28.08.25

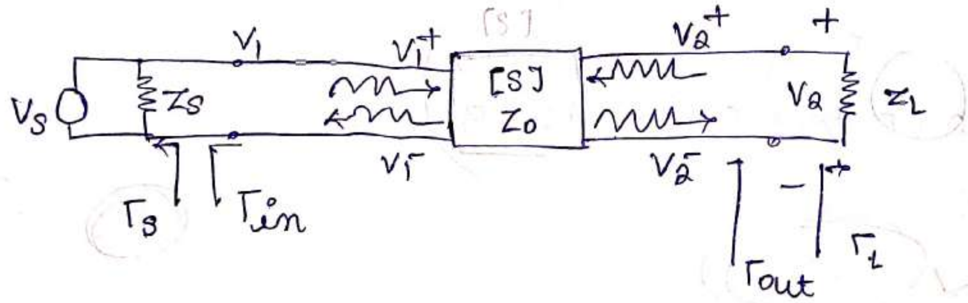
UNIT – II

Microwave Amplifiers and oscillators

1. power Gain
2. Availability Gain (mismatch at i/p change in input side only i/p side)
3. Transducer Gain (Accounts mismatch in both end Tx & Rx)

↓

Design we go with this gain.



For proper match

$\Gamma_{out} = \Gamma_L$

→ Let system is idle → power Gain
o/p is perfectly matched
→ only Zs → Available power gain
change in V_1

→ Accounting Zs and ZL

power Gain:

$$G = \frac{P_L}{P_{in}}$$

power dissipated in load Z_L to power delivered to input of 2 port network. Gain is independant of Z_S and Z_L

Available power Gain

$$G_A = \frac{P_{avn}}{P_{avs}}$$

Ratio of power available from 2 port network to power available from the source. This assumes conjugate matching of both the source and load depends on Z_S and not Z_L

Transducer power gain:

$G_T = P_L / P_{avs}$ is the ratio of power delivered to load to power available from source. This depends on both Z_S and Z_L

Reflection coefficient seen looking toward load

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Reflection coefficient seen looking towards source

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

If any mismatch

$$V_2^+ = \Gamma_L V_2^-$$

In source

$$V_1^+ = \Gamma_S V_1^-$$

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+ = S_{11} V_1^+ + S_{12} \Gamma_L V_2^- \quad \text{—①}$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+ = S_{21} V_1^+ + S_{22} \Gamma_L V_2^- \quad \text{—②}$$

To find

$$\Gamma_{in} = \frac{V_1^-}{V_1^+}$$

From ① $V_1^- = S_{11} V_1^+ + S_{12} V_2^+ = S_{11} V_1^+ + S_{12} \Gamma_L V_2^-$

Sub ② in ① we get

$$V_1^- = S_{11} V_1^+ + S_{12} \Gamma_L (S_{21} V_1^+ + S_{22} \Gamma_L V_2^-)$$

$$V_1^- = S_{11} V_1^+ + S_{12} \Gamma_L S_{21} V_1^+ + S_{12} S_{22} \Gamma_L^2 V_2^-$$

$$V_1^- = V_1^+ (S_{11} + S_{12} S_{21} \Gamma_L) + S_{12} S_{22} \Gamma_L^2 V_2^-$$

From ②

$$V_2^- = S_{21} V_1^+ + S_{22} \Gamma_L V_2^-$$

$$V_2^- (1 - S_{22} \Gamma_L) = S_{21} V_1^+$$

$$\boxed{V_2^- = \frac{S_{21} V_1^+}{1 - S_{22} \Gamma_L}} \quad \text{— ③}$$

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Sub ③ in ①

$$V_1^- = S_{11} V_1^+ + S_{12} \Gamma_L \left(\frac{S_{21} V_1^+}{1 - S_{22}\Gamma_L} \right)$$

$$V_1^- = S_{11} V_1^+ + \frac{S_{21} S_{12} V_1^+ \Gamma_L}{1 - S_{22}\Gamma_L}$$

$$V_1^- = \left(S_{11} + \frac{S_{21} S_{12} \Gamma_L}{1 - S_{22}\Gamma_L} \right) V_1^+$$

$$\boxed{\frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{21} S_{12} \Gamma_L}{1 - S_{22}\Gamma_L}}$$

$$\Gamma_{in} = S_{11} + \frac{S_{21} S_{12} \Gamma_L}{1 - S_{22}\Gamma_L} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

IIIly

$$\Gamma_{out} = \frac{V_2^-}{V_2^+}$$

In ①

$$S_{11} \Gamma_S V_1^- + S_{12} V_2^+ = V_1^-$$

$$V_1^- (1 - S_{11}\Gamma_S) = S_{12} V_2^+$$

$$V_1^- = \left(\frac{S_{12}}{1 - S_{11}\Gamma_S} \right) V_2^+ \quad \text{——— ④}$$

Now in ②

$$V_2^- = S_{21} \Gamma_S V_1^- + S_{22} V_2^+ \quad \text{——— ⑤}$$

Add ④ in ⑤

$$V_2^- = S_{21}\Gamma_S\left(\frac{S_{12}}{1-S_{11}\Gamma_S}\right)V_2^+ + S_{22}V_2^+$$

$$\frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1-S_{11}\Gamma_S}$$

$$\boxed{\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}}$$

At port 1, the total voltage is

$$V_1 = V_1^+ + V_1^-$$

But we know that

$$\Gamma_{in} = \frac{V_1^-}{V_1^+}$$

$V_1^- = \Gamma_{in}V_1^+$ hence

$$V_1 = V_1^+ + \Gamma_{in}V_1^+ = V_1^+(1+\Gamma_{in})$$

Looking at source side the source V_S is divided between Z_S and Z_{in} hence

$$V_1 = V_S \cdot \frac{Z_{in}}{Z_{in}+Z_S}$$

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We also know that

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

Multiply both sides by $Z_{in} + Z_0$

$$\Gamma_{in}(Z_{in} + Z_0) = Z_{in} - Z_0$$

$$\Gamma_{in} Z_{in} + \Gamma_{in} Z_0 = Z_{in} - Z_0$$

$$(\Gamma_{in} - 1) Z_{in} = -(1 + \Gamma_{in}) Z_0$$

Multiply by -1 and solve Z_{in}

$$(1 - \Gamma_{in}) Z_{in} = (1 + \Gamma_{in}) Z_0$$

$$Z_{in} = Z_0 \left(\frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} \right)$$

so

$$V_1 = V_S \frac{Z_{in}}{Z_S + Z_{in}} = V_1^+ + V_1^- = V_1^+ (1 + \Gamma_{in})$$

Now;

$$V_S \cdot \frac{Z_{in}}{Z_S + Z_{in}} = V_1^+ (1 + \Gamma_{in})$$

$$V_1^+ = \frac{V_S \cdot \frac{Z_{in}}{Z_S + Z_{in}}}{1 + \Gamma_{in}} \quad \text{--- (6)}$$

we also know that

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \Rightarrow Z_{in} = Z_0 \left(\frac{1+\Gamma_{in}}{1-\Gamma_{in}}\right) \quad \text{⑦}$$

same for source

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \Rightarrow Z_S = Z_0 \left(\frac{1+\Gamma_S}{1-\Gamma_S}\right) \quad \text{⑧}$$

sub ⑦ & ⑧ in ⑥

$$V_1^+ = \frac{V_S \cdot \frac{Z_{in}}{Z_S + Z_{in}}}{1+\Gamma_{in}}$$

$$V_1^+ = \frac{V_S \cdot Z_0 \left(\frac{1+\Gamma_{in}}{1-\Gamma_{in}}\right) / Z_S + Z_{in}}{1+\Gamma_{in}}$$

$$V_1^+ = \frac{V_S \cdot \left(\frac{Z_0\left(\frac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)}{Z_0\left(\frac{1+\Gamma_S}{1-\Gamma_S}\right) + Z_0\left(\frac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)}\right)}{1+\Gamma_{in}}$$

$$V_1^+ = \frac{V_S \cdot \left(\frac{Z_0\left(\frac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)}{Z_0\left[\left(\frac{1+\Gamma_S}{1-\Gamma_S}\right) + \left(\frac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)\right]}\right)}{1+\Gamma_{in}}$$

$$V_1^+ = \frac{V_S \cdot \left[\dfrac{\left(\dfrac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)}{\left(\dfrac{1+\Gamma_S}{1-\Gamma_S} + \dfrac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)}\right]}{1+\Gamma_{in}}$$

$$V_1^+ = \frac{V_S}{1+\Gamma_{in}} \left[\frac{\left(\dfrac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)}{\left(\dfrac{1+\Gamma_S}{1-\Gamma_S}\right) + \left(\dfrac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)}\right] \quad \text{— (9)}$$

Multiply (9) with $(1-\Gamma_S)(1-\Gamma_{in})$ on both N^r & D^r

$$V_1^+ = \frac{V_S}{1+\Gamma_{in}} \left[\frac{\left(\dfrac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)(1-\Gamma_S)(1-\Gamma_{in})}{\left[\left(\dfrac{1+\Gamma_S}{1-\Gamma_S}\right) + \left(\dfrac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)\right](1-\Gamma_S)(1-\Gamma_{in})}\right]$$

$$V_1^+ = \frac{V_S}{1+\Gamma_{in}} \left[\frac{(1+\Gamma_{in})(1-\Gamma_S)}{(1+\Gamma_S)(1-\Gamma_{in}) + (1+\Gamma_{in})(1-\Gamma_S)}\right]$$

$$V_1^+ = V_S \left[\frac{1-\Gamma_S}{(1+\Gamma_S)(1-\Gamma_{in}) + (1+\Gamma_{in})(1-\Gamma_S)}\right]$$

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① Expanding denr

$$(1+\Gamma_s)(1-\Gamma_{in}) = (1-\Gamma_{in}) + \Gamma_s(1-\Gamma_{in})$$

$$= 1-\Gamma_{in} + \Gamma_s(1-\Gamma_{in})$$

②

$$(1+\Gamma_{in})(1-\Gamma_s) = (1-\Gamma_s) + \Gamma_{in}(1-\Gamma_s)$$

Adding Both

$$(1-\Gamma_{in}) + \Gamma_s(1-\Gamma_{in}) + (1-\Gamma_s) + \Gamma_{in}(1-\Gamma_s)$$

$$1 - \cancel{\Gamma_{in}} + \cancel{\Gamma_s} - \Gamma_s\Gamma_{in} + 1 - \cancel{\Gamma_s} + \cancel{\Gamma_{in}} - \Gamma_{in}\Gamma_s$$

$$2 - 2\Gamma_s\Gamma_{in} = 2(1-\Gamma_s\Gamma_{in}) \quad \text{— ⑩}$$

Sub ⑩ in V_1^+

$$V_1^+ = V_s\left[\frac{1-\Gamma_s}{2(1-\Gamma_s\Gamma_{in})}\right]$$

$$\boxed{V_1^+ = \frac{V_s}{2}\,\frac{1-\Gamma_s}{1-\Gamma_s\Gamma_{in}}} \longrightarrow ⑪$$

By poynting's theorem

$$P_{in} = \frac{1}{2Z_0}|V_1^+|^2(1-|\Gamma_{in}|^2)$$

sub ⑪ here

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$$P_{in} = \frac{|V_S|^2}{8Z_0} \frac{|1-\Gamma_S|^2}{|1-\Gamma_S \Gamma_{in}|^2} (1-|\Gamma_{in}|^2)$$

By poynting's theorem

$$P_L = \frac{|V_2^-|^2}{2Z_0} (1-|\Gamma_L|^2)$$

From (2)

$$V_2^- = S_{21} V_1^+ + S_{22} \Gamma_L V_2^-$$

$$V_2^- (1 - S_{22}\Gamma_L) = S_{21} V_1^+$$

$$\boxed{V_2^- = \frac{S_{21} V_1^+}{1 - S_{22}\Gamma_L}}$$

$$|V_2^-|^2 = \frac{|S_{21}|^2 |V_1^+|^2}{|1-S_{22}\Gamma_L|^2}$$

$$P_L = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 (1-|\Gamma_L|^2) |1-\Gamma_S|^2}{|1-S_{22}\Gamma_L|^2 |1-\Gamma_S\Gamma_{in}|^2}$$

$$G = \frac{P_L}{P_{in}} = \frac{\dfrac{|V_S|^2}{8Z_0} \dfrac{|S_{21}|^2 (1-|\Gamma_L|)^2 (1-\Gamma_S)^2}{|1-S_{22}\Gamma_L|^2 |1-\Gamma_S\Gamma_{in}|^2}}{\dfrac{|V_S|^2}{8Z_0} \dfrac{|1-\Gamma_S|^2 (1-|\Gamma_{in}|^2)}{|1-\Gamma_S\Gamma_{in}|^2}}$$

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$$G_1 = \frac{|\cancel{V_S}|^2}{8\cancel{Z_0}} \frac{|S_{21}|^2(1-|\Gamma_L|^2)(1-\cancel{\Gamma_S})^2}{|1-S_{22}\Gamma_L|^2 \, |1-\Gamma_S \cancel{\Gamma_{in}}|^2}$$

$$\times \frac{8\cancel{Z_0}}{|\cancel{V_S}|^2} \frac{(1-\Gamma_S \cancel{\Gamma_{in}})^2}{(1-\cancel{\Gamma_S})^2(1-|\Gamma_{in}|)^2}$$

$$\boxed{G_1 = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)(1-S_{22}\Gamma_L)^2}} \quad \text{— (12)}$$

proper match, power Gain.

