Assignment 2 - Probability

1. The probability that the sum of the values of 2 die when thrown is equal to 11 is:
    a. 1/18
    b. 1/36
    c. 1/12
    d. 1/9

Solution : Number of total outcomes = 36
Number of favorable outcomes = 2 - (5,6) and (6,5)
Thus probability = 2/36 = 1/18

2. The probability that an ace is drawn on the second draw from a well shuffled pack of cards given that the first one was an ace is:
    a. 3/51
    b. 4/51
    c. 4/52
    d. 3/52

Solution: Number of aces left after the first draw = 3
Number of cards left after the first draw = 51
Thus probability = 3/51

3. A family has two children. Given that one of the children is a boy, what is the probability that both children are boys?
    a. ½
    b. ¼
    c. ⅓
    d. ¾

Solution:
In a family with 2 children there are four possibilities

1) the first child is a boy and the second child is a boy (bb)

2) the first child is a boy and the second child is a girl (bg)

3) the first child is a girl and the second child is a boy (gb)

4) the first child is a girl and the second child is a girl (gg)

Since we are given that at least one child is a boy there are three possibilities: bg, gb, or bb
Out of those three possibilities the only one with two girls is bb. Hence the probability is 1/3.

4. Which of the following statements is true?
    a. Independent events must be mutually exclusive.
    b. The sum of probabilities of mutually exclusive events must be 1.
    c. The sum of probabilities of mutually exclusive and collectively exhaustive events must be 1.
    d. None of the above.

Solution: By definition, mutually exclusive and collectively exhaustive events give the entire sample space. This is thus summed up to a probability of 1.

5. If the random variable X follows the below distribution, what is the value of c?

$$f(x) = cx^3 \text{ from } 0 \text{ to } 1$$

    a. 4
    b. 3
    c. 2
    d. 1

Solution: The distribution upon being integrated between 0 and 1 gives c/4. Here, we have that the value of this integral must be 1. Thus c must be 4.

6. Which of the following statements is true with regards to the probability distribution function f(x) of a random variable X?

    a. f(x) must be less than 1 for all values of x.
    b. f(x) must be non-negative for all values of x.
    c. f(x) cannot exist for negative values of x.
    d. All of the above.

Solution: By definition, only b holds.

7. An image is represented as a vector **x**. We wish to classify the image in one of 3 classes -- a cat, a dog or neither.
   The classification output is represented as a vector **y** as follows. If it is a cat, then **y** =[1 0 0], if it is a dog then **y** =[0 1 0] and if it is neither, then **y** =[0 0 1].
   Someone creates an algorithm that takes in as input the image and output a probability vector **h** -- where each element gives the respective probability. For example, if **h** = [0.7 0.2 0.1], it means that the given image has a probability of 0.7 that it is a cat, 0.2 that it is a dog and 0.1 that it is neither. Which of the following statements is true? (Mark all that are correct)

a. $\sum_k h_k = 1$

b. $h_k = p(y_k = 1|x)$

c. $h_k = p(y_k = 0|x)$

d. Hi $h_k = p(y_k = 1)$

Solution: The sum of all probabilities is 1. Option b follows by definition.

8. If a fair coin is tossed 4 times, what is the expected number of heads?

    a. 1
    b. 2
    c. 3
    d. 4

Solution: Assume 1 is heads and 0 is tails. The expected outcome of the first toss say T1 is ½. Thus, as we can add these events up due to independence and no correlation, we see that the final expected outcome is 2. This corresponds to 2 heads and 2 tails.

9. Given two random variables X and Y, which of the following equations hold true?

    a. E[XY] = E[X]E[Y]
    b. var(X) + var(Y) = var(X+Y)
    c. If X and Y are independent, the covariance of X and Y is zero.
    d. E[X+Y] = E[X] + E[Y]

Solution: By definition, we have c and d.

10. What is the expected value of the random variable X with probability distribution function given below?

$$f(x) = \frac{3}{8}x^2 \text{ from } 0 \text{ to } 2$$

    a. ½
    b. ⅔
    c. 2/9
    d. 3/2

Solution: Upon performing the required integration, we get option d - 3/2.

Answers:

1.a
2.a
3.c
4.c
5.a
6.b
7.a,b
8.b
9.c,d
10.d