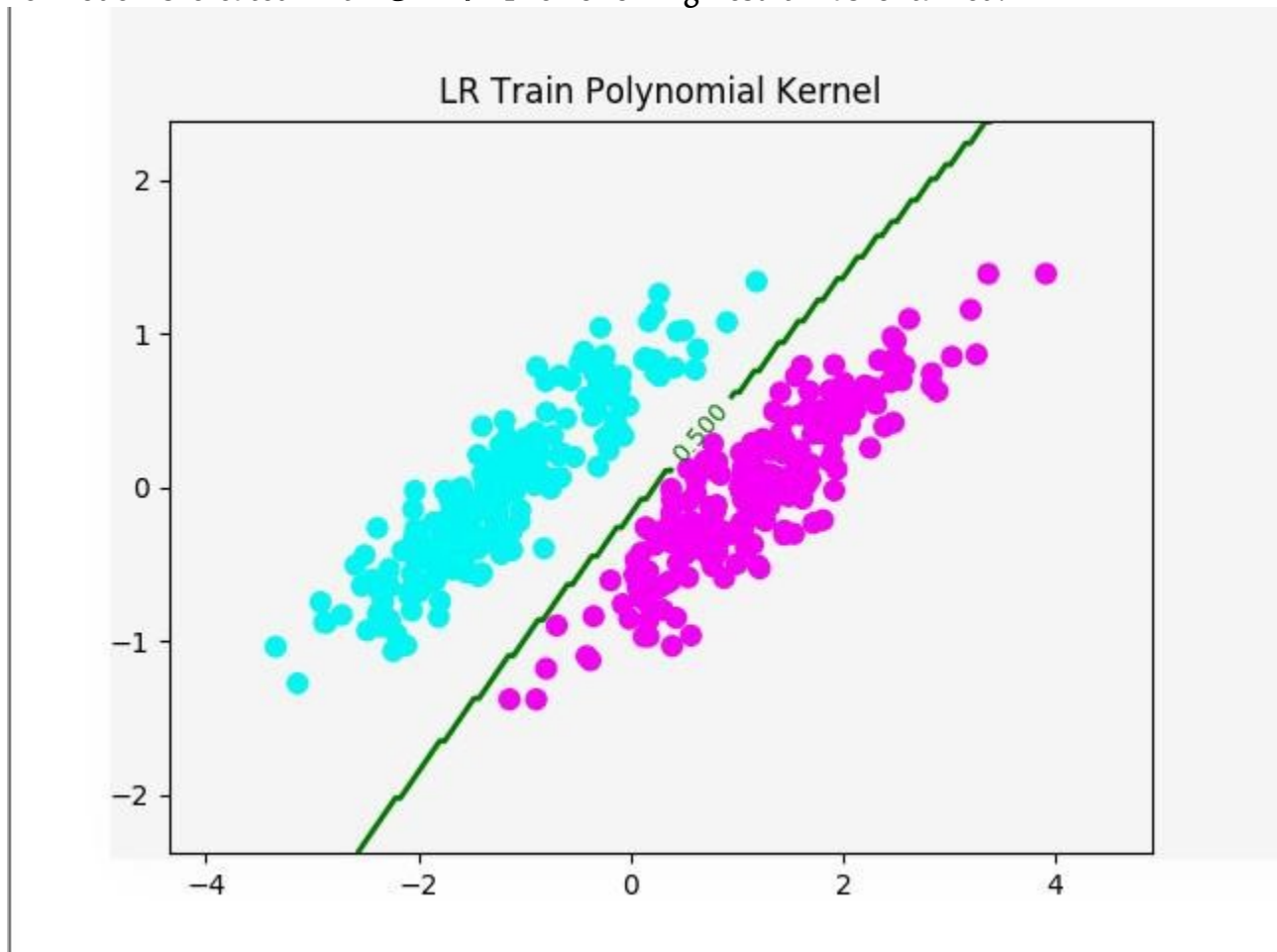


Number of errors: 78

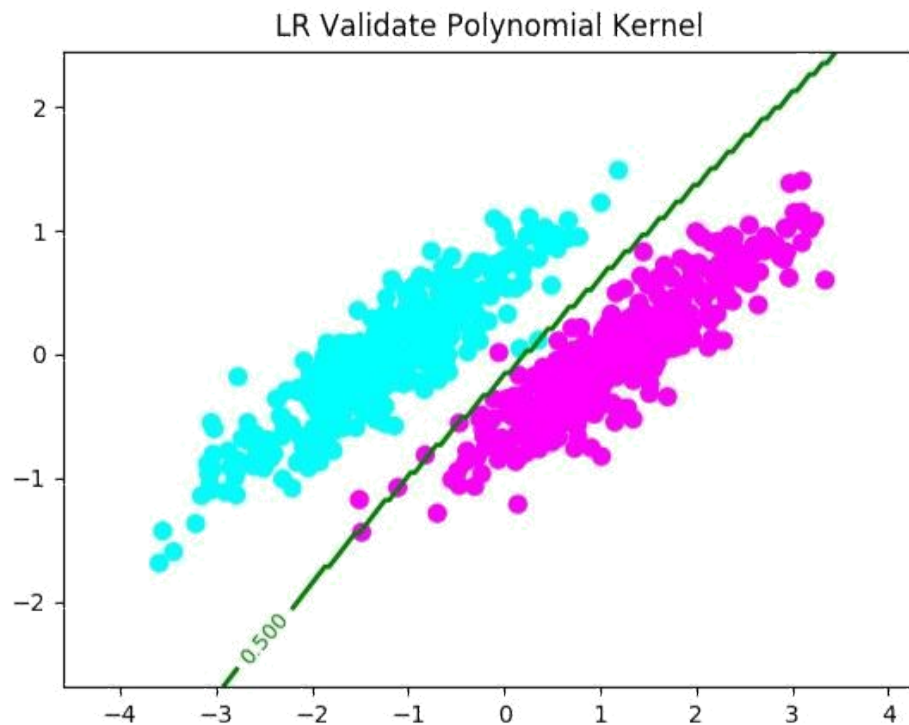
The Gaussian kernel did not produce results for linearly separable and non linearly separable data.

