GATE 2010 CS, 6th Question Analysis

Question 6

The minterm expression of $F(P, Q, R) = PQ + Q\bar{R} + P\bar{R}$

Question Analysis:

Finding minterms of the expression $F = PQ + Q\bar{R} + P\bar{R}$
  Step 1: Write the expression $F = PQ + Q\bar{R} + P\bar{R}$
  Step 2: Expand each term to include all variables P, Q, R

$$PQ = PQ(R + \bar{R}) = PQR + PQ\bar{R}$$
$$Q\bar{R} = (P + \bar{P})Q\bar{R} = PQ\bar{R} + \bar{P}Q\bar{R}$$
$$P\bar{R} = (Q + \bar{Q})P\bar{R} = PQ\bar{R} + P\bar{Q}\bar{R}$$

  Step 3: Combine all terms (removing duplicates) $F = PQR + PQ\bar{R} + \bar{P}Q\bar{R} + P\bar{Q}\bar{R}$
  Note that $\bar{P}Q\bar{R}$ is the same as $\bar{P}Q\bar{R}$.
  Step 4: Identify minterms explicitly from each product

Term	Minterms (binary)		Decimal
PQ	$PQR = 111$,	$PQ\bar{R} = 110$	$7, 6$
$Q\bar{R}$	$PQ\bar{R} = 110$,	$\bar{P}Q\bar{R} = 010$	$6, 2$
$P\bar{R}$	$PQ\bar{R} = 110$,	$P\bar{Q}\bar{R} = 100$	$6, 4$

  Step 5: Unique minterms
  $m_2, \quad m_4, \quad m_6, \quad m_7$
  Step 6: Final expression in sum of minterms notation
  $f = \sum m(2, 4, 6, 7)$

Hardware Implementation

The above problem is implemented and tested in hardware using Arduino UNO board. Here we implemented a FSM using the 7474 IC and blinked the LED as per truth table and verified the expression.

Required Components & Pin Connections

Component	Arduino Pin
Input P	Digital 2
Input Q	Digital 3
Input R	Digital 4
Output a (sevenseg)	Digital 5
Output b (sevenseg)	Digital 6
Output c (sevenseg)	Digital 7
Output d (sevenseg)	Digital 8
Output e (sevenseg)	Digital 9
Output f (sevenseg)	Digital 10
Output g (sevenseg)	Digital 11
Output F (LED)	Digital 13
GND	GND
VCC	5V

S.No	Component
1	Arduino Uno Board
2	Breadboard
3	LEDs (1)
4	Seven segment (1)
5	Resistors: 220Ω (2)
6	Jumper Wires
7	USB Cable

Logic Description

- Let initialize inputs $A = 0, B = 0, C = 0$

- The output expressions from truth table is

- $F = PQ + P\bar{R} + Q\bar{R}$

Code Uploading Steps

1. Create a Platform IO project

2. Write The code in main.cpp in src

3. Run the PIO project with command "pio run". It will compile the code and creates .hex file

4. Copy the .hex file to ArduinoDriod folder

5. connect the Arduino UNO to mobile with OTG cable

6. Upload the hex file using "upload precomplied" option

7. Observe the ouput and verify the expression

Experimental Truth Table

A	B	C	F (LED Output)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Conclusion

- From Experimental Truth Table F will be 1 when $F = P.Q + Q.\overline{R} + P.\overline{R}$.

- This matches option (A) from the original GATE question.

- The hardware experiment confirms the circuit's theoretical logic.