Available power Gain;

Input side matched, thro complex conjugate match

$$P_{in} = \frac{1}{2Z_0}|V_1^+|^2(1-|\Gamma_{in}|^2)$$

$$= \frac{|V_S|^2}{8Z_0} \frac{|1-\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}(1-|\Gamma_{in}|^2)$$

$$P_{avs} = P_{in}\Big|_{\Gamma_{in}=\Gamma_S^*}$$

$$= \frac{|V_S|^2}{8Z_0} \frac{|1-\Gamma_S|^2}{|1-\Gamma_S\Gamma_S^*|^2}(1-|\Gamma_S^*|^2)$$

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Available power Gain

$$G_A = \frac{P_{avn}}{P_{avs}}$$

$$G_T = \frac{P_L}{P_{avs}}$$

$\Gamma_{in} = \Gamma_s^*$

$\Gamma_L = \Gamma_{out}^*$

only Real component remove complex

$$\boxed{P_{avs} = \frac{|V_s|^2}{8Z_0} \frac{|1-\Gamma_s|^2}{(1-|\Gamma_s|^2)}} \quad \text{—— (13)}$$

power available in the network

P_{avn}, maximum power delivered to load

$$P_L = \frac{|V_1^+|^2}{2Z_0} \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2} (1-|\Gamma_L|^2)$$

$$P_L = \frac{|V_s|^2}{8Z_0} \frac{|S_{21}|^2 (1-|\Gamma_L|^2)(1-\Gamma_s|^2}{|1-S_{22}\Gamma_L|^2 |1-\Gamma_s \Gamma_{in}|^2} \quad \text{—— (13.a)}$$

$$P_{avn} = P_L \Big|_{\Gamma_L = \Gamma_{out}^*}$$

we know that

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L} \quad \text{—— (13.b)}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$$

$$(1-\Gamma_S \Gamma_{in}) = 1 - \Gamma_S \left(S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L} \right)$$

$$= \frac{(1-S_{22}\Gamma_L) - \Gamma_S \left[(S_{11}(1-S_{22}\Gamma_L)) + S_{12} S_{21} \Gamma_L \right]}{1 - S_{22}\Gamma_L}$$

$$= \frac{(1-S_{22}\Gamma_L) - S_{11}\Gamma_S (1-S_{22}\Gamma_L) - S_{12} S_{21} \Gamma_S \Gamma_L}{1 - S_{22}\Gamma_L}$$

$$= \frac{(1-S_{22}\Gamma_L)\left[1 - S_{11}\Gamma_S\right] - S_{12} S_{21} \Gamma_S \Gamma_L}{1 - S_{22}\Gamma_L} \quad \text{--- (14)}$$

WKT $\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$

$$\Gamma_{out} = \frac{S_{22}(1-S_{11}\Gamma_S) + S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$$

$$\Gamma_{out}(1-S_{11}\Gamma_S) = S_{22}(1-S_{11}\Gamma_S) + S_{12} S_{21} \Gamma_S$$

$$S_{12} S_{21} \Gamma_S = (1-S_{11}\Gamma_S)\Gamma_{out} - S_{22}(1-S_{11}\Gamma_S) \quad \text{--- (15)}$$

sub (15) in (14)

$$\frac{(1-S_{22}\Gamma_L)(1-S_{11}\Gamma_S) - [(1-S_{11}\Gamma_S)\Gamma_{out} - S_{22}(1-S_{11}\Gamma_S)]\,\Gamma_L}{1-S_{22}\Gamma_L}$$

Factor of $(1-S_{11}\Gamma_S)$

$$\frac{(1-S_{11}\Gamma_S)\left[(1-S_{22}\Gamma_L) - \Gamma_{out}\Gamma_L + S_{22}\Gamma_L\right]}{1-S_{22}\Gamma_L}$$

$$\frac{(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L - \Gamma_{out}\Gamma_L + S_{22}\Gamma_L)}{1-S_{22}\Gamma_L}$$

$$\frac{(1-S_{11}\Gamma_S)(1-\Gamma_{out}\Gamma_L)}{1-S_{22}\Gamma_L} = 1-\Gamma_S\Gamma_{in}$$

NOW $\left.1-\Gamma_S\Gamma_{in}\right|_{\Gamma_L=\Gamma_{out}^*}$

$$= \frac{(1-S_{11}\Gamma_S)(1-\Gamma_{out}\Gamma_{out}^*)}{1-S_{22}\Gamma_{out}^*}$$

$$= \frac{(1-S_{11}\Gamma_S)(1-|\Gamma_{out}|^2)}{1-S_{22}\Gamma_{out}^*}$$

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$$|1-\Gamma_S\Gamma_{in}|^2 = \frac{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)^2}{|1-S_{22}\Gamma_{out}^*|^2} \quad \text{— (16)}$$

sub (16) in (13.a) and $\Gamma_L = \Gamma_{out}^*$

$$P_L = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_{out}|^2)(1-\Gamma_S|^2}{\cancel{|1-S_{22}\Gamma_{out}^*|^2}\cdot\frac{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)^2}{\cancel{|1-S_{22}\Gamma_{out}^*|^2}}}$$

$$P_{avn} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_{out}|^2)(1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2\cdot(1-|\Gamma_{out}|^2)^{\cancel{2}}}$$

$$\boxed{P_{avn} = \frac{|V_S|^2|S_{21}|^2|1-\Gamma_S|^2}{8Z_0|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)}} \quad \text{— (17)}$$

Available power G_A

$$G_A = \frac{P_{avn}}{P_{avs}}$$

$$= \frac{\cancel{|V_S|^2}|S_{21}|^2|1-\cancel{\Gamma_S}|^2}{\cancel{8Z_0}|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)} \times \frac{\cancel{8Z_0}(1-|\Gamma_S|^2)}{|\cancel{V_S}|^2|1-\cancel{\Gamma_S}|^2}$$

$$\boxed{G_A = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2 (1-|\Gamma_{out}|^2)}} \quad \text{(18)}$$

Transducer power Gain G_T

$$G_T = \frac{P_L}{P_{avs}}$$

$$= \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 (1-|\Gamma_L|^2)(1-|\Gamma_S|)^2}{|1-S_{22}\Gamma_L|^2 (1-\Gamma_S\Gamma_{in})^2} \times \frac{8Z_0 (1-|\Gamma_S|^2)}{|V_S|^2 |1-\Gamma_S|^2}$$

$$\boxed{G_T = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2 |1-S_{22}\Gamma_L|^2}} \quad \text{(19)}$$

A special case of G_T when both out and input are matched for zero reflection (in contrast to conjugate matching) then $\Gamma_L = \Gamma_S = 0$ hence

$$G_T = |S_{21}|^2$$

Another case is unilateral Transducer power gain G_{TU}, where $S_{12} = 0$

From (13.b) $\Gamma_{in} = S_{11}$ when $S_{12} = 0$

hence in (19)

$$\boxed{G_{TU} = \frac{|S_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - S_{11}\Gamma_S|^2 \, |1 - S_{22}\Gamma_L|^2}} \quad \text{--- (20)}$$

1. For $Z_0 = 50\,\Omega$ the operating freq. of x^r is 1GHZ and S parameters are

$$S_{11} = 0.5 \angle{-100^\circ}$$

$$S_{12} = 0.25 \angle 80^\circ$$

$$S_{21} = 5 \angle 70^\circ$$

$$S_{22} = 0.5 \angle{-25^\circ}$$

Find power Gain, Available power gain and Transducer power Gain where source impedance is $25\,\Omega$ and load impedance is $40\,\Omega$.

Soln

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{25 - 50}{25 + 50} = -0.333$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{40 - 50}{40 + 50} = -0.111$$

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}$$

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$$\Gamma_{in} = \left((0.5 \angle -100^\circ) + \left(\frac{(0.25 \angle 30^\circ)(5 \angle 70^\circ)(-0.111)}{(1 - ((0.5 \angle -25^\circ)(-0.111)))}\right)\right)$$

$$= -0.06099 - 0.6219j$$

$$\boxed{\Gamma_{in} = 0.62490 \angle -95.601}$$

$$\Gamma_{out} = \frac{S_{22} + S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

$$\Gamma_{out} = \left((0.5 \angle -25^\circ) + \left(\frac{(0.25 \angle 30^\circ)(5 \angle 70^\circ)(-0.333)}{(1 - ((0.5 \angle -100^\circ)(-0.111)))}\right)\right)$$

$$= 0.5486 - 0.61995j$$

$$= 0.8278 \angle -48.489$$

$$\boxed{\Gamma_{out} = 0.8516 \angle -45.698}$$

The power Gain

$$G = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2)|1 - S_{22}\Gamma_L|^2}$$

$$S_{G} = (0.25)^2 = 0.0625$$

$$S_{21} = (5)^2 = 25$$

$$G = \frac{25 \times 0.987679}{0.60949 \times (1.05056)^2}$$

$$G = \frac{25 \times 0.987679}{0.60949 \times 1.10368}$$

$$= \frac{24.6919}{0.672681}$$

$$\boxed{G = 36.7067}$$

$10 \log(36.7067) = 15.647\ dB$

Available power Gain:

$$G_A = \frac{|S_{21}|^2 (1 - |\Gamma_S|^2)}{|1 - S_{11}\Gamma_S|^2 (1 - |\Gamma_{out}|^2)}$$

$$= \frac{25 \times 0.889111}{0.96989 \times 0.27477}$$

$$= \frac{22.2277}{0.266496}$$

$$\boxed{G_A = 83.4072}$$

$10 \log(83.4072) = 19.212\ dB$

Transducer power Gain:

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2 |1-S_{22}\Gamma_L|^2}$$

$$= \frac{25 \times 0.889111 \times 0.987679}{1.002682 \times 1.103680}$$

$$= \frac{21.95390}{1.106640}$$

$\boxed{G_T = 19.838}$ $10 \log(19.833) = 13\ dB$.

01.09.25

stability:

→ Maintaining between i/p & o/p.

→ Based on i/p & o/p matching conditions we check for the Amplifier stability

→ System Gain should not vary based on input and output load ⇒ Amplifier design.

5 Tests

* K-Δ Test (Rollet's condition)
* stability circle - Smith chart
* μ-Test

stability in two ways

* unconditionally stable

$|\Gamma_{in}| < 1$ for any load $|\Gamma_L|$ $|\Gamma_S|$ $\Downarrow$ <1

* unconditionally stable (unstable)

$|\Gamma_{in}| < 1$ range of Γ_S & Γ_L
(Retaining Γ_S & Γ_L in range → stable else not)

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1$$

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1$$

If unilateral $S_{12} = S_{21} = 0$ (No Transition)

$|\Gamma_{in}| = S_{11}$

$|\Gamma_{out}| = S_{22}$

Plane of Γ_L & Γ_S ← Γ_{in} & Γ_{out} accounts on it

K - Δ Test :

For stable system

$$\Delta = S_{11}S_{22} - S_{12}S_{21} < 1$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1$$

If any one satisfies ⇒ marginally stable

(Aim)

* $K > 1$; $\Delta < 1$ ⇒ unconditionally stable

* $K < 1$; $\Delta < 1$ (or) $K > 1$; $\Delta > 1$

* $K < 1$; $\Delta > 1$ ⇒ unstable

→ Fix Bias
→ stability ⇒ check Noise figure

$S_{11} < 1$ & proper match at load ⇒ stable

Range of stability → S-circle

μ – Test :

$$\frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}|} > 1$$

$\mu > 1$ = stable

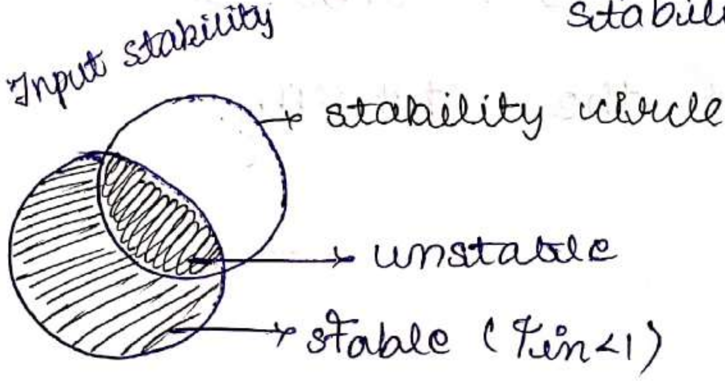
$\mu < 1$ = unstable.

