TIME AND WORK& PIPES AND CISTERNS

In most of the problems on time and work, either of the following basic parameters are to be calculated:

TIME:

- If A can do a piece of work in X days, then A's one day's work $=\frac{1}{x}$ th part of whole work.
- If A's one day's work $=\frac{1}{x}$ the part of whole work, then A can finish the work in X days.
- If A can do a piece of work in X days and B can do it in Y days then A and B working together will do the same work in $\frac{XY}{X+Y}$ days.
- If A, B and C can do a work in X, Y and Z days respectively then all of them working together can finish the work in $\frac{XYZ}{XY+YZ+XZ}$ days.

Example 1:

A can do a piece of work in 5days, and B can do it in 6 days. How long will they take if both work together?

Solution:

A's 1 day's work $=\frac{1}{5}$th part of whole work and

B's 1 day's work $=\frac{1}{6}$th part of whole work

∴ (A+B)'s one day's work $= \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$ th part of whole work.

So, both together will finish the work in $\frac{30}{11}$ days = $2\frac{8}{11}$days.

By Direct Formula:

A+B can do the work on $\frac{5\times6}{5+6}$ days $= \frac{30}{11} = 2\frac{8}{11}$ days.

Example 2:

Two men, Vikas and Vishal, working separately can mow a field in 8 and 12 hours respectively. If they work in stretches of one hour alternately, Vikas beginning at 8 a.m., when will the mowing be finished?

Solution:

In the first hour, Vikas mows $\frac{1}{8}$ of the field.

In the second hour, Vishal mows $\frac{1}{12}$ of the field.

∴ In the first 2 hours, $\frac{1}{8} + \frac{1}{12} = \frac{5}{24}$ of the field is mown.

∴ In 8 hours, $\frac{5}{24} \times 4 = \frac{5}{6}$ of the field is mown.

Now, $(1 - \frac{5}{6} = \frac{1}{6}$ of the field remains to be mown.

In the 9th hour, Vikas mows $\frac{1}{8}$ of the field.

Remaining work $= \frac{1}{6} - \frac{1}{8} = \frac{1}{24}$

∴ Vishal will finish the remaining work in

$\frac{1}{24} + \frac{1}{12}$

or $\frac{1}{2}$ of an hour.

∴ The total time required (8 +$9\frac{1}{2}$ = $17\frac{1}{2}$) or 5.30 pm.

Example 3:

A can do a piece of work in 36 days, B in 54 days and C in 72 days. All the three began the work together on the Dec. 15, 2014, but A left 8 days and B left 12 days before the completion of the work. If C took the reset for a week then in how many days, the work was finished from the day it started?

Solution:

Let the total time taken be x days.

According to the given condition

$\Rightarrow \frac{x-8}{36} + \frac{x-12}{54} + \frac{x}{72} = 1$

$\Rightarrow \frac{6(x-8)+4(x-12)+3x}{216} = 1$

$\Rightarrow \frac{6x-48+4x-48+3x}{216} = 1 \Rightarrow \frac{13x-96}{216} = 1$

$\Rightarrow$ 13x – 96= 216 $\Rightarrow$ 13x = 216 +96 = 312

$\Rightarrow$ x= $\frac{312}{13}$ = 24

Since, C takes the rest for a week, so the number of days in which the work was finished from one day it started = 31 i.e. on 14.01.2015.

Example 4:

A and B can do a certain piece of work in 8 days, B and C can do it in 12 days and C and A can do it in 24 days. How long would each take separately to do it?

Solution:

(A+B)'s one day's work = 1/18,

(A+C)'s one day's work = 1/24,

(B+C)'s one day's work = 1/12,

Now add up all three equations:

2 (A+B+C)'s one day's work = $\frac{1}{18} + \frac{1}{24} + \frac{1}{12} = \frac{13}{72}$

(A+B+C)'s one day's work = $\frac{13}{144}$

A's one day's work = (A+B+C)'s one day's work

-(B+C)'s one day's work = $\frac{13}{144} - \frac{1}{2} = \frac{1}{144}$

Since A completes of the work in 1 day, he will complete 1 work in $\frac{144}{1}$ = 144 days

By similar logic we can find that B needs days and C will require $\frac{144}{5}$ days.

