

# BIT Manipulations

lecture 1

0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

operators

&

$$a=5 \quad b=6$$

$$\begin{array}{r} 101 \\ \wedge 110 \\ \hline 100 = 4 \end{array}$$

$$a=5 \quad b=7 \quad c=8$$

$$a \& b \& c$$

$$\begin{array}{r} 0101 \\ \wedge 0111 \\ \hline 0000 \end{array}$$

$$a \rightarrow 101 \quad 5$$

$$b \rightarrow 111 \quad 7$$

$$\begin{array}{r} 101 \\ \wedge 111 \\ \hline 010 \end{array}$$

$\wedge$  (XOR)

$$5 \quad 7$$

$$\begin{array}{r} 101 \\ \wedge 111 \\ \hline 010 \end{array}$$

$\sim$  (Negation)

$$1 \rightarrow 0$$

$$0 \rightarrow 1$$

Right shift (>>)

$$a=5$$

$$1010$$

$$a >> 1$$

$$101 \Rightarrow 2$$

$$(5 >> 1) \rightarrow 2$$

$$\frac{5}{2} = 2$$

$$a=10$$

$$1010$$

$$a >> 3$$

$$1 \Rightarrow 1$$

$$(10 >> 3) \rightarrow 1$$

$$\frac{10}{2} = \frac{5}{2} = \frac{2}{2} = 1$$

$$\begin{array}{l} 1010 \\ \downarrow \\ 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ \downarrow \\ 0001 \\ \downarrow \\ 1 \times 2^0 \end{array}$$

$$(>> \text{num}) = \frac{\text{decimal no}}{2^{\text{num}}}$$

left shift (<<)

$$(5 << 2)$$

$$(10100)_2$$

$$\downarrow \times 2^2$$

$$(5 \times 2^2)$$

$$(5)_{10} \rightarrow (000 \dots 0101)_2$$

32 bits.

$$\text{int } a = 5$$

4 bytes  
32 bits

$$101$$

$$1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$\times 2^2$$

Q:- arr[] = { 2, 1, 2, 5, 6, 5, 7, 7, 2 }

Array contains N integers  $\rightarrow$  Find The number which occurs only

Sol:-  
 $XOR = 0$   
 for ( $i=0; i < n; i++$ )  
 {  $XOR = XOR \oplus arr[i];$   
 }  
 Print (XOR).

Q:- Swap 2 numbers (imp for interview - without using Third variable).

a = 5

b = 7

①  $a = a \oplus b$

①

②

③

②  $b = a \oplus b$

$a = 5 \oplus 7$

$b = 5 \oplus 7$

$a = 5 \oplus 7$

③  $a = a \oplus b$

$b = 7$

$a = 5 \oplus 7$

$b = 5$

Q:- Given N, print the XOR of all numbers b/w 1 to N

ex:- 5  $1^2 \wedge 3^4 \wedge 5$

ans:-  
 $XOR = 0$   
 for ( $i=1 \rightarrow N$ )  
 $XOR = XOR \oplus i;$   
 Print (XOR)

$O(N)$

N=1  
 N=2  
 N=3  
 N=4  
 N=5  
 N=6  
 N=7  
 N=8

if ( $n \% 4 == 0$ ) print (n)  
 if ( $n \% 4 == 1$ ) print (1)  
 if ( $n \% 4 == 2$ ) print (n+1)  
 if ( $n \% 4 == 3$ ) print (0)  
 $O(1)$

Q:- Given a range (L-R), print the XOR ( $L \oplus L+1 \oplus \dots \oplus R-1 \oplus R$ )

Sol:-  
 $L=2$   
 $R=4$   
 $2^3 \wedge 4$

another way Formula

(3-6)

$XOR(6)$

$XOR(2)$

$\rightarrow 3^4 \wedge 5^6$

$(1^2 \wedge 3^4 \wedge 5^6) \wedge (1^2)$

$3^4 \wedge 5^6$

$XOR(R) \wedge XOR(L-1)$

$\rightarrow O(1)$



Q: even (or) odd

if  $(n \& 1 == 0)$

even

else

odd

more  
time

if  $(n \& 1 == 0)$

even

else

odd

12  $\rightarrow$  1100  $\rightarrow$

13  $\rightarrow$  1101

& 1

0001

1100

11

0000

Q: check if the  $i$ th bit is set (or) not.

$N = 13 \rightarrow (1101)_2$

$i = 2$  yes

$i = 1$  no

mask

$\rightarrow$   
&

4 3 2 1 0  
1 1 0 0 1

0 1 0 0 0

0 1 0 0 0

$(mask \& n)$

$\neq 0$  yes

$(1 < 3)$

mask

$= (1 < i)$

1 0 0 0 1

& 0 1 0 0 0

0 0 0 0 0  $\rightarrow 0$

no

bool set =  $(mask \& n)$

Q: extract  $i$ -th bit of the number

$n = 13$

$i = 3$

(Same as before)

3 2 1 0  
1 1 0 1

if yes 1

no 0

Q: Set the  $i$ th bit of a number.

5 4 3 2 1 0  
1 1 0 0 1 0

$i = 2$

1 1 0 1 1 0

Solve

5 4 3 2 1 0  
1 1 0 0 1 0

mask

0 0 0 1 0 0

or / operation

1 1 0 1 1 0

$mask = (1 < i)$

$n = n | mask$

Q) Clear The  $i$ th bit

5 4 3 2 1 0  
1 1 0 0 1 0

$i=2$

1 1 0 0 1 0

already '0'

$0 \rightarrow 0$

5 4 3 2 1 0  
1 1 0 0 1 0

$i=4$

1 0 0 0 1 0

if  $1 \rightarrow 0$

ans

↓  
1 1 0 0 1 0

& 1 1 1 0 1 1

1 1 0 0 1 0

$\sim(000100)$  mask  $\sim(1 \leq i)$

$(n \& \text{mask})$

Q) Remove The last set bit

(last)  
1 1 0 1 1 0  
1 1 0 1 0 0

$n \& n-1$

12  $\rightarrow$  1100  
11  $\rightarrow$  1011

1000

(last)  
1 1 0 1 1 0  
1 1 0 1 0 0  
1 1 0 0 0 0

Q) Check if number is power of 2

if  $(n \& n-1 == 0)$  ✓

else X

$n$  1000  
 $n-1$  0111

0000

Q) Count the no. of set bits.

14  $\rightarrow$  1110  $\rightarrow$  set bits (3)

cnt = 0;  
while  $(n != 0)$

{

if  $(n \& 1 == 1)$

cnt++;

$n = n \gg 1;$

}

Print(cnt);

14  $\rightarrow$  1110

& 1

0000

1110

$\rightarrow$  0111

0111

& 1

0001

cnt++

= 1

432  
1110  
4  $\rightarrow$  no. of bits.  
O(MSB)  
position of Most significant bit



another way

$n=13$

1101 ①  
1100 ②  
1000 ③  
0000

$O(\text{set bits})$

slightly optimal for some.

$n=15$  1111

both methods

same time complexity.

## lecture 2

Q: N integers  $\rightarrow$  every integer appears twice two integers appear once.

[1, 1, 2, 5, 3, 2, 3, 4, 7, 4]

5, 7 answer

soln

Brute

```
for (i=0; i<n; i++)
```

```
{ cnt = 0
```

```
  for (j=0; j<n; j++)
```

```
    { if (a[j] == a[i]) cnt++;
```

```
      if (cnt == 1) print(a[i]);
```

```
}
```

Map

```
map<int, int> mpp;
```

```
for (i=0; i<n; i++)
```

```
  { mpp[a[i]]++;
```

```
    for (auto it: mpp)
```

```
      { if (it.second == 1)
```

```
        print(it.first);
```

TC  $\rightarrow O(N \log N)$

SC  $\rightarrow O(N)$

mal

logn

unordered map

$O(1)$

$O(N)$  worst.

another

arr  $\rightarrow$  [2, 1, 2, 5, 1, 4, 4, 7, 3, 3]

XOR =  $5 \oplus 7 = 2$

for (i=0  $\rightarrow$  n)

XOR = XOR  $\oplus$  a[i]

$O(N)$

2 1 0  
1 0 1  
1 1 1  
0 0 0

5 (1st index)

was 0

7 (1st index)

was 1

1

5

1

4

4

15/1

2

2

7

3

3

7/1

XOR = 0

for (i=0 → n)

XOR = 2

XOR = XOR ^ a[i];

cnt = 0

while (XOR)

O(32)

{ if (XOR & 1) != 0

break;

else cnt++;

cnt = 1

XOR = XOR >> 1; } index = 1

XOR1 = 0 XOR2 = 0.

for (i=0 → n)

O(N)

{ if ((a[i] & (1 << cnt)) != 0

XOR1 = XOR1 ^ a[i];

else

XOR2 = XOR2 ^ a[i];

}

print (XOR1)

print (XOR2).

Q) Generate all the subsets arr → [3, 2, 4].

Power set

(bit index)

num	2	1	0	
0	0	0	0	→ { }
1	0	0	1	→ { 3 }
2	0	1	0	→ { 2 }
3	0	1	1	→ { 3, 2 }
4	1	0	0	→ { 4 }
5	1	0	1	→ { 3, 4 }
6	1	1	0	→ { 2, 4 }
7	1	1	1	→ { 3, 2, 4 }

8X (2^n) - 1 → (1 << n) - 1

n = 3

(1000) - 1

0111 → 7

for (num = 0 → (1 << n) - 1)

{ ss = [ ];

for (bit = 0 → n - 1)

{ if (num & (1 << bit))

{ ss.add (a[bit])

}

for (auto it : ss)

{ print (it);

}

TC = 2^n \* n

This algo only runs for

n ≤ 16/17/18

> This may be TLE