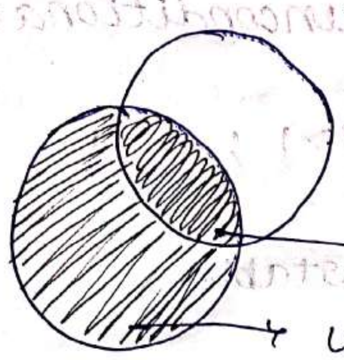
Stability circle :

center C_L ; Radius = R_L ⇒ output stability

center C_S ; Radius R_S ⇒ Input stability

$|S_{11}| < 1$





$$C_S = \frac{(S_{11} - \Delta S_{22}^{*})^{*}}{|S_{11}|^2 - |\Delta|^2}$$

$$R_S = \frac{|S_{12} S_{21}|}{|S_{11}|^2 - |\Delta|^2}$$

$$C_L = \frac{(S_{22} - \Delta S_{11}^{*})^{*}}{|S_{22}|^2 - |\Delta|^2}$$

$$R_L = \frac{|S_{12} S_{21}|}{|S_{22}|^2 - |\Delta|^2}$$

For $S_{11} = 0.869 \angle -159^\circ$ — A

$S_{12} = 0.031 \angle -9^\circ$ — B

$S_{21} = 4.250 \angle 61^\circ$ — C

$S_{22} = 0.507 \angle -117^\circ$ — M

d the stability.

→ cmplx mode

→ Type $0.869 \angle -159^\circ$ ⇒ $\underbrace{\text{Shift + RCL}}_{}$ press A. (storing)

Make others as B, C, D/M.

To call press Alpha + variable

<u>Soln</u>

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$= 0.336182 \angle 95.986$$

a) <u>K – Δ Test</u>

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

$$= \frac{1 - (0.869)^2 - (0.507)^2 + (0.336)^2}{2 \times 0.13175}$$

$$= \frac{1 - (0.869)^2 - (0.507)^2 + (0.336)^2}{0.2635}$$

$$= 0.3821$$

$K = 0.3821 < 1$

$\Delta = 0.336182 < 1$

conditionally stable

b) S- circle

I/p stability circle

$$C_S = \frac{(S_{11} - \Delta S_{22}^{*})^{*}}{|S_{11}|^2 - |\Delta|^2}$$

$$R_S = \frac{|S_{12} S_{21}|}{|S_{11}|^2 - |\Delta|^2}$$

O/p stability circle

$$C_L = \frac{(S_{22} - \Delta S_{11}^{*})^{*}}{|S_{22}|^2 - |\Delta|^2}$$

$$R_L = \frac{|S_{12} S_{21}|}{|S_{22}|^2 - |\Delta|^2}$$

I/P

$C_S = (1.094 \angle -162)^{*} = 1.09 \angle 162^\circ$

$R_S = 0.205$

O/p

$C_L = (1.592 \angle -132.3)^{*} = 1.592 \angle 132.3^\circ$

$R_L = 0.91395$

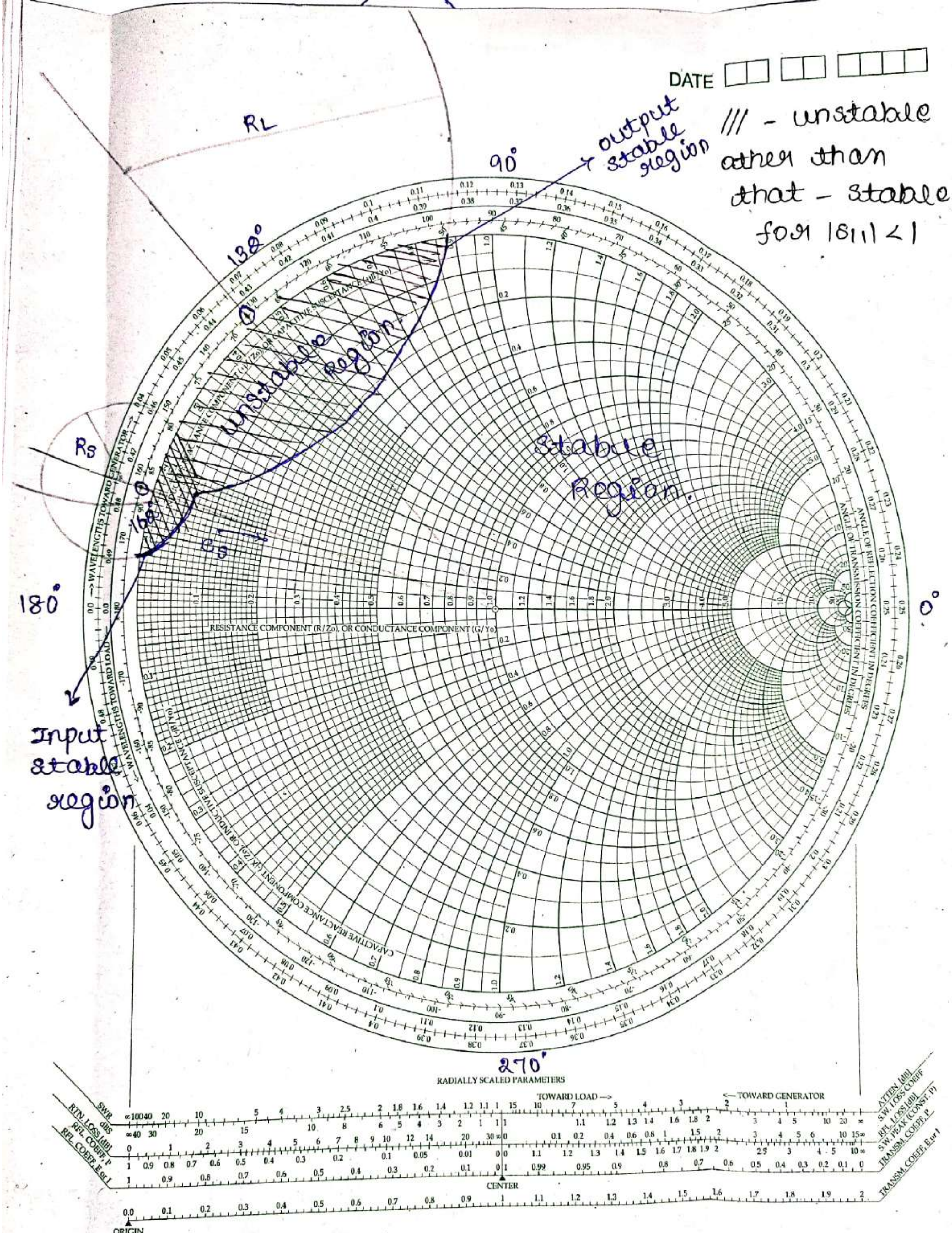
Basic radius = 7.6 cm.

CS $\Rightarrow$ $1.09 \times R = 1.09 \times 7.6 = 8.284$ cm

Rs $\Rightarrow$ $0.205 \times R = 0.205 \times 7.6 = 1.558$ cm

DATE

/// - unstable
other than
that - stable
for $|S_{11}| < 1$



Smith Chart

$G_L = 1.592 \times 7.6 = 12.0992$

$R_L = 0.9139 \times 7.6 = 6.94556$

c) μ - Test

$$= \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12} S_{21}|}$$

$$= \frac{1 - (0.869)^2}{0.2295 + 0.13175}$$

$$= \frac{0.244839}{0.36125}$$

$\mu = 0.6777$

$\mu < 1 \Rightarrow$ unstable

3. For $S_{11} = 1.02 \angle -159^\circ$

$S_{12} = 0.031 \angle -9^\circ$

$S_{21} = 4.250 \angle 61^\circ$

$S_{22} = 1.52 \angle -117^\circ$

check for stability using all 3 Tests.

$\Delta = S_{11} S_{22} - S_{12} S_{21}$

$\Delta = 1.4403 \angle 86.7783$

a) K - Δ Test

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12} S_{21}|}$$

$$K = \frac{1 - 1.0404 - 2.3104 + 2.07446}{2 \times 0.13175}$$

$K = -1.0487$

$K < 1 \quad \Delta > 1$ conditionally stable

μ-Test

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12} S_{21}|}$$

$$\mu = \frac{1 - (1.02)^2}{0.08854 + 0.13175}$$

$$= \frac{-0.0404}{0.22029}$$

$= -0.1833$

$\mu < 1$

conditionally stable

Stability circle:

I/p stability circle:

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}$$

$$C_S = (1.1332 \angle -154)^*$$

$$C_S = (1.1332 \angle -154)^* = 1.1332 \angle 154$$

$$R_S = \frac{|S_{12} S_{21}|}{|S_{11}|^2 - |\Delta|^2} = \frac{0.13175}{1.0340}$$

$$R_S = 0.12740$$

O/p stability circle

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}$$

$$C_L = (0.37528 \angle -170.5)^*$$

$$C_L = 0.37528 \angle 170.5$$

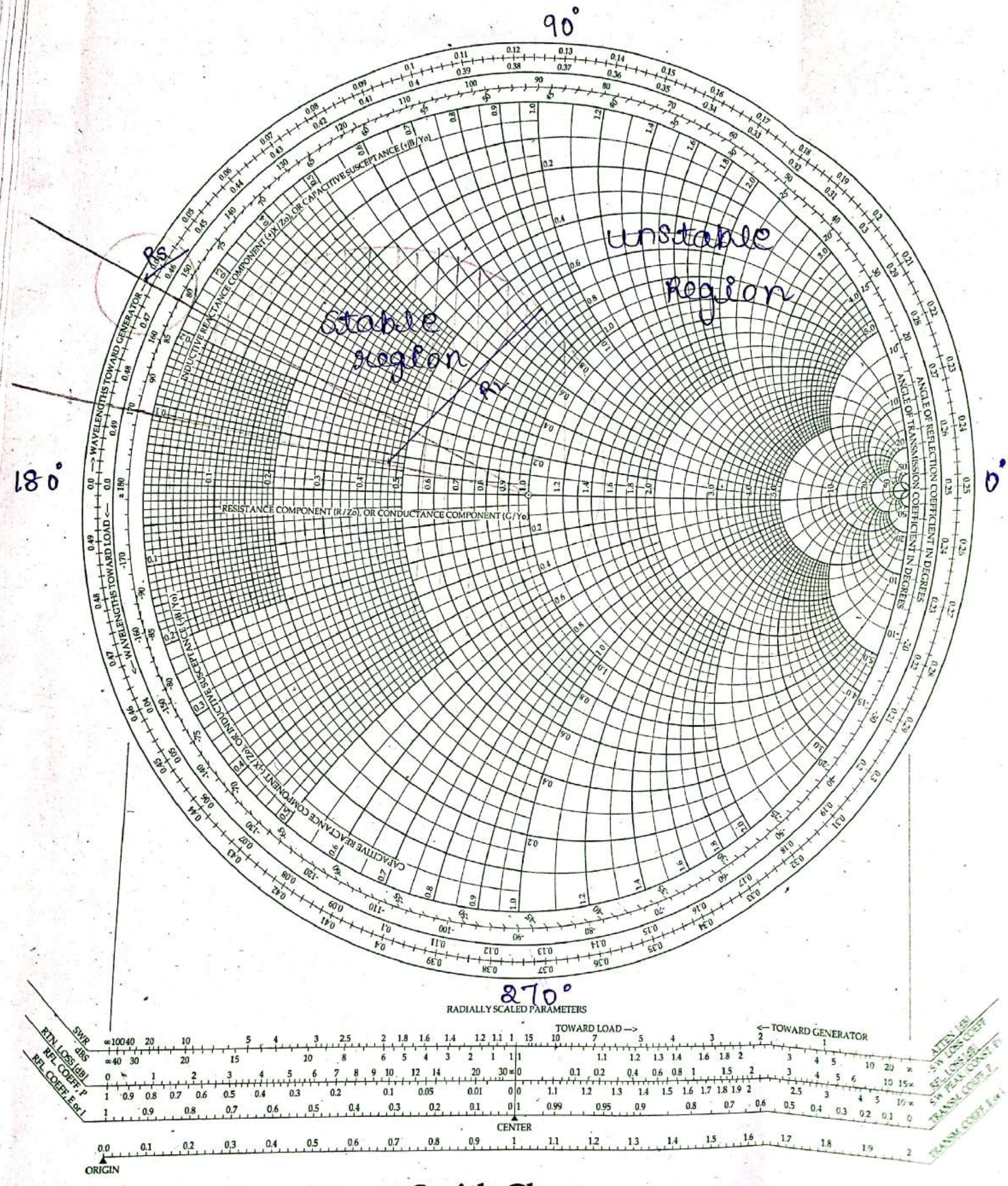
$$R_L = \frac{|S_{12} S_{21}|}{|S_{22}|^2 - |\Delta|^2} = \frac{0.13175}{0.2359}$$

$$R_L = 0.558414$$

$$C_S = 7.6 \times 1.132 = 8.6032$$
$$R_S = 7.6 \times 0.12740 = 0.96824$$

|zin| > 1

DATE



Smith Chart

$C_L = 7.6 \times 0.37528$

$C_L = 2.852128$

$R_L = 0.55841 \times 7.6$

$R_L = 4.2439$.