### 3.1 Effect of C

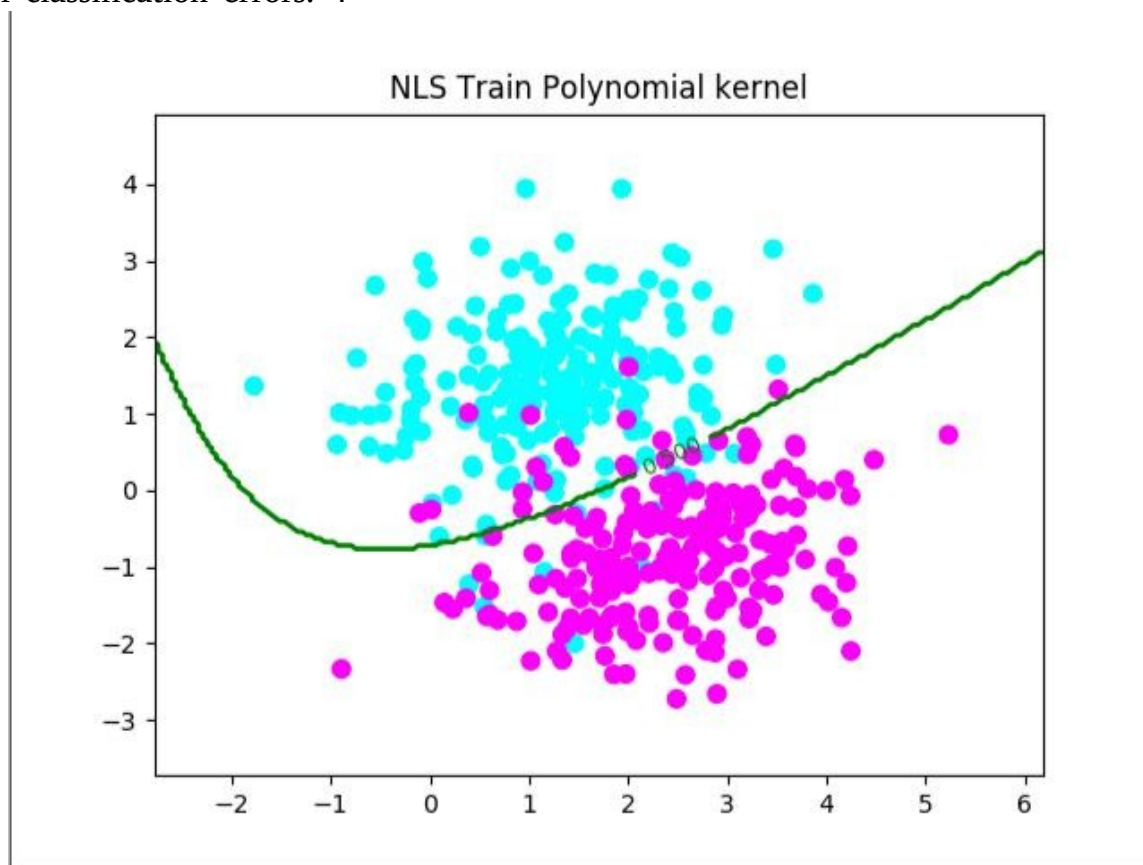
The model is created with  $C = 1$ . The following result was obtained:



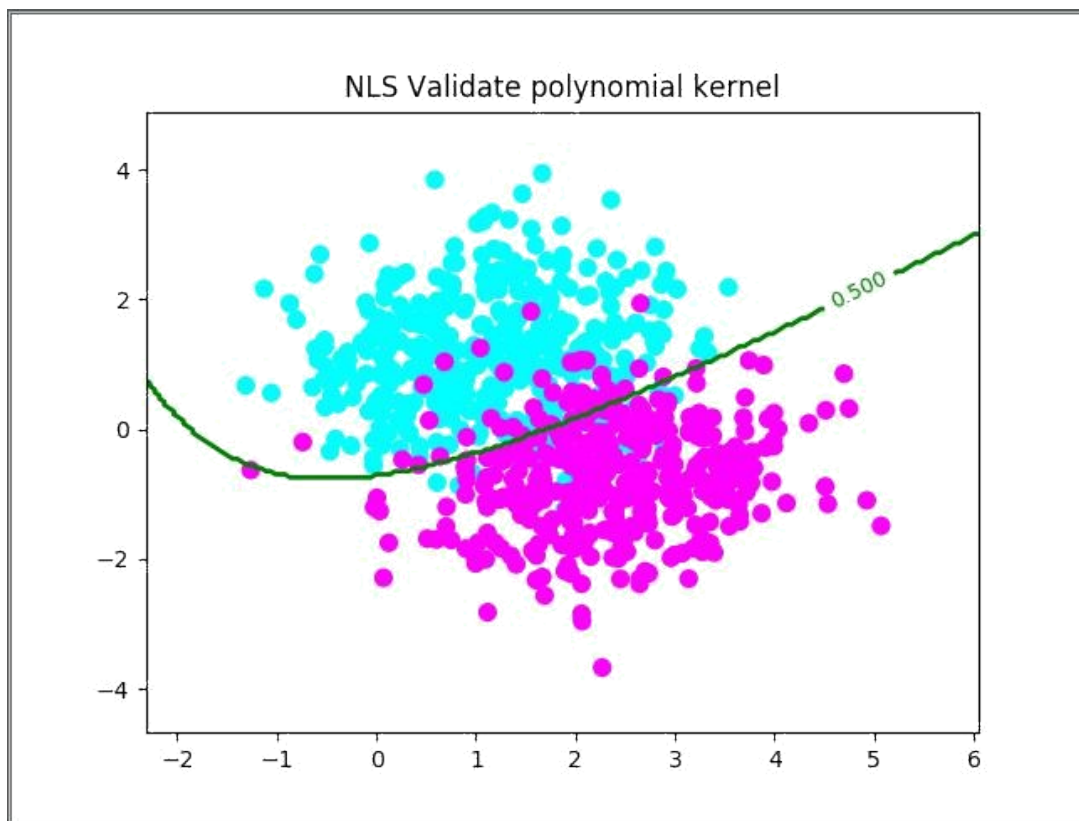
Number of classification errors: 0



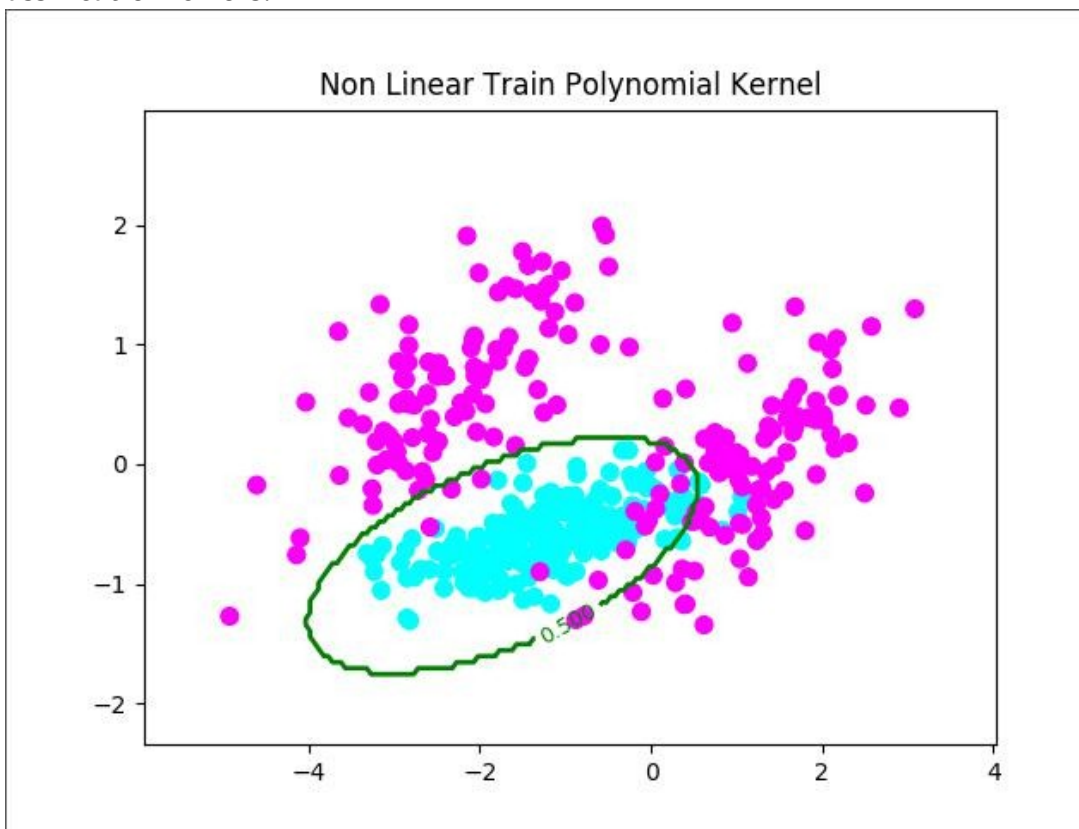
No. of classification errors: 4



