Assignment-9: PROBABILITY AND MONTO CARLO METHODS

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AIM

To generate a random list of numbers and study probability distribution and its applications like integration using monte carlo method.

Problem statement

part a: pseudo random numbers and probability distribution:
1) generate a random set of real numbers and plot its histogram
2) take any two distributions and plot its histogram and find its mean, variance, covariance, skewness and kurtosis.
3)verify central limit theorem
part b: Application of probability distribution
1) calculate the value of pi
2) perform a single integral using monte carlo method
3) perform a double integral using monte carlo method

part c:
1)comment on result
2) Do you think the values of the integral calculated in Q1 (or Q2) of Part B will obey central limit theorem? Substantiate your response.
3) Read about Brownian motion and Random walk and comment.

code organisation

PART A
1)generating random integer list
2)generating two distribution and calculating mean ,variance, covariance, skewness, kurtosis
3)verifying central limit theorem
PART B
1)calculating pi value

2)single integral using monte carlo method

3)double integral using monte carlo method

computational code

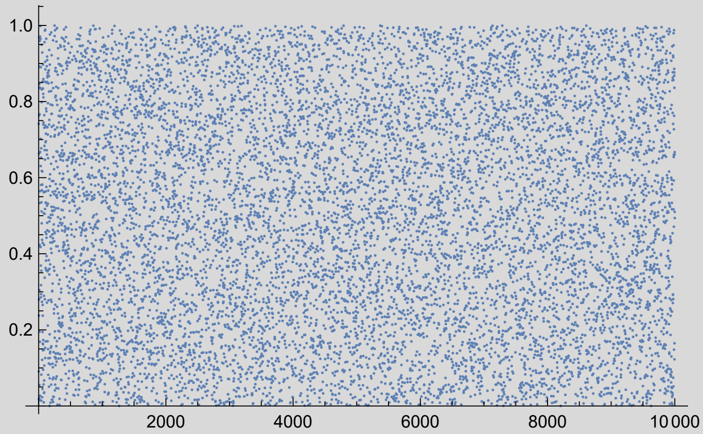
Part-a

generating random integer list

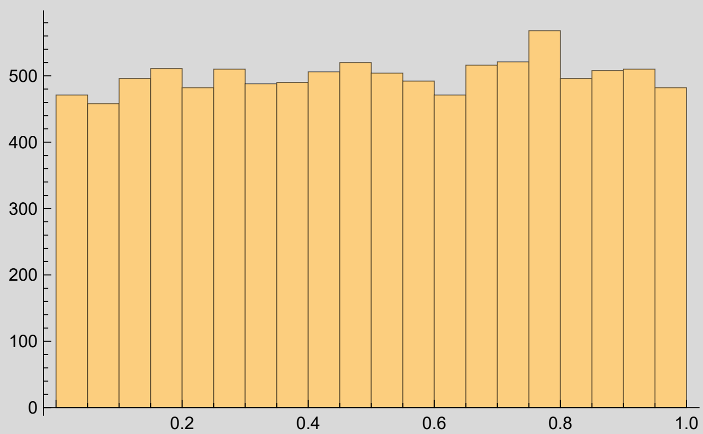
In[105]:=

```
Clear["Global`*"]
ranClf[n_] := Module[{x},
  RecurrenceTable[{x[j + 1] == Abs[Mod[100 Log[x[j]], 1]], x[0] == 0.1}, x, {j, 1, n}]]
ap = ranClf[10 000];
ListPlot[ap]
Histogram[ap]
```

Out[108]=



Out[109]=

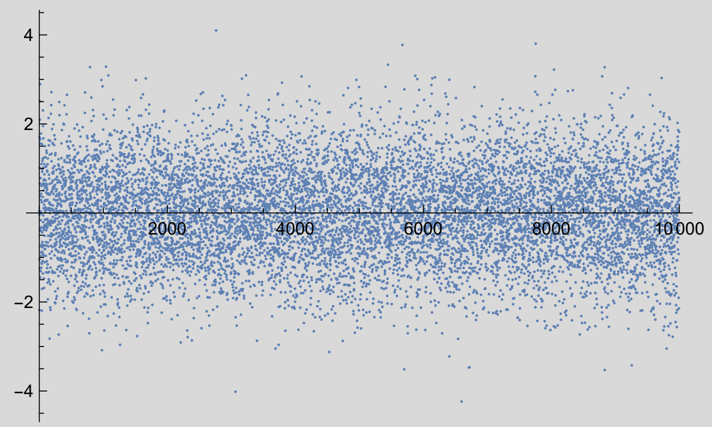


generating two distribution and calculating mean ,variance, covariance, skewness, kurtosis