03.09.25

Higher the input impedance, higher the Gain.

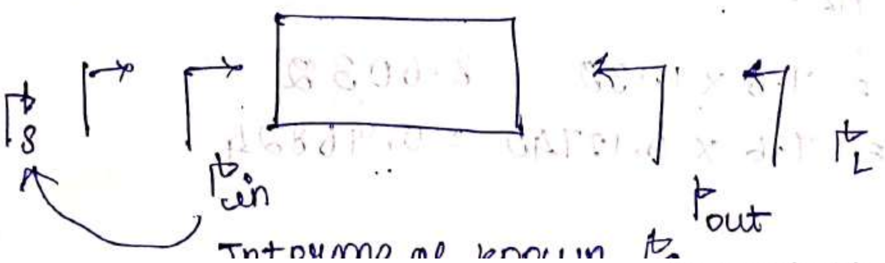
Imp for Design.

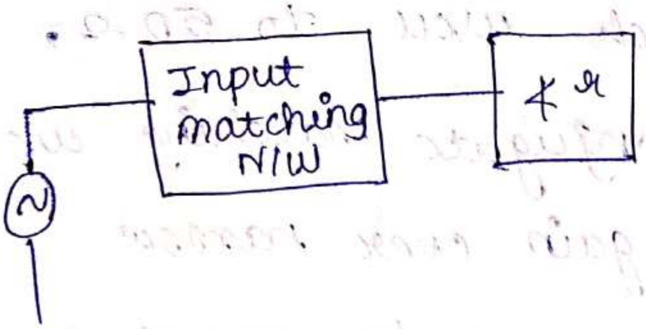
- stability of system.
- Gain of system
- To design proper matching at input and output, By that achieving Better Gain so we tend to match complex conjugate.

$$\boxed{\Gamma_S = \Gamma_{in}^{*}}$$

$$\Gamma_{in} = \Gamma_S^{*}$$

- calculate Γ_S $\quad \Gamma_S = \Gamma_{in}^{*}$.
- once knowing Γ_S and Γ_L we can design.





single stage Transistor Amplifier Design

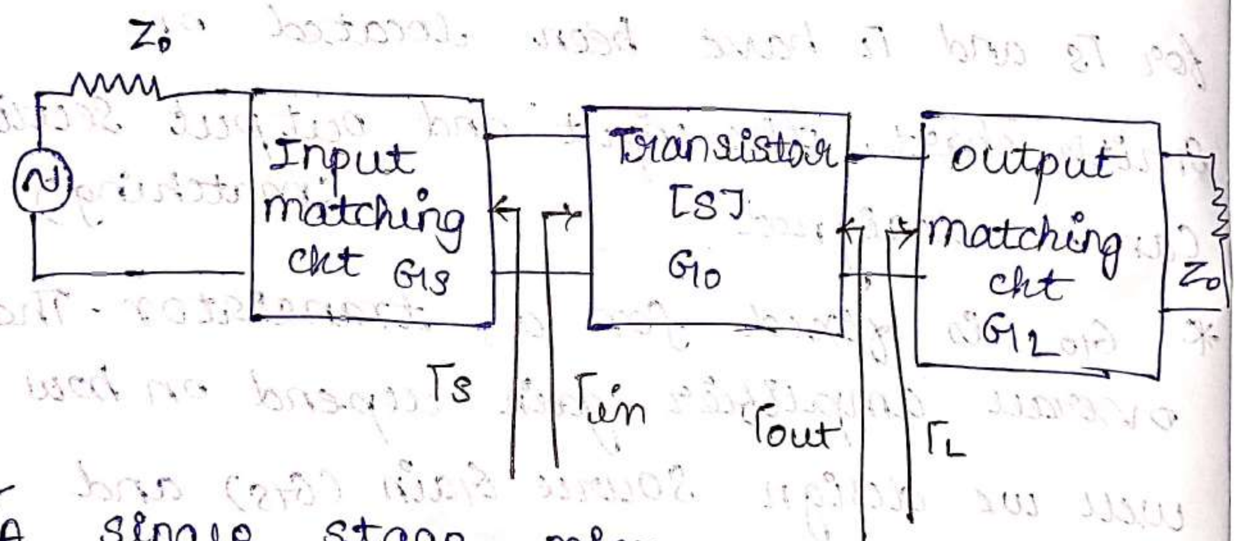
conjugate Matching

$\Gamma_{in} = \Gamma_S^*$ $\Gamma_{out} = \Gamma_L^*$

Matching → Tranducer gain G_T

* After stability of transistor has been determined and stable regions for Γ_S and Γ_L have been located on smith chart, the input and output section can be designed (matching).
* G_0 is fixed for a transistor. The overall amplifier gain depend on how well we design source Gain (G_S) and load gain (G_L). Maximum gain occurs only when we achieve conjugate matching (Input & output impedances are complex conjugates of Transistor's Impedances
* Real Transistor often have large reflection co-efficients ($|S_{11}|, |S_{22}|$) as

they don't match well to 50 Ω,
If we force conjugate matching we get very high gain over narrow frequency band. To make amplifier work over wider Bandwidth, designers reduce gain by choosing a match that is not perfect (less than maximum) which gives balanced and useful frequency response



* A single stage microwave transistor amplifier can be modeled by the above circuit where matching networks are used on both sides of transistor to transform the input and output impedance Z_0 to source and load impedances Z_S and Z_L

* The most useful gain is Transducer power gain which accounts both source and load mismatch

WKT

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2 |1-S_{22}\Gamma_L|^2}$$

we can define seperate effective gain factors for input (source) matching network, the transistor and output (load) matching networks

$$G_S = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}$$

$$G_0 = |S_{21}|^2$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

overall $G_T = G_S\, G_0\, G_L$

The Effective gains G_S and G_L of matching network may be greater than unity, because the unmatched transistor would incur power loss due to reflections at input and output of network transistor and matching section can reduce these losses

If the transistor is unilateral $S_{12} = 0$ then $\Gamma_{in} = S_{11}$, $\Gamma_{out} = S_{22}$

$G_{TU} = G_S G_0 G_L$

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}$$

$$G_0 = |S_{21}|^2$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

The maximum power transfer from input matching network to transistor will occur when

$$\Gamma_{in} = \Gamma_S^* \quad \text{—— (1)}$$

The maximum power transfer from transistor to output matching network will occur when

$$\Gamma_{out} = \Gamma_L^* \quad \text{—— (2)}$$

Assuming lossless matching sections, these conditions maximize overall Transducer gain

$$G_T = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_S \Gamma_{in}|^2} \cdot |S_{21}|^2 \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

Scanned with CamScanner

$\Gamma in = \Gamma_S^*$ in G_S. hence

$$|1 - \Gamma_S \Gamma in|^2 = (1 - |\Gamma_S|^2)^2$$

$$G_S = \frac{1 - |\Gamma_S|^2}{(1 - |\Gamma_S|^2)(1 - |\Gamma_S|^2)} = \frac{1}{1 - |\Gamma_S|^2}$$

$\Gamma out = \Gamma_L^*$ in G_L

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$G_{Tmax} = \frac{1}{1 - |\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

In addition with conjugate matching and lossless matching sections, the input and output ports of 4 n matched to Zo.

In general case with bilateral ($S_{12} \neq 0$) Γin is affected by Γout so input and output sections must be matched simultaneously

wkt $\Gamma in = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}$

$$\Gamma out = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$$

Applying ① and ② above

$$\Gamma_S^* = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$$

To solve for Γ_S

$$\Gamma_S^* = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$(\Gamma_S^*)^* = S_{11}^* + \frac{S_{12}^* S_{21}^* \Gamma_L^*}{1 - S_{22}^* \Gamma_L^*}$$

divide N^r & D^r by Γ_L^*

$$\boxed{\Gamma_S = S_{11}^* + \frac{S_{12}^* S_{21}^*}{1/\Gamma_L^* - S_{22}^*}}$$

Now: $\Gamma_L^* = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$

$$\Gamma_L^* = \frac{S_{22}(1 - S_{11}\Gamma_S) + S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$$

$$\Gamma_L^* = \frac{S_{22} - S_{22} S_{11} \Gamma_S + S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$$

$$= \frac{S_{22} - (S_{11} S_{22} - S_{12} S_{21}) \Gamma_S}{1 - S_{11}\Gamma_S}$$

we know $\Delta = S_{11} S_{22} - S_{12} S_{21}$

$$\boxed{\Gamma_L^* = \frac{S_{22} - \Delta \Gamma_S}{1 - S_{11} \Gamma_S}}$$

sub Γ_L^* in Γ_S

$$\frac{1}{\Gamma_L^*} = \frac{1 - S_{11}\Gamma_S}{S_{22} - \Delta\Gamma_S}$$

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* S_{21}^*}{\frac{1 - S_{11}\Gamma_S}{S_{22} - \Delta\Gamma_S} - S_{22}^*}$$

$$\Gamma_S - S_{11}^* = \frac{S_{12}^* S_{21}^*}{\frac{1 - S_{11}\Gamma_S - S_{22}^*(S_{22} - \Delta\Gamma_S)}{S_{22} - \Delta\Gamma_S}}$$

$$\Gamma_S - S_{11}^* = \frac{S_{12}^* S_{21}^* (S_{22} - \Delta\Gamma_S)}{1 - S_{11}\Gamma_S - S_{22}^*(S_{22} - \Delta\Gamma_S)}$$

$$\Gamma_S - S_{11}^* = \frac{S_{12}^* S_{21}^* (S_{22} - \Delta\Gamma_S)}{1 - S_{11}\Gamma_S - |S_{22}|^2 + S_{22}^* \Delta\Gamma_S} \qquad S_{22}^* \times S_{22} = |S_{22}|^2$$

$$(\Gamma_S - S_{11}^*)(1 - S_{11}\Gamma_S - |S_{22}|^2 + S_{22}^* \Delta\Gamma_S) = S_{12}^* S_{21}^* (S_{22} - \Delta\Gamma_S)$$

$$\Rightarrow \Gamma_S - S_{11}\Gamma_S^2 - \Gamma_S |S_{22}|^2 + S_{22}^* \Delta\Gamma_S^2 - S_{11}^* + S_{11}^* S_{11}\Gamma_S + S_{11}^* |S_{22}|^2 - S_{11}^* S_{22}^* \Delta\Gamma_S$$

$$= S_{12}^* S_{21}^* S_{22} - S_{12}^* S_{21}^* \Delta\Gamma_S$$

$\Rightarrow$ Grouping

$$[(\Delta S_{22}^* - S_{11})]\Gamma_S^2 + [(1 - |S_{22}|^2 + S_{11}^* S_{11} - S_{11}^* S_{22}^* \Delta + S_{12}^* S_{21}^* \Delta)]\Gamma_S$$

$$+ [(-S_{11}^* + S_{11}^* |S_{22}|^2 - S_{12}^* S_{21}^* S_{22})] = 0.$$

Scanned with CamScanner

$$\left[(\Delta S_{22}^{*} - S_{11})\right]\Gamma_S^2 + \Big[(1-|S_{22}|^2) + |S_{11}|^2 - \Delta\big(\underbrace{S_{11}^{*}S_{22}^{*} - S_{12}^{*}S_{21}^{*}}_{\Delta^{*}}\big)\Big]\Gamma_S$$

$$+ \left[S_{11}^{*}(-1+|S_{22}|^2) - S_{12}^{*}S_{21}^{*}S_{22}\right] = 0.$$

$$(\Delta S_{22}^{*} - S_{11})\Gamma_S^2 + (1-|S_{22}|^2 + |S_{11}|^2 - |\Delta|^2)\Gamma_S$$

