Assignment 1_solution

A (colour) 128x128 image is input into an algorithm which outputs a (colour) 16x16 image representing some important portions of the original image. For example, the input could be the image of a lung and the output could be a suspicious tumorlike portion, etc. (We encourage you to think of other examples, as an exercise). Answer questions 1 to 3 for this

1) If the input is turned into a vector x, its length would be?

Ans: 128x128x3 = 49152

2) If the output is turned into a vector y, its length would be?

Ans: 16x16x3 = 768

3) If we write a model of y as y = Ax + b. Then, the total number of elements in the matrix A is?

Ans: 49152x768 = 37748736

4). Let $x = [-10 \ \ 2 \ \ 4 \ \ 8 \ \ 9]^T$. Then, which of these is the greatest?

Ans:

$$L_1 - norm = \sum_{r=1}^{n} |x_r| = (10+2+4+8+9) = 33$$

$$L_2 - norm = \sqrt{\left(\sum_{r=1}^{n} x_r^2\right)}$$

$$= \sqrt{(-10)^2 + 2^2 + 4^2 + 8^2 + 9^2} = \sqrt{100+4+16+64+81} = \sqrt{265} = 16.2788$$

$$L_3 - norm = \left(\sum_{r=1}^{n} |x_r|^3\right)^{\frac{1}{3}}$$

$$= (1000+8+64+512+729)^{\frac{1}{3}} = 13.2249$$

$$L_\infty - norm = max\,(|x_1|,\ |x_2|,\ \cdots,\ |x_n|)$$

$$= max\,(10,\ 2, 4, 8,\ 9) = 10$$

Therefore greatest norm here is $L_1 - norm$

Given ***W*** **= [3 2 7 8 7**
3 2 7 8 7
2 7 3 7 8
4 8 7 4 9
6 7 8 9 5]

5.) Maximum eigenvalue is **28.7679**

6.) Since eigen_max is maximum eigenvalue of **W** then Square of eigen_max will be the maximum eigenvalue of the **W^2.**

Therefore square root of maximum eigenvalue of **W^2** is **28.7679**

7.) Here

$W = transpose(W)$

therefore, singular value of W is absolute value of eigenvalue of W.

Hence, maximum singular value is **28.7679.**

8.) Which of the following is true? Choose all the correct answers

1. $\lambda(M) = \sqrt{(\lambda(M^2))} = svd(M)\ For\ any\ real\ matrix\ M$
2. $\lambda(M) = \sqrt{(\lambda(M^2))} = svd(M)\ For\ any\ real,\ symmetric\ matrix\ M$
3. $\sqrt{(\lambda(M^2))}\ is\ always\ real\ for\ any\ real\ matrix\ M$
4. $\sqrt{(\lambda(M^2))}\ is\ always\ real\ for\ any\ real,\ symmetric\ matrix\ M$

Answer: (2), 4

For 4 -- Eigenvalue of real, symmetric matrix M is always real.

Therefore, eigenvalue of M^2 is $(eigenvalue\ of\ M)^2$ and $\sqrt{eigenvalue}$ is always real.

For 2 -- For symmetric matrices M = transponse(M). So, svd(M) = $\sqrt{(\lambda(MM^T))} = \sqrt{(\lambda(M^2))}$. Also, $\lambda(M) = \sqrt{(\lambda(M^2))}$, provided the square root can be interpreted as either the negative or the positive root.

9.) y_0 = [1 0 0 0 0 $]^T$, b = [0 1 0 0 0$]^T$, $y_n = w \times y_{n-1} + b$

$y_1 = W \times y_0 + b = [1\ 4\ 2\ 4\ 6]$

$||y_1|| = 8.5440$

10.) $y_2 = W \times y_1 + b = [69\ 100\ 112\ 120\ 116]$

$||y_2|| = 234.8638$

$\frac{||y_2||}{||y_0||} = 234.8638$