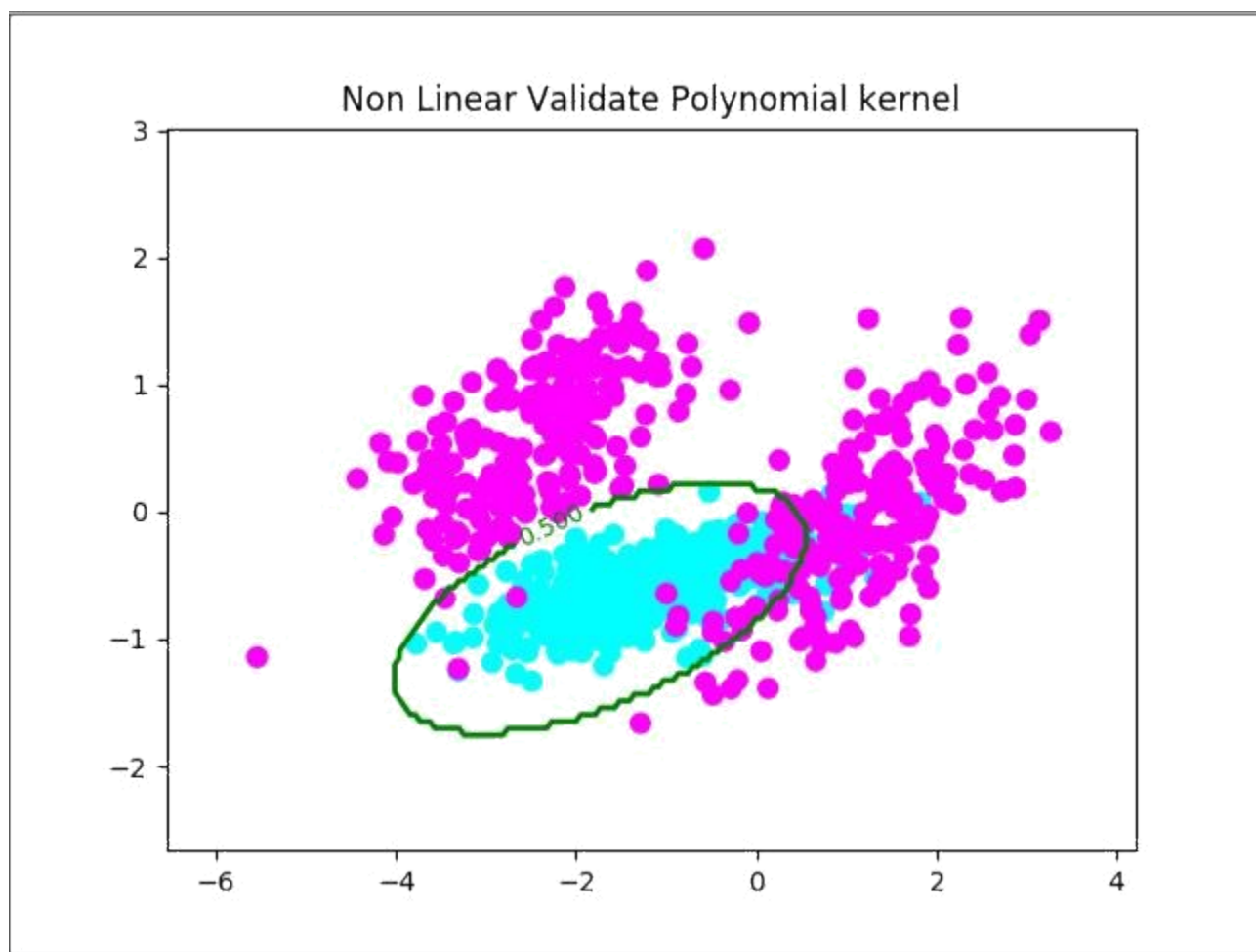
No. of classification errors: 67



No of classification errors: 156



No of classification errors: 67



No of classification errors: 127