4. AVERAGE

The average or mean or arithmetic mean of a number of quantities of the same kind is equal to their sum divided by the number of those quantities

- Arithmetic average is used for all averages like: average income, average profit, average age, average marks etc
- It is defined as the sum total of all volumes of items divided by the total number of items
- In individual series.

  Average = $\frac{sum of observation}{Number of observation}$

  or $\bar{x} = \frac{x_1+x_2+x_3+\cdots+x_n}{n}$
- To calculate the sum of observations, they should be in the same unit

Example 1:

A man purchased 5 toys at the rate of Rs 200 each, 6 toys at the rate of Rs 250 each and 9 toys at the rate of Rs 300 each.

Calculate the average cost of one toy.

Solutions:

Price of 5 toys = 200 x 5 = 1000

Price of 6 toys = 250 x 6 = 1500

Price of 9 toys = 300 x 9 = 2700

Aveage cost of one toy

$= \frac{1000+1500+2700}{20} = \frac{5200}{20} = Rs260\backslash-$

Example 2:

In three numbers, the first is twice the second and thrice the third, if the average of these three numbers is 44, then the first number is:

Solution:

Let the three numbers be x, y and z

Therefore, x = 2y = 3z.

$y = \frac{x}{2}$ and $z = \frac{x}{3}$

Now, $\frac{x+\frac{x}{2}+\frac{x}{3}}{3} = 44$

or $\frac{11x}{18} = 44$ or $x = 72$

Example 3:

The average of five consecutive odd numbers is 61. What is the difference between the highest and lowest numbers?

Solution:

Let the numbers be x, x + 2, x+ 4, x + 6 and x + 8

Then, $\frac{x+(x+2)+(x+4)+(x+6)+(x+8)}{5} = 61$

or 5x + 20 = 305 or x = 57.

So, required difference = (57 + 8) – 57 = 8.

- Average of a group consisting two different groups when their averages are known:

  (a) Let Group A contains m quantities and their average is A and group B contains n quantities and their average is b, then average of group C containing a + b quantities

$$= \frac{ma+mb}{m+n}$$

Example 4:

There are 30 student in a class. The average age of the first 10 students is 12.5 years. The average age of the next 20 students is 13.1 years. The average age of the whole class is:

Solution:

Total age of 10 students = 12.5x10 = 125 years

Total age of 20 students = 13.1x20 = 262 years

Average age of 30 students = $\frac{125+262}{30} = 12.9$ years

Example 5:

The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girl is 15.4 years. The ration of the number of boys to the number of girls in the class is

Solution:

Let the number of boys in a class be x.

Let the number of girls in a class be y.

∴ Sum of the ages of the boys = 16.4 x

Sum of the ages of the girls = 15.4 y

$$\therefore 15.8(x+y) = 16.4x + 15.4y$$

$$\Rightarrow 0.6x = 0.4y \Rightarrow \frac{x}{y} = \frac{2}{3}$$

∴ Required ratio = 2: 3

Shortcut method:

16.4 15.4

15.8

15.8-15.4= 0.4 16.4-15.8= 0.6

∵ required ratio = $\frac{0.4}{0.6} = \frac{2}{3}$

(b) It average of m quantities is a and the average of n quantities out of them is b then the average of remaining group rest of quantities is $\frac{ma-nb}{m-n}$

Example 6:

Average salary of all the 50 employees increasing 5 officers of a company is Rs 850. If the average salary of the officers is Rs 2500. Find the average salary of the remaining staff of the company.

(a) 560 (b) 660

(c) 667 (d) 670

Solution:

Here, m = 50, n =5, a = 850, b = 2500

∴ Average salary of remaining staff = $\frac{ma-nb}{m-n}$

$$= \frac{50 \times 850 - 5 \times 2500}{50-5}$$

$$= \frac{42500 - 12500}{45}$$

= 667 (approx)

WEIGHTED AVERAGE

If we have two or more groups of members whose individual averages are know, then combined average of all the members of all the groups is known as weighted average. Thus if there are k groups having member of number n_1, n_2, n_3… n_k with averages A_1, A_2, A_3, ……. A_k respectively then weighted average.

$$A_w = \frac{n_1A_1 + n_2A_2 + n_3A_3 + \cdots + n_kA_k}{n_1 + n_2 + n_3 + \cdots + n_k}$$