❖ If A and B toether can do a piece of work in X days and A alone can do it in Y days, then B alone can do the work in $\frac{XY}{Y-X}$ days.

Example 5:

A and B together can do a piece of work in 6 days and A alone can do it in 9 days. In how many days can B alone do it?

Solution:

(A+B)'s 1 day's work = $\frac{1}{6}$ th part of the whole work.

A's 1 day's work = $\frac{1}{9}$ th part of the whole work.

$\therefore$ B's 1 day's work = $\frac{1}{6} - \frac{1}{9} = \frac{3-2}{18} = \frac{1}{18}$ th

part of the whole work.

$\therefore$ B alone can do the work in 18^{th} days.

By direct Formula:

B alone can do the whole work in

$\frac{6\times9}{9-6} = \frac{54}{3}$= 18 days

❖ A and B can do a work in 'X' and 'Y' days respectively. They started the work together but A left 'a' days before completion of the work. Then, time taken to finish the work is

$$\frac{Y(X+a)}{X+Y}$$

- ❖ If ‘A’ id ‘a’ times efficient than B and A can finish a work in X days, then working together, they can finish the work in $\frac{aX}{a+1}$ days.
- ❖ If A is ‘a’ times efficient than B and working together they finish a work in Z days then, time taken by A = $\frac{Z(a+1)}{a}$ days, and time taken by B = Z (a+1) days.
- ❖ If A working alone takes ‘x’ days more than A and B together, and B working along takes ‘y’ days more than A and B together then the number of days taken by A and B working together is given by $\sqrt{xy}$)days.

Example 6:

A and B can do alone a job in 6 days and 12 days. They began the work together but 3 days before the completion of job, A leaves off. In how many days will the work be completed?

Solution:

Let work will be completed in x days. Then, work done by A in (x-3) days + work done by B in x days = 1

i.e. $\frac{x-3}{6}+\frac{x}{12}$ =1

$\Rightarrow\frac{3x-6}{12}$ = 1⇒ x =6 days

By Direct Formula:

Required time = $\frac{12(6+3}{12+6}$ = 6 days

Example 7:

A is half good a workman as B and together they finish a job in 14 days. In how many days working alone will B finish the job.

Solution:

Let B can do the work in x days

and A can do the work in 2x days

Then, $\frac{1}{x}+\frac{1}{2x}=\frac{1}{14}$ (given)

$\Rightarrow x=\frac{3}{2}\times 14$ = 21 days

By Direct Formula:

Time taken by b = 14 $\frac{1}{2}+1\big)$=21 days

- ❖ If n men or m women can do a piece of work in X days, then N men and M women together can finish the work in $\frac{nmX}{nM+mN}$ days.

Example 8:

10 men can finish a piece of work in 10 days, where as it takes 12 women to finish it in 10 days. If 15 men and 6 women undertake to complete the work, how many days they will take to complete it?

Solution:

It is clear that 10 men = 12 women or 5 men = 6 men

⇒ 15 men + 6 women = (18 + 6) i.e. 24 women

Now 12 women can completed the work in 10 days

∴ 24 women will do it in 15 days.

By Direct Formula:

Required time =$\frac{10\times12\times10}{10\times6+12\times15}$= 5 days

Example 9:

If 3 men or 4 women can reap a field in 43 days, how long will 7 men and 5 women take to reap it?

Solution:

3 men reap $\frac{1}{43}$ of the field in 1 day.

$\therefore$ 1 men reaps $\frac{1}{43\times3}$ of the field in 1 day.

4 women reap $\frac{1}{43}$of the field in 1 day.

$\therefore$ 1 woman reaps $\frac{1}{43\times4}$ of the field in 1 day.

$\therefore$ 7 men and 5 women reap $\Big(\frac{7}{43\times3}+$ $543\times4=12$ of the field in 1 day.

$\therefore$ 7 men and 5 women will reap the whole field in 12 days.

Alternate method

Required number of days = $\frac{1}{\frac{7}{43\times3}+\frac{5}{43\times4}}$

$= \frac{43\times3\times4}{4\times4+5\times3}$ = 12 days.

Example 10:

If 12 men and 16 boys can do a piece of work in 5 days and 13 men and 24 boys can do it in 4 days, how long will 7 men and 10 boys take to do it?

