ASSIGNMENT-8: RESPONSE OF LINEAR SYSTEM TO ARBITRARTY INPUT

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AIM

To find the responses of different linear systems to an arbitrary input.

PROBLEM STATEMENT

PART A:
We are required to take a rc circuit and the give various inputs like step input, piecewise continuous input and record its responses. Then we required to provide a casual square pulse with vary small width. Record its impulse response as h(t). Find transfer function h(w) .
PART B:
We are required to take a lcr circuit and first record its impulse response for a unitbox pulse. Then we are required to give a casual square input and record its response and then give some arbitrary function and find its response.

code organisation

response=charge on capacitor

Part a: rc circuit with r=500000 c=8*10^-6
1)generating a unit step function
2)solving and getting response for the step input
3)generating piecewise function
4)getting response for piecewise function
5)generating casual square function with small width
6)generating response for casual square function
7)defining h[w]
8)plotting absolute value, phase angle, real part, imaginary part of h[w] vs w.

part b:

1)generating a unitbox and getting impulse response from lcr circuit

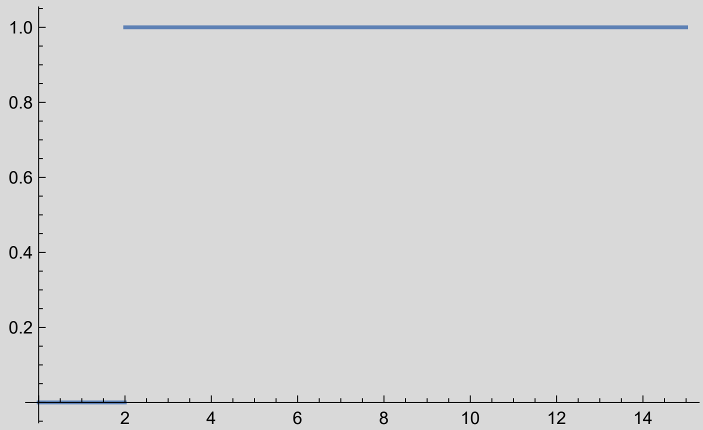
2)generating a casual square function and getting its response.

3)getting lcr circuit response for piecewise continuous input signal.

computational code

In[35]:=
```
v[t_] = UnitStep[t - 2];
Plot[v[t], {t, 0, 15}]
```

Out[36]=



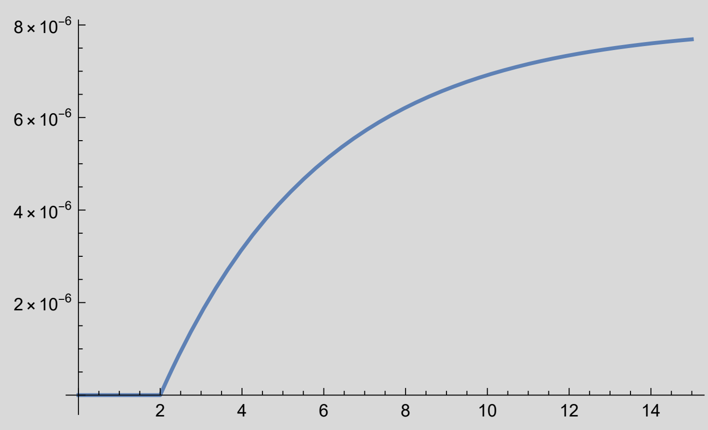
In[38]:=
```
a = NDSolve[{500000 q'[t] + q[t] / (8 * 10^-6) == v[t], q[0] == 0}, q[t], {t, 0, 15}]
```

Out[38]=
```
{{q[t] → InterpolatingFunction[ Domain: {{0., 15.}} Output: scalar ][t]}}
```

In[39]:=
```
Evaluate[q[t] /. a]
```

Out[39]=
```
{InterpolatingFunction[ Domain: {{0., 15.}} Output: scalar ][t]}
```

