4x4 MIMO Alamouti Code

$$\text{Txd Symbols} = \begin{bmatrix} s_1 & s_2^* & s_3^* & s_4 \\ s_2 & -s_1^* & s_4^* & -s_3 \\ s_3 & s_4^* & -s_1^* & -s_2 \\ s_4 & -s_3^* & -s_2^* & s_1 \end{bmatrix} \downarrow \text{Antennas}$$

$\rightarrow$ Time

$h \rightarrow$ Channel matrix with flat fading coefficients

$$h \in \mathbb{C}^{N_t \times N_r} = \mathbb{C}^{4\times 4}$$

$h_{ij} \rightarrow i^{th}$ Tx antenna
j^{th} Rx antenna

1st Time instance

$$\begin{aligned} y_{11} &= h_{11}s_1 + h_{21}\cdot s_2 + h_{31}\cdot s_3 + h_{41}\cdot s_4 \\ y_{21} &= h_{12}\cdot s_1 + h_{22}\cdot s_2 + h_{32}\cdot s_3 + h_{42}\cdot s_4 \\ y_{31} &= h_{13}s_1 + h_{23}s_2 + h_{33}s_3 + h_{43}s_4 \\ y_{41} &= h_{14}s_1 + h_{24}s_2 + h_{34}s_3 + h_{44}\cdot s_4 \end{aligned}$$

$y_{ij} \Rightarrow$ i^{th} receive antenna, j^{th} time instant

$$\underbrace{\begin{bmatrix} y_{11} \\ y_{21} \\ y_{31} \\ y_{41} \end{bmatrix}}_{Y_1} = \underbrace{\begin{bmatrix} h_{11} & h_{21} & h_{31} & h_{41} \\ h_{12} & h_{22} & h_{32} & h_{42} \\ h_{13} & h_{23} & h_{33} & h_{43} \\ h_{14} & h_{24} & h_{34} & h_{44} \end{bmatrix}}_{H_1} \underbrace{\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}}_{S}$$

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2nd Time Instant

$$y_{12} = h_{11} \cdot s_2^* + h_{21} \cdot (-s_1^*) + h_{31}(s_4^*) + h_{41}(-s_3^*)$$

$$y_{12}^* = \begin{bmatrix} -h_{21}^* & h_{11}^* & -h_{41}^* & h_{31}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$$

Illy obtaining the signal at the other antenas

2nd time instant

$$\underbrace{\begin{bmatrix} y_{12}^* \\ y_{22}^* \\ y_{32}^* \\ y_{42}^* \end{bmatrix}}_{Y_2^*} = \underbrace{\begin{bmatrix} -h_{21}^* & h_{11}^* & -h_{41}^* & h_{31}^* \\ -h_{22}^* & h_{12}^* & -h_{42}^* & h_{32}^* \\ -h_{23}^* & h_{13}^* & -h_{43}^* & h_{32}^* \\ h_{24}^* & h_{14}^* & -h_{44}^* & h_{34}^* \end{bmatrix}}_{H_2} \underbrace{\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}}_{S}$$

Obtaining the Hi matrix from channel matrix h & Encoded symbol matrix

$$\begin{bmatrix} s_1 & s_2^* & s_3^* & s_4 \\ s_2 & -s_1^* & s_4^* & -s_3 \\ s_3 & s_4^* & -s_1^* & -s_2 \\ s_4 & -s_3^* & -s_2^* & s_1 \end{bmatrix}$$

antenna ↓ (rows)

time → (columns)

⇒ Taking the 3rd column for the 3rd instant

Original S/I vector

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} \longrightarrow \begin{bmatrix} s_3^* \\ s_4^* \\ -s_1^* \\ -s_2^* \end{bmatrix}$$

Element in (1,1) S_1 is moved to (3,1) & - complex conjugated

i.e., $S_1 @ (1,1) \Rightarrow -S_1^*(3,1)$

A column of H_3 can be obtained by the opposite operation on a column of h, i.e.,

$$H_3[:,1] = -h^*[:,3]$$

Illy all the columns are obtained as

$$H_3 = \begin{bmatrix} -h_{31}^* & -h_{41}^* & h_{11}^* & h_{21}^* \\ -h_{32}^* & -h_{42}^* & h_{12}^* & h_{22}^* \\ -h_{33}^* & -h_{43}^* & h_{13}^* & h_{23}^* \\ -h_{34}^* & -h_{44}^* & h_{14}^* & h_{24}^* \end{bmatrix}$$

$$\therefore \quad Y_3^* = \begin{bmatrix} y_{13}^* \\ y_{23}^* \\ y_{33}^* \\ y_{43}^* \end{bmatrix} = H_3 \cdot \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix}$$

Illy $H_4 = \begin{bmatrix} h_{41} & -h_{31} & -h_{21} & h_{11} \\ h_{42} & -h_{32} & -h_{22} & h_{12} \\ h_{43} & -h_{33} & -h_{23} & h_{13} \\ h_{44} & -h_{34} & -h_{24} & h_{14} \end{bmatrix}$

$$\Rightarrow Y_4 = H_4 \cdot \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix}$$

Composite Rcvd symbols

$$Y = \begin{bmatrix} Y_1 ; & Y_2^* ; & Y_3^* ; & Y_4 \end{bmatrix}_{16\times 1}$$

Composite Channel Matrix

$$H = [H_1 ; H_2 ; H_3 ; H_4]_{16\times 4}$$

$H^{\dagger}_{4\times 16}$ $\Rightarrow$ Pseudo-inverse of H

$$\therefore \quad \hat{S}_{4\times 1} = H^{\dagger}_{4\times 16} \cdot Y_{16\times 4}$$