

07.10.85

UNIT - IV

Filter

- * Removes unwanted signal.
- * To allow/stop set of frequencies.

→ LPF

→ HPF

→ BPF

→ BSF

lower order frequencies;

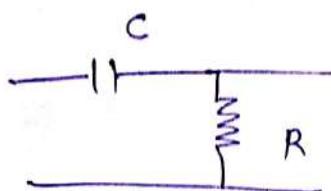
LPF



T or } loops, allows

low frequency on
hence cap takes high
freq.

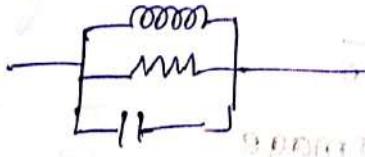
HPF



BPF



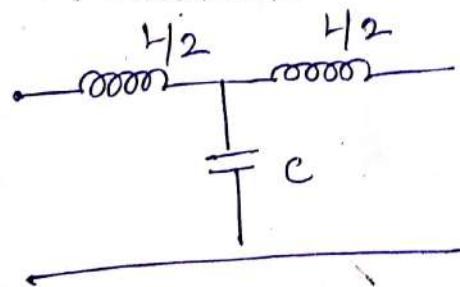
BSF



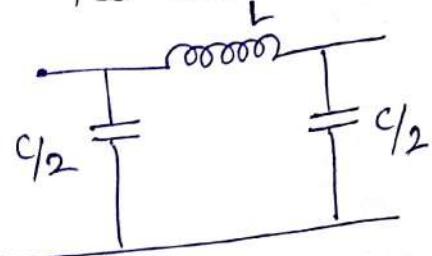
Higher order frequencies

LPF

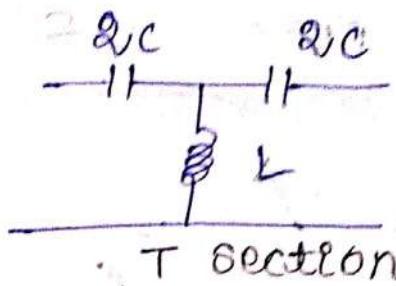
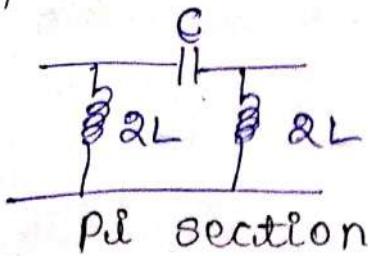
T. section



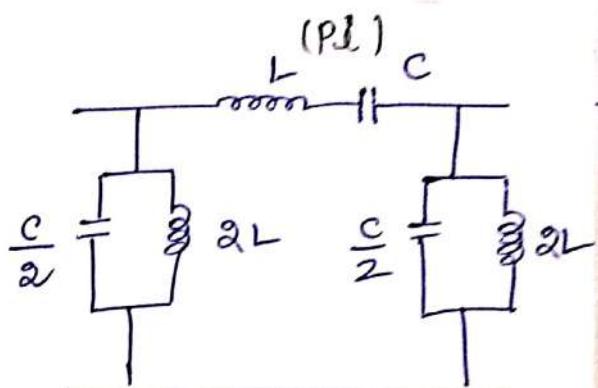
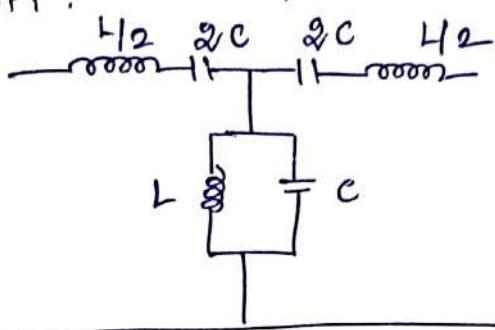
Pi section



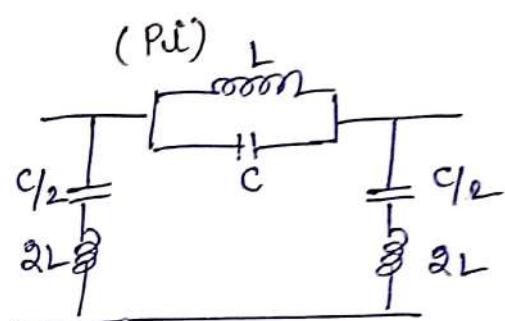
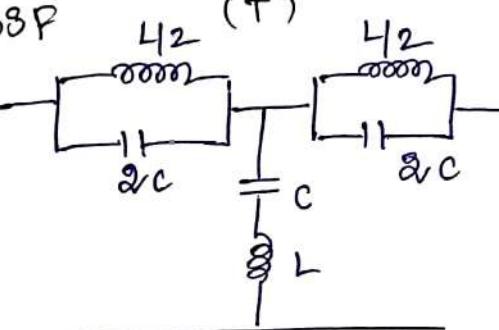
HPF



BPF: (T)



BSP (T)



propagation constant

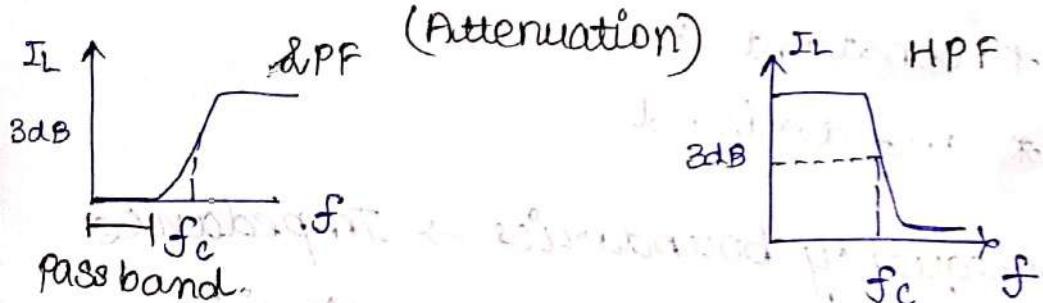
$$\gamma = \alpha + j\beta \quad \alpha = \text{attenuation constant.}$$

$\beta = \text{phase constant}$

pass band $\alpha = 0$

stop band $\gamma = \alpha \uparrow$ (higher quantity)

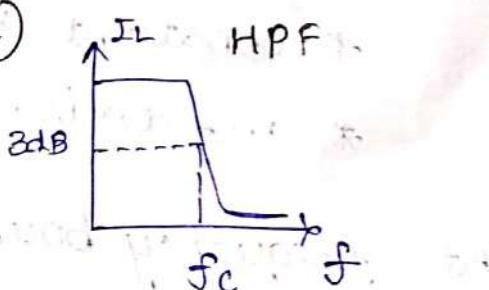
pass band characteristics (wrt insertion loss)



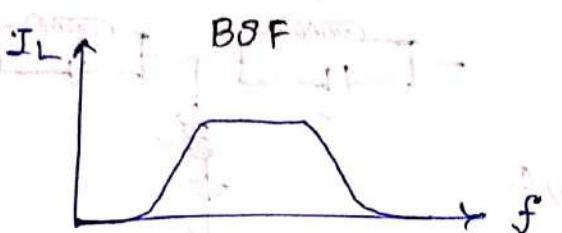
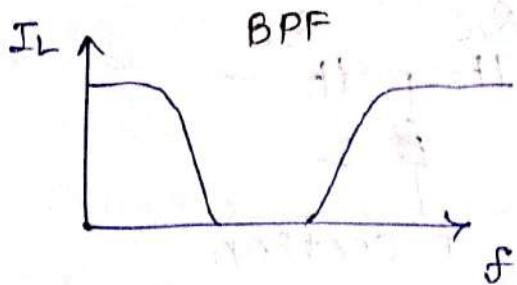
Half power taken (3dB)

Insertion loss less

in pass band

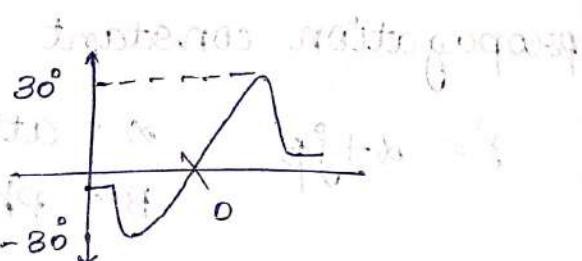
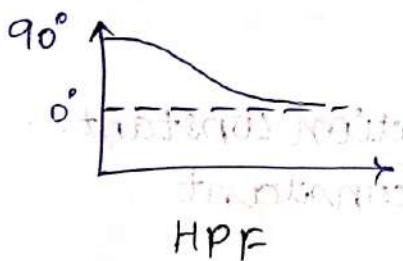
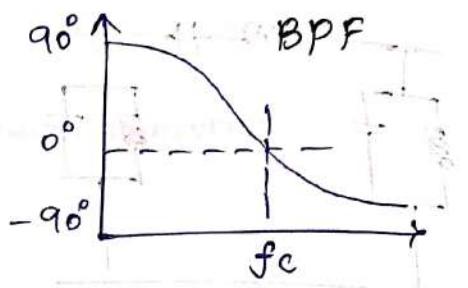
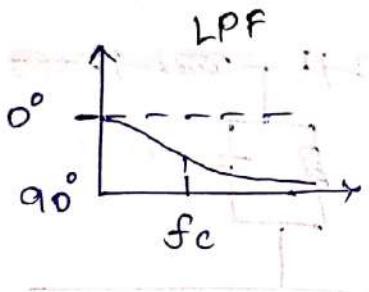


IL is higher



\therefore Realtime loss is more important on purchasing a component if IL w.r.t filter is known.

Phase constant:



With attenuation + phase ~~in BPF band~~ can define pass band characteristics classical methods:

+ Image parameters methods

* constant ($K_{stability}$)

* m - derived

At frequency boundaries \rightarrow Impedance

match \rightarrow No reflections \rightarrow classify

LPF, HPF

- * Insertion loss method
- * Tuning for specific frequency.
- * Involves N/w synthesis for Butterworth and chebyshov.

Impedance Matching :

- * design filter section T / pi so that input - output impedance match at boundaries ensuring minimum reflections.
- Image parameter methods principles are
 - > Image Impedance
 - > propagation constant.

Let $Z_{in} = Z_{out} = Z_I \Rightarrow$ symmetrical network.

To find Impedance at input side considering output is matched with Input Impedance (terminated) in Image.



$$Z_{in} = \frac{V_1}{I_1} \quad \begin{array}{l} \text{part 2 terminated with} \\ Z_{I,2} \end{array}$$

$$Z_{out} = \frac{V_2}{I_1} \quad \begin{array}{l} \text{part 1 terminated with} \\ Z_{I,1} \end{array}$$

far series Impedance $= Z_1$ and,

shunt Impedance $= Z_2$

Far constant - K

T-section:

$$\text{Image Impedance } Z_I = \sqrt{Z_1 Z_2 + Z_1^2/4}$$

$$\cosh(\gamma) = 1 + \frac{Z_1}{2Z_2}$$

Π -section:

$$Z_{\text{img}} = \sqrt{\frac{Z_1}{4} (Z_1 + 4Z_2)}$$

$$\cosh(\gamma) = 1 + \frac{Z_2}{2Z_1}$$

Low pass filter:



series Impedance as $j\omega L$

shunt Impedance as $\frac{1}{j\omega C}$

$$Z_{\text{img}} = 50$$

Relation between Input & output

Impedance

$$K^2 = Z_1 Z_2$$

characteristic Impedance $= K$

$$\omega_c = \frac{1}{\sqrt{LC}}$$

(cut off frequency)

$$K = \sqrt{\frac{L}{C}}$$

$$Z_I = \sqrt{Z_1 Z_2 + Z_1 / 4}$$

$$\omega_c = \sqrt{j\omega_L \cdot \frac{1}{j\omega_C} + \frac{(j\omega_L)^2}{4}}$$

$$\omega_c = \frac{1}{\sqrt{Lc}}$$

$\omega < \omega_c \Rightarrow$ pass band $\alpha = 0$

$\omega > \omega_c \Rightarrow$ stop band $\alpha = \text{present}$.

HPF

$$\omega_c = \frac{1}{\sqrt{Lc}}$$

$\omega < \omega_c =$ stop band

$\omega > \omega_c =$ pass band

BPF

$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}} \quad \omega_2 = \frac{1}{\sqrt{L_2 C_2}}$$

08.10.25

1. design a constant K - T section low pass filter with a cut off frequency of 10MHz with a matched impedance of 50Ω. construct the design to high pass filter

2. design a bandpass filter whose center frequency is 50MHz with a BW of 10MHz terminated with 50Ω impedance.

Design a Band stop filter using the same credential.

