IIITB COMET FOUNDATION

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Batch: COMETFWC036
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MATHEMATICS

Time allotted:3 hrs | Maximum Marks: 100

General Instructions:

1. **All questions are compulsory.**
2. Please check that this question paper contains **13 questions**.
3. Questions **1 – 6** in **Section-A** are very short-answer type questions carrying **1 mark each**.
4. Questions **1-7** in **Section-B** are long-answer I type questions carrying **4 marks each**.

Section - A

Question numbers 1 to 6 carry 1 mark each.

1. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ find,αsatisfying $0 < \alpha < \frac{\pi}{2}$ when $A + A^T = \sqrt{2}I_2$ where$A^T$is transpose of$A$.
2. If A is a 3×3 matrix and $|3A| = k|A|$,then write the value of k.
3. For what values of k the system of linear equations
   $x + y + z = 2$
   $2x + y - z = 3$
   $3x + 2y + kz = 4$
   has a unique solution?
4. Write the sum of intercepts cut off by plane $\vec{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) - 5 = 0$ on the three axes
5. Find λ and μ if $\left(\hat{i} + 3\hat{j} + 9\hat{k}\right) \times \left(3\hat{i} - \lambda\hat{j} + \mu\hat{k}\right) = \vec{0}$
6. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$.

Section - B

Question numbers 1 to 7 carries 4 marks each.

1. Solve for x: $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x$

   OR

   Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) - \tan^{-1}\left(\frac{4x}{1-4x^2}\right) = \tan^{-1} 2x; \quad |2x| < \frac{1}{\sqrt{3}}$

2. **A** typist charges Rs. 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are Rs. 180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only Rs. 2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy? Which values are reflected in this problem?

3. If $f(x) = \begin{cases} \frac{\sin(a+1)x+2\sin x}{x}, & x<0 \\ 2, & x=0 \\ \frac{\sqrt{1+bx}-1}{x}, & x>0 \end{cases}$ is continuous at $x=0$, then find the values of a and b.

4. If $x\cos(a+y)=\cos y$, then prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$ Hence show that $\sin a\frac{d^2y}{dx^2}+\sin(2a+y)\frac{dy}{dx}=0$.

OR

Find $\frac{dy}{dx}$ if

$$y=\sin^{-1}\left(\frac{6x-4\sqrt{1-4x^2}}{5}\right)$$

5. Find the equation of tangents to the curve $y=x^3+2x-4$, which are perpendicular to line $x+14y+3=0$.

6. Find: $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}\,dx$

OR

Find: $\int \frac{x^2+x+1}{(x^2+1)(x+2)}\,dx$

7. Evaluate: $\int_{-2}^{2} \frac{x^2}{1+5x^2}\,dx$