Co-clustering and GMM fitting

- Inducing graph on SC, FC
- Joint spectrum
- GMM fitting

Problem of Co-clustering

- Nodes of a graph are linked to their neighbors in multiple ways.
- Each way becomes a layer of edges.
- Each layer provides a view of the graph.
- Combine information provided by each layer.

Joint Spectrum

- Combine information from multiple graph layers, with the help of a set of joint eigenvectors that are smooth on all the layers, hence capture all their characteristics.

- Find the joint spectrum using graph regularization framework.

- Find spectrum of the more informative layer, then try to regularize the spectrum to align with the calculated spectrum.

Equations for Joint spectrum

to induce a graph using FC, the magnitude of correlation was considered as similarity between ROIs
sign of correlation tells about exitatory or inhibititory nature of ROI

1. First find spectrum of SC u_i (1)
2. Laplacian of graph of FC (2)
3. regularize eigenvectors of FC with those of SC

Formulation

$$\arg\min_{f_i \in \mathbb{R}^n} \left\{ \frac{1}{2} \| f_i - u_i \|_2^2 + \lambda \cdot \Phi_{f_i} \right\} \quad \text{for} \quad i = 2, \ldots, n,$$

$$\Phi_{f_i} = f_i^T L_{\text{sym}}^{(2)} f_i$$

