Assignment-6: ON PHASE TRAJECTORIES

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AIM

To plot the phase trajectories for the given potential

problem statement

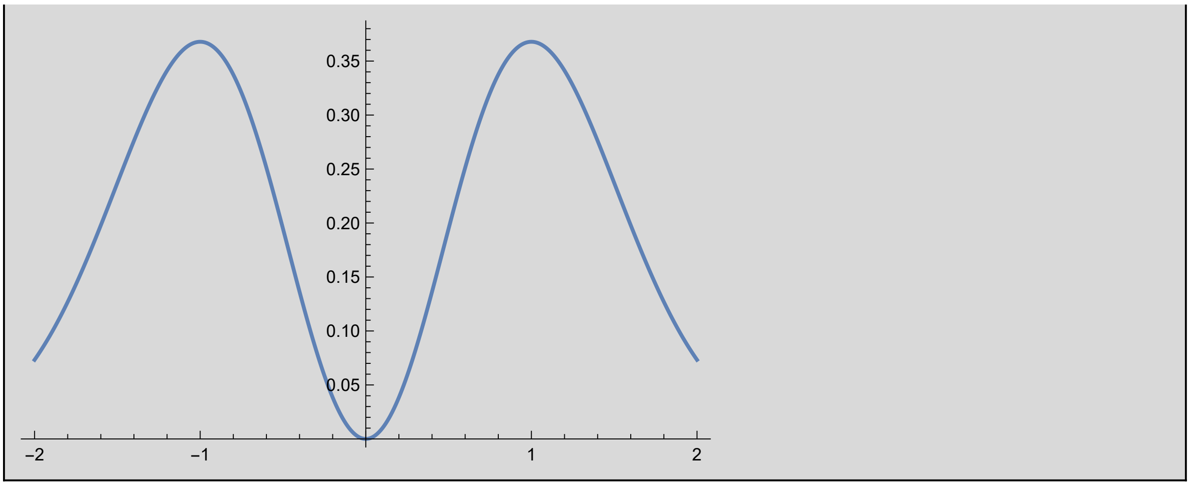
1)plot the given potential energy and find its critical points
2)taylor approximate the function around its critical point and plot its phase portrait around its critical point.
3)draw the full phase portrait
4)find the force and solve equation of motion and plot phase trajectory
5)write equation of motion as two first order coupled differential equation and comment on result.

code organisation

computational code

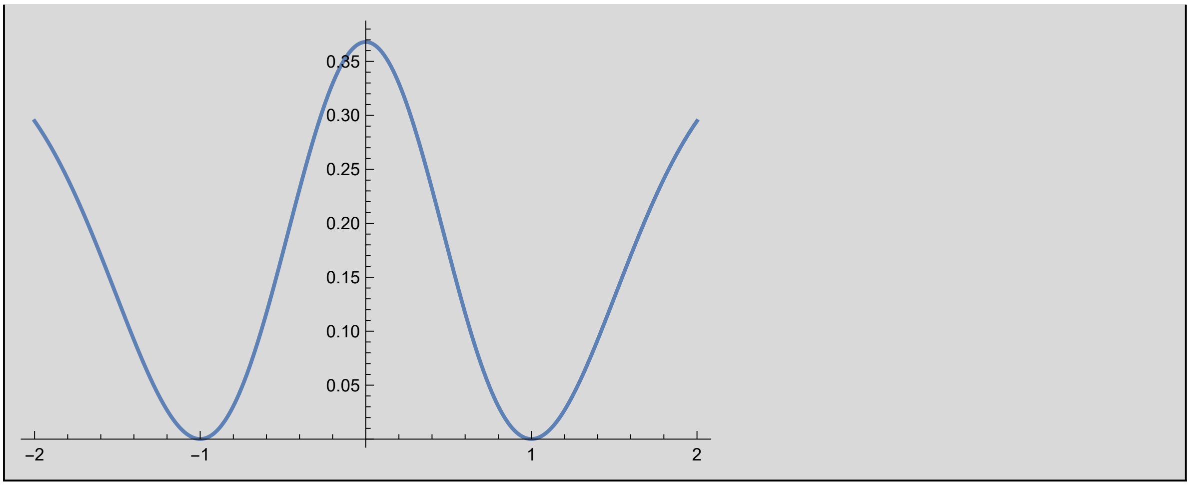
In[14]:=
```
u[x_] := x^2 ⅇ^(-x^2);
data = Table[{x, u[x]}, {x, -2, 2, 0.01}];
ListLinePlot[data]
```

Out[16]=



In[ ]:=
```
k[x_] := 368 / 1000 - u[x]
data1 = Table[{x, k[x]}, {x, -2, 2, 0.01}];
ListLinePlot[data1]
```