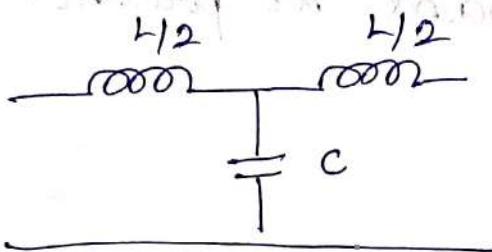
3. construct a low pass and high pass filter using QD credentials

soln

1. T-section:

$$f_c = 10 \text{ MHz} ; R_o = 50 \Omega$$

LPF



$$K = R_o = 50 \Omega$$

$$2\pi f_c = w_c = 2\pi \times 10 \times 10^6 = 62.831 \times 10^6 \text{ rad/s}$$

$$K = Z_1 Z_2 \xrightarrow{\text{series}} \xrightarrow{\text{shunt}}$$

$$= j\omega L \cdot \frac{1}{j\omega C}$$

$$K^2 = \frac{L}{C} \Rightarrow K = \sqrt{\frac{L}{C}} \Rightarrow C = \frac{L}{K^2} \text{ and } L = K^2 C$$

$$w_c = \frac{1}{\sqrt{LC}} \Rightarrow w_c = \frac{1}{\sqrt{K^2 C \cdot C}} = \frac{1}{\sqrt{K^2 C^2}}$$

$$w_c = \frac{1}{K C}$$

$$\boxed{C = \frac{1}{K w_c}}$$

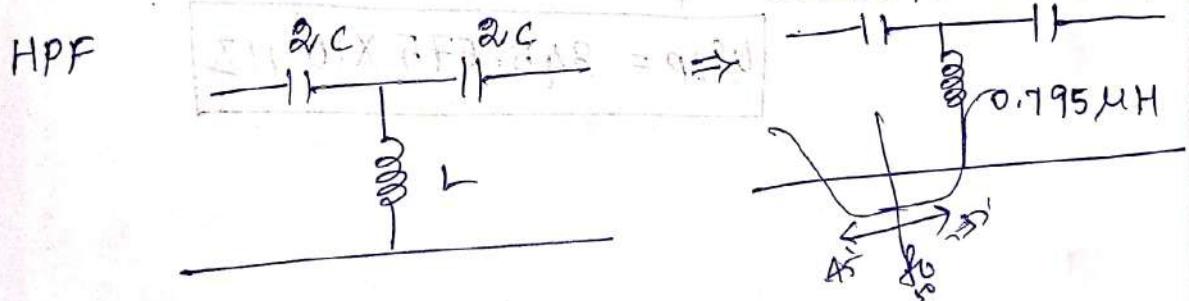
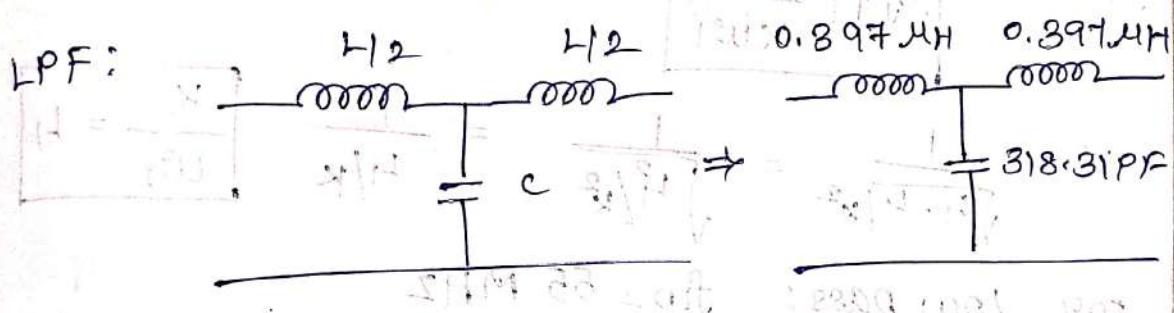
$$w_c = \frac{1}{\sqrt{L \cdot 4/K^2}} = \frac{1}{\sqrt{L^2/K^2}} = \frac{1}{4K}$$

$$w_c = \frac{K}{L} \Rightarrow L = \frac{K}{w_c}$$

$$C = \frac{1}{50 \times 62.831 \times 10^6} = 318.31 \text{ pF}$$

$$L = \frac{50}{62.831 \times 10^6} = 795.785 \text{ nH}$$

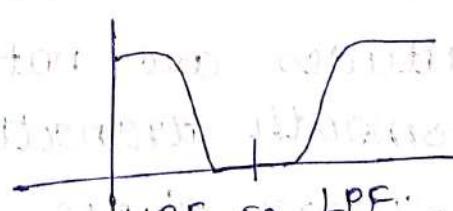
$$L = 0.795 \mu\text{H}$$



a. BPF $f_0 = 50 \text{ MHz}$ $B.W = 10 \text{ MHz}$

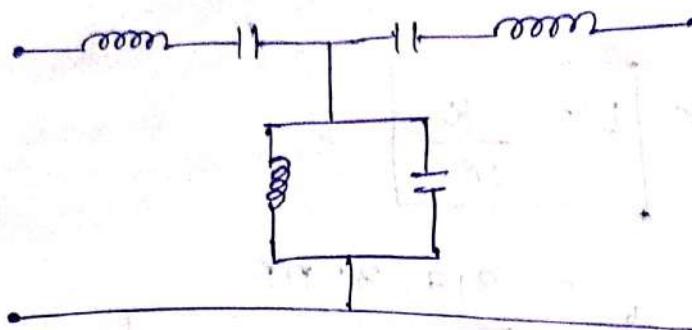
$$f_{chp} = 45 \text{ MHz}$$

$$f_{LP} = 55 \text{ MHz}$$



constant- κ filter (Bandpass)

⇒ T- Section:



$$\kappa^2 = 4c \Rightarrow \kappa = \sqrt{4c} \quad C_1 = \frac{L}{\kappa^2} \quad \text{and} \quad L = \kappa^2 c$$

$$\omega_1 = \frac{1}{\sqrt{Lc_1}} = \frac{1}{\sqrt{\kappa^2 c_1 c_1}} = \frac{1}{\kappa c_1}$$

$$C_1 = \frac{1}{\kappa \omega_1}$$

$$\omega_1 = \frac{1}{\sqrt{L \cdot 4c/\kappa^2}} = \frac{1}{\sqrt{4c/\kappa^2}} = \frac{1}{4c/\kappa} \quad \frac{\kappa}{\omega_1} = 4c$$

For Low pass: $f_{LP} = 55 \text{ MHz}$

$$\omega_{LP} = 2\pi f_{LP} = 2\pi \times 55 \times 10^6$$

$$\omega_{LP} = 345.575 \times 10^6 \text{ rad/s}$$

Limitations on constant κ

- Amplitudes are not easily shaped (no smooth transition)
- Non linear phase nearer to cutoff
- Image Impedance are dependent on frequency
- difficult to match impedance

$$C = \frac{1}{Kw_c}$$

$$C = \frac{1}{50 \times \pi \times 55 \times 10^6} \quad C = 57.874 \text{ PF}$$

$$L = \frac{K}{w_c} = \frac{50}{2\pi \times 55 \times 10^6}$$

$$L = 144.686 \text{ nH}$$

for High pass

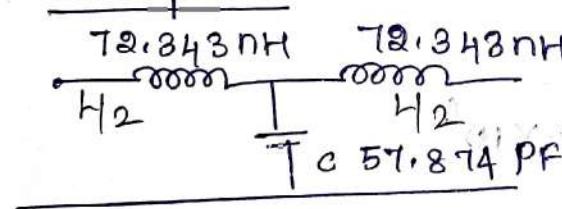
$$C = \frac{1}{Kw_c} = \frac{1}{50 \times 45 \times 10^6 \times 2\pi}$$

$$C = 70.735 \text{ PF}$$

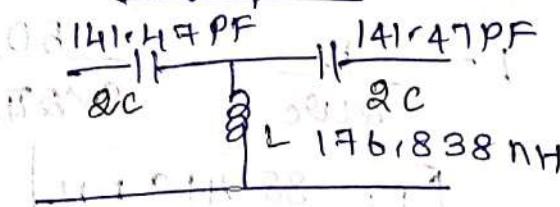
$$L = \frac{K}{w_c} = \frac{50}{2\pi \times 45 \times 10^6}$$

$$L = 176.838 \text{ nH}$$

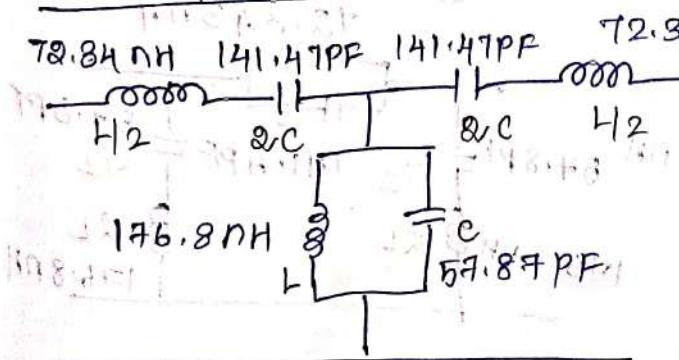
Low pass



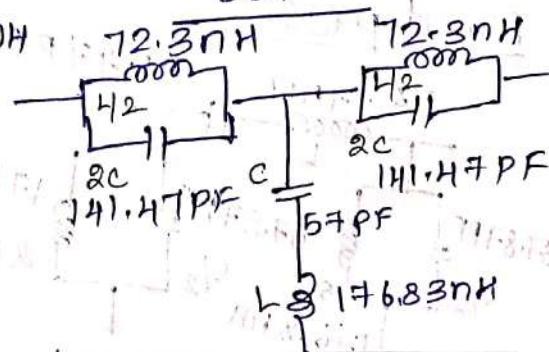
High pass



Band pass



B8F



$\Rightarrow \pi$ section:

$$C = \frac{1}{\omega C}$$

$$L = \frac{\mu}{\omega C}$$

Low pass $f_{LP} = 55 \times 10^6$ Hz.

$$C = \frac{1}{50 \times 2\pi \times 55 \times 10^6}$$

$$C = 115.749 \text{ pF}$$

$$L = \frac{\mu}{\omega C} = \frac{50}{2 \times \pi \times 55 \times 10^6}$$

$$L = 78.343 \text{ nH}$$

$$115.749 \text{ pF}$$

$$78.343 \text{ nH}$$

High pass

$$f_{HP} = 45 \times 10^6 \text{ Hz}$$

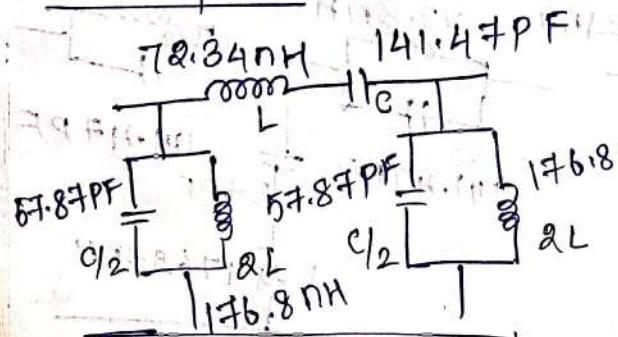
$$C = \frac{1}{\omega C} = \frac{1}{50 \times 2\pi \times 45 \times 10^6}$$

$$C = 141.471 \text{ pF}$$

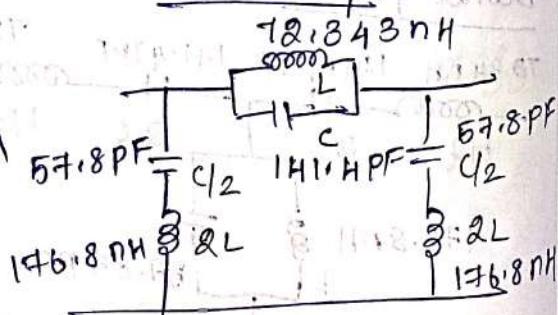
$$L = \frac{\mu}{\omega C} = \frac{50}{2 \times \pi \times 45 \times 10^6}$$

$$L = 88.419 \text{ nH}$$

Band pass



Band stop



3. Pi network:

FOR π section,

$$K^2 = \frac{4L}{C}$$

$$C = \frac{4L}{K^2} \text{ and } 4L = K^2 C$$

FOR C

$$\omega_c = \frac{1}{\sqrt{LC}}$$

$$L = \frac{K^2 C}{4}$$

$$\omega_c = \frac{1}{\sqrt{\frac{K^2 C}{4} \cdot C}} = \boxed{\frac{1}{\sqrt{\frac{K^2 C^2}{4}}} = \frac{1}{\frac{KC}{2}}}$$

$$\omega_c = \frac{\omega}{KC}$$

$$C = \boxed{\frac{\omega}{KC}}$$

FOR L

$$\omega_c = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L \cdot \frac{4L}{K^2}}}$$

$$= \frac{1}{\sqrt{\frac{4L^2}{K^2}}}$$

$$= \frac{1}{\frac{2L}{K}} = \frac{K}{2L}$$

$$\omega_c = \frac{K}{2L}$$

$$L = \boxed{\frac{K}{2\omega_c}}$$

$$f_c = 10 \text{ MHz}$$

$$Z_0 = 50 \Omega$$

$$C = \frac{\lambda}{2\pi Z_0}$$

$$\omega_c = 2\pi f_c = 2\pi \times 10 \times 10^6 = 62,832 \text{ rad/s}$$

$$C = \frac{\lambda}{2\pi \times 62,832 \times 10^6}$$

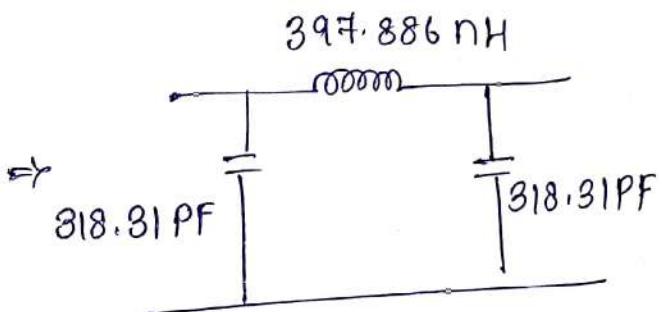
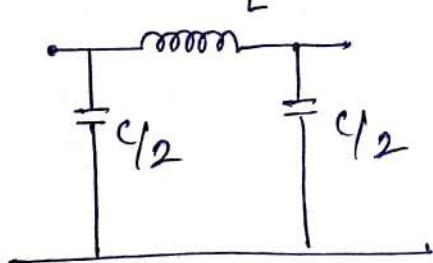
$$C = 636.62 \text{ pF}$$

$$L = \frac{Z_0}{2\pi \omega_c} = \frac{50}{2 \times 62,832 \times 10^6}$$

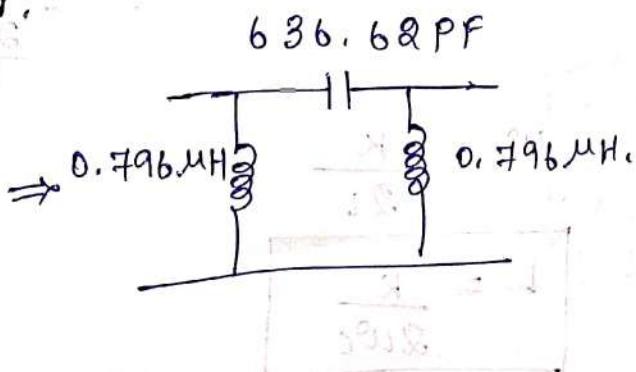
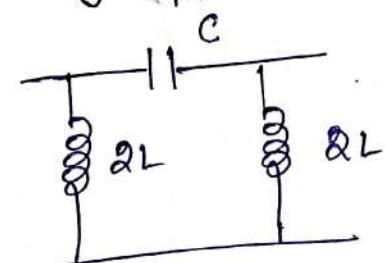
$$L = 397.886 \text{ nH} / 0.398 \mu\text{H}$$

