12.MENSURATION

Mensuration is the branch of mathematics which deals with the study of different geometrical shapes, their areas and volumes in the broadest sense, it is all about the process of measmement. These are two types of geometrical shapes (1) 2D (2) 3D

Perimeter: Perimeter is sum of all the sides. It is measured in cm, m. etc.

Area: The area of any figure is the amount of surface enclosed within its boundary lines. This is measured in square unit like cm^2, m^2, etc.

Volume: If an object is solid, then the space occupied by such an object is called its volume. This is measured in cubic unit like cm^3, m^3, etc.

Basic Conversions:

I. 1 km = 10 hm
1 hm = 10 dam
1 dam = 10m
1 m = 10dm
1 dm = 10 cm
1 cm = 10mm
1 m = 100 cm = 1000 mm
1 km = 1000m

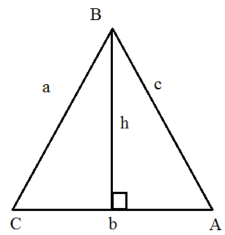
II. 1 km = $\frac{5}{8}$ miles
1 mile = 1.6 km
1 inch = 2.54 cm
1 mile = 1760 yd = 5280 ft.
1 nautical mile (knot) = 6080 ft

III. 100 kg = 1 quintal
10 quintal = 1 tonne
1 kg = 2.2 pounds (approx.)

IV. 1 litre = 1000cc
1 acre = $100m^2$
1 hectare =10000 m^2(100 acre)

PART I: PLANE FIGURES

TRIANGLE B



Perimeter (P) = a+b+c

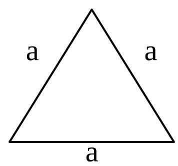
Area (A) = $\sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2}$ and a, band care three sides of the triangle.

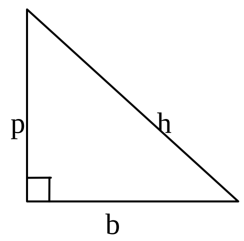
Also A = 1/2×b×h:

Where b →base h→ altitude

a) Equilateral triangle



b) Right triangle



$A = \frac{1}{2}pb \text{ and } h^2 = p^2 + b^2$(Pythagoras triplet)

where p → perpendicular
b → base
h → hypotenuse

Example 1:

Find the area of triangle whose sides are 50 m, 78m, 112m respectively and also find the perpendicular from the opposite angle on the side 112 m.

Solution:

Here a = 50 m, b = 78 m, c = 112m

$s = \frac{1}{2}(50 + 78 + 112) = 120m$

$s - a = 120 - 50 = 70m$

$s - b = 120 - 78 = 42m$

$s - c = 120 - 112 = 8m$

$\because Area = \sqrt{120 \times 70 \times 42 \times 8} = 1680\ sq.m.$

$\therefore$ Area = $\frac{1}{2}$ base × perpendicular

$\therefore Perpendicular = \frac{2\ Area}{Base} = \frac{1680 \times 2}{112} = 30m.$

Example 2:

The base of a triangular field is 880 m and its height 550 m. Find the area of the field. Also calculate the charges for supplying water to the field at the rate of `24.25 per sq. Hectometre

Solution:

Area of the field $= \frac{Base \times Height}{2}$

$= \frac{880 \times 550}{2} = 242000$ sq.m. = 24.20 sq. hm

Cost of supplying water to 1 sq. hm = `24.25

Cost of supplying water to the whole field

= 24.20 × 24.25=`586.85

NOTE:

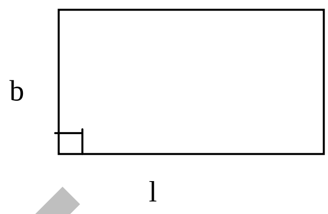
❖ In a rectangle, $\frac{(Perimeter)^2}{4}$ = (diagonal)^2 + 2 × Area