solution

$$f_i^* = \mu (L_{\text{sym}}^{(2)} + \mu I_n)^{-1} u_i,$$

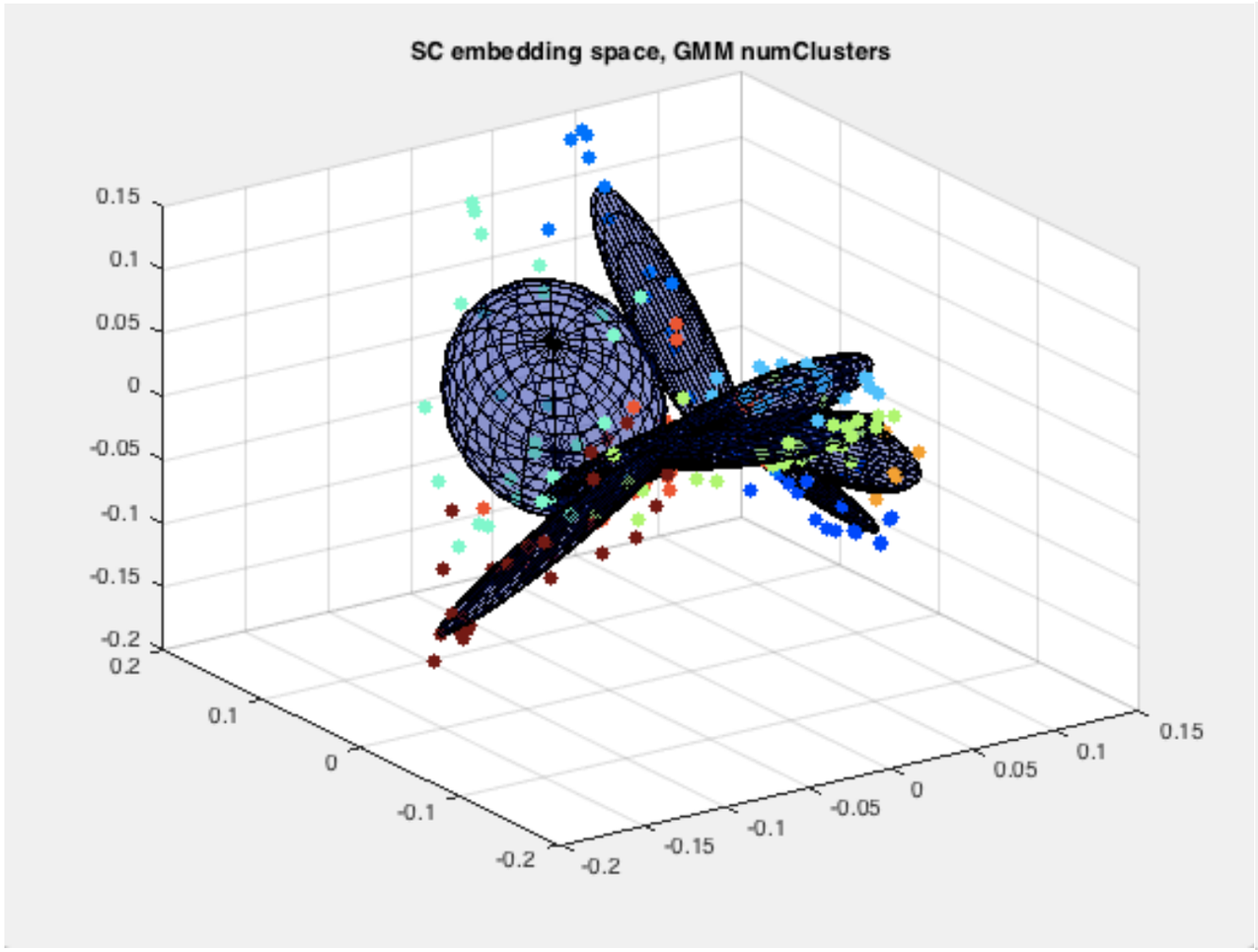
$$\mu = \tfrac{1}{\lambda}.$$

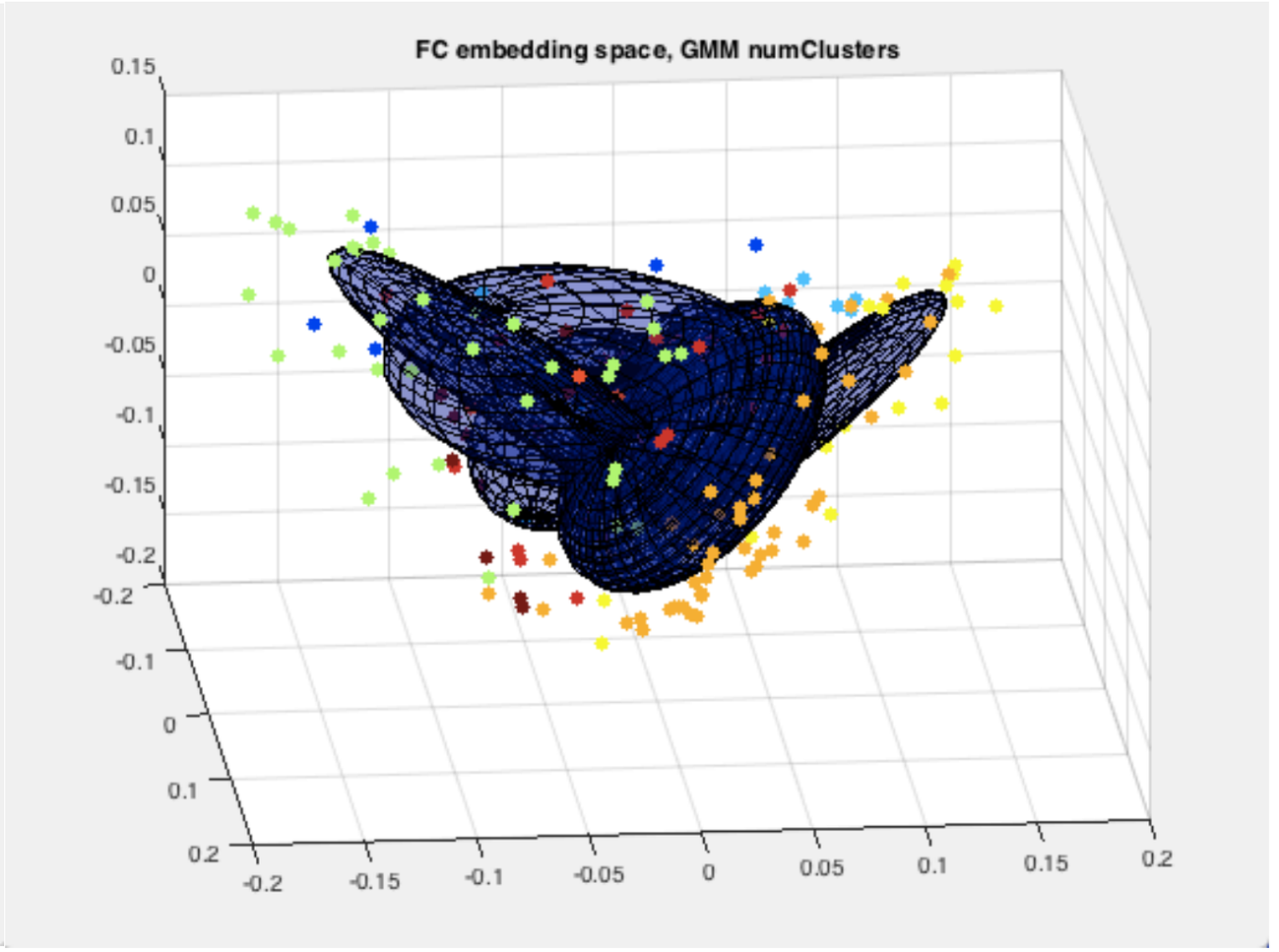
Algorithm 3: Clustering with spectral regularization (**SC-SR**)

1: **Input:**
$W^{(i)}$ $(i = 1, 2)$: $n \times n$ weighted adjacency matrices of two graph layers $\mathcal{G}^{(1)}$ and $\mathcal{G}^{(2)}$
k: Target number of clusters
2: For $\mathcal{G}^{(1)}$, compute the degree matrix $D^{(1)}$.
3: Compute the random walk graph Laplacian $L_{\mathrm{rw}}^{(1)} = (D^{(1)})^{-1}(D^{(1)} - W^{(1)})$.
4: Compute the first k eigenvectors $u_1, \ldots, u_k$ of $L_{\mathrm{rw}}^{(1)}$.
5: Let $U \in \mathbb{R}^{n \times k}$ be the matrix containing $u_1, \ldots, u_k$ as columns.
6: For $i = 2, \ldots, k$, solve the spectral regularization problem in **(11)** for each u_i and replace it with the solution f_i in U to form the new low dimensional embedding U''.
7: Let $y_i \in \mathbb{R}^k$ $(i = 1, \ldots, n)$ be the i-th row of U'' to represent the i-th vertex in the graph.
8: Cluster y_i in $\mathbb{R}^k$ into $C_1, \ldots, C_k$ using the k-means algorithm.
9: **Output:**
$C_1, \ldots, C_k$: The cluster assignment