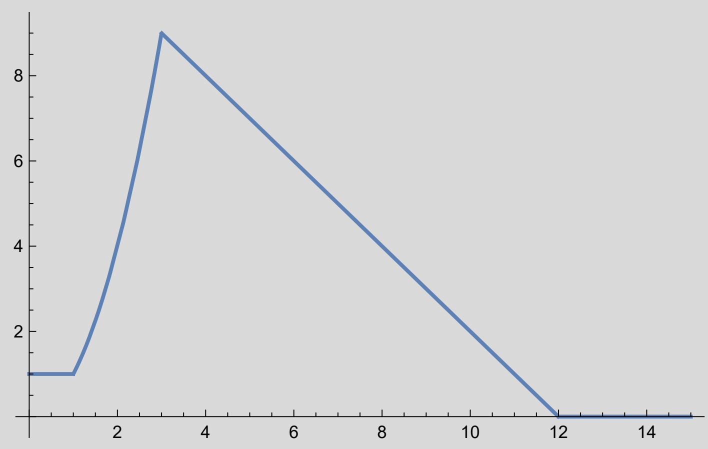
In[40]:=
```
Plot[Evaluate[q[t] /. a], {t, 0, 15}]
```
Out[40]=



In[50]:=
```
v1[t_] = Piecewise[{{1, t ≤ 1}, {t^2, t ≤ 3}, {12 - t, t ≤ 12}, {0, t ≤ 15}}]
```
Out[50]=

$$\begin{cases} 1 & t \le 1 \\ t^2 & t \le 3 \\ 12 - t & t \le 12 \\ 0 & \text{True} \end{cases}$$

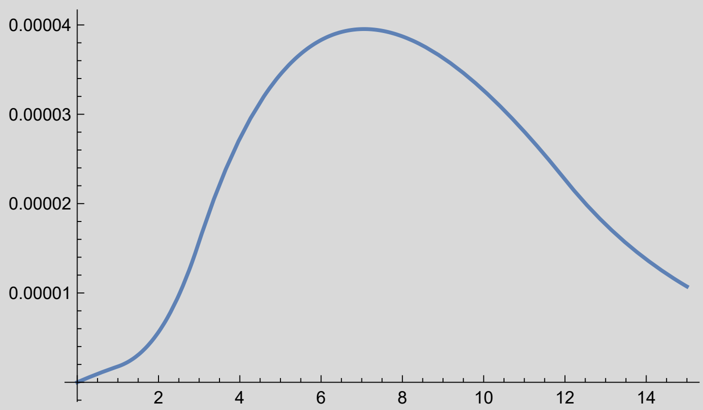
In[51]:=
```
Plot[v1[t], {t, 0, 15}]
```
Out[51]=



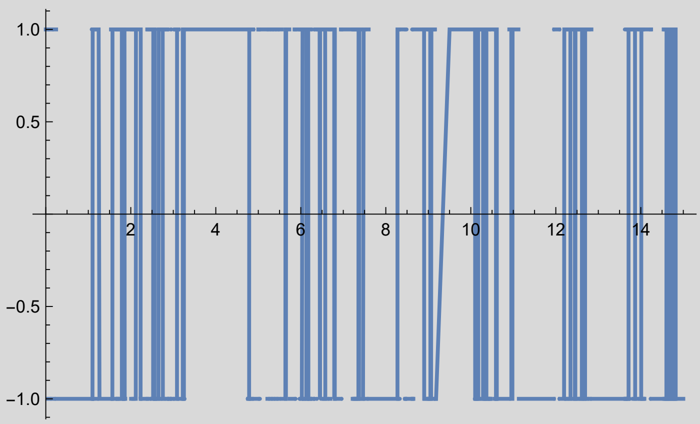
In[52]:=
```
b =
 NDSolve[{500000 q'[t1] + q[t1] / (8 * 10^-6) == v1[t1], q[0] == 0}, q[t1], {t1, 0, 15}]
```
Out[52]=
```
{{q[t1] → InterpolatingFunction[ Domain: {{0., 15.}} Output: scalar ][t1]}}
```