❖ In an isosceles right angled triangle,
Area $= \frac{b}{4}\sqrt{4a^2 - b^2}$
where a is two equal side and b is different side

❖ Inaparallelogram,
Area - Diagonal × length of perpendicular on it

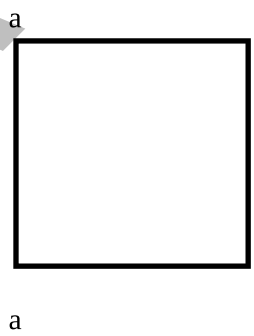
❖ If area of circle is decreased by x%, then the radius of circle is decreased by
$\left(100 - 10\sqrt{100 - x}\right)\%$

RECTANGLE



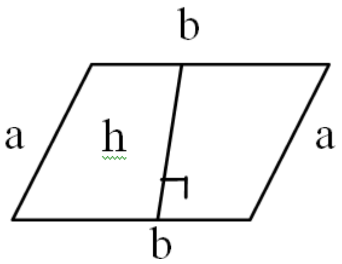
Perimeter - 2 (l + b)
Area = l × b; where l → lengh
b → breadth

SQUARE



Perimeter = 4 × side = 4a
Area = (side)^2 = a^2; where a → side

PARALLELOGRAM



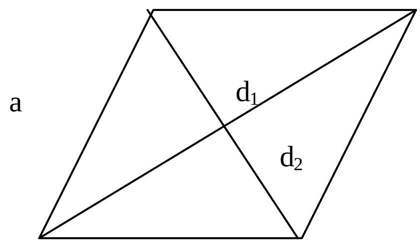
Perimeter = 2 (a+ b)

Area = b × h; where a → breadth

b → base (or length)

h → altitude

RHOMBUS

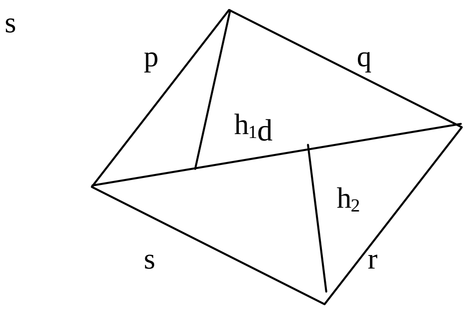


Perimeter = 4a

Area $= \frac{1}{2} d_1 \times d_2$ where a → side and

d_1 and d_2 are diagonals

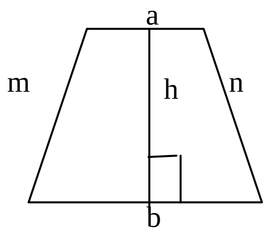
IRREGULAR QUADRILATERAL



Perimeter = p + q + r + s

Area $= \frac{1}{2} \times d \times (h_1 + h_2)$

TRAPEZIUM



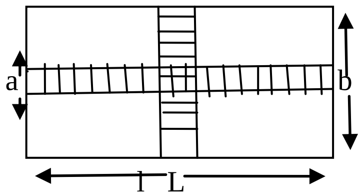
Perimeter = a + b + m + n

Area $= \frac{1}{2}(a + b)h;$ where a and b are two parallel sides;

m and n are two non-parallel sides;

h → perpendicular distance between two parallel sides.

Area of pathways running across the middle of a rectangle

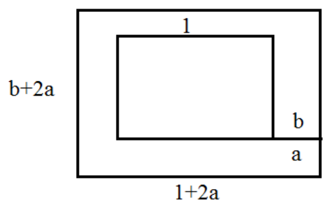


Area = a (l + b) – $a^2$where l → length

b→ breadth

a → width of the pathway

PATHWAYSOUTSIDE



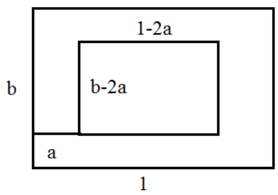
A = (l +2a) (b + 2a) – lb,

where l → length

b → breadth

a → width of the pathway

PATHWAYSINSIDE



A = lb – (l – 2a) (b – 2a); where l → length
b → breadth
a → width of the pathway

Example 3:

A 5100 sq.cm trapezium has the perpendicular distance between the two parallel sides 60 m. If one of the parallel sides be 40m then find the length of the other parallel tide.