Out[ ]=



In[ ]:=
```
u1[x_] = D[u[x], x]
```

Out[ ]=

$$2\, e^{-x^2} x - 2\, e^{-x^2} x^3$$

In[ ]:=
```
cp = SolveValues[Evaluate[u1[x] == 0], x]
```

Out[ ]=

```
{-1, 0, 1}
```

In[17]:=
```
e0[x] := Series[u[x], {x, 0, 2}]
e0[x]
e01[x]:=Normal[e0[x]]
e01[x]
```

Out[18]=

$$x^2 + O[x]^3$$

Out[20]=

$$x^2$$

In[ ]:=
```
e1[x] := Series[u[x], {x, 1, 2}]
e1[x]
e11[x] := Normal[e1[x]]
e11[x]
```

Out[ ]=

$$\frac{1}{e} - \frac{2 (x - 1)^2}{e} + O[x - 1]^3$$

Out[ ]=

$$\frac{1}{e} - \frac{2 (-1 + x)^2}{e}$$

In[ ]:=
```
e2[x] := Series[u[x], {x, -1, 2}]
e2[x]
e12[x] := Normal[e2[x]]
e12[x]
```

Out[ ]=

$$\frac{1}{e} - \frac{2 (x + 1)^2}{e} + O[x + 1]^3$$

Out[ ]=

$$\frac{1}{e} - \frac{2 (1 + x)^2}{e}$$

In[ ]:=
```
t[x, v] = 1 / 2 v^2 + e01[x]
```

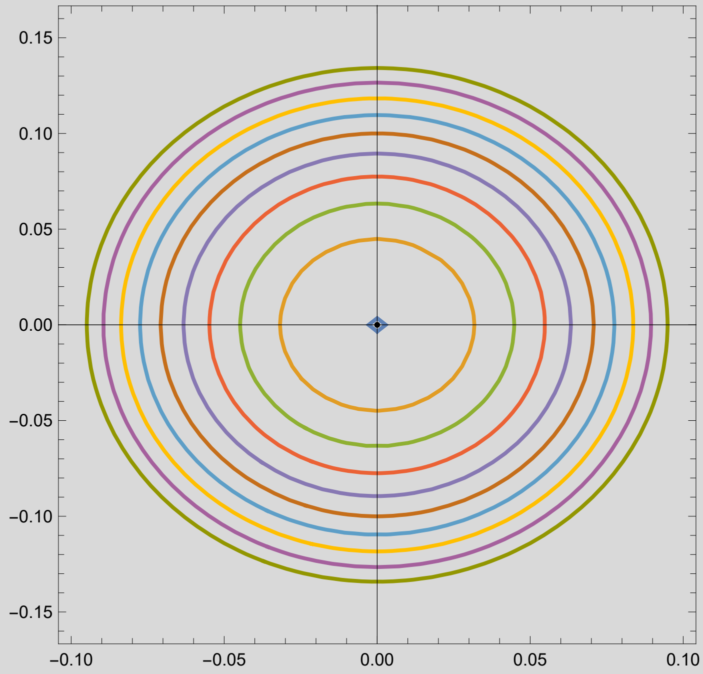
Out[ ]=

$$\frac{v^2}{2} + x^2$$

In[5]:=

In[ ]:=
```
data2 = Table[t[x, v] == a, {a, 0.00001, 0.01, 0.001}];
cp0 = ContourPlot[Evaluate[data2], {x, -0.1, 0.1}, {v, -0.16, 0.16}];
axes0 = {Line[{{0, -0.6}, {0, 0.6}}], Line[{{-0.5, 0}, {0.5, 0}}]};
axNr0 = Graphics[{Thin, Black, axes0}];
p = Graphics[{Point[{0, 0}]}];
Show[cp0, axNr0, p]
```