Dong, Xiaowen, Pascal Frossard, Pierre Vandergheynst, and Nikolai Nefedov. "Clustering with multi-layer graphs: A spectral perspective." Signal Processing, IEEE Transactions on 60, no. 11 (2012): 5820-5831.

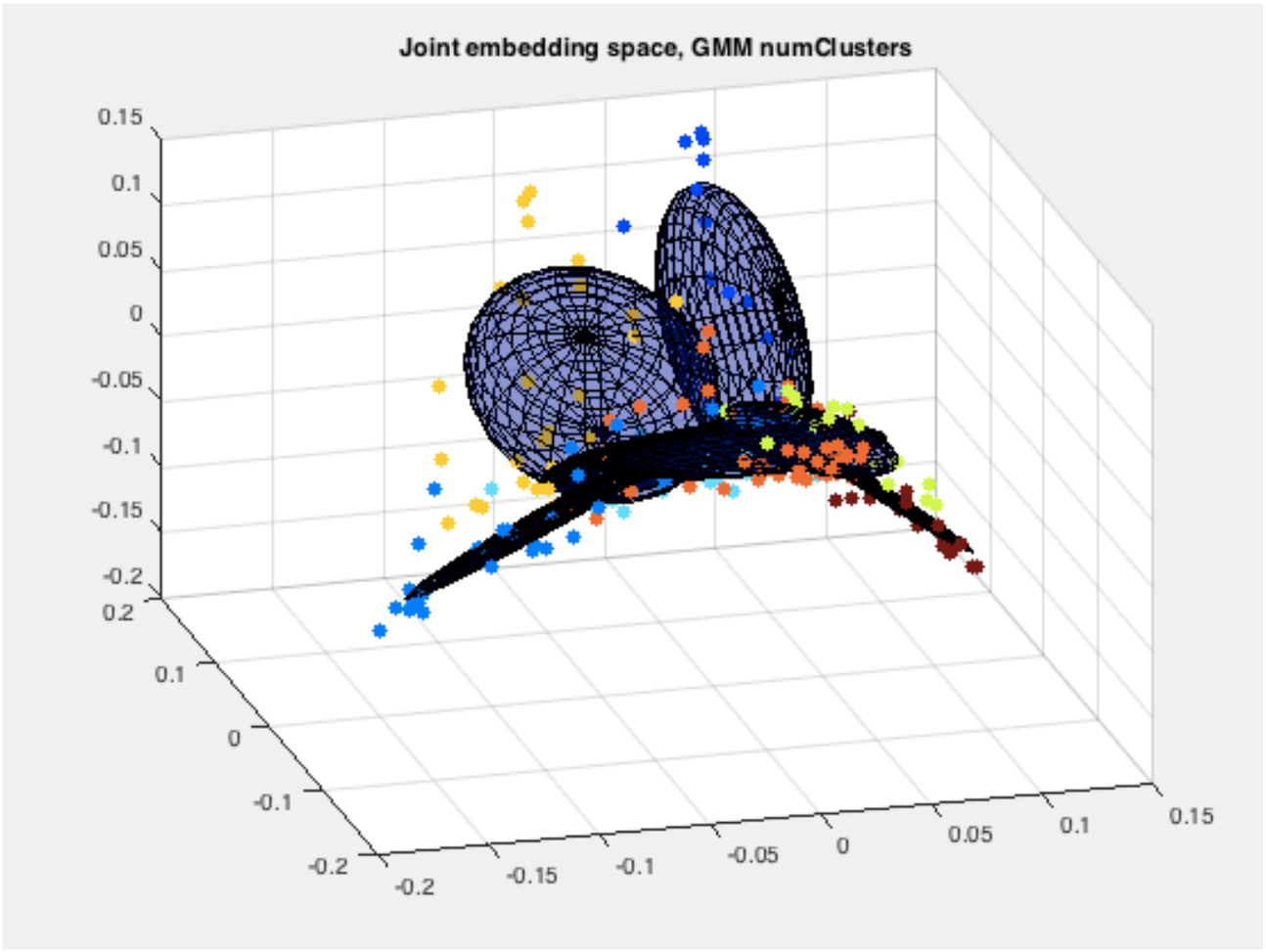
GMM ellipsoids





10 clusters

GMM ellipsoids, contd.



10 clusters

Similar to SC ellipsoids

But

Shape modified by FC ellipsoids also

combination depends on choice of regularization parameter μ