Solution:

12 men and 16 boys can do the work in 5 days

.... (i)

13 men and 24 boys can do the work in 4 days

…. (ii)

Now it is easy to see that if the no. of workers be multiplied by any number, the time must be divided by the same number (derived from: more worker less time).

Hence multiplying the no. of workers in (i) and (ii) by 5 and 4 respectively, we get 5 (12 men + 16 boys) can do the work in 5/5 = 1 day

4 (13 men +24 boys) can do the work in $\frac{4}{4}$=1 day

or, 5 (12m +16b) = 4 (13m + 24b)

or, 60 m + 80b = 52m + 96b

or, 60m – 52m = 96b – 80b

or, 8m = 16b

$\therefore$ 1 men = 2 boys.

Thus, 12 men + 16 boys = 24 boys + 16 boys = 40 boys and 7 men + 10 boys = 14 boys + 10 boys = 24 boys

The question now becomes:

"If 40 boys can do a piece of work in 6 days how long will 24 boys take to do it?"

Using basic formula

we have,

or, $D_2 = \frac{40\times5}{24} = 8\frac{1}{3}$ days

Example 11:

Two men and 7 boys can do a piece of work in 14 days. 3 men and 8 boys can do it in 11 days. In how many days can 8 men and 6 boys do a work 3 times as big as the first?

Solution:

2 men + 7 boys in 14 days ⇒ 28 men + 98 boys in 1 day

3 men + 8 boys in 11 days ⇒ 33 men + 88 boys in 1 day

$\therefore$ 28 men + 98 boys = 33 men +88 boys

Now, 2 men + 7 boys = 11 boys; 8 men + 6 boys = 22 boys

More boys, fewer days; more work, more days.

Boys	Days	Work
11	14	1
22	X	3

$\therefore \frac{x}{14} = \frac{11}{12} \times \frac{3}{1}$ ∴ Number of days = 21 days

Example 12:

Kaberi takes twice as much time as Kanti and thrice as much as Kalpana to finish a place of work. They together finish the work in one day. Find the time taken by each of them to finish the work.

Solution:

Here, the alone time of kaberi is related to the alone times of other two persons, so assume the alone time of kaberia =x,

Then, alone time of Kanti $=\frac{x}{2}$ and of Kalpana $= \frac{x}{3}$

Kaberi's 1 day work + Kanti's 1 day work + Kalpana's 1 day work = combined 1 days work

$\Rightarrow \frac{1}{x} + \frac{1}{x/2} + \frac{1}{x/3} = \frac{1}{1} \Rightarrow x = 6$

∴ Alone time for Kaberi = 6 days, for Kanti = 6/2 = 3 days, Kalpana = $\frac{6}{3}$ = 2days

Example 13:

1 man or 2 women or 3 boys can do a work in 44 days. Then in how many days will 1 man, 1 woman and 1 boy do the work?

Solution:

Number of required days

$= \frac{1}{\frac{1}{44\times1}+\frac{1}{44\times2}+\frac{1}{44\times3}} = \frac{44\times1\times2\times3}{6+3+2} = 24$ days

- ❖ If 'M_1' persons can do 'W_1' works in 'D_1' days and 'M_2' persons can do 'W_2' works in 'D_2' days then $M_1 D_1 W_2 = M_2 D_2 W_1$

If T_1 and T_2 are the working hours for the two groups then $M_1 D_1 W_2 T_1 = M_2 D_2 W_1 T_2$

Similarly,

$M_1 D_1 W_2 T_1 E_1 = M_2 D_2 W_1 T_2 E_2$, where E_1 and E_2 are the efficiencies of the two groups.

- ❖ If the number of men to do a job is changed in the ratio a: b, then the time required to do the work will be in the ratio b: a, assuming the amount of work done by each of them in the given time is the same, or they are identical.
- ❖ A is K times as good a worker as B and takes X days less than B to finish the work. Then the amount of time required by A and B working together is $\frac{K\times X}{K^2-1}$ days.
- ❖ If A n times as efficient then B, i.e. A has n times as much capacity to do work as B, A will take $\frac{1}{n}$ of the time taken by B to do the same amount of work.

Example 14:

5 men prepare 10 toys in 6 days working 6 hrs a day. Then in how many days can 12 men prepare 16 toys working 8 hrs a day?