$$+(-)\left[S_{11}^{*}(1-|S_{22}|^2) + S_{12}^{*}S_{21}^{*}S_{22}\right] = 0.$$

$$(\Delta S_{22}^{*} - S_{11})\Gamma_S^2 + (1-|S_{22}|^2 + |S_{11}|^2 - |\Delta|^2)\Gamma_S$$

$$- S_{11}^{*} - S_{11}^{*}S_{22}^{*}S_{22} + S_{12}^{*}S_{21}^{*}S_{22}$$

$$(\Delta S_{22}^{*} - S_{11})\Gamma_S^2 + (1-|S_{22}|^2 + |S_{11}|^2 - |\Delta|^2)\Gamma_S$$

$$-\Big(S_{11}^{*} - \big(S_{22}\big[\underbrace{S_{11}^{*}S_{22}^{*} - S_{12}^{*}S_{21}^{*}}_{\Delta^{*}}\big]\big)\Big)$$

$$(\Delta S_{22}^{*} - S_{11})\Gamma_S^2 + (1-|S_{22}|^2 + |S_{11}|^2 - |\Delta|^2)\Gamma_S$$

$$-(S_{11}^{*} - S_{22}\Delta^{*})$$

(−) common

$$(S_{11} - \Delta S_{22}^{*})\Gamma_S^2 + (|\Delta|^2 - |S_{11}|^2 + |S_{22}|^2 - 1)\Gamma_S$$

$$+ (S_{11}^{*} - \Delta^{*}S_{22}) = 0.$$

Solution to Quadratic Equation

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

similarly for Γ_L

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

Now $B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$

$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$

$C_1 = S_{11} - \Delta S_{22}^*$

$C_2 = S_{22} - \Delta S_{11}^*$

we know that

$A = S_{11} - \Delta S_{22}^*$

$B = |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2 - 1$

$C = S_{11}^* - \Delta^* S_{22}$

$C = (S_{11} - \Delta S_{22}^*)^* = A^*$

So constant is complex conjugate of Quadratic coefficient

Let $C_1 = A = S_{11} - \Delta S_{22}^*$

hence $C = C_1^* = A^*$

wkt $\Gamma_S = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$C = A^*$ hence

$$\Gamma_S = \frac{-B \pm \sqrt{B^2 - 4(A)(A^*)}}{2A}$$

$$\Gamma_S = \frac{-B \pm \sqrt{B^2 - 4|A|^2}}{2A}$$

But $A = C_1$ hence

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

Always $K>1$, Thus unconditionally stable devices can always be conjugately matched for maximum gain and potentially unstable devices can be conjugately matched if $K>1$ and $|\Delta|<1$

when $S_{12} = 0$; $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2} |S_{21}|^2 \frac{1}{1-|S_{22}|^2}$$

If the Tr. is unconditionally stable $K>1$ then

$$G_{T\max} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2-1}\right)$$

$G_{T\max}$ = matched gain.

G_{msg} = Maximum stable gain

$$G_{msg} = \frac{|S_{21}|}{|S_{12}|}$$

4. Design an Amplifier for maximum gain at 4 GHZ using Impedance matching network. calculate

and plot input return loss and gain from 3 GHZ to 5 GHZ. The Transistor is a GaAS MESFET with the following S parameters with $Z_0 = 50\Omega$

For 3 GHZ

$S_{11} = 0.80 \angle -89^\circ$

$S_{12} = 0.03 \angle 56^\circ$

$S_{21} = 2.86 \angle 99^\circ$

$S_{22} = 0.76 \angle -41^\circ$

For 4 GHZ

$S_{11} = 0.72 \angle -116^\circ$

$S_{12} = 0.03 \angle 57^\circ$

$S_{21} = 2.60 \angle 76^\circ$

$S_{22} = 0.73 \angle -54^\circ$

For 5 GHZ

$S_{11} = 0.66 \angle -142^\circ$

$S_{12} = 0.03 \angle 62^\circ$

$S_{21} = 2.39 \angle 54^\circ$

$S_{22} = 0.72 \angle -68^\circ$

calculate i) K and ∇

ii) Determine Γ_S and Γ_L

iii) Find Gain

iv) Draw Impedance matching network

i)
For 3 GHZ

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\,|S_{12} S_{21}|}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$\Delta = 0.5916 \angle -121.947$$

$$K = \frac{0.13239}{2 \times 0.0868}$$

$K = 0.7715 \angle 0.$

$\Delta = 0.592 < 1$

$K = 0.7715 < 1$

Hence conditionally stable

For 4GHZ.

$\Delta = S_{11}S_{22} - S_{12}S_{21}$

$\Delta = 0.48775 \angle -162.28$

$$K = \frac{0.18635}{2 \times 0.078}$$

$K = 1.19455$

$\Delta < 1$ & $K > 1$ Hence unconditionally stable.

For 5GHZ

$\Delta = S_{11}S_{22} - S_{12}S_{21}$

$\Delta = 0.41768 \angle 155.508$

$$K = \frac{0.22045}{2 \times 0.0717}$$

$K = 11.537$

$K > 1$ & $\Delta < 1$ hence unconditional stable

we can proceed with design at 4 GHZ.

ii) To find maximum Gain we design matching sections for a conjugate match to the transistor hence $\Gamma_S = \Gamma_{in}^*$ and $\Gamma_L = \Gamma_{out}^*$

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta S_{22}^*$$

$$C_2 = S_{22} - \Delta S_{11}^*$$

FOR 4 GHZ

$B_1 = 0.7479$

$B_2 = 0.7769$

$C_1 = 0.72\angle{-116^\circ} - 0.355875\ \angle{-108.28} = 0.370448\angle{-123}$

$C_2 = 0.73\angle{-54^\circ} - 0.351\angle{-46.28} = 0.38507\ \angle{-61.033}$

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

$$\Gamma_S = \frac{0.7479 \pm \sqrt{(0.7479)^2 - 4(0.370448)^2}}{2 \times 0.370448 \angle -123.4}$$

$$\Gamma_S = \frac{0.7479 \pm 0.102115}{0.740896 \angle -123.4}$$

For stability take negative sign

$$\Gamma_S = \frac{0.645785}{0.740896 \angle -123.4}$$

$$\boxed{\Gamma_S = 0.872 \angle 123.4^\circ}$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$\Gamma_L = \frac{0.7769 \pm \sqrt{(0.7769)^2 - 4(0.38507)^2}}{2 \times 0.38507 \angle -61.033}$$

$$= \frac{0.7769 \pm 0.102264}{\underbrace{0.37298 + 1.01j}\ 0.77014 \angle -61.033}$$

Take negative sign

$$\Gamma_L = \frac{0.674636}{0.77014 \angle -61.03}$$

$$\boxed{\Gamma_L = 0.876 \angle 61.03}$$

iii) Gain.

$$G_S = \frac{1}{1 - |\Gamma_S|^2} = \frac{1}{1 - (0.872)^2}$$

$$G_S = 4.1733$$

$$G_S|_{dB} = 6.2048 \text{ dB.}$$

$$G_0 = |S_{21}|^2 = (2.60)^2 = 6.76 = 8.299 \text{dB}$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} = \frac{1 - (0.876)^2}{\left(1 - [0.73 \angle -54)(0.876 \angle 61.03]\right)^2}$$

$$= \frac{1 - (0.876)^2}{|0.3736 \angle -12.091|^2}$$

$$= \frac{0.232624}{0.139576} = 1.67$$

$$G_L|_{dB} = 2.218 \text{ dB.}$$

$$G_T = G_S + G_0 + G_L$$

$$G_T = 4.1733 + 6.76 + 1.67 = 12.6033$$

$$G_T|_{dB} = 6.2048 + 8.299 + 2.218 = 16.7 \text{dB.}$$

Lumped Networks

1. Make sure one L and C

capacitor Zs

$-1.789 \rightarrow 0.4$

$0.4-(-1.789) = 2.189j = jB$

$X_c = \frac{1}{B} = \frac{1}{2.189j} = -0.4568j$

Normalised hence $X_c = -0.4568j \times 50$

$= -22.8414j$

Scanned with CamScanner

$$X_C = \frac{1}{j\omega C} = -22.8414j$$

$$X_C = \frac{-j}{\omega C} = -22.8414j$$

$$X_C = \frac{1}{\omega C} = 22.8414$$

$$X_C = \frac{1}{2\pi f \times C} = 22.8414$$

$$\frac{1}{2\pi f C} = 22.8414$$

$$\frac{1}{2\pi \times 4 \times 10^9} = 22.8414C$$

$$3.978873 \times 10^{-11} = 22.8414C$$

$$22.8414C = 3.978873 \times 10^{-11}$$

$$\boxed{C = 1.7419\ pF}$$

Inductor Z_L

$$-2 \to 0$$

$$0 - (-2) = 2j$$

$$j\omega L = 2j \times 50 \text{ (normalised)}$$

$$j\omega L = 100j$$

$$2\pi f L = 100$$

$$2\pi \times 4 \times 10^9 \times L = 100$$

$$100 = 2\pi \times 4 \times 10^9 L$$

$$\boxed{L = 3.9788\ nH}$$

For Z_L

capacitor:

$-0.583j \rightarrow 0.22j$

$0.22 - (-0.583) = 0.803j \Rightarrow B$

$X_C = \frac{1}{B} = -1.2453j$.