In[53]:=
```
Evaluate[q[t] /. b]
```
Out[53]=
```
{q[t]}
```

In[54]:=
```
Plot[Evaluate[q[t1] /. b], {t1, 0, 15}]
```
Out[54]=



In[55]:=
```
v2[t_] = SquareWave[ 10^5 t];
Plot[v2[t], {t, 0, 15}]
c = NDSolve[{500000 q'[t] + q[t] / (8 * 10^-6) == v2[t], q[0] == 0}, q[t], {t, 0, 15}]
```
Out[56]=

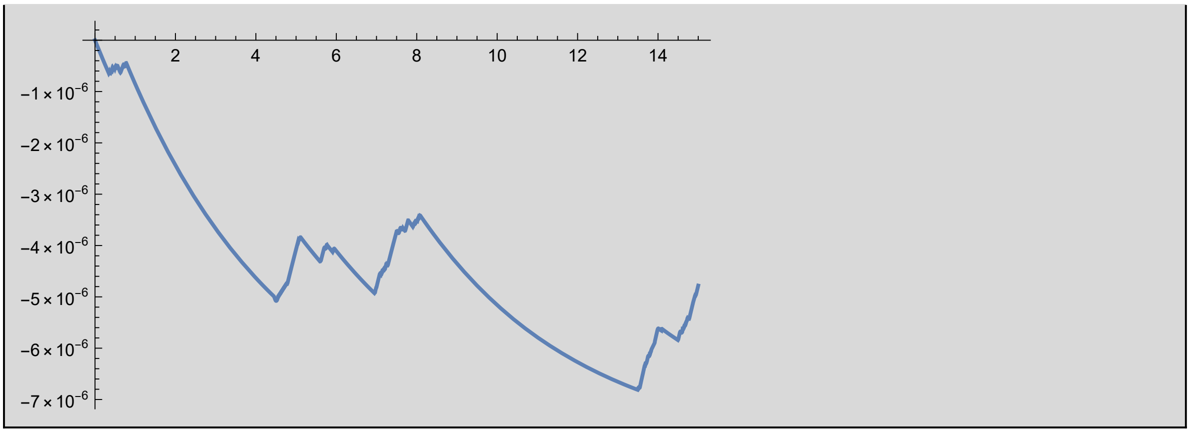


Out[57]=
```
{{q[t] → InterpolatingFunction[ Domain: {{0., 15.}} Output: scalar ][t]}}
```