Solution:

Since, A $= \frac{1}{2}(a+b)h$

$\Rightarrow 5100 = \frac{1}{2}(40+x) \times 60$

$\Rightarrow 170 = 40 + x$

∴ other parallel side = 170 – 40 = 130m

Example 4:

A rectangular grassy plot is 112m by 78 m. It has a gravel path 2.5m wide all round it on the inside. Find the area of the path and the cost of constructing it at `2 per square metre?

Solution:

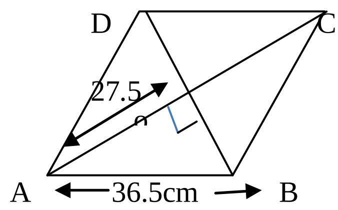
A = lb – (l – 2a) (b – 2a)
= 112 × 78 – (112 – 5) (71 – 5)
=112 × 78 – 107 × 73 – 8736 – 7811
= 925 sq.m
∴ Cost of construction = rate × area
= 2 × 925 = `1850

Example 5:

The perimeter of a rhomb us is 146 cm and one of its diagonals is 55 cm. Find the other diagonal and the area of the rhombus.

Solution:

Let ABCD be the rhombus in which AC = 55 cm.



A 36.5cm B

and $AB = \frac{146}{4} = 36.5cm$

Also, $AO = \frac{55}{2} = 27.5cm$

$\therefore BO = \sqrt{(36.5)^2 - (27.5)^2} = 24cm$

Hence, the other diagonal BD = 48 cm

Now, Area of the rhombus $= \frac{1}{2} AC \times BD$

$= \frac{1}{2} \times 55 \times 48 = 1320\ sq.cm.$

Example 6:

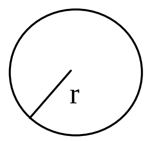
Find the area of a quadrilateral piece of ground, one of whose diagonals is 60 m long and the perpendicular from the other two vertices are 38 and 22m respectively.

Solution:

Area $= \frac{1}{2} \times d \times (h_1 + h_2$

$= \frac{1}{2} \times 60(38+22) = 1800\ sq.m$

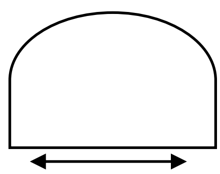
CIRCLE



Perimeter (Circumference) = 2πr = πd
Area = πr^2; where r → radius
d → diameter

SEMICIRCLE

π=22/7 or 3.14

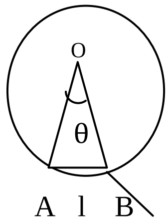


r

Perimeter = πr + 2r

Area $= \frac{1}{2} \times \pi r^2$

SECTOR OFACTRCLE



Segment

Area of sector $OAB = \frac{\theta}{360} \times \pi r^2$

Length of an arc (l) $= \frac{\theta}{360} \times 2\pi$

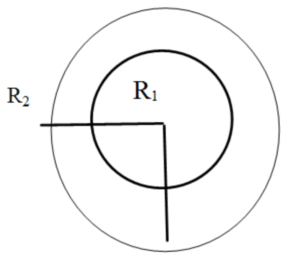
Area of segment = Area of sector – Area of triangle OAB

$\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

Perimeter of segment – length of the arc+ length of segment

$AB = \frac{\pi r\theta}{180} + 2r \sin\frac{\theta}{2}$

RING



Area of ring $= \pi(R_2^2 - R_1^2)$

Example 7:

A wire is looped in die form of a circle of radius 28 cm. It is re-bent into a square form. Determine the length of a side of the square.