Out[ ]=

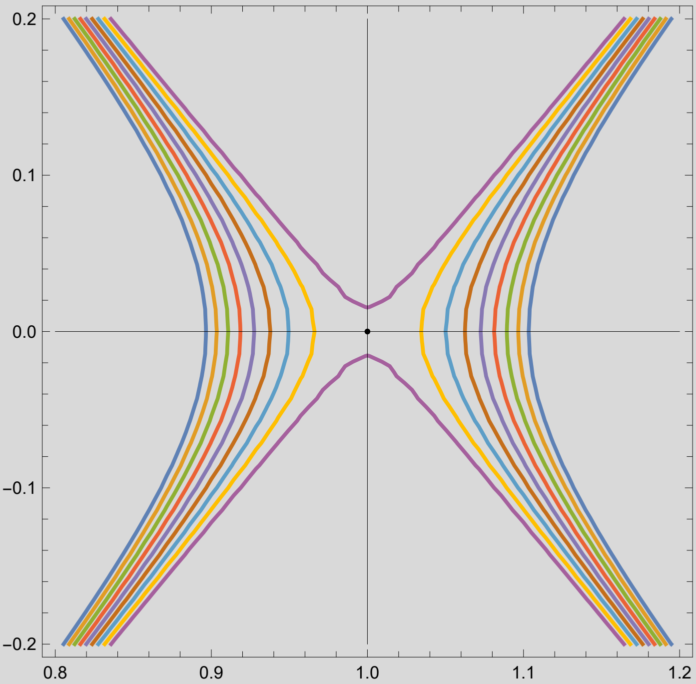


In[ ]:=
```
t3[x_, v_] = 1 / 2 (v) ^2 + e11[x]
data3 = Table[t3[x, v] == a, {a, 0.360, 0.368, 0.001}];
cp1 = ContourPlot[Evaluate[data3], {x, 0.8, 1.2}, {v, -0.2, 0.2}];
axes0 = {Line[{{1, -0.2}, {1, 0.2}}], Line[{{0.8, 0}, {1.2, 0}}]};
axNr0 = Graphics[{Thin, Black, axes0}];
p = Graphics[{Point[{1, 0}]}];
Show[cp1, axNr0, p]
```

Out[ ]=

$$\frac{1}{e} + \frac{v^2}{2} - \frac{2\,(-1+x)^2}{e}$$

Out[ ]=

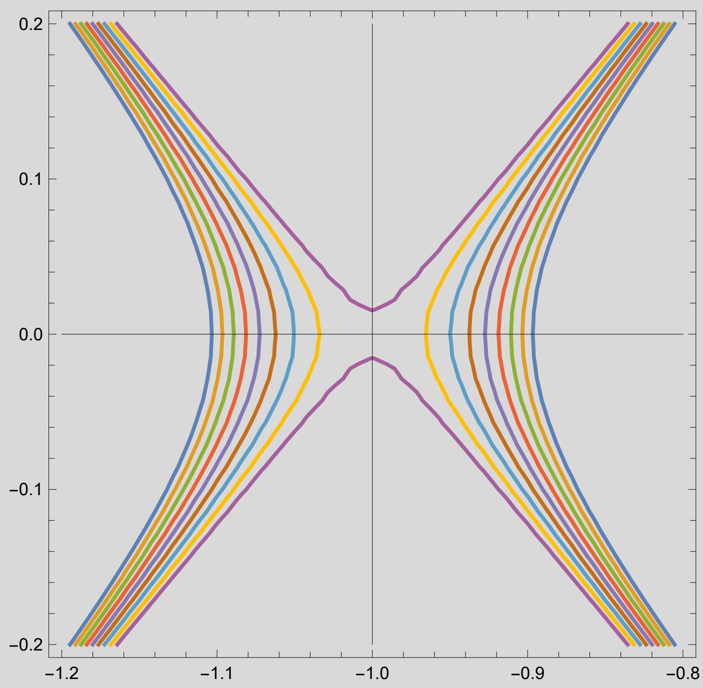


In[•]:=
```
t4[x_, v_] = 1 / 2 (v) ^2 + e12[x]
data4 = Table[t4[x, v] == a, {a, 0.360, 0.368, 0.001}];
cp2 = ContourPlot[Evaluate[data4], {x, -0.8, -1.2}, {v, -0.2, 0.2}];
axes0 = {Line[{{-1, -0.2}, {-1, 0.2}}], Line[{{-0.8, 0}, {-1.2, 0}}]};
axNr0 = Graphics[{Thin, Black, axes0}];
p = Graphics[{Point[{1, 0}]}];
Show[cp2, axNr0, p]
```

Out[•]=

$$\frac{1}{e} + \frac{v^2}{2} - \frac{2\,(1+x)^2}{e}$$

Out[•]=



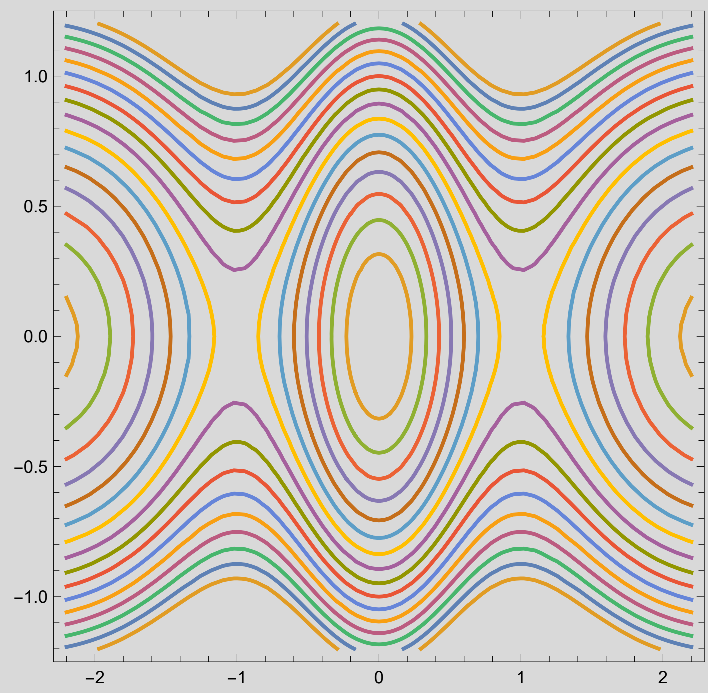
In[32]:=
```
t2[x_, v_] = 1 / 2 (v) ^2 + u[x]
```

Out[32]=

$$\frac{v^2}{2} + e^{-x^2} x^2$$

In[33]:=
```
data3 = Evaluate[Table[t2[x, v] == b, {b, 0, 0.8, 0.05}]];
sh7 = ContourPlot[Evaluate[data3], {x, -2.2, 2.2}, {v, -1.2, 1.2}]
```

Out[34]=



In[21]:=
```
f[x_] = -D[u[x], x]

soln = NDSolve[{x''[t] + 2 e^(-x[t]^2) x[t] - 2 e^(-x[t]^2) x[t]^3 == 0, x[0] == 0, x'[0] == 0.4},
  x[t], {t, 0, 100}]

s[t_] = x[t] /. Flatten[soln]
```

Out[21]=

$$-2\, e^{-x^2} x + 2\, e^{-x^2} x^3$$

Out[22]=

{{x[t] → InterpolatingFunction[Domain: {{0., 100.}} Output: scalar][t]}}

Out[23]=

InterpolatingFunction[Domain: {{0., 100.}} Output: scalar][t]

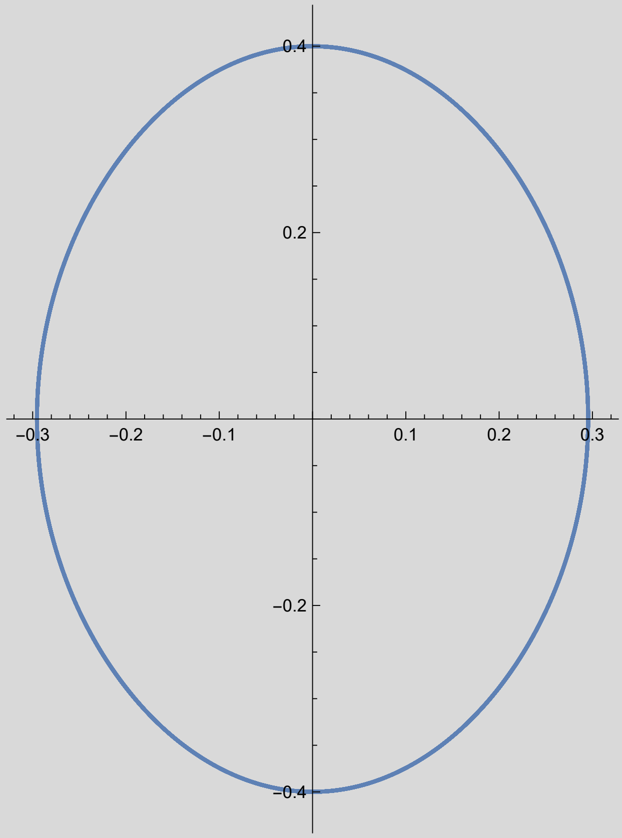
In[6]:=
```
soln1 = NDSolve[{x''[t] + 2 e^(-x[t]^2) x[t] - 2 e^(-x[t]^2) x[t]^3 == 0, x'[0] == 0.4, x[0] == 0},
  x'[t], {t, 0, 100}]
s2[t_] = x'[t] /. Flatten[soln1]
```

Out[6]= {{x′[t] → InterpolatingFunction[Domain: {{0., 100.}} Output: scalar][t]}}

Out[7]= InterpolatingFunction[Domain: {{0., 100.}} Output: scalar][t]

```
sh3 = ParametricPlot[{s[t], s2[t]}, {t, 0, 100}, PlotRange → All]
```

Out[13]=



In[•]:=
```
t5[v_, x_] = 1 / 2 (v) ^2 + u[x]
```

Out[•]= $\frac{v^2}{2} + e^{-x^2} x^2$

In[∘]:=
```
eq1 = dv / dt == D[t5[v, x], x]
```
Out[∘]=

$$\frac{dv}{dt} == 2\, e^{-x^2}\, x - 2\, e^{-x^2}\, x^3$$

In[∘]:=
```
eq2 = dx / dt == D[t5[v, x], v]
```
Out[∘]=

$$\frac{dx}{dt} == v$$

In[∘]:=
```
eq1 // Simplify
```
Out[∘]=

$$v == \frac{dv}{dt}$$

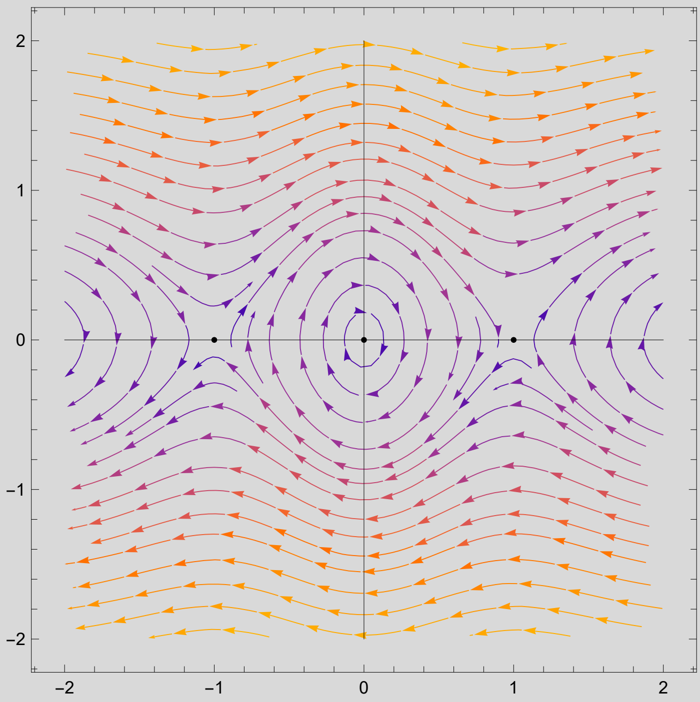
In[∘]:=
```
eq2 // Simplify
```
Out[∘]=

$$\frac{dx}{dt} == -2\, e^{-x^2}\, x\, \left(-1 + x^2\right)$$

In[24]:=

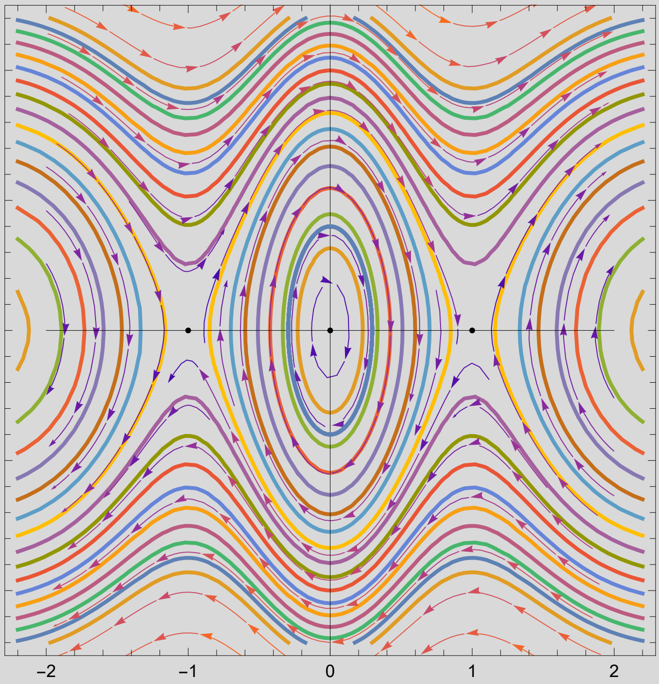
```
field = {-v, 2 ⅇ^(-x^2) x - 2 ⅇ^(-x^2) x^3};
Stp = StreamPlot[{v, -(2 ⅇ^(-x^2) x - 2 ⅇ^(-x^2) x^3)}, {x, -2, 2}, {v, -2, 2}];
axes0 = {Line[{{0, 2}, {0, -2}}], Line[{{2, 0}, {-2, 0}}]};
axNr0 = Graphics[{Thin, Black, axes0}];
p = Graphics[{Point[{1, 0}]}];
p1 = Graphics[{Point[{0, 0}]}];
p2 = Graphics[{Point[{-1, 0}]}];
sh8 = Show[Stp, axNr0, p, p1, p2]
```

Out[31]=



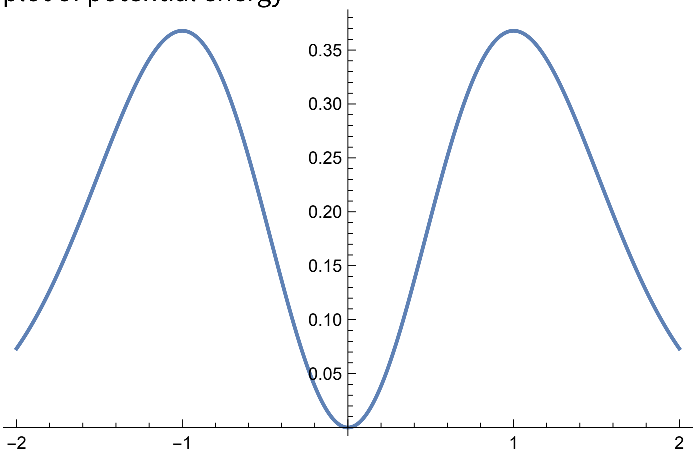
In[35]:=
```
Show[sh7, sh8, sh3]
```

Out[35]=

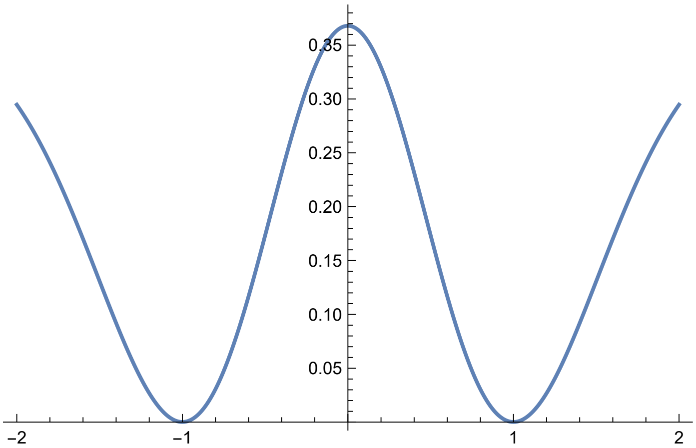


result:

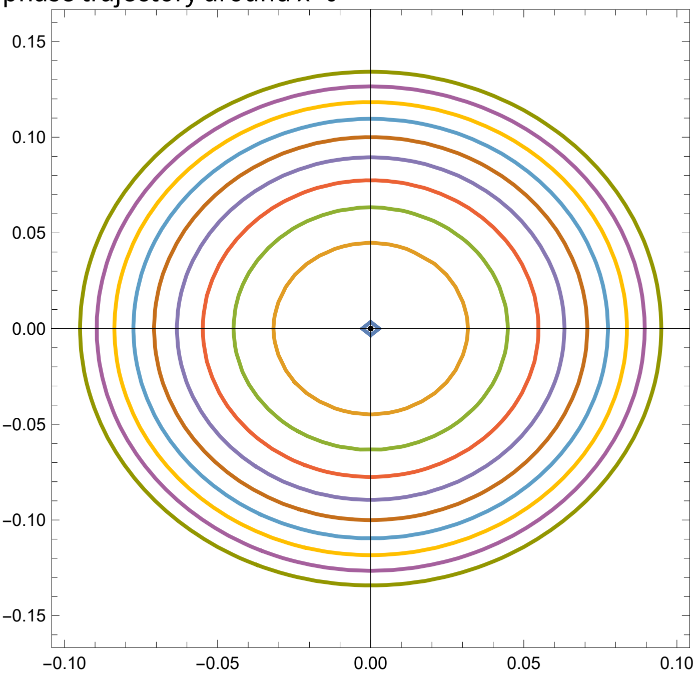
plot of potential energy



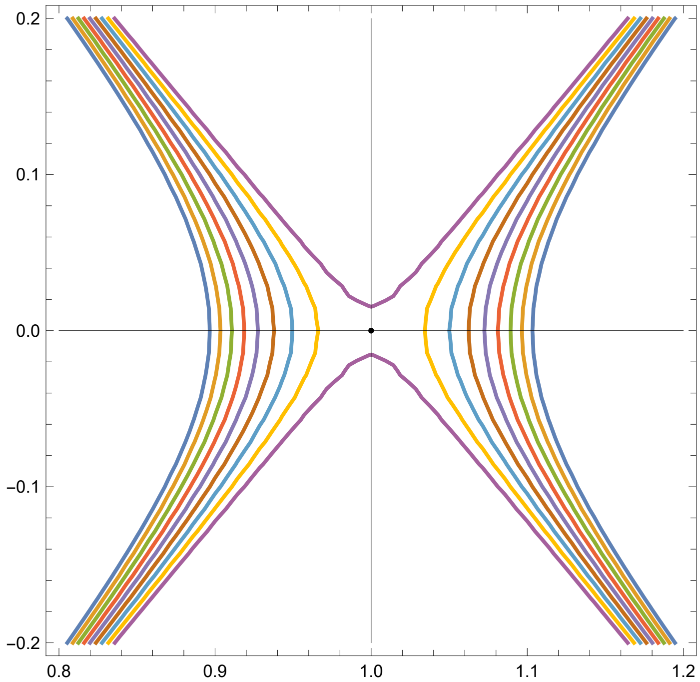
plot of kinetic energy



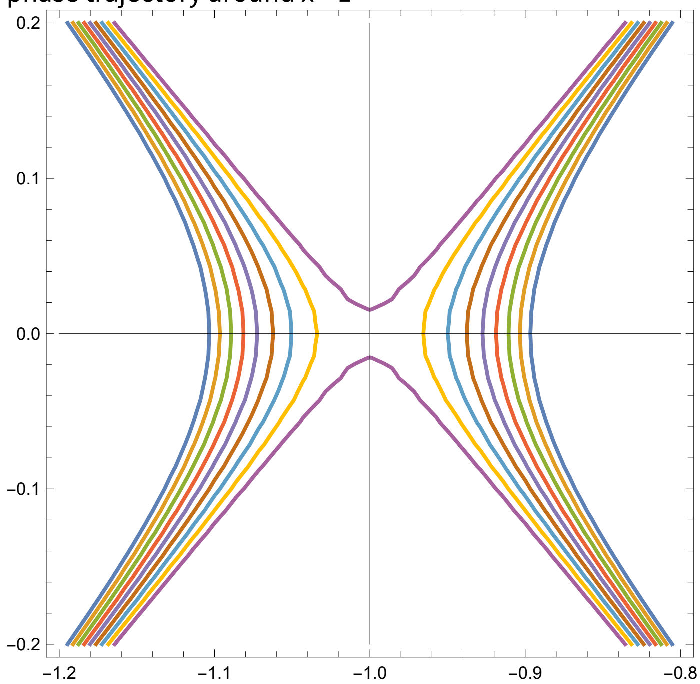
phase trajectory around x=0



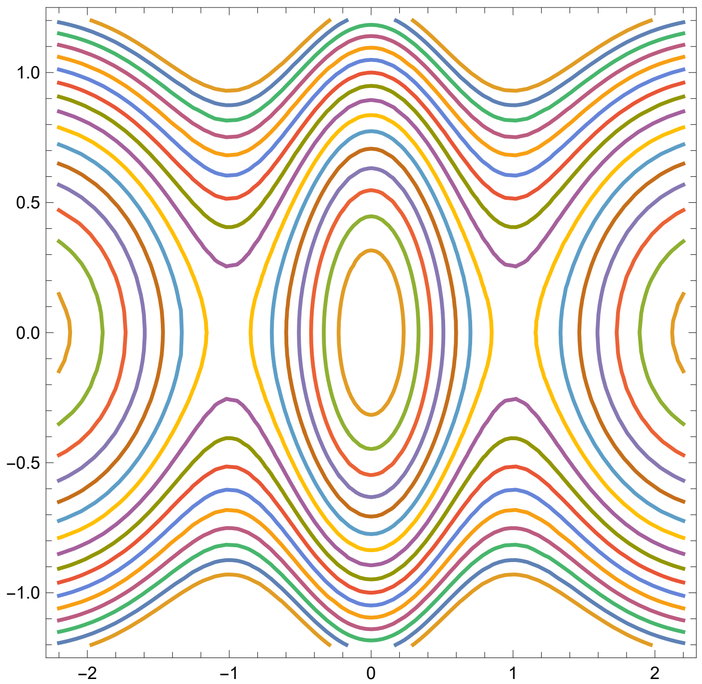
phase trajectory around x-1



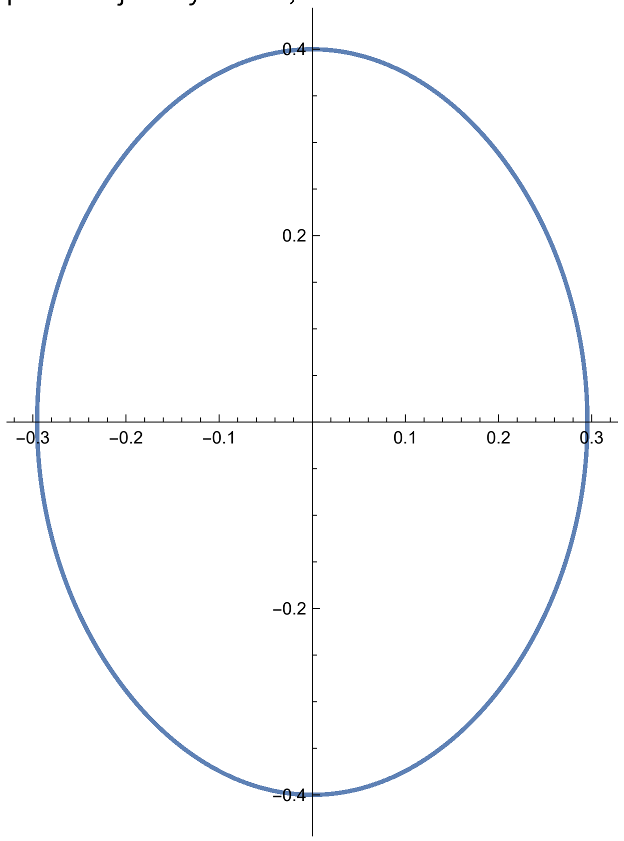
phase trajectory around x=-1



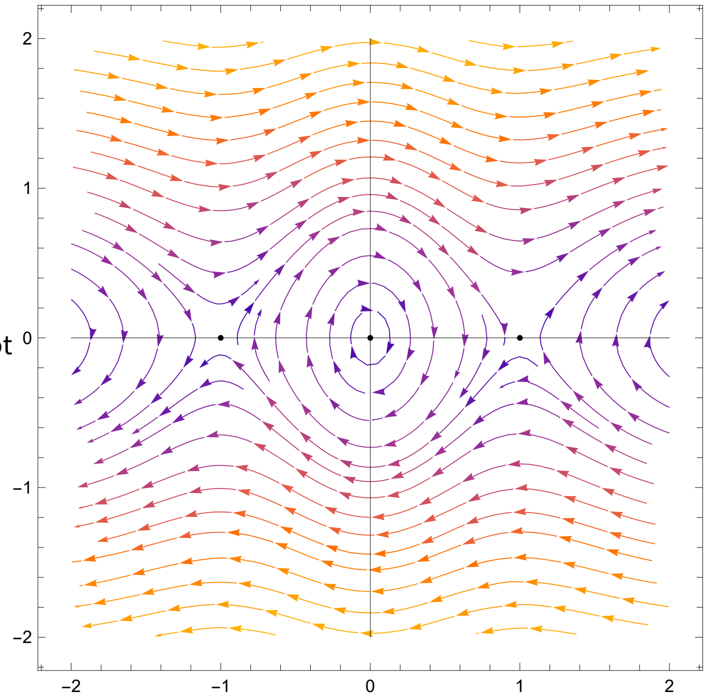
phase trajectory



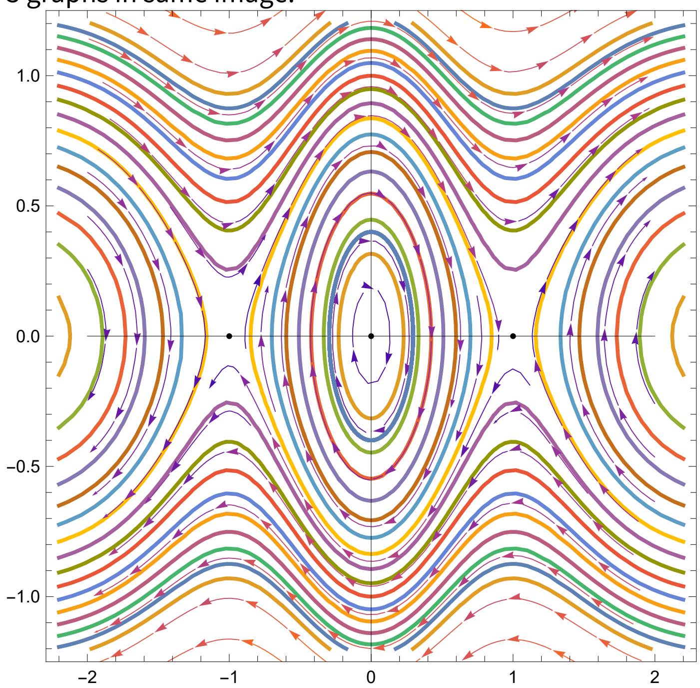
phase trajectory for x=0,v=0.4 at t=0



streamline plot



3 graphs in same image:



comments and discussion

In this assignment i learnt to plot the phase trajectories. The points of stable equilibrium is x=0. The phase trajectory around it is concentric ellipse in clockwise direction. The points of unstable equilibrium is x=+1 or x=-1. The phase trajectory around those points are hyperbola.if the kinetic energy of the particle is greater 0.368 the particle never comes to rest. I took a case where at x=0,v=0.4. The phase trajectory for this is an ellipse travelling in clockwise direction.

references

mathematica stack exchange
mathematica documentation