Normalised $x_C = -62.266j$

$\frac{-\cancel{j}}{\omega C} = \cancel{+}62.266\cancel{j}$

$\frac{1}{2\pi f X_C} = 62.266$

$\frac{1}{2\pi \times 4 \times 10^9} = 62.266C$

$3.978873 \times 10^{-11} = 62.266C$

$62.266C = 3.978873 \times 10^{-11}$

$\boxed{C = 0.6390\,pF}$

Inductor

$-5j \rightarrow 0j$

$0 - (-5j) = 5j \times 50 = 250j$

$\cancel{j}\omega L = 250\cancel{j}$

$2\pi \times 4 \times 10^9 L = 250$

$\boxed{L = 9.947\,nH}$

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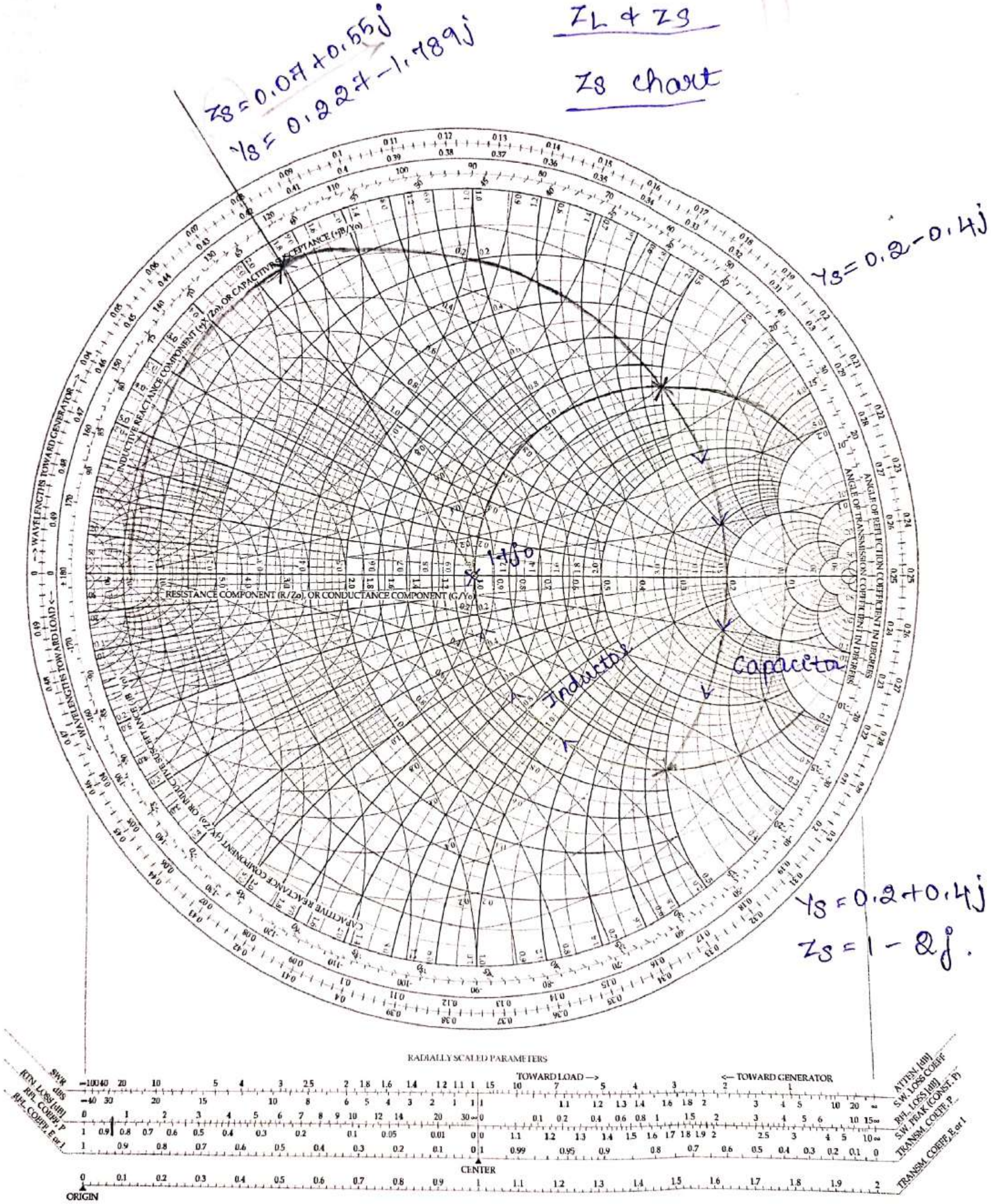
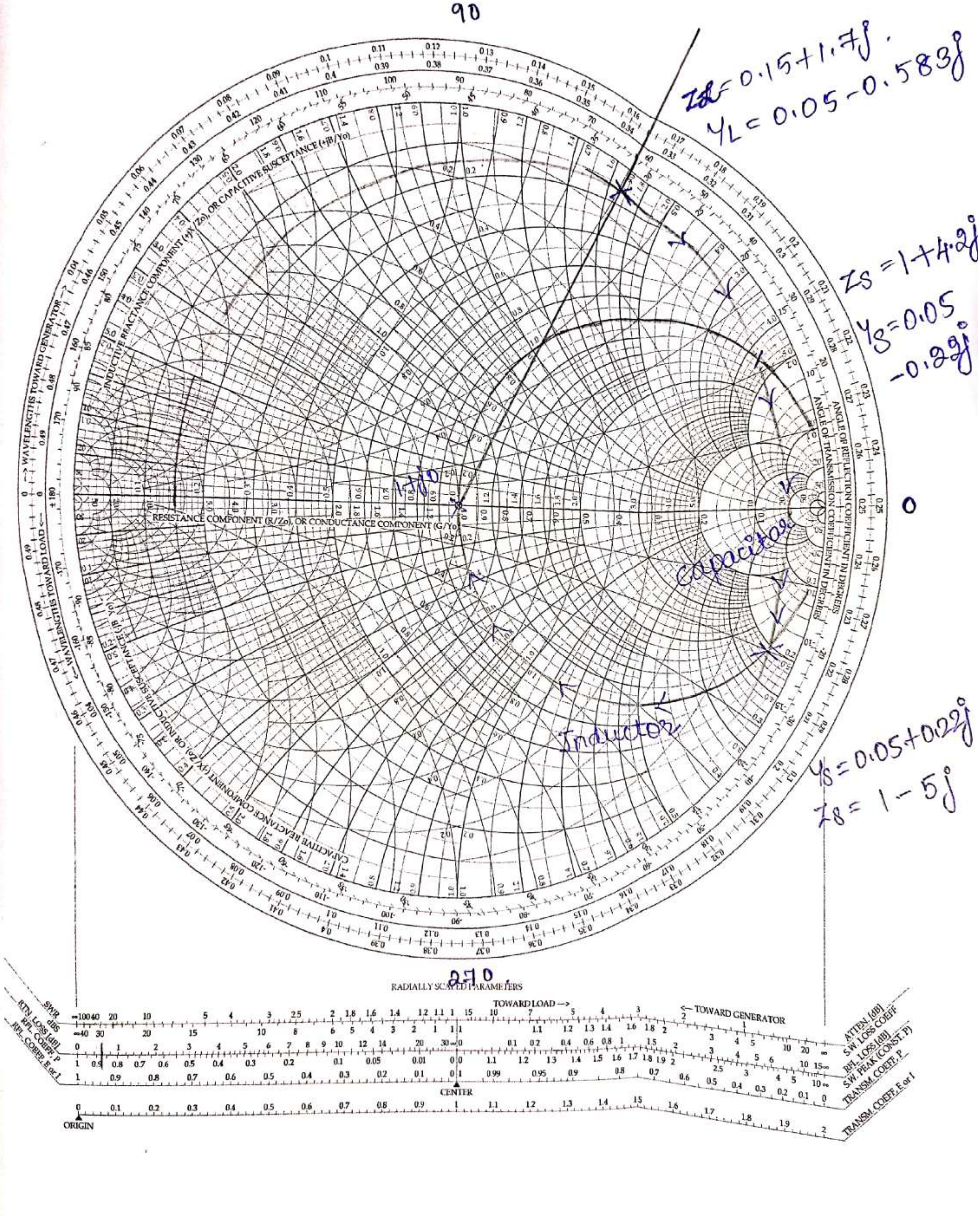
Lumped Networks:

Z_L & Z_S

Z_S chart



Z_L chart



90
$Z_L = 0.15+1.7j$.
$Y_L = 0.05-0.583j$
$Z_S = 1+4.2j$
$Y_S = 0.05 -0.22j$
$Y_S = 0.05+0.22j$
$Z_S = 1-5j$
0
1+j0
capacitor
Inductor
270.
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I
CENTER
ORIGIN

08.09.25 (optimum Γ_S, Γ_L picked up) (Stability

Noise circle → Gain circle → Γ_S circle)

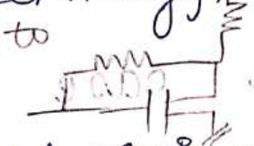
choice of Biasing method (tuned for optimum Γ_S, Γ_L)

Low Noise Amplifier – Topology:

→ common source LNA } Based on CMOS Technology

→ common Gate LNA }

due to



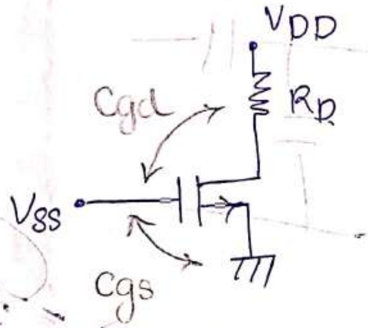
* Resistive load

* Resistive feedback load (match Gain and stability)

* Inductive load → power consumption (less) (useful)

In Resistive load → more prone to power consumption not easy to achieve gain factor

common source – Resistive load



Gain (A) = $g_m R_D$

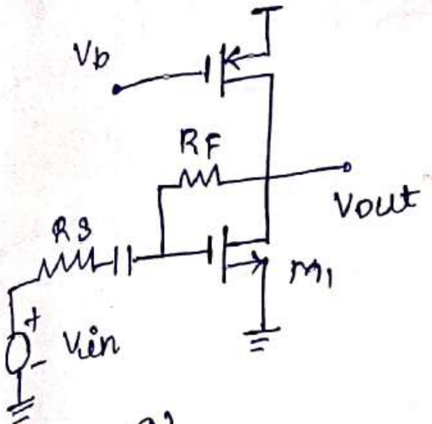
g_m = Transconductance

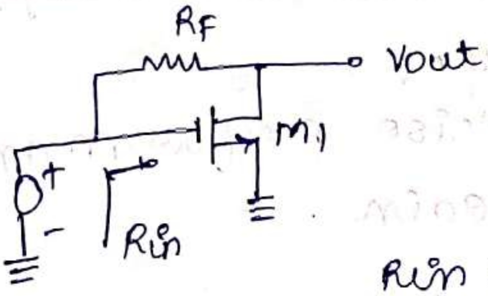
Due to Biasing – parasitic capacitance created on its own, To compensate we go with Impedance matching network

Generally → Degenerative

→ shunt reactive

common source - Resistive feedback load:





$R_{in} = \frac{1}{g_{m1}}$

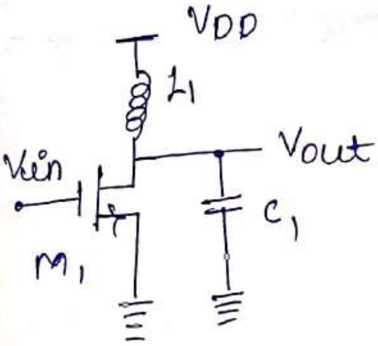
a)

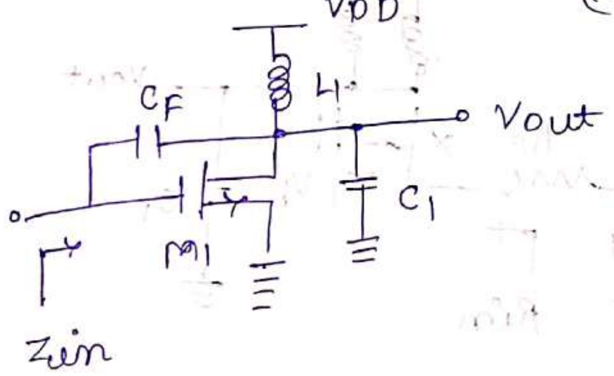
$V_x - g_{m1} V_x R_F = V_{out}$

$A_v = -\frac{R_F}{R_S}$ $\qquad$ $\frac{V_{out}}{V_x} = 1 - \frac{R_F}{R_S}$

Alternative: Inductive loads

common source - Inductive load $NF = 1.3$ - $(1.5 - 3)$ ✓





a) $\qquad$ b)

* Inductive load replaces a resistive load → High impedance at operating frequency → High Gain without noise penalty of a resistor

* Input matching achieved via gate inductance or source degeneration.

* voltage gain = $g_m L_1 \omega$

* Feedback stabilizes the amplifier and improves linearity.

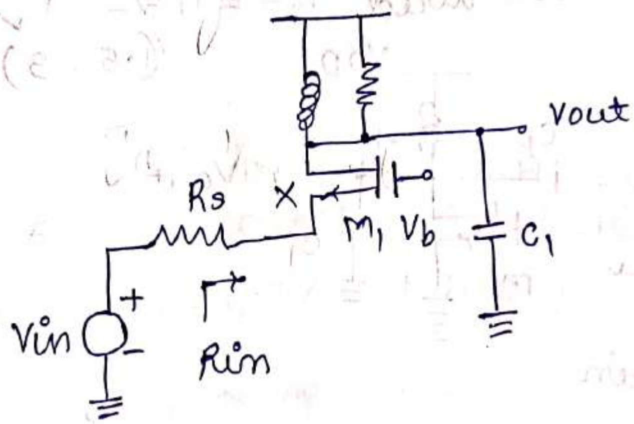
Advantage:

- Good Noise performance
- High gain

Disadvantage:

- Narrow Bandwidth
- poor stability in absence of matching
- Sensitive in parasitic capacitance.

Common Gate with Inductive Load



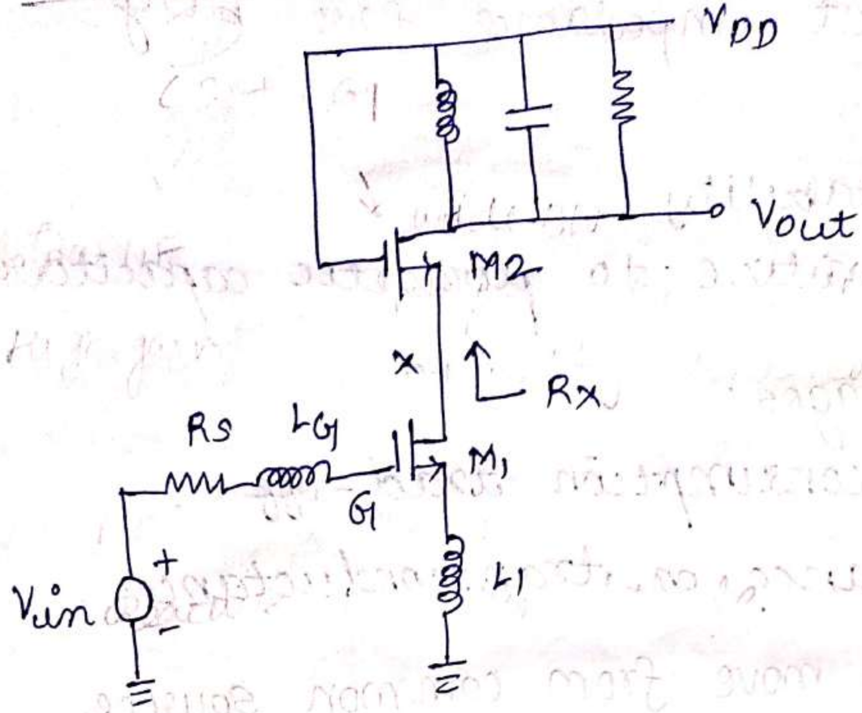
* Gate terminal is AC grounded, the input signal is applied to the source terminal and output taken from drain
* The Input impedance is low
* High stability
* Broadband
* Easy impedance matching

Advantages:
* Low input impedance for Easy matching
* High stability
* Low sensitive to parasitic capacitors

Disadvantages:
* power consumption trad-off
* Dependance on transconductance.