Design:

i) Low pass filter:



ii) High pass filter:



constant K-section

1) T-section:

$$K = \sqrt{z_1 z_2 + z_1^2/4}$$

constant section width

$$K = \sqrt{j_w L \cdot \frac{1}{j_w c} + \frac{(j_w L)^2}{A}}$$

$$K = \sqrt{\frac{L}{c} + \frac{(-\omega^2 L^2)}{A}}$$

$$K^2 = \frac{L}{c} - \frac{\omega^2 L^2}{A} \quad \omega \ll \omega_c$$

$K^2 = 4c$

2) Π -section:

$$K = \sqrt{\frac{z_1}{A} (z_1 + Hz_2)}$$

$$= \sqrt{\frac{z_1^2}{A} + A z_1 z_2}$$

$$= \sqrt{-\frac{\omega^2 L^2}{4} + \frac{HL}{c}}$$

$$K = \sqrt{HL/c}$$

$K^2 = \frac{HL}{c}$

8.10.25

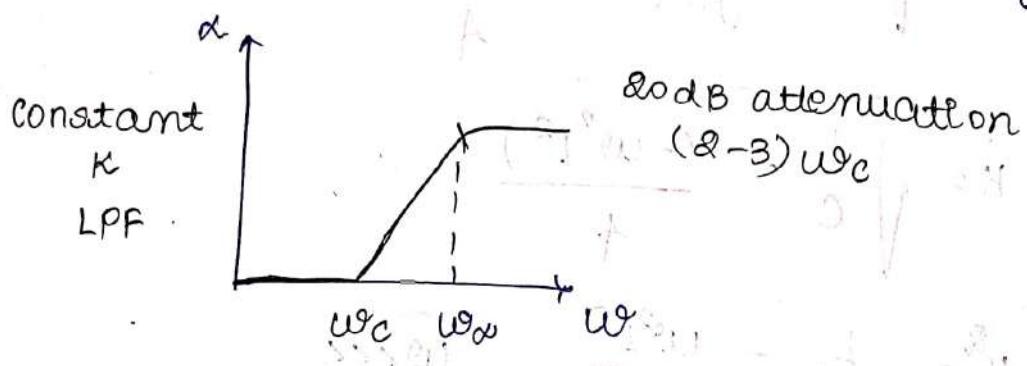
Notes on Filters

m-derived filter:

constant K-filter

1. Non-constant Image Impedance
(Frequency \propto Image Impedance)

2. slow attenuation rate near cut-off



To address these drawbacks \rightarrow m-derived LPF T section:

a) constant K

$$Z_1 = j\omega L$$

$$Z_2 = \frac{1}{j\omega C}$$

For m-derived:

$$z'_1 = mz_1$$

To obtain Image Impedance now over Z_2' should be

we know that

$$Z_{1T} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$$= \sqrt{z'_1 z'_2 + \frac{(z'_1)^2}{4}}$$

$$z_1' z_2' + \frac{(z_1')^2}{4} = z_1 z_2 + \frac{z_1^2}{4}$$

$$z_1' z_2' = z_1 z_2 + \frac{z_1^2}{4} - \frac{(z_1')^2}{4}$$

$$(m z_1) z_2' = z_1 z_2 + \frac{z_1^2}{4} - \frac{m^2 z_1^2}{4}$$

$$m z_1 z_2' = z_1 z_2 + \frac{z_1^2}{4} (1 - m^2)$$

$$z_2' = \frac{z_1 z_2}{m z_1} + \frac{z_1^2}{4} (1 - m^2)$$

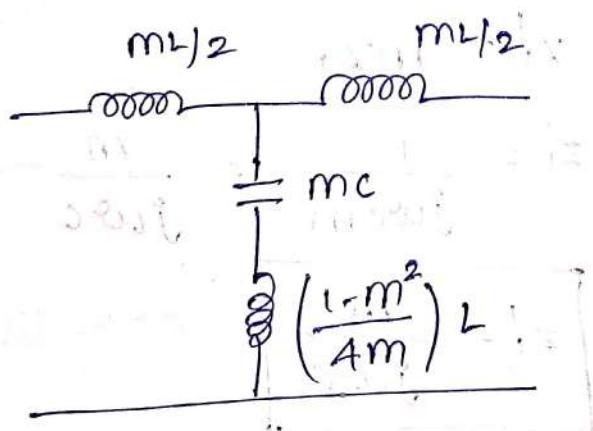
$$z_2' = \frac{z_2}{m} + \frac{z_1 (1 - m^2)}{4m}$$

$$z_1' = m z_1 = m j w L$$

$$z_2' = \frac{1}{j w cm} + j w L \frac{(1 - m^2)}{4m} \quad (\text{Bath } L \text{ & } c) \text{ component.}$$

m-derived

T-section:



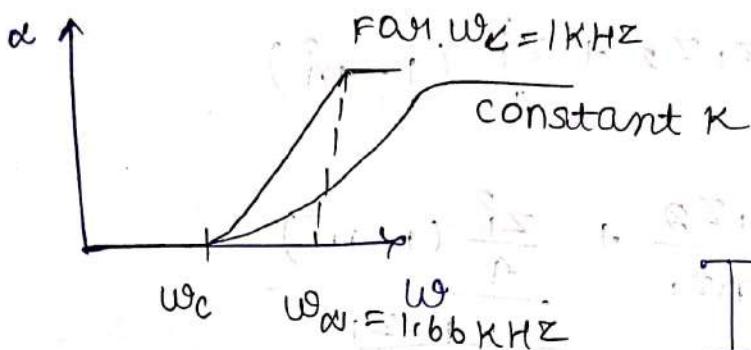
$$0 < m < 1$$

If $m=1$ then constant K filter

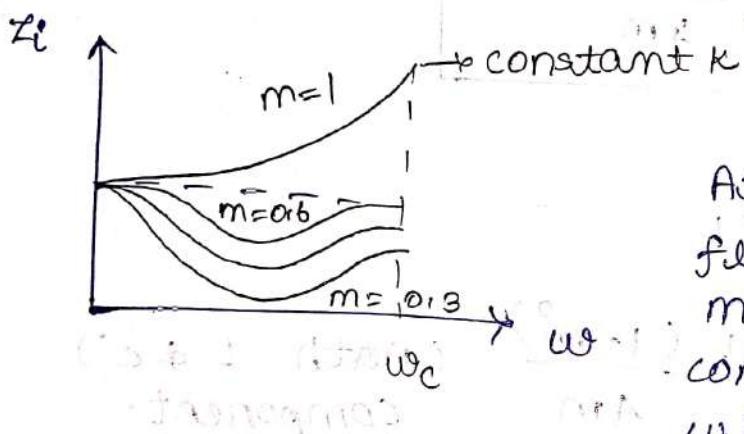
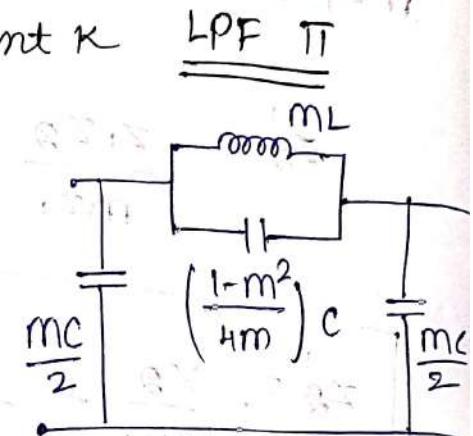
$$\omega_o = \frac{\omega_c}{\sqrt{1-m^2}} \quad m < 1$$

If $\omega_c = 1 \text{ kHz}$ and $m = 0.8$ then

$$\omega_o = \frac{1 \text{ kHz}}{\sqrt{1-(0.8)^2}} = 1.66 \text{ kHz}$$



$m = 0.6$ for real time



High pass filter

For constant k

$$Z_1 = \frac{1}{j\omega c}$$

$$Z_2 = j\omega L$$

negative feedback

At pass band,
fluctuations when:
 $m = 0.3$ but
compensated by $m = 0.6$
with short circuit
(better)
in attenuation

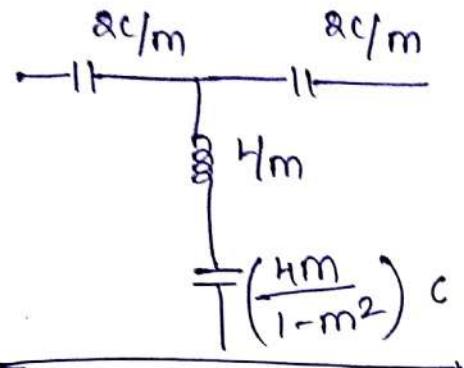
$$Z_1' = m Z_1$$

$$Z_1' = \frac{1}{j\omega cm} = \frac{m}{j\omega c}$$

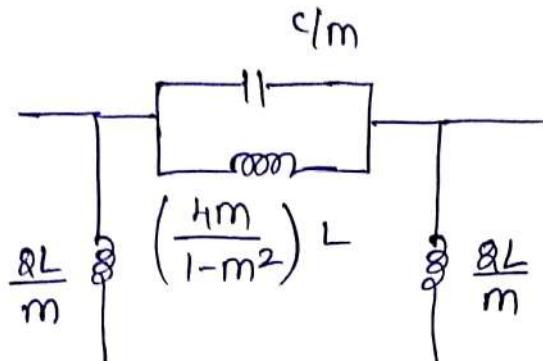
$$Z_1' = \frac{m}{j\omega c}$$

$$Z_0' = \frac{g_{WL}}{m} + \frac{1}{g_{WC}} \left(\frac{1-m^2}{4m} \right)$$

For m^2 -derived



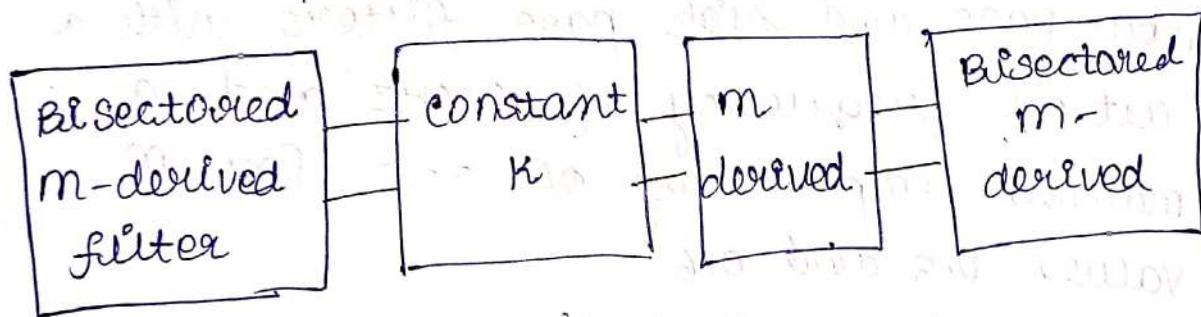
T-section



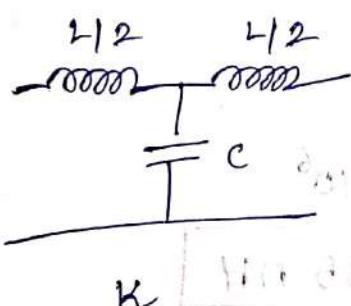
Pi section

composite filter:

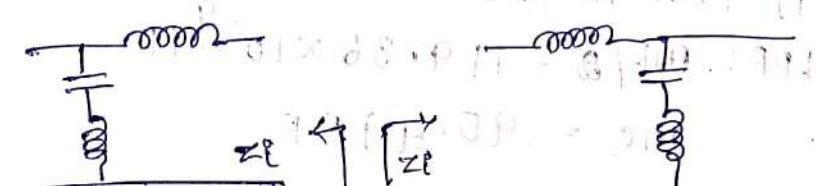
constant κ with m-derived along
with Impedance matching network.

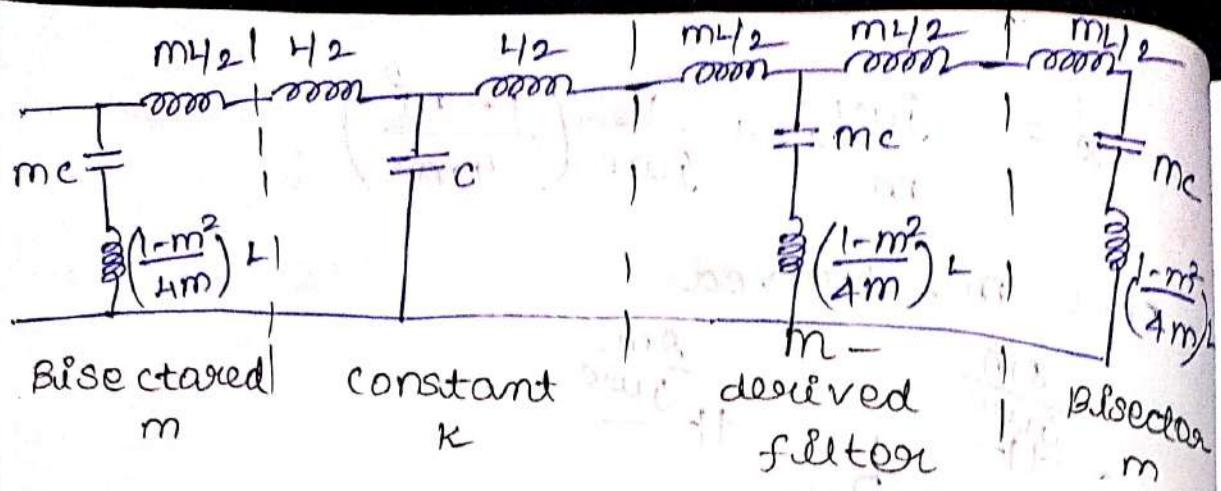


composite low pass filter

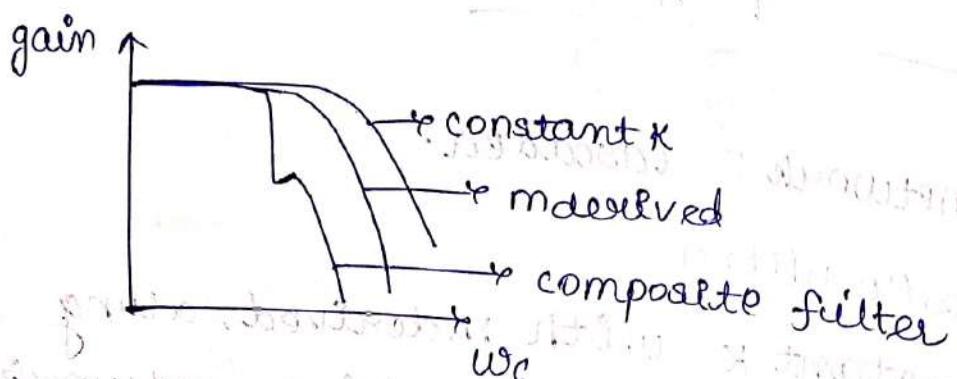


Bisected m





sharp cut off - freq un feasible way
for composite



4. Design a m -derived T and pi section low pass and high pass filters with a cut off frequency of 10 MHz and a matched impedance of 50Ω for m values 0.3 and 0.6.