Solution:

This example has an extra variable 'time' (hrs a day), so the 'basic-formula' can't work in this case. An extended formula is being given:

$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$

Here, $5 \times 6 \times 6 \times 16 = 12 \times D_2 \times 8 \times 10$

$\therefore D_2 = \frac{5\times6\times6\times16}{12\times8\times10} = 3$ days

Example 15:

A and B can do a work in 45days and 40 days respectively. They began the work together, but A left after some time & B finished the remaining work in 23 days. After how many days did A leave?

B finished the remaining work in 23 days.

$\therefore$ Work done by B in 23 days = $\frac{23}{40}$ work

$\therefore$ A+B do together 1 - $\frac{23}{40} = \frac{17}{40}$ work

Now, A+B do 1 work in $\frac{40\times45}{40+45} = \frac{40\times45}{85}$ days

$\therefore$ A +B do $\frac{17}{40}$ work in $\frac{40\times45}{85} \times \frac{17}{40} = 9$ days.

Alternate method:

If we ignore the intermediate steps, we can write a direct formula as: $\frac{40\times45}{40+45}\ \frac{40-23}{40}$ = 9 days.

Example 16:

Two friends take a piece of work for ` 960. One alone could do it in 12 days, the other in 16 days with the assistance of an expert they finish it in 4 days. How much remuneration the expert should get?

Solution:

First friend's 4 day's work = $\frac{4}{12} = \frac{1}{3}$

(Since, the work is finished in 4 days, when expert assists)

Second friend's 4 day's work = $\frac{4}{16} = \frac{1}{4}$

The expert's 4 day's work = 1 - $\frac{1}{3} + \frac{1}{4} \ = \frac{5}{12}$

Now, total wages of Rupee Foradian 960 is to be distributed among two friends and the expert in proportion to the amount of work done by each of them.

So, 960 is to be divided in the proportion of

$\frac{1}{3} : \frac{1}{4} : \frac{5}{12}$ or 4:3:5

$\therefore$ Share of expert = $\frac{5}{12}$ × 960 =` 400

Hence, the expert should get ` 400.

Example 17:

A certain number of men can do a work in 60 days. If there were 8 men more it could be finished in 10 days less. How many men are there?

Solution:

Let there be x men originally.

(x+8) men can finish the work in (60-10) = 50 days.

Now, 8 men can do in 50 days what x men do in 10 days, then by basic formula we have

$\therefore$ x = $\frac{8\times50}{10}$ = 40 men.

Alternate method:

We have:

x men to the work in 60 days and (x+8) men do the work in (60 – 10) = 50 days.

Then by "basic formula", 60x = 50 (x+8)

$\therefore x = \frac{50\times8}{10}$ = 40 men.

Example 18:

Two coal loading machines each working 12 hours per day for 8 days handles 9,000 tonnes of coal with an efficiency of 90 %. While 3 other coal loading machines at an efficiency of 80% set to handle 12,000 tonnes of coal in 6 days. Find how many hours per day each should work.

Solution:

Here $\frac{N_1\times D_1\times R_1\times E_1}{W_1} = \frac{N_2\times D_2\times R_2\times E_2}{W_2}$

$N_1 = 2$, R_1 =12h/day; $N_2 = 3$, R_2 =?

$E_1 = \frac{90}{100}$ W_1= 9,000;

$E_2 = \frac{80}{100}$ W_2= 12,000

$\Rightarrow \frac{2\times8\times12\times90}{9,000\times100} = \frac{3\times6\times R_2\times80}{12,000\times100}$

⇒ R_2 = 16h/day.

∴ Each machine should work 16h/ day.

WORK AND WAGES

❖ Wages are distributed in proportion to the work done in indirect proportion to the time taken by the individual

Example 19:

A, B, and C can do a work in 6, 8 and 12 days respectively. Doing that work together they get an amount of Rs. 1350. What is the share of B in that amount?

Solution:

A's one day's work = $\frac{1}{6}$

B's one day's work = $\frac{1}{8}$

C's one day's work = $\frac{1}{12}$

A's share : B's share : C's share

$= \frac{1}{6} : \frac{1}{8} : \frac{1}{12}$

Multiplying each ratio by the L.C.M. of their denominators, the ratio become 4: 3: 2

∴ B's share = $\frac{1350\times3}{9}$ = `450

Example 20:

If 6 men working 8 hours a day earn ` 1680 per week, then how much will 9 men working 6 hours a day earn per week?