In[58]:=

```
Plot[Evaluate[q[t] /. c], {t, 0, 15}]
```

Out[58]=

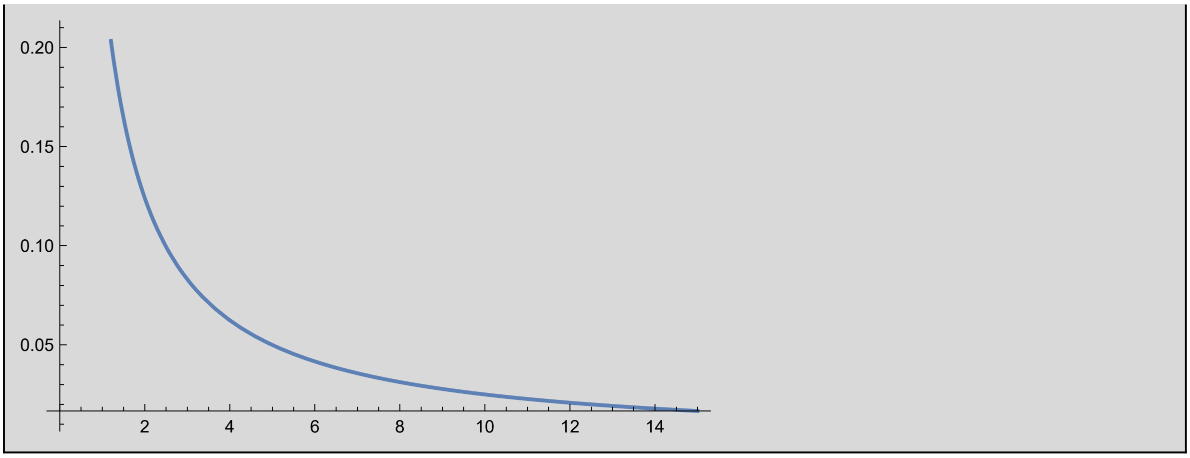


In[61]:=

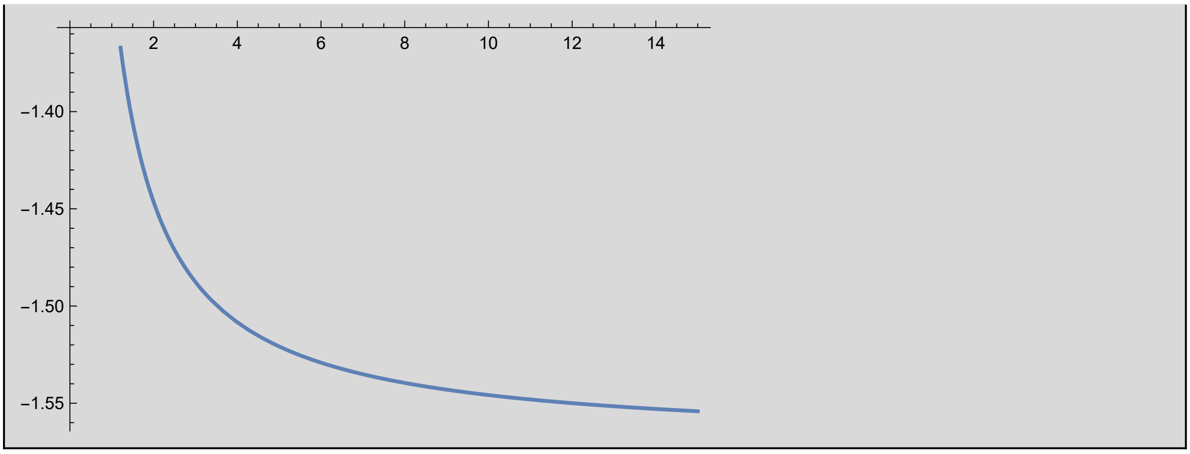
```
r = 500000;
c = 8 * 10^-6;
tc = r * c;
h[w_] = 1 / (1 + I w tc);

Plot[Abs[h[w]], {w, 0, 15}]
Plot[Arg[h[w]], {w, 0, 15}]
```