Low pass filter - T section

T - section:

$$C = \frac{1}{K w_c} \quad L = \frac{K}{w_c}$$

$$C = \frac{1}{50 \times 2\pi \times 10 \times 10^6}$$

$$C = 318.309 \text{ pF}$$

$$L = \frac{50}{2\pi \times 10 \times 10^6}$$

$$L = 795.785 \text{ nH}$$

i) For $m = 0.3$

$$\text{LPF: } m_{L/2} = 119.36 \times 10^{-9} \text{ H}$$

$$m_c = 95.49 \text{ pF}$$

$$\left(\frac{1-m^2}{4m}\right) L = \left(\frac{1-0.3^2}{4 \times 0.3}\right) L = 54.3123 \text{ nH}$$

$$\text{HPF: } \frac{\partial C}{m} = 8122.06 \text{ PF} \quad L/m = 2658.61 \text{ nH}$$

$$\left(\frac{4m}{1-m^2}\right) C = \left(\frac{4 \times 0.3}{1-0.3^2}\right) C = 419.748 \text{ PF}$$

iii) FOR $m=0.6$

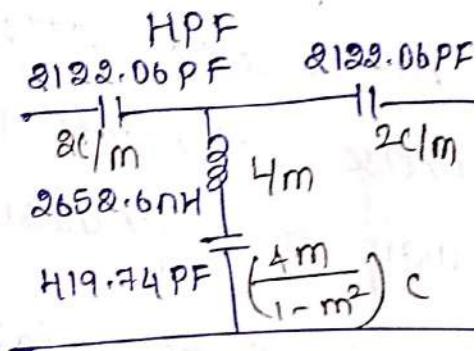
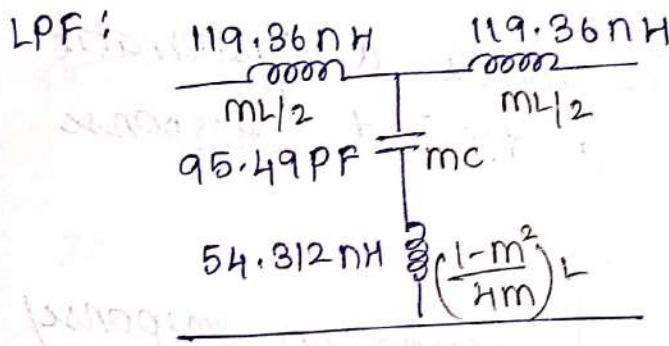
$$\text{LPF: } m L/2 = 238.73 \text{ nH} \quad m C = 190.98 \text{ PF}$$

$$\left(\frac{1-m^2}{4m}\right) L = \left(\frac{1-0.6^2}{4 \times 0.6}\right) L = 76.395 \text{ nH}$$

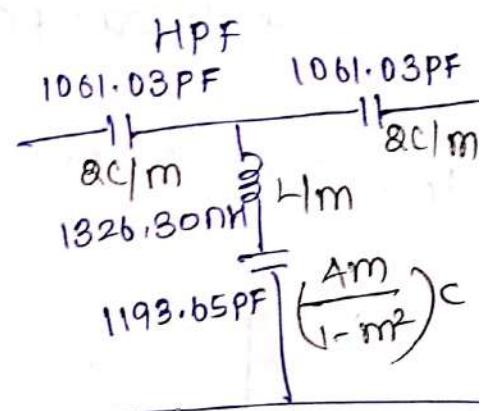
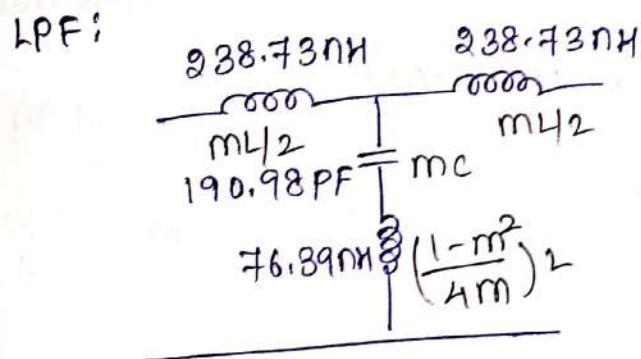
$$\text{HPF: } \frac{\partial C}{m} = 1061.03 \text{ PF} \quad L/m = 1326.30 \text{ nH}$$

$$\left(\frac{4m}{1-m^2}\right) C = \left(\frac{4 \times 0.6}{1-0.6^2}\right) C = 1193.658 \text{ PF}$$

FOR $m=0.3$



FOR $m=0.6$



14.10.25

Filter design:

- pass band → zero insertion loss
- stop band → infinite attenuation
- pass band → linear phase → to avoid signal distortion

In Images parameter method; there is no methodical way of improving the design

Insertion loss method:

→ high degree of control over the pass band and stop band amplitude and phase characteristics with a systematic way to synthesize a desired response

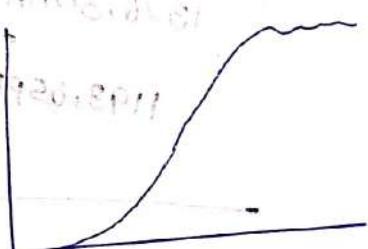
* Binomial

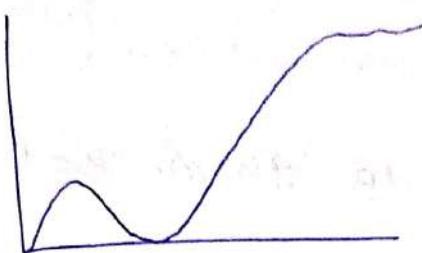
* Chebyshev

* Linear phase

} depends on response polynomial

Fair flat response → binomial





For sharp cutoff
chebyshev.

ripples \rightarrow chebyshev.

define insertion loss and then with
matching L, C components.

for linear phase and flat response
we go for linear phase method.

Insertion loss:

$$IL = 10 \log (\text{power loss ratio})$$

$$\text{power loss ratio} = \frac{P_{in}}{P_{load}}$$

In terms of reflection coefficients

$$= \frac{1}{1 - |r|^2} \quad r = \frac{z_{in} - z_0}{z_{in} + z_0}$$

binomial (flat response):

$$\text{power loss ratio } PLR = 1 + K^2 \left(\frac{\omega}{\omega_c} \right)^{2N}$$

N = order of filter (highest degree
of pole \rightarrow order
of system)

At the edge of boundary ($\omega = \omega_c$)

$$PLR = 1 + K^2$$

$K = 0$ (ideal case)
 $K = 1$ (3dB loss expected)

$$(PLR)_{dB} = 3$$

For reference of -3 dB then $\kappa=1$

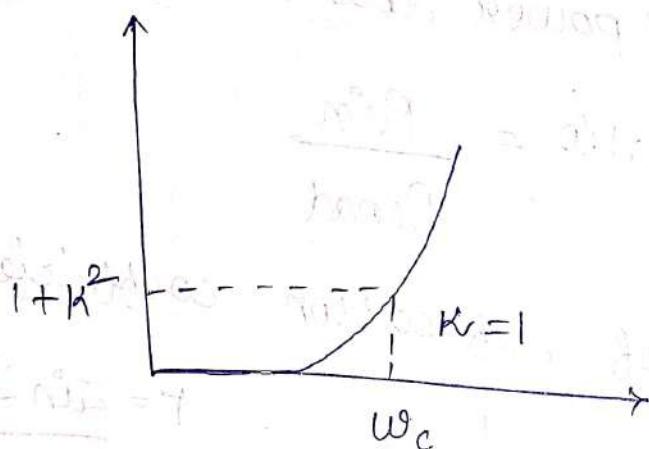
If $\kappa=1$ then

$$PLR = 1 + \left(\frac{\omega}{\omega_c} \right)^{2N}$$

if $\omega \gg \omega_c$

\Rightarrow 20 dB/decade

if $\omega \ll \omega_c = 0$.



Chebyshev Response:

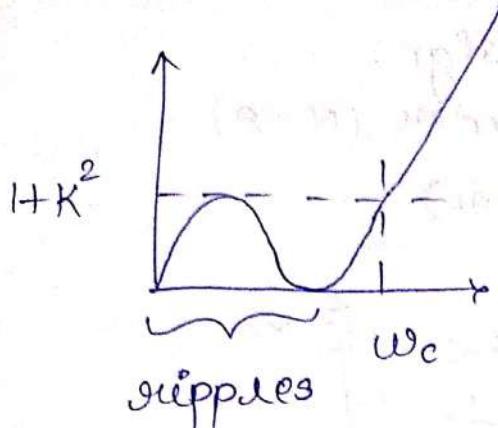
$$PLR = 1 + \kappa^2 T_N^2 \left(\frac{\omega}{\omega_c} \right)$$

$$PLR = 1 + T_N^2 \left(\frac{\omega}{\omega_c} \right) \quad 3 \text{ dB reference } \kappa=1$$

At Edge of Boundary $\omega = \omega_c$

$$PLR = 1 + T_N^2 (1)$$

$= 20 \text{ dB/decade}$



Equiripple in passband response

Linear phase response:

$$\phi(w) = Aw \left[1 + P \left(\frac{w}{w_c} \right)^{2N} \right]$$

$P = \text{constant}$

$$\text{group delay: } \tau_d = \frac{d(\phi(w))}{dw}$$

$$\tau_d = A \left[1 + P(2N+1) \left(\frac{w}{w_c} \right)^{2N} \right]$$

phase change depends on order and constant.

Amplitude vary: $20N$ slope/decade
design steps involved in: Insertion loss method:

→ filter specification

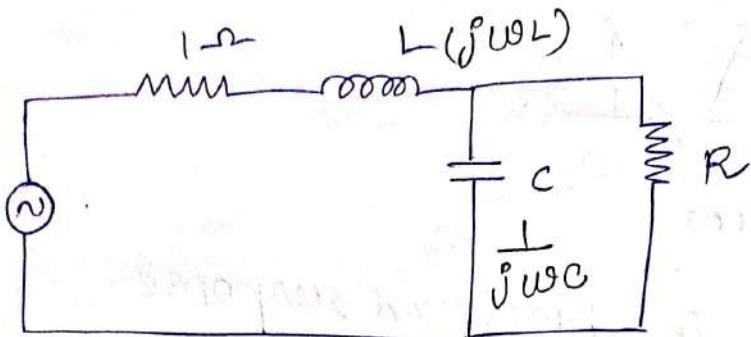
→ low pass filter design

→ scaling of frequency/conversion

→ Implementation.

Low pass filter design:

Let it be second order ($N=2$)



$$\omega_c = 1 \text{ rad/sec}$$

Acceptable range of loss = 3 dB

$$so \boxed{k=1}$$

Binomial method:

$$(PLR) = 1 + k^2 \left(\frac{\omega}{\omega_c} \right)^{2N} = \frac{1}{1 - \tau^2}$$

Suppose $k=1$ then

$$= 1 + \left(\frac{\omega}{\omega_c} \right)^{2N} = 1 + \left(\frac{\omega}{\omega_c} \right)^4$$

Butterworth

$$(PLR) = 1 + \omega^4 \quad \text{--- (1)}$$

$$Z_{in} = j\omega L + \frac{R(1-j\omega RC)}{1+\omega^2 R^2 C^2}$$

$Z_0 = R = 1 \Omega = \text{normalised impedance}$
(50Ω)

$$\hat{\tau} = \frac{Z_{in} - 1}{Z_{in} + 1} = \frac{(Z_{in} + 1)^2}{2(Z_{in} + Z_{in}^*)}$$

$$PLR = \frac{1}{1 - |\Gamma|^2}$$

$$= 1 + \frac{1}{4R} \left[(1-R)^2 + (R^2C^2 + L^2 - 2LCR^2)W^2 + (L^2C^2 R^2)W^4 \right] \quad \text{--- (2)}$$

If no frequency component through

① then $\omega = 0$ and $PLR = 1$

If $R = 1$ then from ② $PLR = 1$

Equate ω^2 in ① and ②

$$RC^2 + L^2 - 2LCR^2 = 0$$

wkt $\boxed{R = 1}$

$$C^2 + L^2 - 2LC = 0$$

$$(C-L)^2 = 0$$

$\boxed{C = L} \quad \text{--- (3)}$

Equating ω^4 from ② and ①

$$L^2 C^2 R^2 = 1/4 \quad \frac{1}{4} L^2 C^2 R^2 = 1$$

$$L^2 C^2 = 1/4$$

$$L^2 C^2 = 4$$

From ③

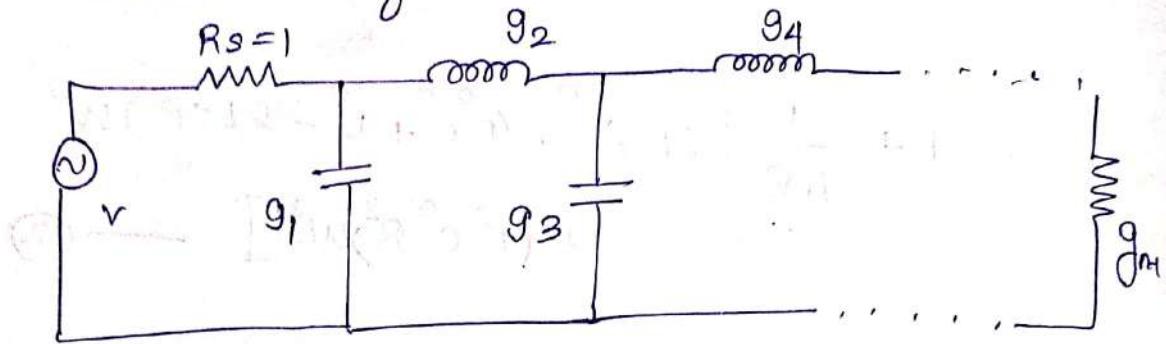
$$L^4 = 4$$

$$L = \sqrt{2} = C$$

$$L = C = \frac{1}{\sqrt{2}}$$

$\boxed{L = C = 1.414}$

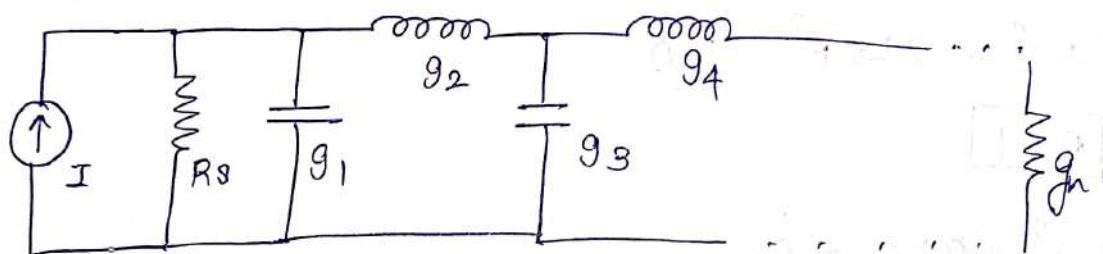
For n - order it goes in a ladder way



Irrespective of order n; R=1

In chebyshev
odd number = 1

Even number = 1, 9, 8



Filter Transformation:

- Impedance Transformation;
- Frequency Transformation
- Low pass filter converted to High pass, Bandpass and Band stop.