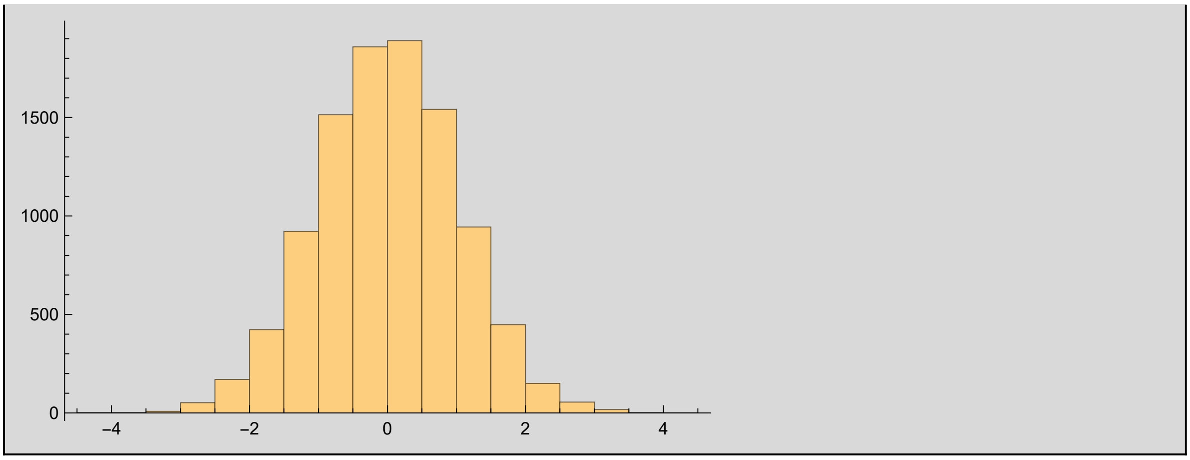
In[87]:=

```
SeedRandom[1243]; (*Resets the seed*)
d1 = RandomVariate[NormalDistribution[0, 1], 10^4];
ListPlot[d1]
Histogram[d1]
Mean[d1]
Variance[d1]
Covariance[d1]
Kurtosis[d1]
Skewness[d1]
SeedRandom[12 345];
d2 = RandomVariate[ExponentialDistribution[2], 10^4];
ListPlot[d2]
Histogram[d2]
Mean[d2]
Variance[d2]
Covariance[d2]
Kurtosis[d2]
Skewness[d2]
```

