IIITB COMET FOUNDATION

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Batch: COMETFWC036
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# MATHEMATICS

Time allotted:3 hrs | Maximum Marks: 100

## General Instuctions:

(i) All Questions are compulsory

(ii) Please check that this question paper contains 26 questions

(iii) Questions 1-6 in Section-A are very short-answer type questions carrying 1 mark each

(iv) Questions 7-19 in Section-B are long-answer I type questions carrying 4 marks each

(v) Questions 20-26 in Section-C are long-answer II type questions carrying 6 marks each

(vi) Please write down the serial number of the question before attempting it

### SECTION-A

**Questions numbers** 1-6 **carry** 1 **mark each**

**1.** If $x \in \mathbb{N}$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8,$ then find the value of $x$ .

**2.** Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix equation: $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

**3.** Write the number of all possible matrices of order 2 x 2 with each entry $1, 2$ or 3.

**4.** Write the position vector of the point which divides the join of points with position vectors $3\vec{a} - 2\vec{b}$ and $2\vec{a} + 3\vec{b}$ in the ratio $2 : 1$.

**5.** Write the number of vectors of unit length perpendicular to both the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$.

**6.** Find the vector equation of the plane with intercepts 3, $-4$ and 2 on $x$, $y$ and $z$-axis respectively.

## SECTION-B

**Questions numbers 7-19 carry 4 marks each**

**7.** Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the $XZ$ plane. Also find the angle which this line makes with the $XZ$ plane.

**8.** The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vectors parallel to its diagonals.Using the diagonal vectors, find the area of the parallelogram.

**9.** In a game, a man wins Rs.5/- for getting a number greater than 4 and loses Rs.1/- otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.

**OR**

A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

**10.** Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to $x$.

**OR**

If $y = 2\cos(\log x) + 3\sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

**11.** If $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t(1 - \cos 2t)$, find $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{4}$.

**12.** The equation of tangent at $(2, 3)$ on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of $a$ and $b$.

**13.** Find: $\int \frac{x^2}{x^4+x^2-2}\,dx$