① save the following reccurence relation.

a) $x(n) = x(n-1) + 5$ for $n>1$ with $x(1) = 0$

1) write down the first two terms to identify the pattern

$x(1) = 0$

$x(2) = x(1) + 5 = 5$

$x(3) = x(2) + 5 = 10$

$x(4) = x(3) + 5 = 15$

2) identify the pattern (or) general item

→ The first item $x(1) = 0$

The common difference $d = 5$

The general formula for nth term of an Ap is.

$x(n) = x(n-1)d$

substituting the given values

$x(n) = 0 + (n-1) \cdot 5 = (n-1)$

The solution is

$x(n) = 5(n-1)$

b) $x(n) = 3x(n-1)$ Pon $n>1$ with $x(1) = 4$.

1) write down the first two terms to identify pattern

$x(1) = 4$

$x(2) = 3x(1) = 3 \cdot 4 = 12$

$x(3) = 3x(2) = 36$

$x(4) = 3x(3) = 108$

2) identify the general terms

→ The first term $x(1) = 4$

The common ratio $r = 3$.

The general formula for nth term of a gp is $x(n) = x(1) \cdot r^{n-1}$

substituting the given values.

$$x(n) = 4 \cdot 3^{n-1}$$

The solution is $x(n) = 4 \cdot 3^{n-1}$

c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$

For $n = 2^k$, we can write reccurence in terms of k

1) substitute $n = 2^k$ in the reccurence

$$x(2^k) = 3(2^{k-1}) + 2^k$$

2) write down the first few terms to identify pattern

$$x(1) = 1$$
$$x(2) = x(2^1) = x(1) + 2 = 1 + 2 = 3$$
$$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$$
$$x(8) = x(2^3) = x(4) + 8 = 7 + 8 = 15$$

3) identify the general term by finding the pattern

we observe that:

$$x(2^k) = x(2^{k-1}) + 2^k$$

we sum the series:-

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \cdots$$

since $x(1) = 12$

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \cdots$$

The geometric series with the term $a = 2$ and the last term 2^k except for the additional +1 term. The sum of geometric series s with ration $r = 2$ is given by $s = \frac{a^{r^n} - 1}{r - 1}$

where $a = 2$, $r = 2$ and $n = k$

lets determine the value of $\log_b a$

$$\log_b a = \log_3 2$$

using the properties of logarithm

$$\log_3 2 = \frac{\log 2}{\log 3}$$

now we compare $F(n) = cn$ with $\log_3 2$

$$F(n) = O(n)$$

$$n = n$$

since $\log_3 2$ we are in third case of master's theorem

$F(n) = O(n^c)$ with $c > \log_b a$

The solution is $T(n) = O(F(n)) = O(cn) = O(n)$

3. consider the following recurrence algorithm:

```
min(A(0... n-2)]
if n=1 return A[0]
else temp=min(A(0--- n-2)]
    if temp<=A(n-1) return temp
   else return A[n-1]
```

a) what does this algorithm compute?

The given algorithm min[A[0---n-2]) computes the minimum value in array 'A' from index '0' for n-1 it does this by recursively finding the minimum value in sub array A[0--- n-2] and then comparing it with the last element A[n-1] to determine the overall maximum value.

b. setup recurrence relation for algorithm basic count & solve it.

The solution is $T(n) = n$

This means the algorithm perform n basic operations for an input array of size n.

2. write down the first few terms to identify the pattern

$x(1) = 1$

$x(3) = 3^1 = x(1)+1 = 1+1 = 2$

$x(9) = x(3^2) = x(3)+1 = 2+1 = 3$

$x(27) = x(3^3) = x(9)+1 = 3+1 = 4$

3. identify the general term:

we observe that

$$x(3^k) = x(3^{k-1})+1$$

sum up the series

$x(3^k) = 1+1+1+---$

$x(3^k) = k+1$

The solution is $x(3^k) = k+1$

2. Evaluate following recurrence complexity.

i) $T(n) = T(n/2)+1$ where $n = 2k$ for all $k \geq 0$

The recurrence relation can be solved using iteration method.

1) substitute $n = 2^k$ in the recurrence

2) iterate the recurrence.

For $k=0$: $T(2^0) = T(1) = T(1)$

$k=1$: $T(2^1) = T(1)+1$

$k=2$: $T(2^2) = T(0) = T(n)+1 = T(1)+2+1 = T(1)+2$

$k=3$: $T(2^3) = T(8) = T(n)+1 = (T(1)+2)+1 = T(1)+3$.

3) generalize the pattern

$T(2^k) = T(1)+k$

since $n = 2^k$, $k = \log_2 n$

$T(n) = T(2^k) = T(1) + \log n$

4) Assume $T(1)$ is a constant c

$T(n) = c + \log_2 n$

The solution is $T(n) = O(\log n)$.

ii) $T(n) = T(n/3) + T(2n/3) + cn$ where c is constant and n is input size

The recurrence can be solved using the master's theorem for divide and conquer recurrence of the form

$$T(n) = aT(n/b) + f(n)$$

where $a=2, b=3$ and $f(n) = cn$

4) Analyze the order of growth

i) $f(n) = 2n^2+5$ and $g(n) = 7n$ use the $\Omega\, g(n)$ notation.

To analyze the order of growth and use the Ω notation, we need to compute the given function $f(n)$ and $g(n)$

given functions $f(n) = 2n^2+5$, $g(n) = 7n$

...er of growth using $\Omega(g(n))$ notation.

...he notation $\Omega(g(n))$ describes a lower bound on the growth rate that for sufficiently large $n \geq f(n)$, grows at least as far as $g(n)$

$$f(n) = c \cdot g(n)$$

less analyze $f(n) = 2n^2+5$ with respect to $g(n) = 7n$

1) identify dominant terms:-

-> The dominant term in $f(n)$ is $2n^2$ since it grows faster

-> The dominant term in $f(n)$ is $2n^2$, $g(n)$ is $7n$

...) establish the inequality.

-> we want to find constants c and n_0 such that:

$$2n^2+5 \geq c \cdot 7n \text{ for all } n \geq n_0$$

3) simplify the inequality

-> ignore the lower order teams for larger

$2n^2 \geq 7cn$

Divide both sides by n

$2n \geq 7c$

solve for n:

$n \geq 7c/2$

4. choose constant's

let $c=1$, $n \geq \frac{7 \cdot 1}{2} = 3.5$

for $n \geq n$, the inequality holds:

$2n^2 + 5 \geq 7n$ for all $n \geq n_0$

we have shown that there exist constant $c=1$ and $n_0 = n$ such that for all $n \geq n_0$

$2n^2+5 \geq 7n$

Thus we can conclude that :

$F(n) = 2n^2+5 = \Omega(7n)$

in Ω notation, the dominant term $2n^2$ in $f(n)$ clearly grows faster than n hence $F(n) = \Omega(n^2)$

However, for the specific comparison asked $F(n) = \Omega(7n)$ is also correct showing that $F(n)$ grows atleast as fast as $7n$