Example 7:

The average monthly expenditure of a family was Rs 2200 during the first 3 months; Rs 2250 during the next 4 months and Rs 3120 during the last 5 months of a year. If the total saving during the year were Rs 1260, then the average monthly income was

Solution:

Total annual income

$$= 3 \times 2200 + 4 \times 2250 + 5 \times 3120 + 1260$$

$$= 6600 + 9000 + 15600 + 1260 = 32460$$

∴ Average monthly income

$$= \frac{32460}{12} = Rs\ 2705$$

❖ If X is the average $x_1 + x_2 + x_3 + \cdots + x_n$ then

(a) The average of $x_1 + a, x_2 + a, x_3 + a, \ldots x_n + a$ is $X + a$

(b) The average of $x_1 - a, x_2 - a, x_3 - a, \ldots x_n - a$ is $X - a$

(c) The average of $ax_1, ax_2, \ldots ax_n$ is aX, provide $a \neq 0$

(d) The average of $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots \frac{x_n}{a}$ is $\frac{x}{a}$ provided a ≠0

❖ If, in a group, one or more new quantities are added or excluded, then the new quantity or sum of added or excluded quantities = [Change in no. of quantities x original average]± [Change in average x final no. of quantities]

Take +ve sing.if quantities added and
Take –ve sing if quantities removed.

Example 8:

The average weight of 29 students in a class is 48 kg. If the weight of the teacher is included, the average weight rises by 500 g. Find the weight of the teacher.

Solution:

Here, weight of the teacher is added and final average of the group increases.

∴ Change in average is (+)ve, using the formula

Sum of the quantities added

$$= \left(\begin{matrix}\text{Change in no. of quantities} \\ \times \\ \text{original average}\end{matrix}\right) + \left(\begin{matrix}\text{Change in average} \\ \times \\ \text{final no. of quantities}\end{matrix}\right)$$

⇒ weight of teacher = $(1 \times 48) + 0.5 \times 30) = 63$kg

∴ weight of teacher is 63 kg.

Example 9:

The average age of 40 students in class is 15 years. When 10 new students are admitted, the average is increased by 0.2 year. Find the average age of the new students.

Solution:

Here, 10 new students are admitted.

∴ change in average is +ve. Using the Formula

Sum of the quantities added

$$= \left(\begin{matrix}\text{Change in no. of quantities} \\ \times \\ \text{original average}\end{matrix}\right) + \left(\begin{matrix}\text{Change in average} \\ \times \\ \text{final no. of quantities}\end{matrix}\right)$$

⇒ Sum of the weight of 10 new students admitted

$= (10 \times 15) + (0.2 \times 50) = 160$ kg

∴ Average age of 10 new students = $\frac{s_a}{n_a} = \frac{160}{10} = 16$

∴ Average age of 10 new students is 16 years.

Example 10:

A train covers the first 160 km at a speed of 120 km/h, another 160 km at 140 km/h, and the last 160 km at 80 km/h, Find the average speed of the train for the entire journey.

Solution:

Average speed $= \frac{3xyz}{xy+yz+zx}$

$= \frac{3\times120\times140\times80}{120\times140+140\times80+80\times120}$

$= \frac{360\times140\times80}{1600+11200+9600} = \frac{4032000}{37600}$

$= 107\frac{11}{47}$ km/h

❖ If a person cover A km at a speed of x km/h,B km at a speed of y km/hr and c km at a speed of z km/h, the average speed during the entire journey is $\frac{A+B+C}{\frac{A}{x}+\frac{B}{Y}+\frac{C}{z}}$ km/h

Example 11:

A person cover 9 km at a speed of 3 km/h,25 km at a speed of 5 km/h and 30 kmat a speed of 10 km/h. Find the average speed for the entire journey.

The average speed = $\frac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}$

$$= \frac{9+25+30}{\frac{9}{3}+\frac{25}{5}+\frac{30}{10}}$$

$= \frac{64}{11} = 5\frac{9}{11}$ km/h

❖ If a person covers At part of the distance at x km/h. Bth part of the distance at y km/h and the remaining Cth part at z km/h, then the average speed during the entire journey is $\frac{1}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\Big)$km/h

Example 12:

A train covers 50% of the journey at 30 km/h. 25% of the journey at 25 km/h and the remaining at 20 km/h. Find the average speed of the train during the entire journey.

Solution:

The average speed

$$= \frac{100}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}$$

$$= \frac{100}{\frac{50}{30}+\frac{25}{25}+\frac{25}{20}}$$

(Here, A=50,B=25 and C=25)

$$= \frac{100}{47/12} = \frac{1200}{47}$$

$= 25\frac{25}{47}$ km/h

❖ If a certain distance is covered at a mph and an equal distance at b kmph, then the average speed during whole Journey =$\frac{2ab}{a+b}$kmph.

Example 13:

A motorist travels to a place 150 km away at an average speed of 50km/hr and returns at 30 km/hr. His average speed for the whole journey in km/hr is

Solution:

Average speed

$= \frac{2ab}{a+b}$ = km/hr

$$= \frac{2\times 50\times 30}{50+30}\Big)\, km/hr$$

$= 37.5$km/hr.

GEOMETRIC MEAN OR GEOMETRIC AVERAGE

Geometric mean of $x_1, x_2, \dots . x_n$ is denoted by

G. M = $\overline{x_1 \times x_2 \times \dots \times x_n}$

Example 14:

The production of a company for three successive years has increased by 10% 20 % and 40% respectively what is the average increase of production.

Solution:

G. M. = $(10 \times 20 \times 10)^{1/3} = 20\%$

Example 15:

The mean of the marks secured by 25 students of section A of class X is 47, that of 35 students of section B is 51 and that 30 students of section C is 53. Find the combined mean of the marks of students of three sections of class X.

Solution:

Mean of the marks of 25 students of XA = 47

∴ Sum of the marks of 25 students

$= 25 \times 47 = 1175$ ... (i)

Mean of the marks of 35 students of XB = 51

∴ Sum of the marks of 35 students

$= 35 \times 51 = 1785$ ... (ii)

Mean of the marks of 30 students of XC = 53

∴ Sum of the marks of 30 students =

$= 30 \times 53 = 1590$ ... (iii)

Adding (i), (ii), and (iii)

Sum of the marks of $(25 + 35 + 30)$ i.e. 90

student

= 1175 + 1785 + 1590 = 4550

Thus the combined mean of the marks of students of three sections = $\frac{4550}{90} =$ 50.56

Example 16:

Find the A.M. of the sequence 1, 2, 3, ..., 100

Solution:

We have sum of first n natural numbers $= \frac{n}{2}(n+1)$

here n =100

$\Rightarrow Sum = \frac{100}{2} \times 101 = 101 \times 50$

$$\Rightarrow AM = \frac{Sum}{100} = \frac{101 \times 50}{100} = 50.5$$

Example 17:

A sequence of seven consecutive integers is given. The average of the first five given integers is n. Find the average of all the seven integers.

Solution:

Let the seven consecutive integers be $x, x+1, x+2, \ldots . x+6$

The sum of the first five is

$x + x + 1 + x + 2 + x + 3 + x + 4 = 5x + 10$

The average of these five is $\frac{5x+10}{5} = x + 2 = n$

The average of the seven will be

$\frac{5x+10+x+5+x+6}{7} = \frac{7x+21}{7} = x + 3$

As x + 2 = n, so x + 3 = x + 2 + 1 = n + 1

Example 18:

The average of 11 results is 50. If the average of first six results is 49 and that of last six results is 52, find the sixth result.

Solution:

Average of 11 results

1 2 3 4 5 6 7 8 9 10 11

Average of last 6 results = 52

Average of last 6 results = 49

It is quite obvious that the sixth result is included twice, once in the first six results and second in the last six results.

∴ Value of the sixth result = (Sum of first six results) + (Sum of last six results) – Sum of 11 results

$= 6 \times 49 + 6 \times 52 - 11 \times 50 = 56$

Example 19:

Typist A can type a sheet in 5 minutes, typist B is 6 minutes and typist C in 8 minutes. The average number of sheets typed per hour per typist is .... sheets.

Solution:

A types 12 sheets in 1 hour

B types 10 sheets in 1 hour

C types 7.5 sheets in 1 hour

Average number of sheets types per hour per typist

$= \frac{12+10+7.5}{3} = \frac{29.5}{3} = 9.83$