Out[89]=



Out[90]=



Out[91]=

0.00943222

Out[92]=

1.01312

Out[93]=

1.01312

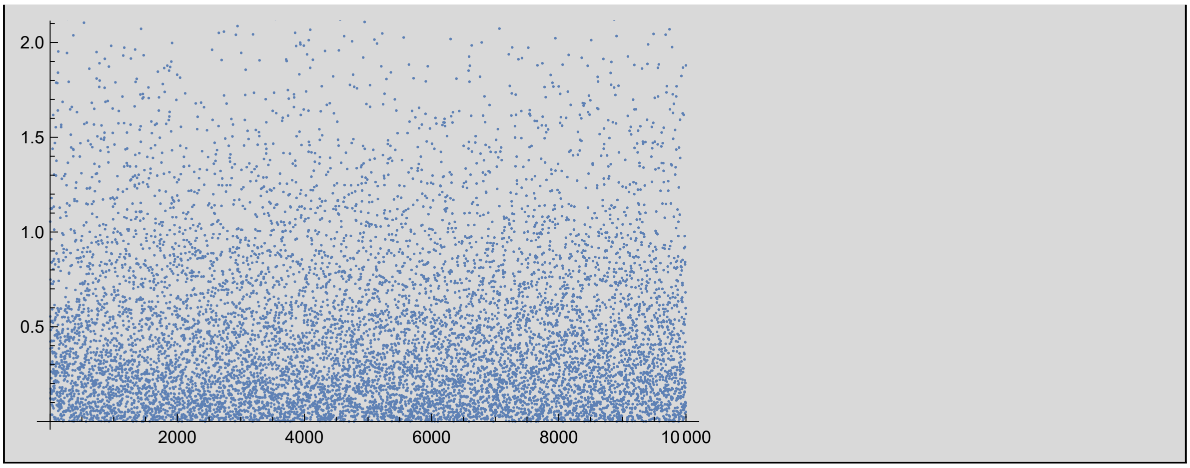
Out[94]=

3.01875

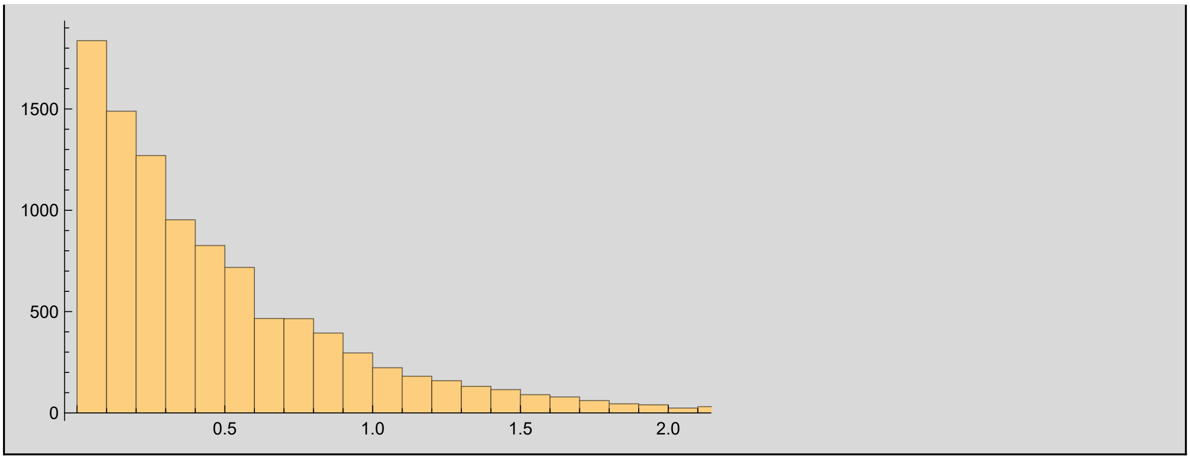
Out[95]=

-0.000762521

Out[98]=



Out[99]=



Out[100]=

0.490789

Out[101]=

0.237728

Out[102]=

0.237728

Out[103]=

8.63159

Out[104]=

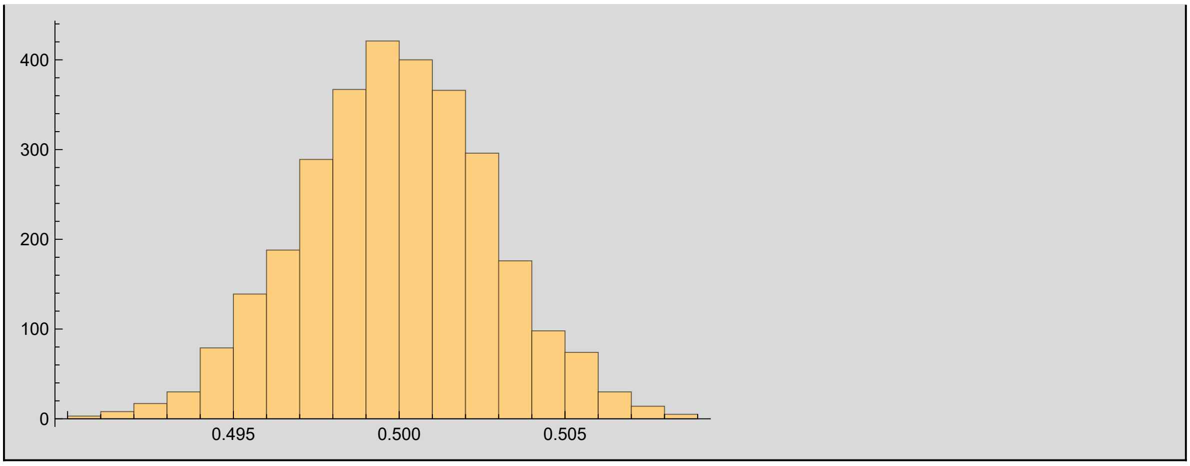
1.9502

verifying central limit theorem

In[82]:=
```
d3[n_] := RandomVariate[UniformDistribution[{0, 1}], n]
j = 1;
s = {};

While[j ≤ 3000,
 AppendTo[s, Mean[d3[10 000]]];
 j++]
Histogram[s]
```