Impedance Transformation:

$$R_S = R_0$$

$$R'_S = R_0$$

far Butterworth with $R_0 = 1$
for all (at least)

$$R'_L = R_0 R_L$$

If $R_{0 \rightarrow 2}$ is varied
then new R_L found

$$L' = R_0 L$$

$$C' = \frac{C}{R_0}$$

Due to Impedance change

frequency Transformation: change in frequency

$$PLR'(w) = PLR \left(\frac{w}{w_c} \right)$$

$$jX_L = j\omega L$$

$$jB_K = \frac{1}{j\omega c}$$

$$\left. \begin{array}{l} w=w_c=1 \\ w \leftarrow \frac{w}{w_c} \end{array} \right\}$$

$$w' = 1/w_c$$

Due to change

other than $w_c = 1 \text{ rad/s}$
and $w=1$

$$jX_L = j\omega L' = j \left(\frac{w}{w_c} \right) L$$

$$jB_K = \frac{1}{j\omega c'} = j \left(\frac{w}{w_c} \right) C_K$$

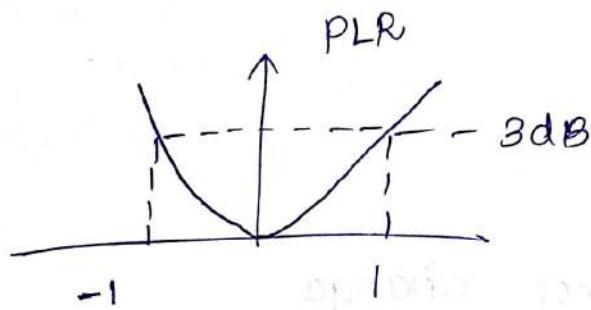
$$jB_K = j\omega c' = j \left(\frac{w}{w_c} \right) C_K$$

Impedance & frequency change:

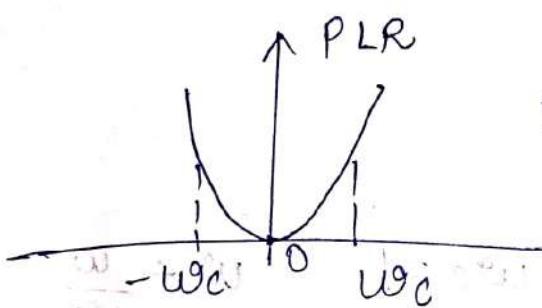
$$j\omega L' = j\left(\frac{\omega}{\omega_c}\right) L R_0$$

$$j\omega C' = j\left(\frac{\omega}{\omega_c}\right) \cdot \frac{C}{R_0}$$

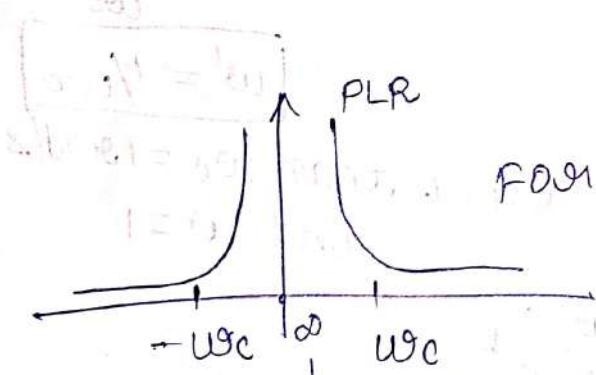
conversion from LPF to HPF



due to frequency translation



for LPF



for HPF

Frequency Translation:

$$\text{LPF: } \omega \leftarrow \frac{\omega}{\omega_c} = 1 \leftarrow \frac{1}{\omega_c} \quad \omega = 1$$

$$\text{HPF: } \frac{-\omega_c}{\omega} \quad \text{for } \omega = 1 \text{ then } -\omega_c$$

Impedance Translation Frequency

LPF:

$$L_K' = \frac{L_K R_0}{w_c}$$

\$L_K, C_K\$ from
\$g_1, g_2\$ in table

$$C_K' = \frac{C_K}{R_0 w_c}$$

\$R_0\$: last value
in table

w_c : new frequency

HPF:

series and shunt changes

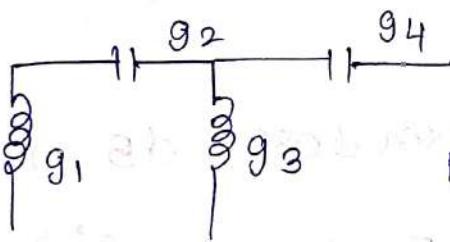
$$C_K' = \frac{R_0 (w_c L_K)}{w_c C_K} \quad \left[jX_K = j\left(\frac{w_c}{w}\right) L = \frac{1}{j w C_K} \right]$$

Reverse since HPF

$$L_K' = \frac{R_0}{w_c C_K}$$

IT no changes

For HPF



But we have \$g_1\$ &
\$g_2\$ as (\$C\$ and \$L\$)

(changing component
value alone)

- 15.10.25
1. Design maximally flat LPF with a cut off frequency of 20Hz impedance of 50Ω and atleast 15dB insertion loss at 30Hz compare with an equiripple of 3dB and linear phase filter has the same order.

To design filter:

→ LPF

→ flat, Equiripple and linear phase given

$$f_0 = 2 \text{ GHz}$$

$$w = \underbrace{3 \text{ GHz}}^f \times \pi; I_L = 15 \text{ dB} \quad (\text{Insertion loss})$$

$$Z = 50 \Omega$$

$$Z_S = 50 \Omega$$

i) Butterworth filter: a) LPF

Table 8.3

$$L_K = \frac{R_O L_K}{w_C} \quad C_K = \frac{C_K}{R_O w_C} \quad \text{Fig 8.26}$$

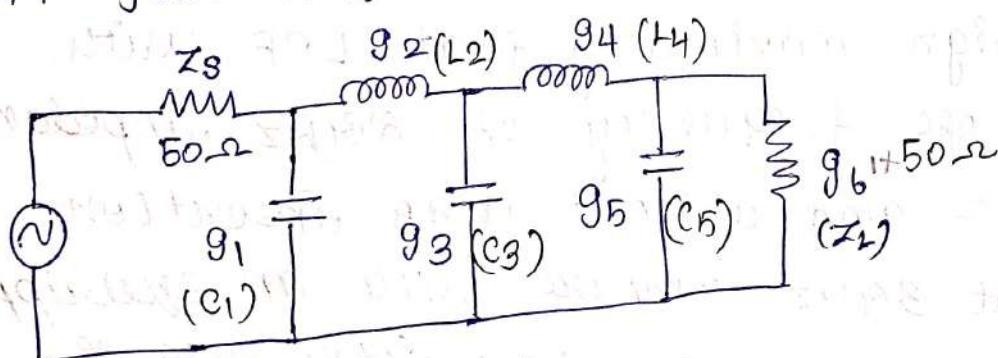
N → depends on IL

$$\frac{w}{w_C} - 1 = \frac{3 \times 10^9}{2 \times 10^9} - 1 = 0.5$$

Attenuation (Insertion loss) dB = 15 dB

comes between 4 and 5 go to higher value hence $\boxed{n=5}$

LPF for n=5



From Table 8.3 (pozai)

$$n=5$$

$$g_1 = 0.6180$$

$$= C_1$$

$$g_2 = 1.6180$$

$$= L_2$$

$$g_3 = 8.0000 = C_3$$

$$g_4 = 1.6180 = L_4$$

$$g_5 = 0.6180 = C_5$$

$$g_6 = 1.0000$$

$$g_6 \times 50 = 50 \Omega \text{ hence last } R = 50 \Omega$$

For g_1

$$C_K' = \frac{C_K}{R_0 \times w_c} = \frac{0.6180}{50 \times 2\pi \times 2 \times 10^9} = 0.983 \text{ PF}$$

For g_2

$$L_K' = \frac{R_0 L_K}{w_c} = \frac{50 \times 1.6180}{2\pi \times 2 \times 10^9} = 6.437 \text{ nH}$$

For g_3

$$C_K' = \frac{C_K}{R_0 \times w_c} = \frac{8}{50 \times 2\pi \times 2 \times 10^9} = 3.183 \text{ PF}$$

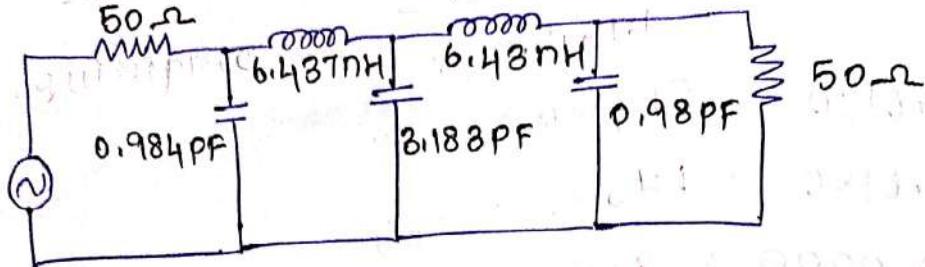
For g_4

$$L_K' = \frac{R_0 L_K}{w_c} = \frac{50 \times 1.6180}{2\pi \times 2 \times 10^9} = 6.43 \text{ nH}$$

For g_5

$$C_K' = \frac{C_K}{R_0 \times w_c} = \frac{0.6180}{50 \times 2\pi \times 2 \times 10^9} = 0.98 \text{ PF}$$

Butterworth Low pass filter:



b) High pass Filter:

Now, $g_1 = 0.6180$ From maximally flat table

$$g_2 = 1.6180$$

$$g_3 = \infty$$

$$g_4 = 1.6180$$

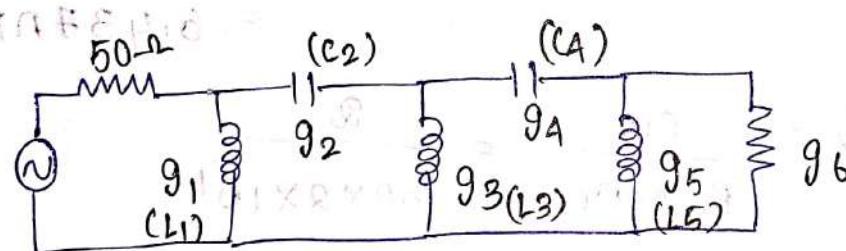
$$L_1 = g_5 = 0.6180$$

$$C_K = \frac{R_0 \times 10^9}{8\pi f \times 10^9} = \frac{50}{8\pi \times 10^9 \times 10^9} = 10^{-10}$$

$$C_K = \frac{1}{R_0 \omega_c L_K}$$

Take respective
(g_n) values from
LPF

$$H_{HPF}(s) = \frac{R_0 \times 10^9}{\omega_c C_K} =$$



For g_1

$$L_1 = \frac{R_0}{\omega_c C_K} = \frac{50}{8\pi \times 2 \times 10^9 \times 0.6180}$$

$$L_1 = 6.438 \text{ nH}$$

For g_2

$$C_2 = \frac{1}{R_0 \omega_c L_2} = \frac{1}{50 \times 2\pi \times 2 \times 10^9 \times 1.6180}$$

$$= 0.984 \text{ pF}$$

for 93

$$L_3 = \frac{R_o}{w_c C_K} = \frac{50}{2\pi \times 2 \times 10^9 \times 2} = 1.989 \text{ nH.}$$

for 94

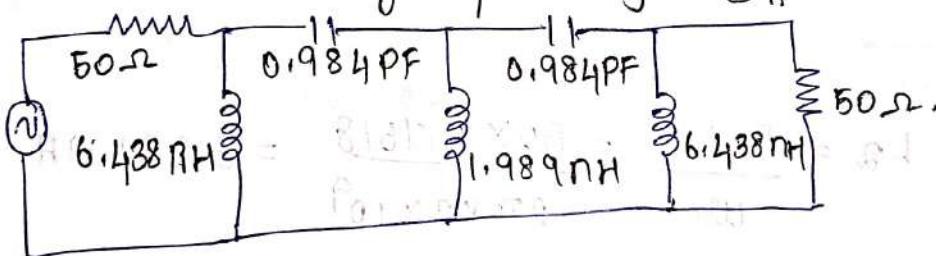
$$C_4 = \frac{1}{R_o w_c L_K} = \frac{1}{50 \times 2\pi \times 2 \times 10^9 \times 1.6180} = 0.984 \text{ PF.}$$

for 95

$$L_5 = \frac{R_o}{w_c C_K} = \frac{50}{2\pi \times 2 \times 10^9 \times 0.6180} = 6.438 \text{ nH.}$$

$$Z_6 = 1 \times 50 = 50 \Omega$$

Butterworth High pass filter:



ii) Chebyshev filter: (From Table 8.4)

a) LPF:

For a Equiripple of 3dB

From the Table,

since, same order $n=5$

$$g_1 = 3.4817$$

$$g_2 = 0.7618$$

$$g_3 = 4.5381$$

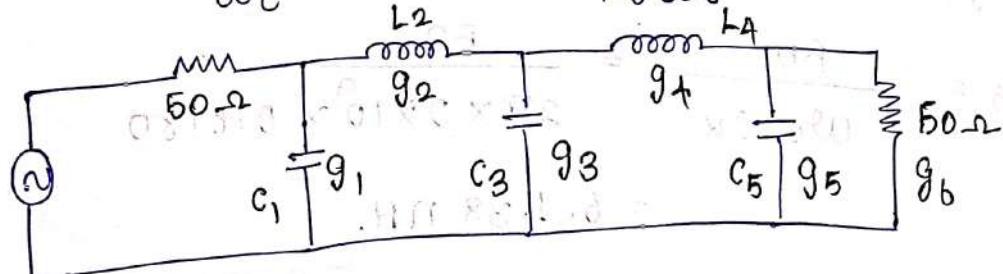
$$g_4 = 0.7618$$

$$g_5 = 3.4817$$

$$g_6 = 1$$

$$g_6 \times 50 = 50 \Omega$$

$$L_K = \frac{R_0 L_K}{w_C} ; C_K = \frac{C_K}{R_0 w_C}$$



$$\text{For } g_1 \quad C_1 = \frac{C_K}{R_0 w_C} = \frac{3.4817}{50 \times 2\pi \times 2 \times 10^9} = 5.541 \text{ pF}$$

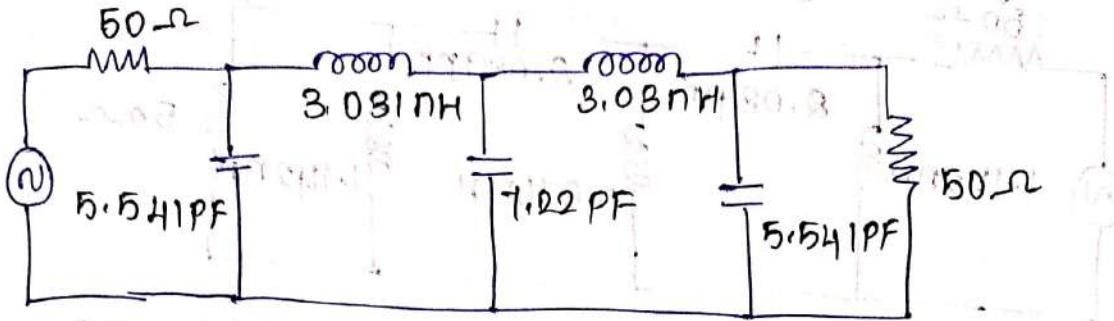
$$\text{For } g_2 \quad L_2 = \frac{R_0 L_K}{w_C} = \frac{50 \times 0.7618}{2\pi \times 2 \times 10^9} = 3.031 \text{ nH}$$

$$\text{For } g_3 \quad C_3 = \frac{C_K}{R_0 w_C} = \frac{4.5381}{50 \times 2\pi \times 2 \times 10^9} = 7.222 \text{ pF}$$

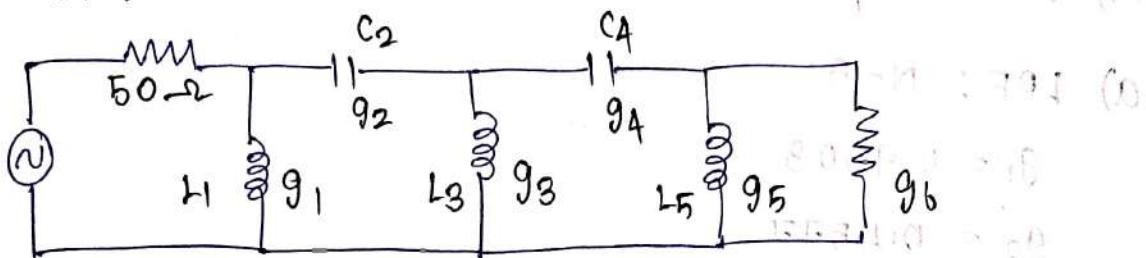
$$\text{For } g_4 \quad L_4 = \frac{R_0 L_K}{w_C} = \frac{50 \times 0.7618}{2\pi \times 2 \times 10^9} = 3.03 \text{ nH}$$

$$\text{For } g_5 \quad C_5 = \frac{C_K}{R_0 w_C} = \frac{3.4817}{50 \times 2\pi \times 2 \times 10^9} = 5.541 \text{ pF}$$

chebyshov low pass filter: (not yet calculated)



b) HPF:



$$C_K' = \frac{1}{R_0 \omega_c L_K} ; \quad L_K' = \frac{R_0}{\omega_c C_K}$$

for g1

$$L_1 = \frac{R_0}{\omega_c C_{K1}} = \frac{50}{2\pi \times 2 \times 10^9 \times 3.4817} = 1.142 \text{ nH}$$

$$L_1 = 1.142 \text{ nH}$$

for g2

$$C_2 = \frac{1}{R_0 \omega_c L_K} = \frac{1}{50 \times 2\pi \times 2 \times 10^9 \times 0.7618} = 2.08 \text{ pF}$$

for g3

$$L_3 = \frac{R_0}{\omega_c C_K} = \frac{50}{2\pi \times 2 \times 10^9 \times 1.5381} = 0.876 \text{ nH}$$

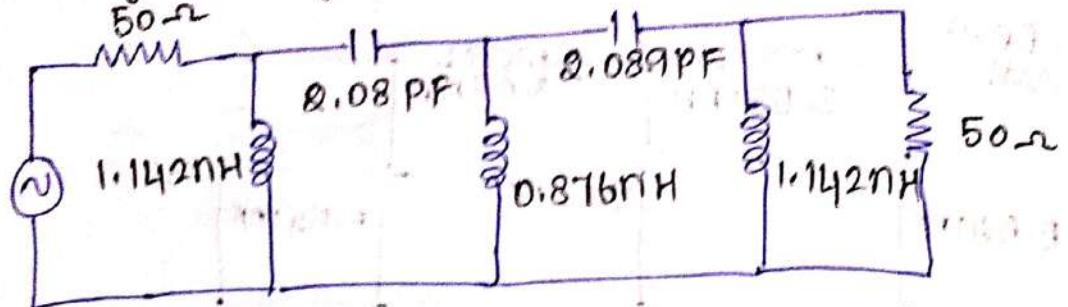
for g4

$$C_4 = \frac{1}{R_0 \omega_c L_K} = \frac{1}{50 \times 2\pi \times 2 \times 10^9 \times 0.7618} = 2.089 \text{ pF}$$

for g5

$$L_5 = \frac{R_0}{\omega_c C_K} = \frac{50}{2\pi \times 2 \times 10^9 \times 3.4817} = 1.142 \text{ nH}$$

Chebyshev Highpass Filter:



iii) Linear phase filters (from Table 8.5)

a) LPF: N=5

$$g_1 = 0.9303$$

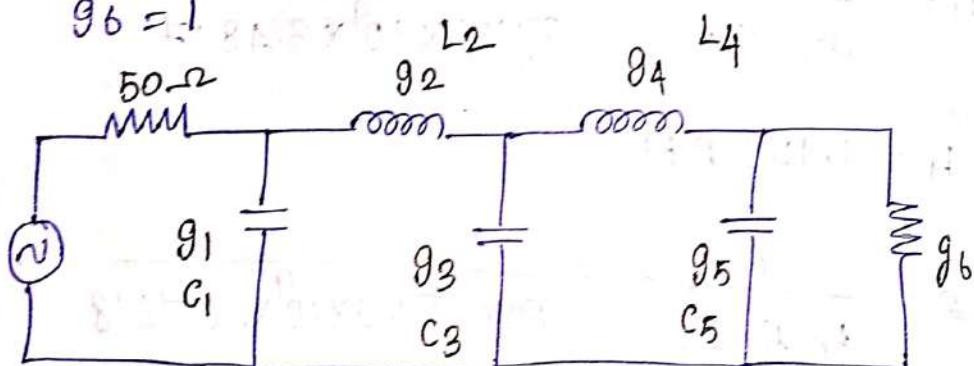
$$g_2 = 0.4577$$

$$g_3 = 0.3312$$

$$g_4 = 0.2090$$

$$g_5 = 0.0718$$

$$g_6 = 1$$



$$L_K^1 = \frac{R_0 L_K}{w_0 C}$$

$$C_K^1 = \frac{C_K}{R_0 w_0 C}$$

For g_1

$$C_1 = \frac{C_K}{R_0 w_0 C} = \frac{0.9303}{50 \times 2\pi \times 2 \times 10^9} = 1.4806 \text{ PF}$$

For g_2

$$L_{g_2} = \frac{R_0 L_K}{w_0 C} = \frac{50 \times 0.4577}{2\pi \times 2 \times 10^9} = 1.821 \text{ nH}$$

for 93

$$C_3 = \frac{CK}{R_0 \omega_C} = \frac{0.3312}{50 \times 2\pi \times 2 \times 10^9} = 0.1527 \text{ pF}$$

for 94

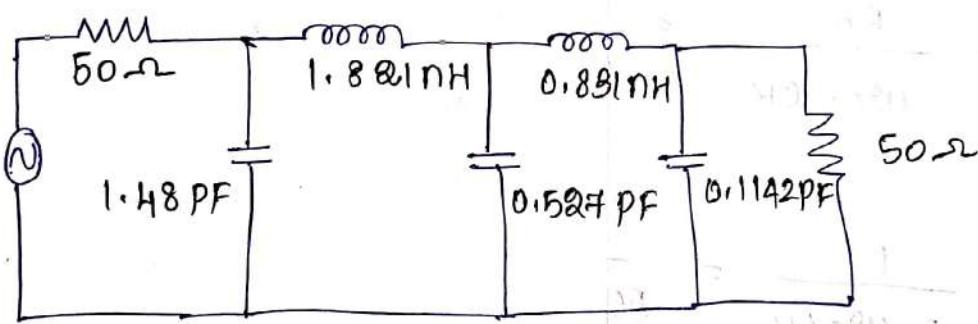
$$L_4 = \frac{R_0 K}{\omega_C} = \frac{50 \times 0.2090}{2\pi \times 2 \times 10^9} = 0.831 \text{ nH}$$

for 95

$$C_5 = \frac{CK}{R_0 \omega_C} = \frac{0.0718}{50 \times 2\pi \times 2 \times 10^9} = 0.1142 \text{ pF}$$

$$g_6 \times 50 = 50 \Omega$$

linear phase Low pass filter:



b) HPF:

$$g_1 = 0.9303$$

$$g_2 = 0.4577$$

$$g_3 = 0.3312$$

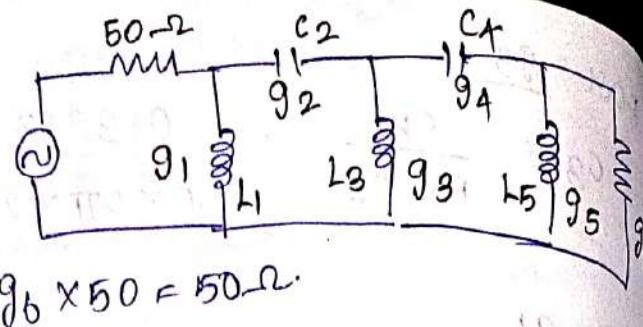
$$g_4 = 0.2090$$

$$g_5 = 0.0718$$

$$g_6 = 1$$

$$C_K = \frac{1}{\omega_C R_0 L_K}$$

$$L_K = \frac{R_0}{\omega_C C_K}$$



For g_1

$$L_1 = \frac{R_0}{\omega_C \cdot C_K} = \frac{50}{2\pi \times 2 \times 10 \times 0.9303} = 4.876 \text{ nH}$$

For g_2

$$C_2 = \frac{1}{R_0 \omega_C L_K} = \frac{1}{50 \times 2\pi \times 2 \times 10 \times 0.4577} = 3.477 \text{ pF}$$

For g_3

$$L_3 = \frac{R_0}{\omega_C \cdot C_K} = \frac{50}{2\pi \times 2 \times 10 \times 0.3312} = 12.01 \text{ nH}$$

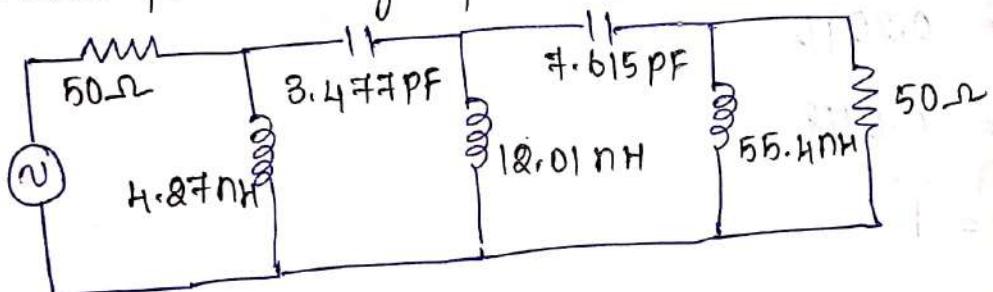
For g_4

$$C_4 = \frac{1}{R_0 \omega_C L_K} = \frac{1}{50 \times 2\pi \times 2 \times 10 \times 0.2090} = 4.615 \text{ pF}$$

For g_5

$$L_5 = \frac{R_0}{\omega_C \cdot C_K} = \frac{50}{2\pi \times 2 \times 10 \times 0.0718} = 55.41 \text{ nH}$$

linear phase High pass filter:



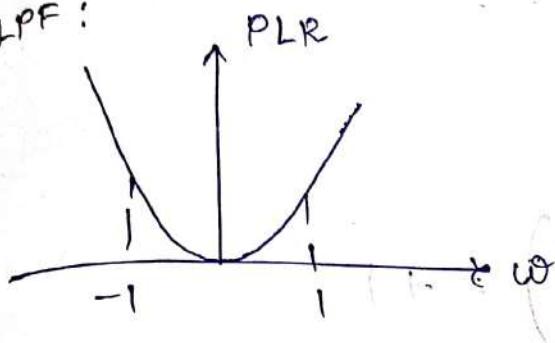
16.10.85

Frequency Translation:

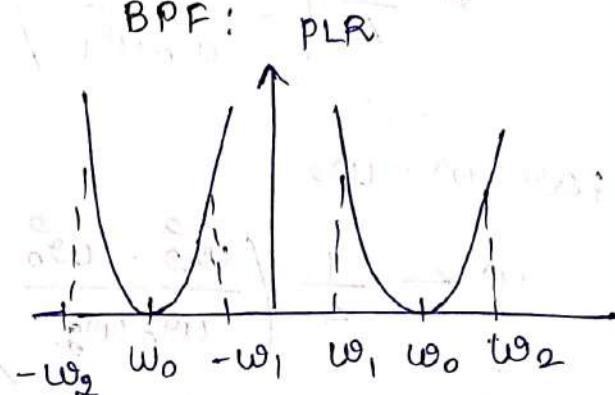
LPF to BPF and BBF

$$\omega_c = 1 ; K = 1$$

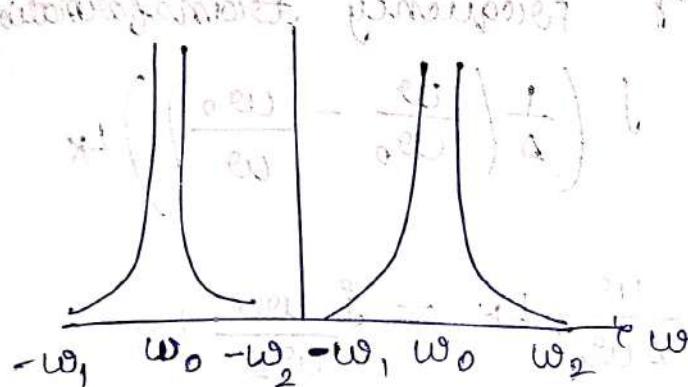
LPF:



BPF: PLR



BBF:



$$w \leftarrow \frac{\omega_0}{\omega_2 - \omega_1} \left(\frac{w}{\omega_0} - \frac{\omega_0}{w} \right)$$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

$$w \leftarrow \frac{1}{\Delta} \left(\frac{w}{\omega_0} - \frac{\omega_0}{w} \right)$$

$$\text{If } w = \omega_0$$

$$w \leftarrow 0 \text{ (LPF)}$$

For $\omega = \omega_1$

$$\omega \leftarrow \frac{1}{\Delta} \left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1} \right)$$

$$\Rightarrow \frac{1}{\Delta} \left(\frac{\omega_1^2 - \omega_0^2}{\omega_0 \omega_1} \right) = -1$$

For $\omega = \omega_2$

$$\omega \leftarrow \frac{1}{\Delta} \left(\frac{\omega_2^2 - \omega_0^2}{\omega_0 \omega_2} \right) = +1$$

For LPF generally

$$\begin{aligned} jX_K &= j\omega L_K = j \left(\frac{1}{\Delta} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \right) L_K \\ &= j \frac{\omega}{\Delta \omega_0} L_K - j \frac{\omega_0}{\Delta \omega_0} L_K \end{aligned}$$

LPF \rightarrow HPF

$$j \frac{\omega_0}{\Delta \omega_0} L_K = \frac{1}{j\omega C_K}$$

$$= j\omega L_K' - j \frac{1}{\omega C_K} + j \frac{\omega}{\Delta \omega_0} L_K$$

series form

For L_K'

$$\frac{j\omega}{\Delta \omega_0} L_K = j\omega L_K'$$

$$L_K' = R_0 \frac{L_K}{\Delta \omega_0}$$

$$\frac{-j\omega_0}{\Delta\omega} L_K = \frac{-j}{\omega_{CK}}$$

$$C_K' = \frac{\Delta}{R_0 \omega_0 L_K}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

shunt arm:

$$jB_K = j\omega_{CK}$$

$$= j \left(\frac{1}{\Delta} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \right) C_K$$

$$\frac{j\omega}{\Delta\omega_0} C_K - \frac{j\omega_0}{\Delta\omega} C_K$$

Equate:

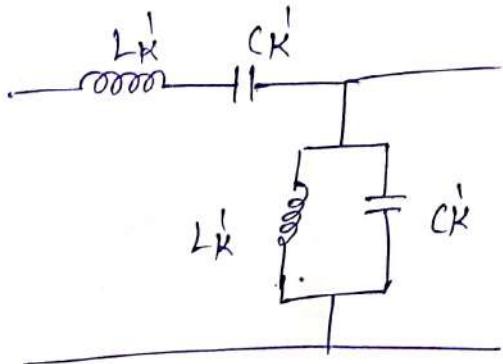
$$j\omega_{CK}' = j/\omega_{CK}$$

$$\frac{j\omega_{CK}}{\Delta\omega_0} = j\omega_{CK}'$$

$$C_K' = \frac{C_K}{\Delta\omega_0 R_0}$$

$$\frac{-j\omega_0}{\Delta\omega} C_K = \frac{-j}{\omega_{CK}'}$$

$$L_K' = \frac{\Delta R_0}{\omega_{CK}}$$



Band stop filter:

$$\omega \leftarrow -\frac{1}{\Delta} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1}$$

series arm:

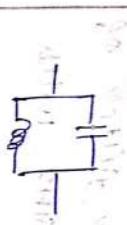
$$L_K = \frac{\Delta L_K R_0}{\omega_0}$$

$$C_K = \frac{1}{\omega_0 \Delta L_K \cdot R_0}$$

shunt arm

$$L_K' = \frac{R_0}{\omega_0 \Delta C_K}$$

$$C_K' = \frac{\Delta C_K}{R_0 \omega_0}$$

	series	shunt	series structure	shunt structure
Lowpass filter:	$L_K' = \frac{L_K}{\omega_0 C} \cdot R_0$	$C_K' = \frac{C_K}{R_0 \omega_0}$	— —	$\frac{1}{1 + \frac{R_0}{L_K' \omega_0}}$
High pass filter:	$C_K' = \frac{1}{R_0 \omega_0 L_K}$	$L_K' = \frac{R_0}{\omega_0 C_K}$	—II—	$\frac{1}{1 + \frac{R_0}{C_K' \omega_0}}$
Bandpass filter:	$L_K' = \frac{L_K R_0}{\Delta \omega_0}$ (LPF) $C_K' = \frac{\Delta}{R_0 \omega_0 L_K}$ (HPF)	$L_K' = \frac{\Delta R_0}{\omega_0 C_K}$ $C_K' = \frac{C_K}{\Delta \omega_0 R_0}$ (LPF)	— — —II—	
Bandstop filter:	$L_K' = \frac{\Delta L_K R_0}{\omega_0}$ $C_K' = \frac{1}{\omega_0 \Delta L_K R_0}$	$L_K' = \frac{R_0}{\omega_0 \Delta C_K}$ $C_K' = \frac{\Delta C_K}{R_0 \omega_0}$	— — —II—	

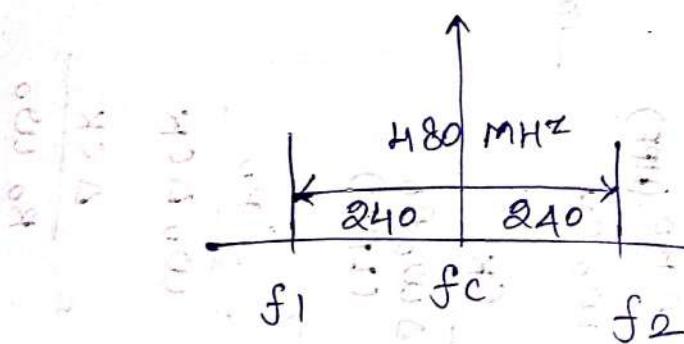
2. consider order of filter is 4, to design BPF save for Butterworth and chebyshov filter of 0.5 dB ripple $f_c = 2.4 \text{ GHz}$ with Impedance B_w ratio of 20%.

Given: $N = 4$

$$(2.4 \times 10^9) \times 20/100$$

$$f_c = 2.4 \text{ GHz} \Rightarrow 480 \text{ MHz}$$

$$f_c = 480 \text{ MHz}$$



$$\begin{array}{r} 2.4 \text{ GHz} \\ 0.48 \text{ GHz} \\ \hline 2.16 \text{ GHz} \end{array}$$

$$\begin{array}{r} 2.4 \text{ GHz} \\ 4.8 \text{ GHz} \\ \hline \end{array}$$

$$\begin{array}{r} 2.4 \text{ GHz} \\ 0.48 \text{ GHz} \\ \hline 2.04 \text{ GHz} \end{array}$$

$$\begin{array}{l} f_1 = 2.16 \text{ GHz} \\ f_2 = 2.64 \text{ GHz} \end{array}$$

$$\begin{array}{l} \omega_c = 2\pi \times 2.4 \times 10^9 \\ \omega_c = 15.07 \times 10^9 \text{ rad/s} \end{array}$$

$$\omega_1 = 2\pi \times 2.16 \times 10^9 = 13.5716 \times 10^9 \text{ rad/s}$$

$$\omega_2 = 2\pi \times 2.64 \times 10^9 = 16.5876 \times 10^9 \text{ rad/s}$$

$$\begin{aligned} \omega_0 &= \sqrt{\omega_1 \omega_2} = 15.004 \times 10^9 \text{ rad/s} \\ &= 15 \times 10^9 \text{ rad/s.} \end{aligned}$$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

$$\Delta = \frac{(16.5876 - 13.5716) \times 10^9}{15 \times 10^9}$$

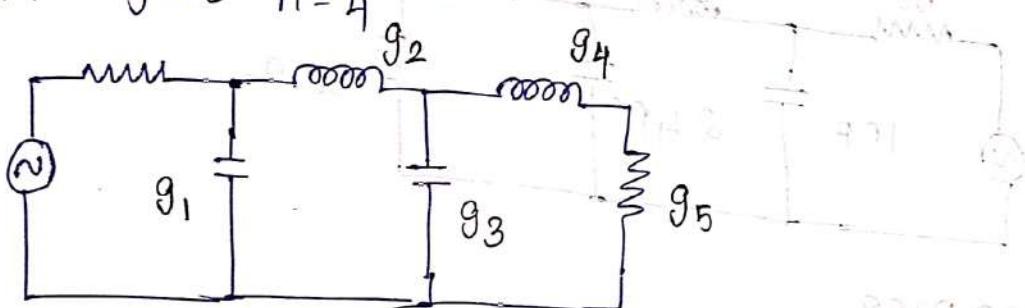
$$\boxed{\Delta = 0.801}$$

i) Butterworth filter:

low pass:

$$L_K^1 = \frac{R_o L_K}{\omega_c} \quad C_K^1 = \frac{C_K}{R_o \omega_c}$$

HPF for $n=4$



$$g_1 = 0.7654 \quad g_5 = 1$$

$$g_2 = 1.8478$$

$$g_3 = 1.8478$$

$$g_4 = 0.7654$$

For g_1

$$C_K^1 = \frac{C_K}{R_o \omega_c} = \frac{0.7654}{50 \times 2\pi \times 0.4 \times 10^9}$$

$$C_K^1 = 1.015 \times 10^{-12} F$$

For g_2

$$L_K^1 = \frac{R_o L_K}{\omega_c} = \frac{50 \times 1.8478}{2\pi \times 0.4 \times 10^9}$$

$$= 6.126 \text{ nH}$$

Part 93

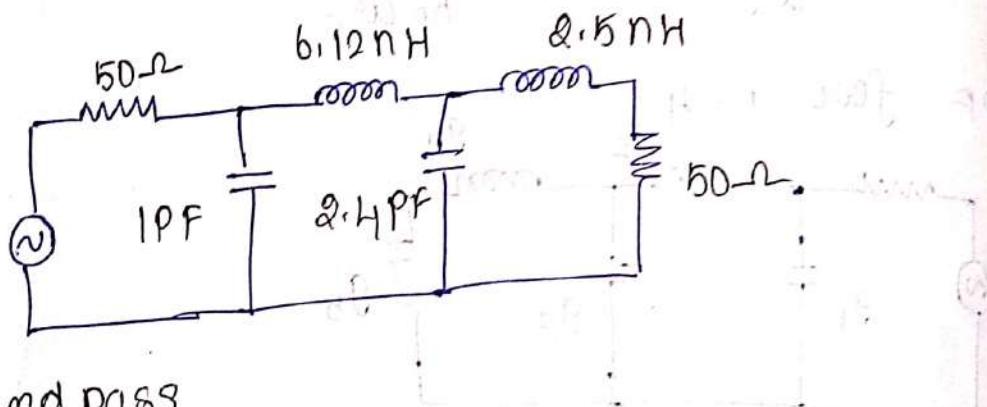
$$C_K' = \frac{C_K}{R_0 w_c} = \frac{1.8478}{50 \times 2\pi \times 0.4 \times 10^9}$$

$$C_K' = 0.4507 \text{ pF}$$

Part 94

$$L_K' = \frac{R_0 L_K}{w_c} = \frac{50 \times 0.7654}{2\pi \times 2.4 \times 10^9}$$

$$L_K' = 2.5378 \text{ nH}$$



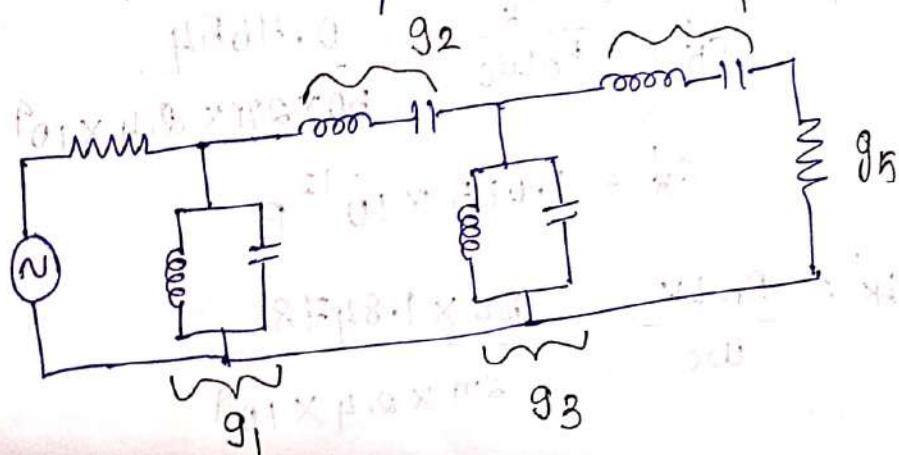
Band pass

$$g_1 = 0.7654$$

$$g_2 = 1.8478 \quad g_5 = 1$$

$$g_3 = 1.8478$$

$$g_4 = 0.7654$$



series

$$L_K' = \frac{L_K R_0}{\Delta w_0}$$

$$L_K' = \frac{\Delta R_0}{w_0 C_K}$$

$$C_K' = \frac{A}{R_0 w_0 L_K}$$

$$C_K' = \frac{C_K}{\Delta w_0 R_0}$$

Shunt (g₁)

$$L_K' = \frac{\Delta R_0}{w_0 C_K} = \frac{0.201 \times 50}{2\pi \times 90 \times 2.4 \times 10^9 \times 0.7654}$$

$$L_K' = 0.8704 \text{ nH}$$

$$C_K' = \frac{C_K}{R_0 w_0} = \frac{100 \times 0.7654}{0.201 \times 2\pi \times 10^9 \times 2.4 \times 10^9 \times 50}$$

$$C_K' = 5.050 \text{ pF}$$

Series (g₂)

$$L_K' = \frac{L_K R_0}{\Delta w_0} = \frac{1.8478 \times 50}{0.201 \times 2\pi \times 2.4 \times 10^9}$$

$$L_K' = 30.481 \text{ nH}$$

$$C_K' = \frac{A}{R_0 w_0 L_K} = \frac{0.201}{50 \times 2\pi \times 2.4 \times 10^9 \times 1.8478}$$

$$C_K' = 0.14427 \text{ pF}$$

Shunt (g₃)

$$L_K' = \frac{\Delta R_0}{w_0 C_K} = \frac{0.201 \times 50}{2\pi \times 2.4 \times 10^9 \times 1.8478}$$

$$L_K' = 0.3606 \text{ nH}$$

$$C_K = \frac{C_K}{\Delta W_0 R_0}$$

$$C_K = \frac{1.8478}{0.201 \times 2\pi \times 2.4 \times 10^9 \times 50}$$

$$C_K = 12.1926 \text{ pF}$$

Series (g4)