Solution:

6m	8hours	`16800
9m	6hours	?

$1680 \times \frac{6}{8} \times \frac{9}{6}$ = `1890

Alternate method:

As earnings are proportional to the work done, we have

$\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2} \Rightarrow \frac{6\times8}{1680} = \frac{9\times6}{W_2} \Rightarrow W_2$

=`1890

Example 21:

A can do a piece of work in 15 days and B in 20 days. They finished the work with the assistance of C in 5 days and got ` 45 as their wages, find the share for each in the wages.

Solution:

A did in 5 days 1/3 of the work,

B did in 5 days 1/4 of the work.

C did in 5 days $1 - \frac{1}{3} + \frac{1}{4} = \frac{5}{12}$ of the work

Since A, B, C did in 5 days 1/3, 1/4, 5/12 of the work respectively.

A's share = ` 45× $\frac{1}{3}$ = ` 15

B's share = ` 45× $\frac{1}{4}$ = `11.25

C's share = ` 45× $\frac{5}{12}$ = ` 18.75

Example 22:

If 8 men, working 9 hours per can build a wall 18 meter long. 2 meters wide and 12 meters high in 10 days, how many men will be required to build a wall 32 meters long, 3 meters wide and 9 meters high by working 6 hours a day in 8 days?

Solution:

This method is a substitute for the conventional method and can be safely employed for most of the problems.

Step 1: Assume the thing to be found as 'X'

Step 2: In the first place look for X's counterpart.

e.g. in the above example, X =no. of men

So X's counterpart = No. of men, given = 8.

So write X = 8x....

Now see the direct and indirect variation or see by which operation more men will be required & by which fewer.

We have $X=8\times\frac{32}{18}\times\frac{3}{2}\times\frac{9}{12}\times\frac{10}{8}\times\frac{9}{6}=$ 30 men

Example 23:

If 5 engines consume 6 tonnes of coal when each runs 9 hrs per day, how much coal will be needed for 8 engines, each running 10 hrs. per day, it being given that 3 engines of the former type consume as much as 4 engines of latter type?

Solution:

We have $X = 6\times\frac{8}{5}\times\frac{10}{9}\times\frac{3}{4}$ = 8 tons

Explanation:

(1) More engines more coal (>1)

(2) More time, more coal (>1)

(3) Latter consumes less coal than former (<1).

In case of men working we have more time, less men (<1) but here we have more time, more coal (1).

Here let W = 6 tonnes ≡ 5× 9× 4/3 engine hours and let X ≡ 8 × 10 × 1 engine hours.

or X ≡ 6 tons × $\frac{8\times10\times1}{5\times9\times(4/3)}$= 8 tons

Example 24:

A garrison of 1500 men is provisioned for 60 days. After 25 days the garrison is reinforced by 500 men, how long will the remaining provisions last?

Solution:

Since the garrison is reinforced by 500 men therefore then are (1500 + 500) or 2000 men now,

since 60 – 25 = 35 days.

⇒ The provisions left would last 1500 men 35 days

⇒ Provisions left would last 1 man 35× 1500 days

⇒ Provisions left would last 2000 men

$35\times\frac{1500}{2000}$ = 26.25 days

Alternate method:

1500×60= (1500×25) + (2000×X)

90000 – 37500 = 2000X

X = 26.25 days.

Example 25:

40 men can cut 60 trees is 8 hrs. If 8 men leaves the job how many trees will be cut in 12 hours?

Solution:

40 men – working 8 hrs – cut 60 trees

or, 1 man – working 1 hr – cuts $\frac{60}{40\times8}$ trees

Thus, 32- working 12 hrs – cut $\frac{60\times32\times12}{40\times8}$ = 73 trees.

Using basic concepts:

M_1 = 40, D_1 = 8 (As days and hrs both demote time)

W_1 = 60 (cutting of trees is taken as work)

M_2 = 40-8 = 32, D_2 = 12, W_2 =?

Putting the values in the formula

$M_1 D_1 W_2 = M_2 D_2 W_1$

We have, $40 \times 8 \times W_2 = 32 \times 12 \times 60$

or, $W_2 = \frac{32\times12\times60}{40\times8} = 72$ trees.

Example 26:

Two women, Ganga and Sarswati, working separately can mow a field in 8 and 12 hrs respectively, If they work in stretches of one hour alternately, Ganga beginning at 9 a.m., when will the mowing be finished?

Solution:

In the first hour Ganga mows $\frac{1}{8}$ of the field.

In the second hour Saraswati mows $\frac{1}{12}$ of the fields.

∴ In the first 2 hrs $\frac{1}{8} + \frac{1}{12} = \frac{5}{24}$ of the field is mown.

∴ In 8 hrs $\frac{5}{24} \times 4 = \frac{5}{6}$ of the field is mown...(i)

Now, $\left(1 - \frac{5}{6}\right. = \frac{1}{6}$ of the remains to be mown. In the 9^{th} hour Ganga mows $\frac{1}{8}$ of the field.

∴ Saraswathi will finished the mowing of $\frac{1}{6} - \frac{1}{8} = \frac{1}{24}$ of the field in $\left(\frac{1}{24} + \right.$ 112 or 12 of an hours.

∴ the total time required is $\left(8 + 1 + \frac{1}{2}\right.$ or $9\frac{1}{2}$ hrs.

Thus, the work will be finish at $9 + 9\frac{1}{2} = 18\frac{1}{2}$ or $6\frac{1}{2}$ p.m.

Example 27:

I can finish a work in 15 days at 8 hrs a day. You can finish it in $6\frac{2}{3}$ days at 9 hrs a day. Find in how many days we can finish it working together 10 hrs a day.

Solution:

First suppose each of us works for only one hr a day.

Then I can finish the work in 15×8 = 120 days and you can finish the work in $\frac{20}{3}\times9 = 60$ days.

But here we are given that we do the work 10 hrs a day. Then clearly we can finish the work in 4 days.

Example 28:

A can do a work in 6 days. B takes 8 days to complete it. C takes as long as A and B would take working together. How long will it take B and C to complete the work together?

Solution:

(A+B) can do the work in $\frac{6\times8}{6+8} = \frac{24}{7}$ days.

∴ C takes $\frac{24}{7}$ days to complete the work.

∴ (B+C) takes $\frac{\frac{24}{7}\times8}{\frac{24}{7}+8} = \frac{24\times8}{24+56} = 2\frac{2}{5}$ days.

Example 29:

A group of 20 cows can graze a field 3 acres in size in 10 days. How many cows can graze a field twice as large in 8 days?

Solution:

Here, first of all, let us see how work can be defined. It is obvious that work can be measured as "acres grazed".

In the first case, there were 20 cows in the group.

They had to work for 10 days to do the work which we call W (which = 3)

⇒ 20×10 =3 ..... (1)

Do not be worried about the numerical values on either side. The point is that logically this equation is consistent as the LHS indicates "Cow days" and the RHS indicates "Acres", both of which are correct ways of measuring work done.

Now the field is twice as large. Hence the new equation is

$\Rightarrow$ C × 8 = 6 .... (ii)

Just divide (ii) by (i) to get the answer.

$$\frac{8C}{200} = \frac{6}{3}$$

$\Rightarrow$ 8C = 2× 200 $\Rightarrow$ C = $\frac{400}{8}$=50 cows.

Hence, there were 50 cows in the second group.

PIPE AND CISTERNS

The same principle of Time and Work is employed to solve the problems on Pipes and Cisterns. The only difference is that in this case, the work done is in terms of filling or emptying a cistern (tank) and the time taken is the time taken by a pipe or a leak (creak) to fill or empty a cistern respectively.

Inlet: A pipe connected with a tank (or a cistern or a reservoir is called in inlet, if it fills it.

Outlet: A pipe connected with a tank is called an outlet, if it empties it.

- ❖ If a pipe can fill a tank in x hours, then the part filled in 1 hour = $\frac{1}{x}$
- ❖ If a pipe can empty a tank in y hours, then the part filled in 1 full tank emptied in 1 hour =$\frac{1}{y}$.
- ❖ If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then the net part filled in 1 hour, when both the pipes are opened = $\frac{1}{x} - \frac{1}{y}\Big)$.