Out[86]=



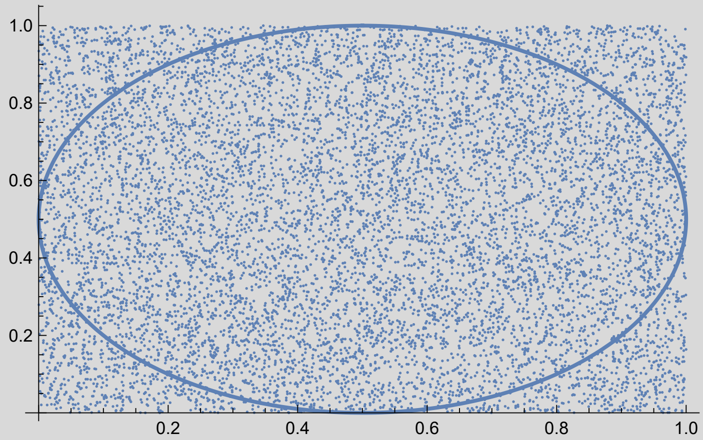
part b

calculating pi value

In[110]:=

```
dat = RandomReal[{0, 1}, {10000, 2}];
a = ListPlot[dat];
b = ParametricPlot[{1 / 2 + Sin[t] / 2, 1 / 2 + Cos[t] / 2}, {t, 0, 2 π}];
Show[a, b]
j = 1;
co = 0;
While[j ≤ 10000,
 If[dat[[j]][[1]] ^2 + dat[[j]][[2]] ^2 ≤ 1, co = co + 1, Indeterminate];
 j++
]
pi = 4.0 * co / 10000;
Print["The estimated value of π is: ", pi]
```

Out[113]=




```
The estimated value of π is: 3.108
```

single integral using monte carlo method

In[119]:=

```
f[x_] = e^x
points = RandomReal[{1, 10}, 10^4];
fs = f[points];
in = (10 - 1) Mean[fs];
Print[" ∫_1^10 e^x dx : ", in]
```

Out[119]=

e^x

$\int_1^{10}$ e^x dx : 22074.

double integral using monte carlo method

In[124]:=

```
f[x_, y_] := x^2 + Sin[y]
pointsx = RandomReal[{1, 10}, 10^4];
pointsy = RandomReal[{0, 1}, 10^4];
fs1 = f[pointsx, pointsy];
ind = (10 - 1) (1 - 0) Mean[fs1];
Print["∫_0^1 ∫_1^10 (x^2+siny) ⅆxⅆy: ", ind]
```

$\int_0^1 \int_1^{10}$ (x^2+siny) ⅆxⅆy: 338.693

RESULT

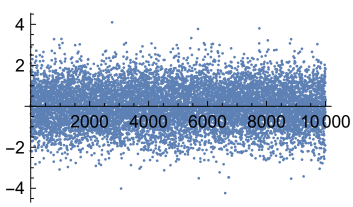
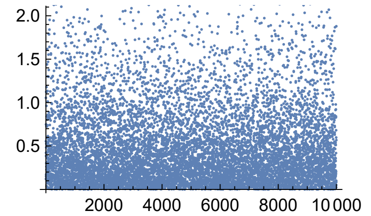
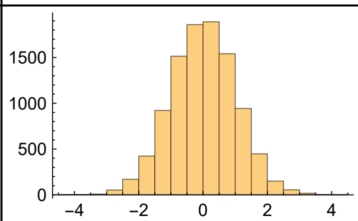
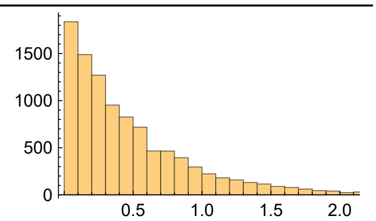
PART A:

1)generated list of real numbers

a)list log plot:output 108

b)histogram: output 109

2)

□	normal distribution	exponential distribution
list plot		
histogram		
mean	0.00943222	0.490789
variance	1.01312	0.237728
covariance	1.01312	0.237728
skewness	3.01875	8.63159
kurtosis	-0.000762521	1.9502

3)central limit theorem verification: output 86

Here i took a uniform distribution. It followed central limit theorem and gave a normal distribution after computing many means.

PART B:

1)The estimated value of π is: 3.108(output 113)

2)single integral using monte carlo method: output 119

3)double integral using monte carlo method: input 124

DISCUSSION AND COMMENTS

1) This assignment was useful in learning about probability distributions. We also performed certain applications of it. The value of estimated pi and the single and double integral were very close to the analytical values. This will be useful in performing complex calculations. It will also be useful in other fields like statistical analysis and machine learning.
2)Do you think the values of the integral calculated in Q1 (or Q2) of Part B will obey central limit theorem? Substantiate your response.
The Central Limit Theorem states that the distribution of the sum (or average) of a large number of independent, identically distributed random variables approaches a normal distribution, regardless of the underlying distribution of those random variables. As the monte carlo integration method does calculation using collection of the value of a function using random variables, integration will be a operation on large number of random data. Hence central limit theorem will be obeyed by the values of integral calculated above. If we calculate the integral for large number of times and plot the histogram, we will get a normal distribution.
3)Read about Brownian motion and Random walk and comment.
Brownian motion: It is the random change in the coordinates of a particle within a fluid. It is defined as the random motion of particles within the fluid.
Random Walk: It is a mathematical concept which defines a series of steps or movement are done where each step is decided randomly. Random walks are used to model various phenomena in science, mathematics, finance, and other fields.
Both Brownian motion and random walk are a concept based on random variables.

REFERENCES

MATHEMATICA DOCUMENTATION
MATHEMATICA STACK EXCHANGE