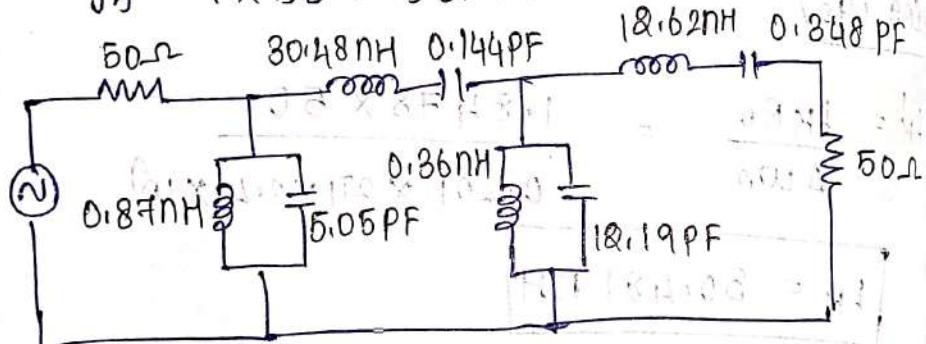
$$L_K = \frac{L_K R_0}{\Delta W_0} = \frac{0.7654 \times 50}{0.201 \times 2\pi \times 2.4 \times 10^9}$$

$$L_K = 12.6261 \text{ nH}$$

$$C_K = \frac{\Delta}{R_0 W_0 L_K} = \frac{0.201}{50 \times 2\pi \times 10^9 \times 2.4 \times 10^9 \times 0.765}$$

$$C_K = 0.3482 \text{ pF}$$

$$g_5 = 1 \times 50 = 50 \Omega$$



ii) chebyshev filter: (0.5 dB ripple)

$$g_1 = 1.6703$$

$$g_2 = 1.1926$$

$$g_3 = 2.3661$$

$$g_4 = 0.8419$$

$$g_5 = 1.9841$$

shunt (g1)

$$L_K^1 = \frac{\Delta R_0}{\omega_0 C_K} = \frac{0.201 \times 50}{2\pi \times 2.4 \times 10^9 \times 1.6703} = 0.899 \text{ nH}$$

$$C_K^1 = \frac{C_K}{\Delta \omega_0 R_0} = \frac{1.6703}{0.201 \times 2\pi \times 2.4 \times 10^9 \times 50} = 11.02 \text{ pF}$$

series (g2)

$$L_K^1 = \frac{L_K R_0}{\Delta \omega_0} = \frac{1.1926 \times 50}{2\pi \times 2.4 \times 10^9 \times 0.201} = 19.67 \text{ nH}$$

$$C_K^1 = \frac{\Delta}{R_0 \omega_0 L_K} = \frac{0.201}{50 \times 2\pi \times 2.4 \times 10^9 \times 1.1926} = 0.82 \text{ pF}$$

shunt (g3)

$$L_K^1 = \frac{\Delta R_0}{\omega_0 C_K} = \frac{0.201 \times 50}{2\pi \times 2.4 \times 10^9 \times 2.3661} = 0.281 \text{ nH}$$

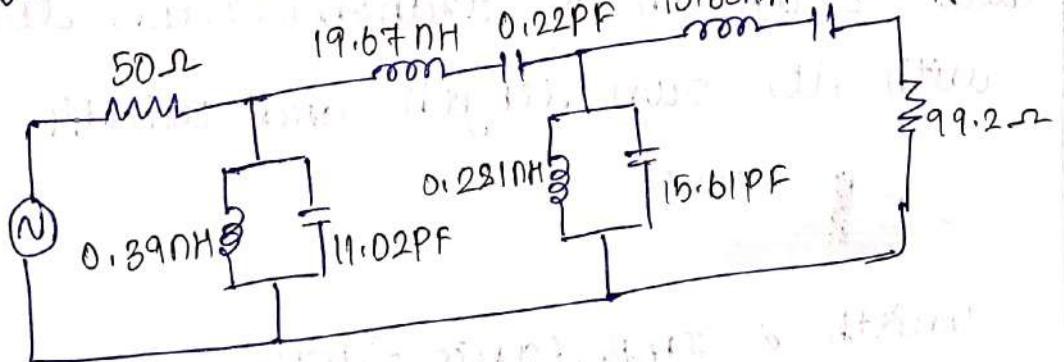
$$C_K^1 = \frac{C_K}{\Delta \omega_0 R_0} = \frac{2.3661}{0.201 \times 2\pi \times 2.4 \times 10^9 \times 50} = 15.61 \text{ pF}$$

series (g4)

$$L_K^1 = \frac{L_K R_0}{\Delta \omega_0} = \frac{0.8419 \times 50}{0.201 \times 2\pi \times 2.4 \times 10^9} = 13.88 \text{ nH}$$

$$C_K^1 = \frac{C_K}{R_0 \omega_0 L_K} = \frac{0.201}{50 \times 2\pi \times 2.4 \times 10^9 \times 0.8419} = 0.316 \text{ pF}$$

$$g_5 = 1.9841 \times 50 = 99.205 \Omega$$



25.10.25

- Limited range of lumped components
- connection line: frequency variation

To overcome this we go for filter implementation

→ Richard's transformation

→ Kuroda's transformation

→ Translating $\omega \rightarrow \omega_2$ (ohm)

$$j\omega_2 = j\tan(\beta L)$$

$$= j \tan\left(\frac{\omega}{V_p} L\right)$$

$$jX_L = j\omega L = j\omega_2 L = L \tan(\beta L)$$

$$j\beta c = j\omega c = j\omega_2 c = c \tan(\beta L)$$

$L \tan(\beta L) \rightarrow$ representation of short

circuit stub, impedance $L \tan(\beta L)$

$c \Rightarrow$ open circuited stub, impedance $\frac{1}{L}$

stub: Extension of transmission line

with its own length and width



width & impedance factor

Length depends on frequency

Let $\omega_c = 1$ | Impedance = 1

$$\tan \phi = -\frac{Z}{1} = -1$$

$$\phi = \tan^{-1}(-1) = \pi/4$$

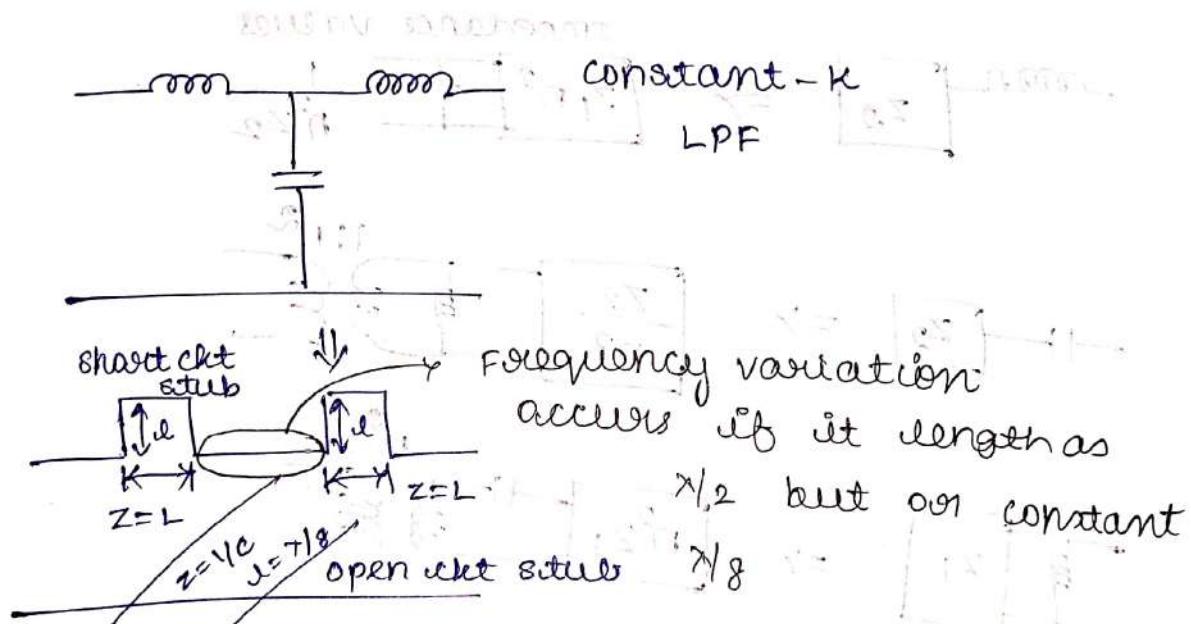
$$d = \frac{\pi/4}{\beta}$$

$$d = \frac{\pi/4}{8\pi/\lambda} \Rightarrow \frac{\pi/4}{8\pi} \times \frac{\lambda}{2\pi}$$

$$d = \frac{\lambda}{8} = \boxed{\text{Normalised Impedance}}$$

Repeats every $4\omega_c$.

frequency changes, in terms of λ
it varies (length varies)



Kuroda's transformation:

- physically separate Tx line stubs
- Transform series stub to shunt stub and shunt stub to series stub

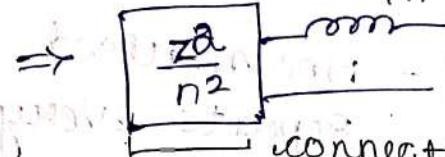
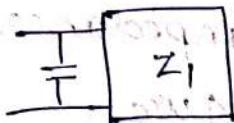
→ change impractical characteristic impedance into more realistic

1)



physical separation

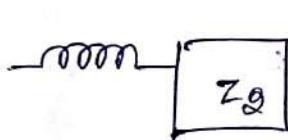
2)



connecting line $\frac{Z_2}{n^2}$

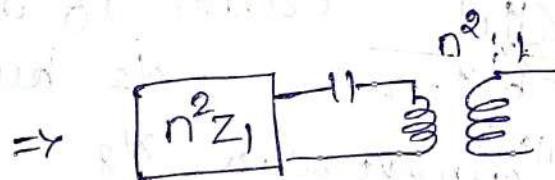
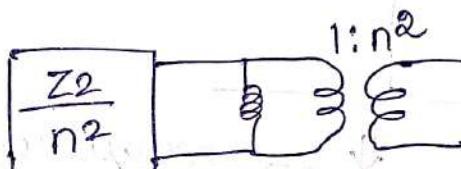
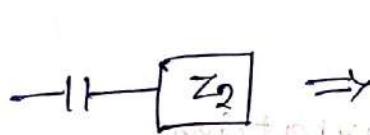
$$n^2 = 1 + \frac{Z_2}{Z_1}$$

can nullify frequency variations

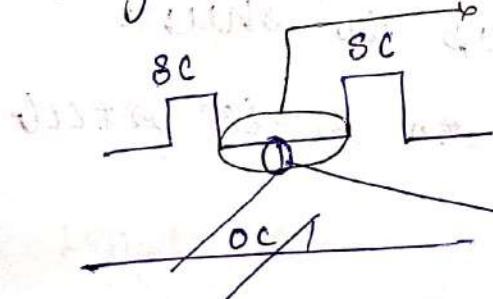


Impedance values

$$\frac{Z_1 n^2}{Z_2} = \frac{1}{n^2 Z_2}$$



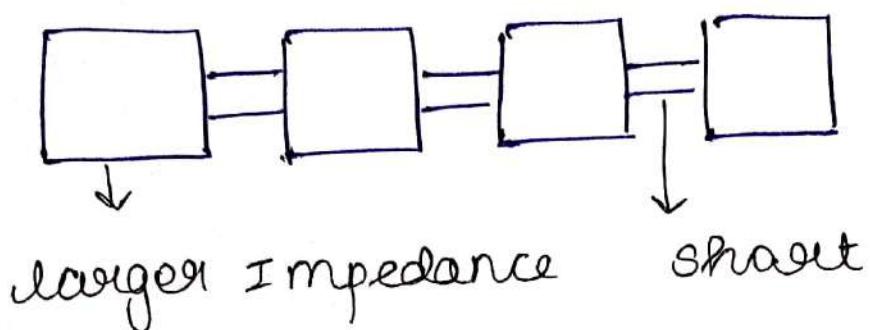
Take impedance value and compute
length between impedances.



→ series impedance check
2nd one impedance
 $Z_1 n^2$ and compute L

→ for shunt capacitor
take Z_2 / n^2

stepped Impedance:



Higher Impedance = L

smaller Impedance = C

$$\beta l = \frac{L R_0}{Z_0}$$

$$\beta l = \frac{C Z_L}{R_0}$$

Coupled line BP filter design.