  ∴ Time taken to fill the tank, when both the pipes are opened =$\frac{xy}{y-x}$.
- ❖ If a pipe can fill a tank in x hours and another can fill the same tank in y hours, then time taken to fill the tank = $\frac{xy}{y+x}$, when both the pipes are opened.
- ❖ If a pipe fills a tank in x hours and another fills the same tank is y hours, but a third one empties the full tank in z hours, and all of them are opened together, then net part filled in 1 hr = $\frac{1}{x} + \frac{1}{y} - \frac{1}{z}$

  ∴ Time taken to fill the tank= $\frac{xyz}{yz+xz-xy}$hours.
- ❖ A pipe can fill a tank in x hrs. Due to a leak in the bottom it is filled in y hrs. If the tank is full, the time taken by the leak to empty the tank = $\frac{xy}{y-x}$hrs.
- ❖ A cistern has a leak which can empty it in X hours. A pipe which admits Y litres of water per hour into the cistern is turned on and now the cistern is emptied in Z hours. Then the capacity of the cistern is $\frac{X+Y+Z}{Z-X}$litres.
- ❖ A cistern is filled by three pipes whose diameters are X cm., Y cm. and Z cm. respectively (where X < Y < Z). Three pipes are running together. If the largest pipe alone will fill it in P minutes and the amount of water flowing in by each pipe is proportional to the square of its diameter, then the time in which the cistern will be filled by the three pipes is $\frac{PZ^2}{X^2+Y^2+Z^2}$ minutes.

- If one filling pipe A is n times faster and takes X minutes less time than the other filling pipe B, then the time they will take to fill a cistern, if both the pipes are opened together, is $\left[\frac{\text{nX}}{(\text{n}^2-1)}\right]$ minutes. A will fill the cistern in $\frac{\text{X}}{\text{n}-1}$ minutes and B will taken to fill the cistern $\frac{\text{nX}}{\text{n}-1}$ minutes.
  Here, A is the faster filling pipe and B is the slower one.
- Two filling pipes A and B opened together can fill a cistern in t minutes. If the first filling pipe A alone takes X minutes more or less than t and the second fill pipe B along takes Y minutes more or less than t minutes, then t is given by $[t = \sqrt{xy}]$minutes.

Example 30:

A pipe can fill a cistern in 6 hours. Due to a leak in its bottom, it is filled in 7 hours. When the cistern is full, in how much time will it be emptied by the leak?

Solution:

Part of the capacity of the cistern emptied by the leak in one hour = $\frac{1}{6} - \frac{1}{7}\Big) = \frac{1}{42}$ of the cistern.

The whole cistern will be emptied in 42 hours.

Example 31:

Three pipes A, B, and C fill a cistern in 6 hrs. After working together for 2 hrs, C is closed and A and B fill the cistern in 8 hrs. Then find the time in which the cistern can be filled by pipe C.

Solution:

A+B+C can fill in 1 hr =$\frac{1}{6}$ of cistern.

A+B+C can fill in 2 hr =$\frac{2}{6}$ = 1/3 of cistern.

Remaining part = $\Big(1 - \frac{1}{3} = \frac{2}{3}$ is filled by A +B in 8 hrs.

∴ (A+B) can fill the cistern in $\frac{8\times3}{2}$ = 12 hrs.

Since (A+B+C) can fill the cistern in 6 hrs.

∴ C = (A+B+C) – (A+B) can fill the cistern in $\frac{12\times6}{12-6}$ hours = 12 hours.

Example 32:

Pipe A can fill a tank in 20 hours while pipe B alone can fill it in 30 hours and pipe C can empty the full tank in 40 hours. If all the pipes are opened together, how much time will be needed to make the tank full?

Solution:

By direct formula,

The tank will be fill in $=\frac{20\times30\times40}{30\times40+20\times40-20\times30}$

$= \frac{120}{7} = 17\frac{1}{7}$ hrs.

Example 33:

Three pipes A, B and C can fill a tank in 6 minutes, 8 minutes and 12 minutes, respectively. The pipe C is closed 6 minute before the tank is filled. In what time will the tank be full?

Solution:

Let it takes t minutes to completely fill the tank.

Now, $\frac{t}{6} + \frac{t}{8} + \frac{t-6}{12}$=1

or $\frac{4t+3t+2t-12}{24}$ = 1

or $9t - 12 = 24$

or $9t = 36 \Rightarrow t = 4$ min.

Example 34:

If three taps are opened together, a tank is filled in 12 hrs. One of the taps can fill it in 10 hrs and another in 15 hrs. How does the third tap work?

Solution:

We have to find the nature of the third tap, whether it is a filler or a waste pipe.

Let it be filler pipe which fills in x hrs.

Then, $\frac{10\times15\times x}{10\times15+10x+15x} = 12$

or, $150x = 150 \times 12 + 25x \times 12$

or, $-150x = 1800 \therefore x = -12$

-ve sign shows that the third pipe is a waste pipe which vacates the tank in 12 hrs.

Example 35:

4 pipes can fill a reservoir in 15, 20, 30 and 60 hours respectively. The first was opened at 6 am, second at 7 am third at 8 am and fourth at 9 am. When will the reservoir be full?

Solution:

Let the time be t hours after 6 am.

$\therefore \frac{1}{15} \times t + \frac{(t-1}{20} + \frac{(t-2)}{30} + \frac{(t-3)}{60} = 1$

$\therefore 4t + 3(t-1) + 2(t-2) + (t-3) = 60$

$\Rightarrow t = 7$ hours

$\Rightarrow$ It is filled at 1 pm

Example 36:

A and B can fill a cistern in 7.5 minutes and 5 minutes respectively and C can carry off 14 litres per minute. If the cistern is already full and all the three pipes are opened, then it is emptied in 1 hour. How many litres can it hold?

Solution:

If the capacity is L litres, water filled in 1 hour = Water removed in 1 hour.

$L + \frac{L}{7\frac{1}{2}} \times 60 + \frac{L}{5} \times 60 = 14 \times 60$

$\therefore L + \frac{2L}{15} \times 60 + 12L = 14\times60 \Rightarrow$

$L + 8L + 12L = 14\times60$

$\Rightarrow 21L = 14 \times 60$ or $L = 40$ litres.

So the capacity of the cistern is 40 litres.

Example 37:

A cistern can be filled by two taps A and B in 25 minutes and 30 minutes respectively can be emptied by a third in 15 minutes. If all taps are turned on at the same moment, what part of the cistern will remain unfilled at the end of 100 minutes?

Solution:

We have $\frac{1}{25} + \frac{1}{30} - \frac{1}{15} = \frac{1}{150}$ part filled in 1 minute.

Hence, $1 - 100 \frac{1}{150} = 1/3$rd of the tank is unfilled after 100 minutes.

Example 38:

A barrel full of beer has 2 taps one midway, which draw a litre in 6 minutes and the other at the bottom, which draws a litre in 4 minutes. The lower tap is lower normally used after the level of beer in the barrel is lower than midway. The capacity of the barrel is 36 litres. A new assistant opens the lower tap when the barrel is full and draws out some before the usual tome. For how long was the beer drawn out by the new assistant?

Solution:

The top tab is operational till 18 litres is drawn out.

∴ Time after which the lower tap is usually open = 18×6 = 108 minutes

∴ Litres drawn = 84/6 = 14 litres

∴ 18-14=4 litres were drawn by the new assistant.

∴ Time = 4 × 4 = 16 minutes

Example 39:

A cistern can be filled by two pipes filling separately in 12 and 16 min. respectively. Both pipes are opened together for a certain time but being clogged, only 7/8 of the full quantity of water flows through the former and only 5/6 through the latter pipe. The obstructions, however, being suddenly removed, the cistern is filled in 3 min. from that moment. How long was it before the full flow began?

Solution:

Both the pipes A and B can fill $\frac{1}{12}+\frac{1}{16}=\frac{7}{48}$ of the cistern in one minute, when there is no obstruction. With obstruction, both the pipes can fill $\frac{1}{12}\times\frac{7}{8}+\frac{1}{16}\times\frac{5}{6}=\frac{7}{96}+\frac{5}{96}=\frac{1}{8}$ of the cistern in one minute.

Let the obstructions were suddenly removed after x minutes.

∴ With obstruction, $\frac{x}{8}$ of the cistern could be

filled in x minutes and so the remaining $1-\frac{x}{8}=\frac{8-x}{8}$ of the cistern was filled without obstruction in 3 minutes, i.e. In one minute, $\frac{8-x}{24}$ of the cistern was filled.

$\Rightarrow\frac{8-x}{24}=\frac{7}{48}\Rightarrow$ 16-2x = 7⇒ x = 4.5