Solution:

(a) Length of the wire - Perimeter of the circle

$= 2\pi \times 28$

$= 176 \text{ cm}^2$

Side of the square $= \frac{176}{4} = 44cm$

Example 8:

The radius of a wheel is 42 cm. How many revolutions will it make in going 26.4 km?

Solution:

Distance travelled in one revolution = Circumference of the wheel= $2\pi r = 2 \times \frac{22}{7} \times 42cm = 264\ cm$

∴ No. of revolutions required to travel 26.4 km $= \frac{26.4 \times 1000 \times 100}{264} = 10000$

Example 9:

Find the area of sector of a circle whose radius is 6 cm when

(a) the angle at the centre is 35°

(b) when the length of arc is 22 cm

Solution:

(a) Area of sector

$$= \pi r^2 . \frac{\theta}{360°} = \frac{22}{7} \times 6 \times 6 \times \frac{35}{360} cm^2$$

$$= 11\ sq.cm$$

(b) Here length of arc l = 22 cm.

$\therefore 2\pi r \frac{\theta}{360°} = 22cm$

Area of sector $= \pi r^2 . \frac{\theta}{360°} = \frac{1}{2} r . 2\pi r \frac{\theta}{360°}$

$= \frac{1}{2} rl = \frac{1}{2} \times 6 \times 22 sq.cm = 66 sq.cm.$

Example 10:

The radius of a circular wheel is $1\frac{3}{4}$ m. How many revolutions will it make in travelling 11 km?

Solution:

Distance to be travelled $= 11km = 11000m$

Radius of the wheel $= 1\frac{3}{4}m = \frac{7}{4}m$

∴ Circumference of the wheel= $2 \times \frac{22}{7} \times \frac{7}{4} = 11m$

∴ In travelling 11 m, wheel makes 1 revolution.

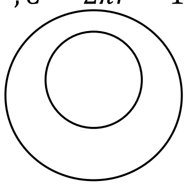
∴In travelling 11000 m the wheel makes $\frac{1}{11} \times 11000$ revolutions. i.e. 1000 revolutions.

Example 11:

The circumference of a circular garden is 1012m. Find the area of outsider road of 3.5m width runs around it. Calculate the area of this road and find the cost of gravelling the road at $32 per 100 sqm.

Solution:

$A = \pi r^2, C = 2\pi r = 1012$



$\Rightarrow r = 1012 \times \frac{1}{2} \times \frac{7}{22} = 161m$

$\therefore$ Area of garden $= \frac{22}{7} \times 161 \times 161 =$

$81466 sq.m$

$\therefore$Area of the road = area of bigger circle - area of the garden

Now, radius of bigger circle = 161 + 3.5 =

$\dfrac{329}{2} m$

Area of bigger circle $= \frac{22}{7} \times \frac{329}{2} \times \frac{329}{2} =$

$85046 \dfrac{1}{2} sq.m$

Thus, area of the road $= 85046 \frac{1}{2} - 81466 =$

$3580 \dfrac{1}{2} sqm.$

Hence, cost $= Rs. \frac{7161}{2} \times \frac{32}{100} = \$1145.76$

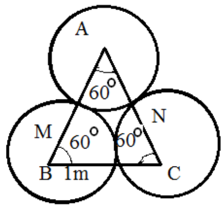
Example 12:

There is an equilateral triangle of which each side is 2m. With all the three corners as centres, circles each of radius 1 mare described.

(i) Calculate the area common to all the circles and the

(ii) Find the area of the remaining portion of the triangle.