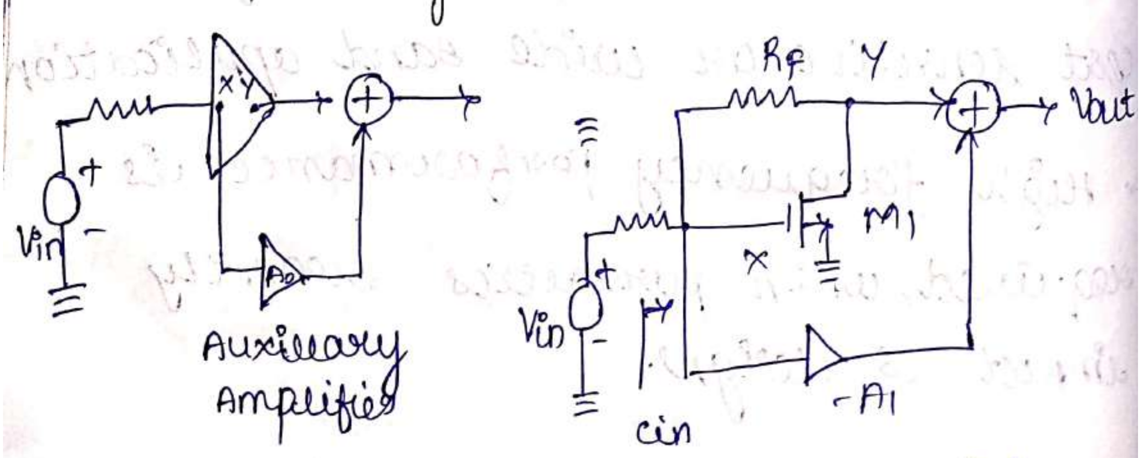
Reason to move from common source to common Gate:
* Broadband operation is needed (Eg UWB, SDR)
* stability is critical especially at high frequencies
* Linearity is prioritized over noise figure in interference-heavy environment
* Design simplicity and reduced component count are desired particularly in cost sensitive or wide band application
* High frequency performance is required, when parasitics heavily impact CS design.

Cascode LNA:



* The cascode LNA is maximum used in RF-front end designs.
* It combines the strength of common source (CS) and common gate (CG) stages to achieve balance a low noise, high stability
* It is a two stage amplifier consisting of a CS transistor followed by CG transistor.

Noise cancelling LNA



$A_1 = 1 + R_F/R_S$

Reactance cancelling LNA:

- Reactance cancelling involves adding reactive components that are equal in magnitude but opposite in phase to unwanted reactances which neutralises the effects.

- Source degeneration Inductance:
  Inductance in series with source of transistor which cancel out input capacitance of transistor to improve input impedance matching.

09.09.25

Γ_{in} = gain, NF (Balance) we see for optimum Γ_{in}

- What Might be Γ_{in} to achieve Gain & NF
- check for stability
- If conditional find out range using smith chart

Noise circle: C_{Noise} = complex in Nature.

$$C_{Noise} = \frac{\Gamma_{opt}}{1+N}$$

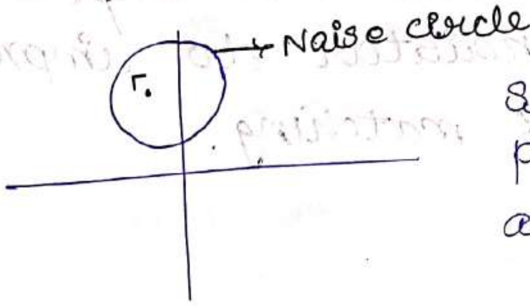
$$r_{noise} = \frac{\sqrt{N[(N+1) - |\Gamma_{opt}|^2]}}{1+N}$$

$$N = \frac{(F_{target} - F_{min})\,|1 + \Gamma_{opt}|^2}{4 r_n}$$

$$(NF)_{target}\ dB = 10 \log F_{target}$$

F_{min} = minimum NF possible to achieve through device

$$\boxed{\frac{R_n}{50} = r_n}$$



Since inside possible to achieve NF.

5. For a frequency of 2GHZ and $Z_0 = 50\Omega$, draw a Noise circle for 1dB, 2dB and 3dB employing HEMT and min NF is 0.6 dB $(NF_{min}) = 0.6\,dB$ and $R_n = 4\Omega$, possible optimum refⁿ coeff.

$\Gamma_{opt} = 0.30 \angle{-45°}$

$\Gamma_{opt} = 0.212 - j0.212$

R_n: Internal resistance

r_n: Normalised Internal resistance.

and s parameters are

$S_{11} = 0.46 \angle{-35°}$

$S_{12} = 0$

S_{21} = ~~[illegible]~~ 26 dB

$S_{22} = 0.50 \angle{-10°}$

F_{target} = 1, 2, 3 dB

Gain target = 10dB, 15, 20dB

Soln

Let F_{target} (NFtarget) $= 2dB \Rightarrow$ Noise Figure

$$F_{target} = 10^{2/10} = 1.58$$

$$NF_{min} = 0.6 dB$$

$$F_{min} = 10^{0.06} = 1.148$$

$$N = \frac{(1.58 - 1.14)\,|1 + (0.212 - j0.212)|^2}{4 \times 4/50}$$

$$N = \frac{(0.44)(1.2305)^2}{4 \times 0.08} = \frac{0.44 \times 1.5141}{0.32}$$

$$\boxed{N = 2.081}$$

$$C_{Noise} = \frac{0.30 \angle -45^\circ}{1 + 2.08} = 0.0974 \angle -45^\circ$$

$$R_{Noise} = \frac{\sqrt{N(N+1) - |\Gamma_{opt}|^2}}{1+N}$$

$$= \frac{\sqrt{2.08(3.08) - |0.3|^2}}{1 + 2.08}$$

$$= \frac{\sqrt{6.4064 - 0.09}}{3.08} = \frac{2.51324}{3.08}$$

$R_{Noise} = 0.81598$ (measure through Not in scale, RF P in smithchart

Scanned with CamScanner

Gain circle

$$g_s = \frac{G}{G_{opt}} \qquad G_{Target} = 20\,dB = 100$$

$$G_{opt} = S_{21} = 26\,dB = 398.107$$

$$g_s = \frac{100}{398.107} = 0.25118875$$

$$C_G = \frac{S_{11}^* g_s}{1-[(1-g_s)|S_{11}|^2]} = \frac{(0.46\angle 35)(0.25118875)}{1-[(1-0.25118875)(0.46)^2]}$$

$$= \frac{0.09465 + 0.06627j}{0.841551539}$$

$$= 0.11247 + 0.07874 = 0.137\angle 35$$

$$R_s = \frac{\sqrt{1-g_s}\,(1-|S_{11}|^2)}{1-[(1-g_s)|S_{11}|^2]} = \frac{\sqrt{1-0.25118875}\,(1-(0.46)^2)}{0.841551539}$$

$$R_s = \frac{0.8653388 \times 0.7884}{0.841551539}$$

$$\boxed{R_s = 0.81068}$$

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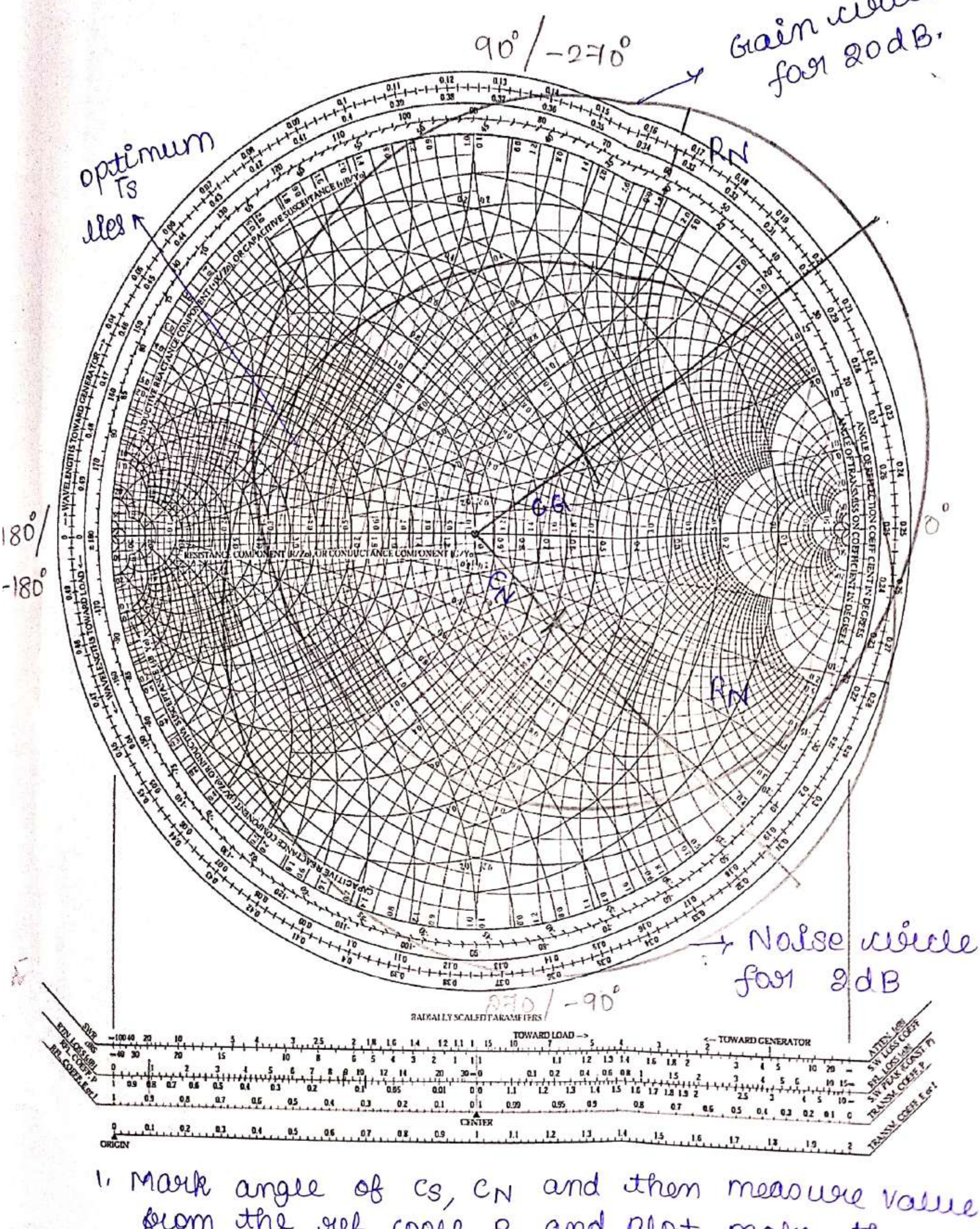
Low Noise Amplifier



1. Mark angle of C_S, C_N and then measure value from the ref coeff P and plot, make the new point as center

2. With the radius R_S & R_N, measure from ref coeff and draw circle from new center (Intersect)

Stability:

i) K-Δ Test

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$\Delta = 0.23 \angle -45^\circ$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} =$$

K = not defined.

10.09.25

Power Amplifier:

Device in stable mode

→ Linearity maintainence

→ n^{th} stage before Antenna

Homogeneous principle

$$y = nx$$

$$\alpha \cdot y = \alpha n x$$

DC power gets modulated based on the input signal → Amplification.