Out[65]=

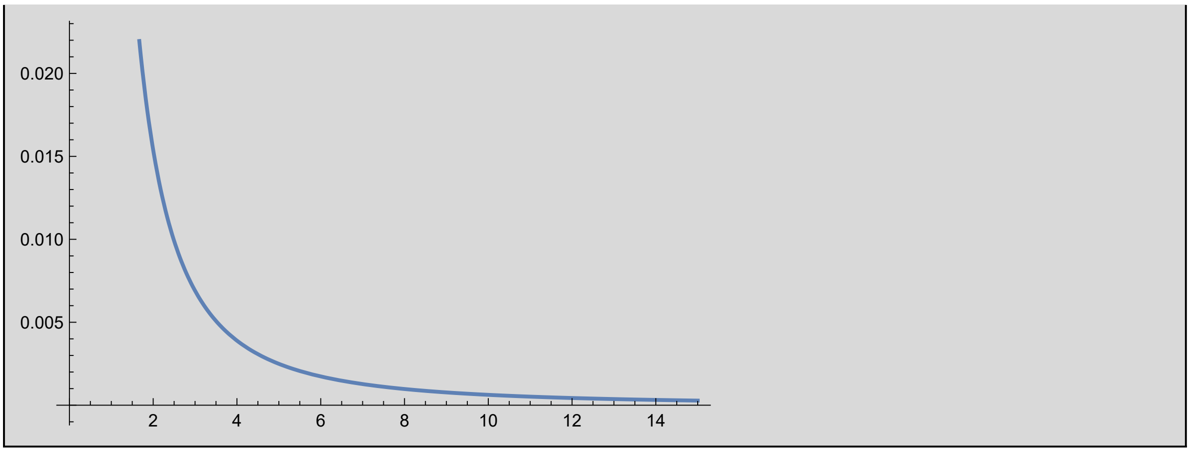


Out[66]=



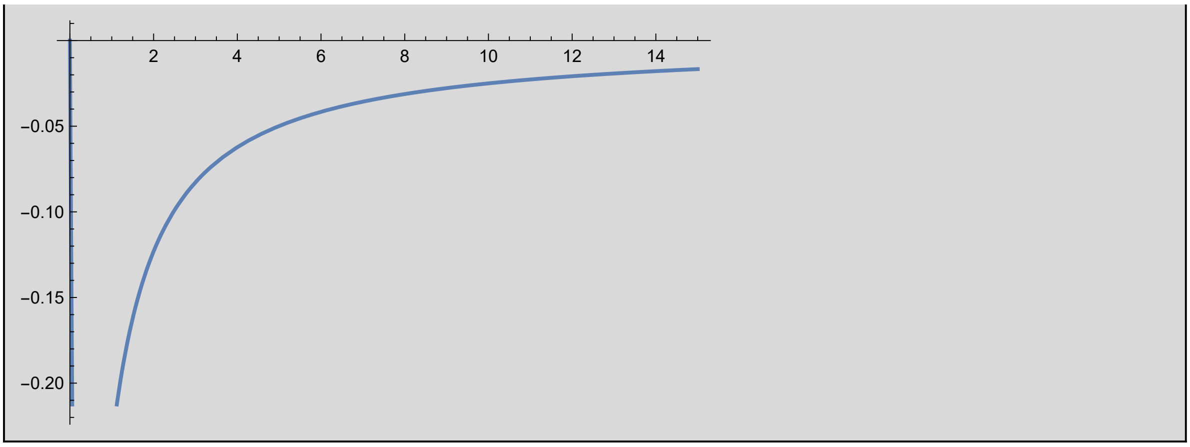
In[46]:=
```
Plot[Re[h[w]], {w, 0, 15}]
```

Out[46]=



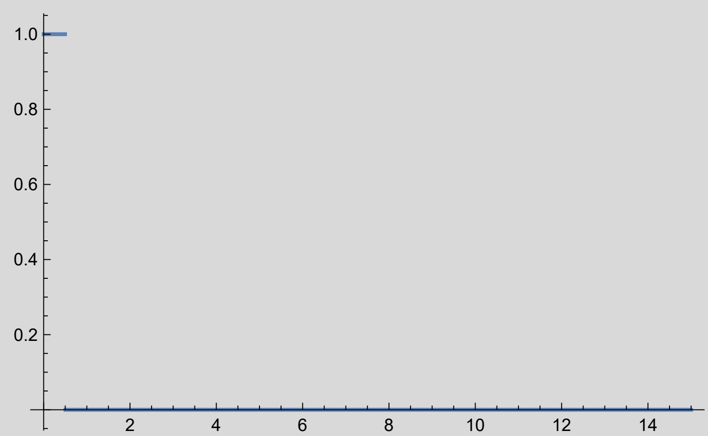
In[47]:=
```
Plot[Im[h[w]], {w, 0, 15}]
```

Out[47]=



In[24]:=
```
v4[t_] = UnitBox[t];
Plot[v4[t], {t, 0, 15}]
lc = NDSolve[{u''[t] + 10^6 u'[t] + u[t] / (2 * 10^-6) == v4[t],
   u[0] == 0, u'[0] == 0}, u, {t, 0, 40}]
Plot[Evaluate[u[t] /. lc], {t, 0, 40}]
```

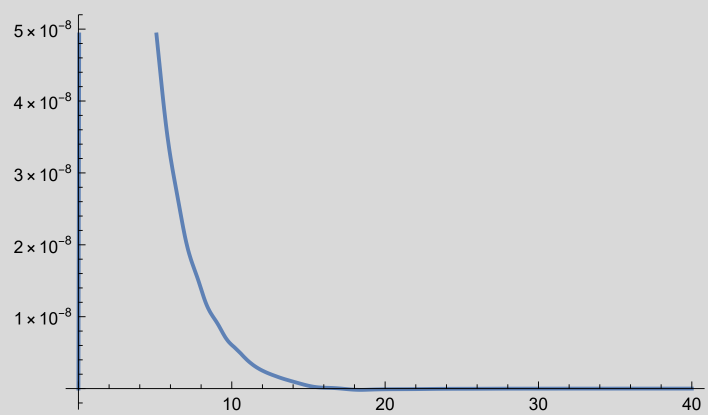
Out[25]=



Out[26]=

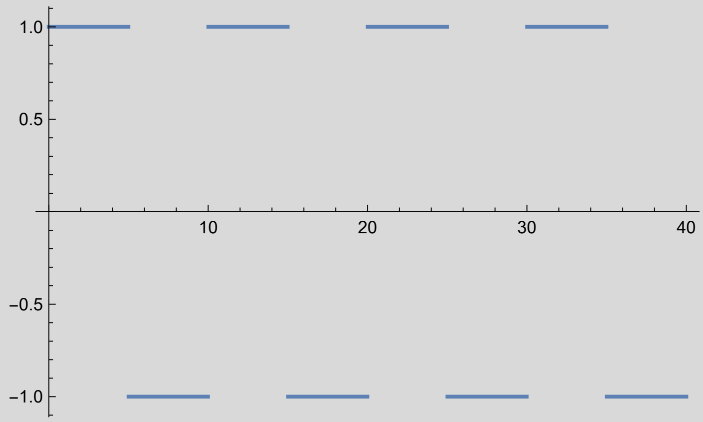
```
{{u → InterpolatingFunction[ Domain: {{0., 40.}} Output: scalar ]}}
```

Out[27]=



In[28]:=
```
v3[t_] = SquareWave[t / 10];
Plot[v3[t], {t, 0, 40}]
d = NDSolve[{u''[t] + 10^6 u'[t] + u[t] / (2 * 10^-6) == 100 v3[t],
    u[0] == 0, u'[0] == 0}, u, {t, 0, 40}]
```

Out[29]=

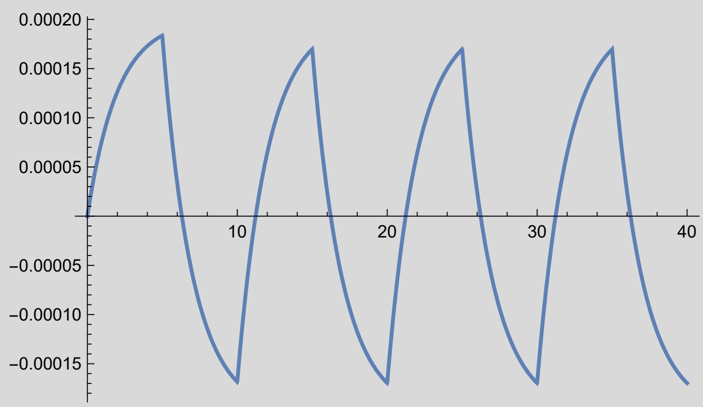


Out[30]=

{{u → InterpolatingFunction[Domain: {{0., 40.}} Output: scalar]}}

In[48]:=
```
Plot[Evaluate[u[t] /. d], {t, 0, 40}]
```

Out[48]=

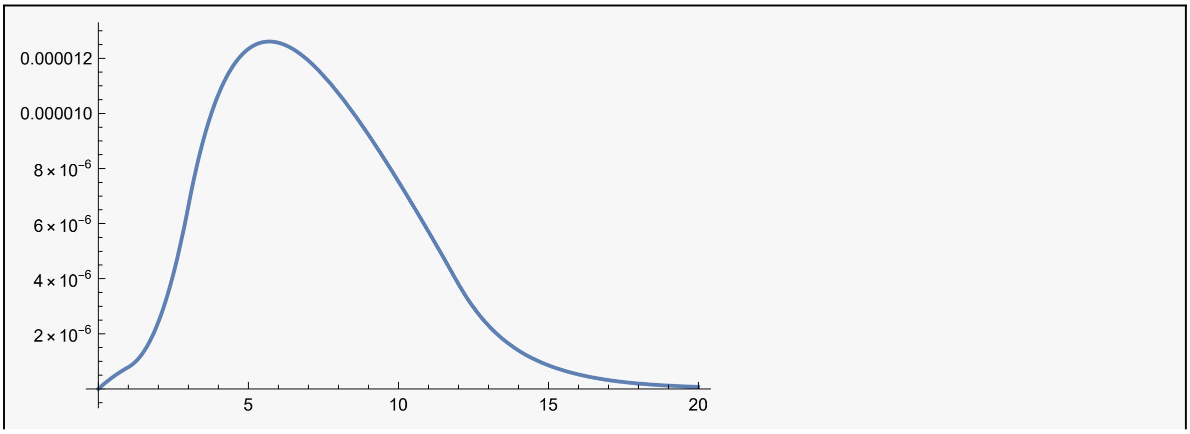


In[59]:=
```
e = NDSolve[{u''[t] + 10^6 u'[t] + u[t] / (2 * 10^-6) == v1[t],
    u[0] == 0, u'[0] == 0}, u, {t, 0, 20}]
```

Out[59]=

{{u → InterpolatingFunction[Domain: {{0., 20.}} Output: scalar]}}

In[60]:=
```
Plot[Evaluate[u[t] /. e], {t, 0, 20}]
```



Result

PART A

1)input signal:output 36

response:output 40

2)input signal:output 51

response:output54

3)input signal :output56

response:output 58

H[w]:

w-frequency

absolute value vs w:output 65

phase angle vs w:output 66

real part vs w: output 46

imaginary part vs w: output 47

PART B

1)input signal :output 25

response:output 27

2)input signal:output 29

response: output 48

3)input signal: output 51

response: output 60

DISCUSSION

CONCLUSION

1)The relation between h(t) and H(w): h(t) is the impulse response obtained from an rc circuit when a casual square wave with width of five order less than the time constant r c. H(w) is the function related to frequency and time constant. It will produce complex values. These two functions are related by Fourier transformation. The Fourier transform is a transform that converts a function in a particular domain to another. Here h(t) is in time domain and H(w) is in frequency domain. Both are related by:

$$h[t]=\int_{-\text{infinity}}^{+\text{infinity}} H[w]*e^{-i wt}\,dw$$

$$H[w]=\int_{-\text{infinity}}^{+\text{infinity}} h[t]*e^{i wt}\,dt$$

2)Finding response to a given input using h(t): If we have found the impulse response of a system then the response to an arbitrary input can be found by convolution. By this concept, we can combine the impulse function and an arbitrary input to find its output by:

o(t)= i(t)*h(t)

where i(t):arbitrary input

o(t):output for arbitrary input

3) The second result makes our job easier as we now don’t need to solve complex differential equations to find the output of an arbitrary input. For this having the impulse response itself is sufficient.

COMMENTS

This assignment was very interesting while working. This assignment will be very useful in future electrical engineering courses dealing with signals and systems. The response of linear systems are fundamental concept in engineering and this assignment helped me to get a basic understanding of this concept.

REFERENCES:

MATHEMATICA STACK EXCHANGE
MATHEMATICA DOCUMENTATION
wikipedia