Solution:



Area of each sector $= \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times$

1×1

$= \frac{1}{6} \times \frac{22}{7} = \frac{11}{21} m^2$

Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$

$= \frac{\sqrt{3}}{4} \times 2 \times 2 = \sqrt{3} m^2$

i) Common area = 3 × Area of each sector

$= 3 \times \dfrac{11}{21} = \dfrac{11}{7} = 1.57 m^2$

ii) Area of the remaining portion of the triangle = Ar. of equilateral triangle – 3 (Ar. of each sector)

$\sqrt{3} - 1.57 = 1.73 - 1.57 = 0.16 m^2$

PART-II: SOLID FIGURE

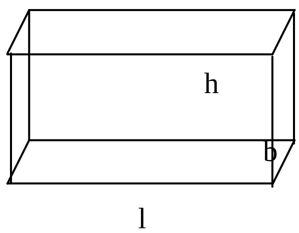
CUBOID

A cuboid is a three dimensional box.

Total surface area of a cuboid = 2 (lb + bh + lh)

Volume of the cuboid = lbh

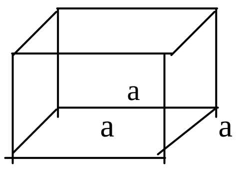
Length of diagonal $\sqrt{l^2 + b^2 + h^2}$



$\boxed{\text{Area of four walls } = 2(l + b) \times h}$

Rectangular Parallelpiped box. It is same as cuboid. Formally a polyhedron for which all faces are rectangles.

CUBE

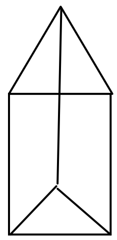


a

A cube is a cuboid which has all its edges equal.
Total surface area of a cube = $6a^2$
Volume of longest the cube = a^3
Length of longest diagonal = $\sqrt{3}a$

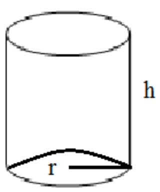
RIGHT PRISM

A prism is a solid which can have any polygon at both its ends.
Lateral or curved surface area = Perimeter of base × height
Total surface area = Lateral surface area + 2 (area of the end)
Volume = Area of base × height



RIGHT CIRCULAR CYLINDER

It is a solid which has both its ends in the form of a circle, lateral surface area = $2\pi rh$
Total surface area = $2\pi r\ (r + h)$
Volume = $\pi r^2 h$; where r is radius of the base and h is height



PYRAMID

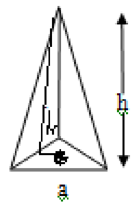
A pyramid is a solid which can have any polygon at its base and its edges converge to single apex.

Lateral or curved surface area

$$= \frac{1}{2}(\text{perimeter of base}) \times \text{slant height} = \frac{1}{2}pl$$

Total surface area = lateral surface area + area of the base
Volume 1/3 (area of the base) × height

1. Triangular Pyramid:



(i) Area of the lateral surface of the pyramid

$$= \frac{1}{2} \times perimeter \times slantheight$$

$$= \frac{1}{2} \times 3a \times l = \frac{3}{2}al$$

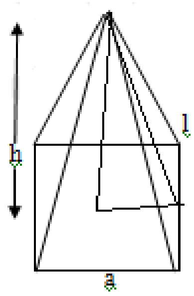
(ii) Volume $= \frac{1}{3} \times h \times$ area of base $= \frac{1}{3} \times h \times \frac{\sqrt{3}}{4}a^2$

$$= \frac{ha^2}{4\sqrt{3}}$$

(iii) Total Area of the pyramid $= \frac{1}{2}3al + \frac{\sqrt{3}}{4}a^2$

2. Square Pyramid:

(i) Volume $= \frac{1}{3}h \times$ area of base $= \frac{1}{3}h \times a^2$



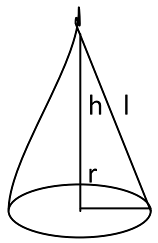
(ii) Lateral surface area $= \frac{1}{2} \times$ perimeter × slant height $= \frac{1}{2} \times 4a \times l = 2al$

(iii) Total area of the pyramid = $2al + a^2 = a\ (2l + a)$

RIGHTCIRCULAR CONE

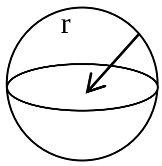
It is a solid which has a circle as its base and a slanting lateral surface that converges at the apex.

Lateral surface area = πrl
Total surface area = πr (l + r)
Volume == $\frac{1}{3}\pi r^2 h$; where r: radius of the base
h: height
l: slant height

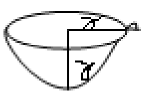


SPHERE

It is a solid in the form of a ball with radius r.
Lateral surface area = Total surface area = $4\pi r^2$
Volume = $\frac{4}{3}\pi r^3$; where r is radius.



HEMISPHERE



It is a solid half of the sphere.
Lateral surface area = $2\pi r^2$
Total surface area = $3\pi r^2$
Volume= $\frac{2}{3}\pi r^3$; where r is radius

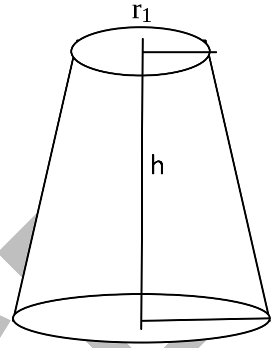
FRUSTUM OFA CONE

When a cone cut the left over part is called the frustum of the cone.
Curved surface of area = πl ($r_1 + r_2$)

Total surface area = πl ($r_1 + r_2$) + $\pi r_1^2 + \pi r_2^2$

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

Volume $= \frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$
Where r_1 and r_2:radii
h:height



Example 13:

The sum of length, breadth and height of a room is 19m. The length of the diagonal is 11m. Find he cost of painting the total surface area of the room at the rate of `10 per m^2.

Solution:

Let length, breadth and height of the room be l, b and h, respectively. Then,

l + b + h = 19 … (i)

and $\sqrt{l^2 + b^2 + h^2} = 11$

$\Rightarrow l^2 + b^2 + h^2 = 121$ … (ii)

Area of the surface to be painted

$= 2(lb + bh + hl)$

$(l + b + h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + hl$

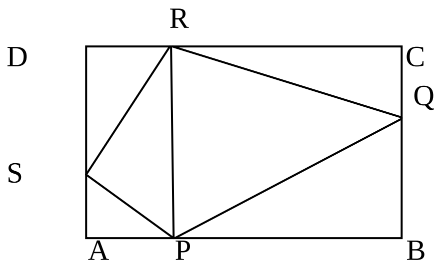
$\Rightarrow 2(lb + bh + hl) = (19)^2 - 121 = 361 - 121 = 240$

Surface area of the room = $240m^2$

Cost of painting the required area = 10 × 240 = `2400

Example 14:

ABCD is a parallelogram. P, Q, R and S are points on sides AB, BC, CD and DA, respectively such that AP = DR. If the area of the, rectangle ABCD is 16cm^2, then find the area of the quadrilateral PQRS.



Solution:

Area of the quadrilateral PQRS

Area of ΔSPR + Area of ΔPQR

$=\frac{1}{2}\times PR\times AP+\frac{1}{2}\times PR\times PB$

$=\frac{1}{2}\times PR(AP+PB)=\frac{1}{2}\times AD\times AB$

$(\because PR=AD \ and\ AP+PB=AB)$

$=\frac{1}{2}\times Area\ of\ rectangle\ ABCD=\frac{1}{2}\times 16=8cm^2$

Example 15:

A road roller of diameter 1.75 m and length 1 m has to press a ground of area 1100 sqm. How many revolutions does it make?

Solution:

Area covered in one revolution = curved surface area

Number of revolutions$=\frac{Total\ \ area\ \ to\ be\ pressed}{Curved\ \ surface\ \ area}$

$=\frac{1100}{2\pi rh}=\frac{1100}{2\times\frac{22}{7}\times\frac{1.75}{2}\times 1}$

= 200

Example 16:

The annual rainfall at a place is 43 cm. Find the weight in metric tonnes of the annual rain falling there on a hectare of land, taking the weight of water to be 1 metric tonne to the cubic metre.

Solution:

Area of land = 10000 sqm

Volume of rainfall $=\frac{10000\times 43}{100}=4300m^3$

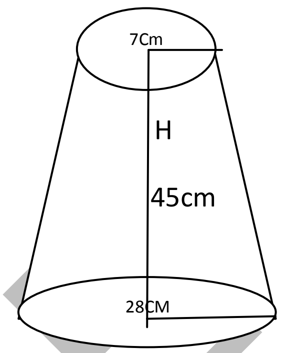
Weight of water = 4300 × 1 m tonnes = 4300 m tonnes

Example 17:

The height of a bucket is 45 cm. The radii of the two circular ends are 28 cm and 7 cm, respectively. Find the volume of the bucket

Solution:

Here r_1 = 7cm, r_2 = 28cm and h = 45cm



Volume of the bucket $=\frac{1}{3}\pi h(r_1^2+r_2^2+r_1r_2)$

Hence, the required volume

$=\frac{1}{3}\times\frac{22}{7}\times 45(28^2+7^2+28\times 7)=48510cm^2$

Example 18:

A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the external diameter be 50 cm and the length of the tube be 140 cm, find the number of cubic cm of iron in it.

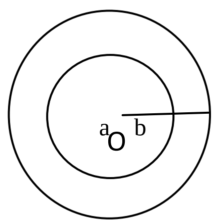
Solution:

Height = 140 cm

External diameter = 50 cm

External radius = 25 cm

O AB



Also, internal radius OA = OB – AB = 25 – 2 = 23 cm

∴Volume of iron = $V_{external}$ - $V_{internal}$

$= \frac{22}{7} \times 140(25^2 - 23^2) = 42240$ cu. cm.

Example 19:

A cylindrical bath tub of radius 12cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. What is the radius of the ball?

Solution:

Volume of the spherical ball = volume of the water displaced.

$\Rightarrow \frac{4}{3}\pi r^3 = \pi(12)^2 \times 6.75$

$\Rightarrow r^3 = \frac{144 \times 6.75 \times 3}{4} = 729$

or r = 9 cm

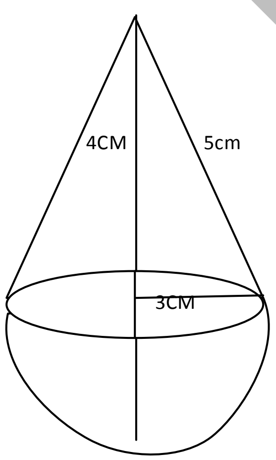
Example 20:

A toy is in the form of a cone mounted on a hemisphere with the same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. Determine the surface area of the toy. (Use π = 3.14).

Solution:

The radius of the hemisphere $= \frac{1}{2} \times = 3cm$

Now, slant height of cone $= \sqrt{3^2 + 4^2} = 5cm$



The surface area of die toy

= Curved surface of the conical portion + Curved surface of the hemisphere

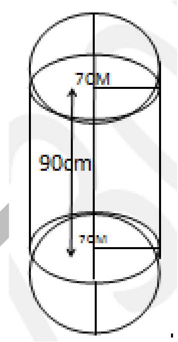
$= (\pi \times 3 \times 5 + 2\pi \times 3^2)cm^2$

$= 3.14 \times 3\ (5 + 6)\ cm^2 = 103.62\ cm^2$

Example 21:

A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and die radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of Re. 1 per dm^2.

Solution:



Let the height of the cylinder be h cm.

Then h + 7+7 = 104

$\Rightarrow$ h = 90

Surface area of the solid

= 2 × curved surface area of hemisphere + curved surface area of the cylinder

$= \left(2 \times 2 \times \frac{22}{7} \times 7 \times 7 + 2 \times \frac{22}{7} \times 7 \times\right.$

90cm2

$= 616 + 3960\ cm^2 = 4576cm^2$

Cost of polishing the surface of the solid

$= \frac{4576 \times 1}{100} = \$45.76$

Example 22:

A regular hexagonal prism has perimeter of its base as 600 cm and height equal to 200 cm. How many litres of petrol can it hold 7 Find the weight of petrol if density is 0.8 gm/cc.

Solution:

Side of hexagon$= \frac{Perimeter}{Number\ \ of\ sides} = \frac{600}{6} = 100cm$

Area of regular hexagon $= \frac{3\sqrt{3}}{2} \times 100 \times 100 = 25950\ sq.cm$

Volume = Base area × height

= 25930 × 200 = 5190000 cu.cm. = 5.19 cu.m.

Weight of petrol = Volume × Density

= 519000 × 0.8 gm/cc

$= 4152000\ gm = 4152\ kg.$

Example 23:

A right pyramid, 12 cm high, has a square base each side of which is 10 cm. Find the volume of the pyramid.

Solution:

Area of the base = 10 × 10 = 100 sq.cm.

Height =12 cm

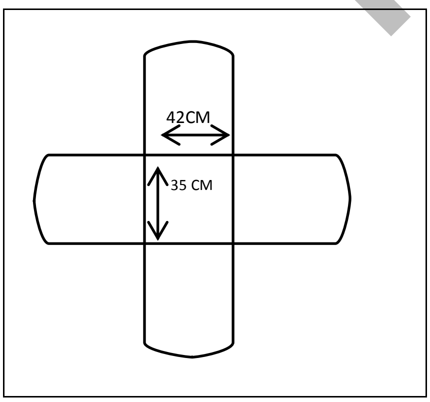
∴ Volume of the pyramid $= \frac{1}{3} \times 100 \times 12$ =400cu.cm.

Example 24:

Semi-circular lawns are attached to both the edges of a rectangular field measuring 42 m × 35m. The area of the total fieldis:

(a) $3818.5m^2$ (b) $8318m^2$
(c) $5813m^2$ (d) $1358m^2$

Solution:



(a) Area of the field

$= 42 \times 35 + 2 \times \frac{1}{2} \times \frac{22}{7} \times (21)^2 + 2 \times \frac{1}{2} \times \frac{22}{7} \times (17.5)^2$

$= 1470 + 1386 + 962.5 = 3818.5m^2$

Example 25:

A frustum of a right circular cone has a diameter of base 10 cm, top of 6 cm, and a height of 5 cm, find the area of its whole surface and volume.

Solution:

Here r_1 = 5 cm, r_2 = 3 cm and h = 5 cm.

$\therefore l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{5^2 + (5-3)^2} = \sqrt{29}cm = 5.385cm$

∴Whole surface of the frustum

$= \pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

$= \frac{22}{7} \times 5.385(5+3) + \frac{22}{7} \times 5^2 + \frac{22}{7} \times 3^2 = 242.25\ sq.cm.$

Volume $= \frac{\pi h}{3}\ r_1^2 + r_1 r_2 + r_2^2$

$= \frac{22}{7} \times \frac{5}{3}[5^2 + 5 \times 3 + 3^2] = 256.67\ cu.cm$

Example 26:

A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of one end, and the other end as its base. The volume of the cylinder, hemisphere and the cone are, respectively in the ratio:

(a) 2:3:2 (b) 3:2:1
(c) 3:1:2 (d) 1:2:3

Solution:

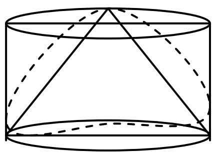
(b) We have,

radius of the hemisphere = raidus of the cone

= height of the cone

= height of the cylinder = r (say)

Then, ratio of the volumes of cylinder, hemisphere and cone



$= \pi r^3 : \frac{2}{3}\pi r^3 : \frac{1}{3}\pi r^3 = 1 : \frac{2}{3} : \frac{1}{3} = 3:2:1$