A^r modulates the input that DC power with rf input(DC power gets superimposed)

→ Filtering output

→ Harmonics always present

power A^r

→ (RF controls current flow in on-off basics)

Conventional power A^r

→ (RF controlls the current continuously)

→ linear mode/ Active region

Switched mode power A^r (ON/OFF) (saturation and cutoff)

Harmonically tuned power A^r

→ (can control the Harmonics)

→ self impedance matching network

conventional η (mod)↓↓ linearity ↑

- class A (50% η duty cycle Phase angle 180°)
- class B (phase angle > 90° ⇒ $\eta = 80\%$)
- class C (phase angle < 90° ⇒ $\eta = 90\%$
- class AB

→ cross over distortions.

switched mode (prefer Inductive load) η (mod) power ↓ linearity (mod)

- class D (Balanced Amplifiers)
- class E (single ended)

Harmonically Tuned (prefer Inductive load) η ↑ (harmonics are tuned)

- F (odd no. of harmonics)
- F^{-1} (Even harmonics)
- J (2nd harmonics)

wider BW.
power ↓
linearity (mod)

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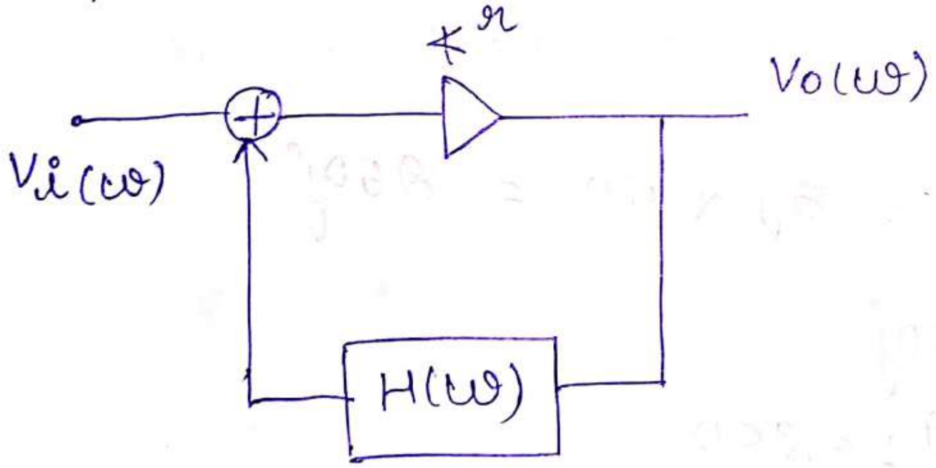
15.09.25

Microwave Oscillators:

- Non linear circuits
- Absence of input there is a possibility of output.
- It converts DC power to AC waveforms
- Amplifier circuit which has frequency dependant feedback which is given as Input



$$V_o(\omega) = A V_i(\omega) + V_o(\omega) H(\omega) V_i(\omega)$$

$$\boxed{\frac{V_o(\omega)}{V_i(\omega)} = \frac{A}{1 - AH(\omega)}}$$

$$V_o(\omega) = \frac{A}{1 - AH(\omega)} \cdot V_i(\omega)$$

$$V_i(\omega) = AV_i(\omega) = V_o(\omega) - AV_o(\omega)H(\omega)$$

$$\boxed{V_o(\omega) = AV_i(\omega) + AV_o(\omega).H(\omega)}$$

on making characteristic Equation zero.

Nyquist criteria $\boxed{1 - AH(\omega) = 0}$ $\Rightarrow$ At any one of frequency zero i/p

$\downarrow$ &

Barkhausen $A\beta = 1$ still o/p present.

If $V_i(\omega) = 0$

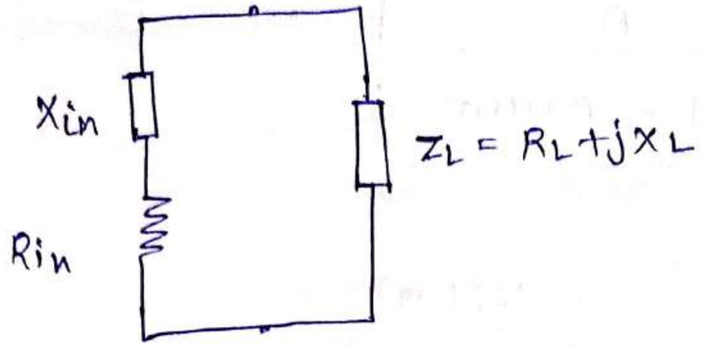
$V_o(\omega) = AV_oH(\omega)$

$$\boxed{V_o(1 - AH(\omega)) = 0}$$

→ Ensure that stability criteria damping at one point becomes zero and becomes stable

one part Negative resistance oscillator

GUNN
IMPACT } Negative resistance



$1 + H(j\omega)A = 0$

NR Device → Amplier cir → compute Γ_{in}, Γ_{out} ↓ Γ_L find stability circle

Impedance matching Network ← stability region with 50Ω imped.

$\Gamma_{out} \leftrightarrow \Gamma_L$

lumped (or) stub

Apply KVL for the circuit

$Z_{in} = X_{in} + R_{in}$

Impedance ⇒ Depends on I/V (ct. / Volt.) & frequency

$Z_{in}(I, j\omega) = X_i(I, j\omega) + R_{in}(I, j\omega)$

$Z_L = R_L + jX_L$

KVL

$$I(R_{in} + X_{in}) + IZ_L = 0 \Rightarrow I(Z_{in} + Z_L) = 0$$

If $Z_L + Z_{in} = 0$

then $\boxed{Z_L = -Z_{in}}$

In RF I should be present hence $I \neq 0$ hence $Z_L + Z_{in} = 0$.

$$\boxed{\begin{aligned} R_L + R_{in} &= 0 \\ X_L + X_{in} &= 0 \end{aligned}}$$

In ckt $R_L \uparrow$ then $R_{in} \downarrow$ hence balanced to yield 0.

If R+ve $\Rightarrow$ dissipation occurs

R-ve $\Rightarrow$ Energy source

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = \frac{-Z_{in} - Z_0}{-Z_{in} + Z_0}$$

$$\Gamma_L = \frac{-(Z_0 + Z_{in})}{-(Z_{in} - Z_0)} = \frac{-(Z_0 + Z_{in})}{Z_0 - Z_{in}}$$

$$\boxed{\Gamma_L = \frac{-1}{\Gamma_{in}}}$$

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For unstable ckt.

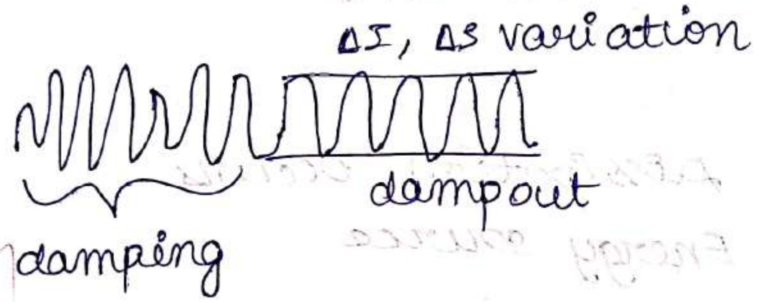
R_{in} dep on I & $j\omega$ freq.

$R_{in} + R_L < 0.$

$R_{in}(I, j\omega) + R_L < 0$

To make this stable, if slight variation in current (ΔI) and frequency Δs ($s = \alpha + j\omega$) retains the same point.

ΔI & $\Delta s \rightarrow$ leads to damp out, reduces damping (oscillation) makes out stable



$$\boxed{\frac{\partial Re(Z_{in})}{\partial I}\,\frac{\partial Im(Z_L)}{\partial \omega} - \frac{\partial Im(Z_{in})}{\partial I}\cdot\frac{\partial Re(Z_L)}{\partial \omega} > 0}$$

$$\left.\begin{aligned} R_s &= -\frac{R_{in}}{3} \\ X_s &= -X_{in} \end{aligned}\right\} \begin{array}{l} \underline{\text{source}} \\ \text{possibility of} \\ \text{osc}^n. \end{array}$$

$R_s \downarrow\downarrow$ possible to construct high osc. circuit

$\Gamma_s = -\Gamma_{in}$ (proper Termination)

1. Suppose 1 port osc^r which uses negative resistance device whose $\Gamma_{in} = 1.25\angle 40^\circ$, $Z_0 = 50\,\Omega$ for freq of 6 GHZ. What should be the load matching Network.

$1.120229\angle 76.81^\circ$

$0.82430\angle -11.2417$

Soln: $\Gamma_{in} = \dfrac{Z_{in} - Z_0}{Z_{in} + Z_0}$

$Z_{in} = Z_0 \cdot \dfrac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$

1.9575

$2.116036\angle 22.315$

$0.80464\angle -86.976$

$Z_{in} = -44 + j123$

Load

$Z_L = -Z_{in}$

$$\boxed{Z_L = 44 - j123}$$

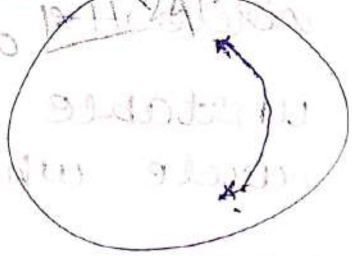
Nyquist

$$\boxed{\Gamma_s \Gamma_{in} = 1}$$

$1.202298\angle 76.810$

$0.82430\angle -11.2417$

Gunn



$\dfrac{1}{R_s} = R_{in}$

V↑ I↓

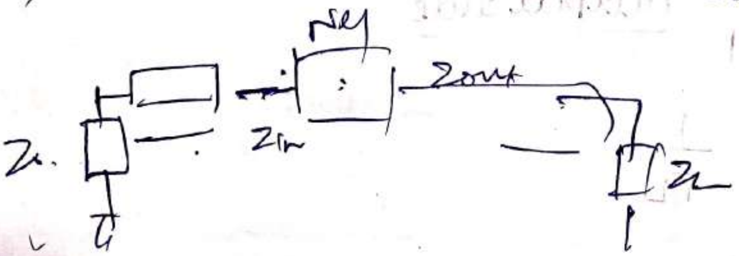
↳ provides

Negative resistance → Achieve Rs ↓↓

↑↑

↓

damp out.



17.09.25

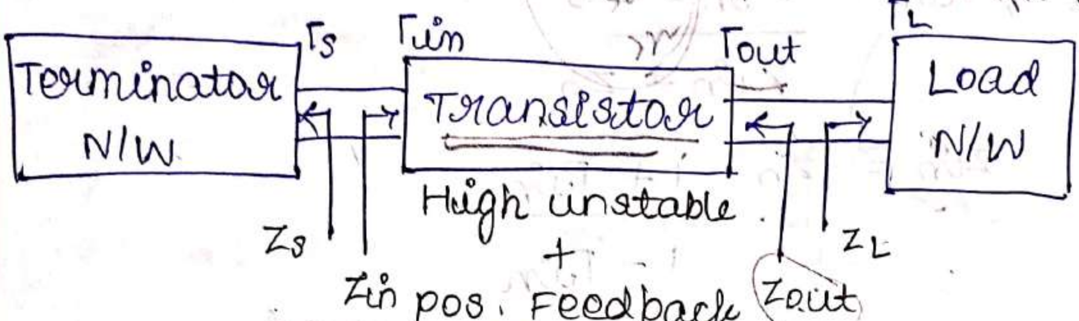
Transistor oscillator

1. Highly unstable device designed.

MOSFET - common source

2. Identify Γ_L

S_{11} S_{12} S_{21} S_{22} C_{out} R_{out}



High unstable + pos. Feedback

$\downarrow$

Lumped elements

stubs

Dielectric Resonator

$\Gamma_L = |S_{11}|$ 2_L

To ensure unstability, input impedance unstable

→ stability circle → Γ_L plane

if $\Gamma_L < 1$, unstable point outside circle where Γ_{in} (unstable)

Condn of oscillation

$\Gamma_S \Gamma_{in} = 1$ ✓

$\Gamma_{in} = \frac{1}{\Gamma_S}$ ✓

output side

$\Gamma_L \Gamma_{out} = 1$

$\Gamma_{out} = \frac{1}{\Gamma_L}$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = \frac{1}{\Gamma_S}$$

$$\frac{1}{\Gamma_S} = \frac{S_{11} - S_{11}S_{22}\Gamma_L + S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$= \frac{S_{11} - \Gamma_L(S_{11}S_{22} - S_{12}S_{21})}{1 - S_{22}\Gamma_L}$$

$$\frac{1}{\Gamma_S} = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L} \qquad \Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$1 - S_{22}\Gamma_L = S_{11}\Gamma_S - \Delta\Gamma_L\Gamma_S$$

$$\Gamma_L[S_{22} - \Delta\Gamma_S] = 1 - \Gamma_S S_{11}$$

$$\boxed{\Gamma_L = \frac{1 - \Gamma_S S_{11}}{S_{22} - \Delta\Gamma_S}}$$

$\Gamma_{in} \Rightarrow Z_{in} \Rightarrow R_{in} + jX_{in} \qquad X_S = -X_{in}$

$$\downarrow$$

$$R_S = \frac{-R_{in}}{3}$$

Output $\Rightarrow Z_L = -Z_{out}$

Dielectric Resonators: