H.C.F & L.C.M

FACTORS

A number may be made by multiplying two or more other numbers together. The numbers that are multiplied together are called factors of the final number.

Factors of 12 = 1,2,3,4,6, and 12.

All the numbers have a factor of one.

Common factor: A common factor of two or more given numbers is a number which divides each given number completely. Common factor of 12 and 18 are 1,2,3,6.

Co-prime numbers: Two or more numbers that do not have a common factor are known as co-prime or relatively prime.

For example: 4 and 15 are Co-prime numbers.

Highest common factor: The highest common factor (H.C.F.) of two or more numbers is the greatest number which divides each of them exactly. It is also known as greatest common divisor (GCD.).

H.C.F. can be calculated by:

(i) Prime factorisation method

(ii) Division method

(i) H.C.F. by prime factorisation method:

Example 1:

Find the H.C.F. of 40 and 60 by prime factorisation method.

Solution:

Prime factors of 40.

2	40
2	20
2	10
5	5
	1

$\therefore 40 = 2 \times 2 \times 2 \times 5$

Prime factors of 60

2	60
2	30
3	15
5	5
	1

$\therefore 60 = 2 \times 2 \times 3 \times 5$

Hence H.C.F. = $2 \times 2 \times 5 = 20$.

Divide the greater number by the smaller number, divide the divisor by the remainder and so on, until no remainder is left. The last divisor in the required H.C.F,

Finding the H.C.F of more than two number We find the H.C.F of any two say P. How find the H.C.F of P and the third number and so on. The last H.C.F will be the required ,H.C.F.

(ii) H.C.F by Division method:

❖ Divide the greater number by the smaller number, divide the divisor by the remainder divide die remainder by the next remainder and so on, until no remainder is left, the last divisor is the required H.C.F.

❖ Finding the H.C.F of more than two number: we find the H.C.F of any two say P. Now find the H.C.F of P and the third number, and so on. The last H.C.F will be the required H.C.F.

H.C.F. of two numbers by division method:

Example 2:

Find the H.C.F. of 140 and 200 by division method.

Solution:

140) 200 (1
140
60) 140 (2
120
20) 60 (3
60
x

∴ H.C.F. of 140 and 200 = 20.

H.C.F. of three numbers by division method:

Example 3:

Find the H.C.F. of 324,630 and 342 by division method.

Solution:

324) 630 (1
324
306) 324 (1
306
18) 306 (17
306
x

18) 342 (19
342
x

∴H.C.F. of324, 630 and 342 is 18.

H.C.F of polynomials:

When two or more-polynomials are factorised, the product of common factor is known as HCF of these polynomials.

e.g Lets find the HCF of $16x^3 (x - 1)^3 (x+ 1)$ and $4xy(x+ 1)^2 (x- 1)$

Now, $16x^3 (x - l)^3 (x+1)$

$=2\times2\times2\times2\times x\times x\times x\times(x – l)\times (x - 1) \times(x - 1) \times(x+ 1)$ and $4xy(x + l)^2(x- 1) = 2 \times 2xy (x + 1) (x +1) (x – 1)$

$\therefore$H.C.F. $= 2 \times 2\times x \times (x + 1) (x – 1) = 4x (x^2 –1)$

MULTIPLES

Multiples of a number are all those numbers which can be divided completely by the given number.

For example, Multiples of 5 are 5,10,15,20 etc.

Common multiples: Common multiples of two or more numbers are the numbers which can be exactly divided by each of the given number.

For example, Multiples of 3 are 3,6,9,12,15,18,21,24 etc. and Multiples of4 are 4,8,12,16,20,24,28 etc.

Common multiples of 3 and 4 are 12,24 etc.

Least common multiple: The least common multiple (L.C.M.) of two or more numbers is the smallest number which is exactly divisible by each of them

L.C.M. can be calculated by:

(i) Prime factorisation method

(ii) Division method

(I) L.C.M. by prime factorisation method:

Example 4:

Find the L.C.M. of 12 and 20 by prime factorization method.

Solution:

$12= 2\times2\times3$and$20 = 2\times2\times5$

$\therefore$L.C.M $=2 \times 2 \times 3 \times 5 = 60$.

(ii) L.C.M. by division method:

Example 5:

Find the L.C.M. of 14,56,91 and 84.

Solution:

2	14, 56, 91, 84
2	7, 28, 91, 42
7	7, 14, 91, 21
	1, 2, 13, 3

∴ L.C.M = 2×2×7×2×13×3 = 2184.

L.CM of polynomials:

When two or more polynomials are factorised, the product of the factors with highest powers is the lowest common multiple (LCM) of the polynomials.

Eg. Consider the polynomials (x^3- 8) and (x^2- 4)

$x^3 – 8 = (x – 2) (x^2 + 2x + 4)$

and $x^2 – 4 = (x+2) (x-2)$

∴ L.C.M = $(x – 2) (x + 2) (x^2 + 2x +4)$

$= (x +2) (x^3 – 8)$

H.C.F and L.C.M of FRACTIONS

First express the given fractions in their lowest terms.

Then,

H.C.F. = $\frac{\text{H.C.F.of numerators}}{\text{L.C.M.of denominators}}$

L.C.M. = $\frac{\text{L.C.M.of numerators}}{\text{H.C.F.of denominators}}$

Example 6:

Find the H.C.F. and L.C.M. of $4\frac{1}{2}, \frac{6}{2}, 10\frac{1}{2}$.

Solution:

Here, $4\frac{1}{2} = \frac{9}{2}, \frac{6}{2} = 3, 10\frac{1}{2} = \frac{21}{2}$.

H.C.F. = $\frac{\text{H.C.F.of 9,3,21}}{\text{L.C.M.of 2,1,2}} = \frac{3}{2} = 1\frac{1}{2}$

L.C.M. = $\frac{\text{L.C.M.of 9,3,21}}{\text{H.C.F.of 2,1,2}} = \frac{63}{1}$ =63.

H.C.F. AND L.C.M. OF DECIMAL NUMBERS:

H.C.F of decimal numbers:

STEP I: Find the H.C.F of given numbers without decimal.

STEP II: Put the decimal point from right to left according to the maximum decimal places among the given numbers.

LCM of decimal numbers;

STEP I: Find the L.C.M. of given numbers without decimal.

STEP II: Put the decimal point from right to left according to the minimum decimal places among the given numbers.

Example 7:

Find the H.C.F. and L.C.M. of 0.6,9.6 and 0.36.

Solution:

H.C.F of60,96 and 36 = 12

∴Required HCF = 0.12

L.C.Mof60,96and36 = 1440

∴ Required L.C.M =144.0

The product of two numbers:

H.C.F. of numbers × L.C.M. of numbers = Product of numbers

Example 8:

If H.C.F. and L.C.M. of two numbers are 3 and 60 respectively and. one number is 12 then find the other number.

Solution:

Let the other number be x.

Product of numbers = H.C.F. × L.C.M.

x × 12 = 3 × 60

$x = \frac{3\times60}{12} = 15$

REMEMBER

- The greatest number that will exactly divide x,y, z= H.C.F of x, y and z.
- The greatest number that will divide x, y and z leaving remainders a, b and c respectively =H.C.F of (x- a), (y -b) and (z - c).
- The least number which is exactly divisible by x, y and z. = L.C.M of x, y and z.
- The least number which when divided by x, y and z leaves the remainder a, b and c respectively= L.C.M of(x, y and z) - R where R=(x - a)=(y - b) = (z - c)
- The least number which when divided by x, y and z leaves the same remainder r in each case = L.C.M of (x, y and z)+r
- The greatest number that will divide x, y and z leaving the same remainder in each case = H.C.F of (x - y), (y - z) and (z- x).
- When two numbers P and Q are exactly divisible by a third number r. Then p + q, p - q and pq is also divisible by r.

Example 9:

The H.C.F of two numbers, each having three digits, is 17 and their L.C.M is 714.

Five sum of the numbers will be:

Solution:

Let the number be 17x and 17y where x and y are co-prime. L.C.M of 17x and 17y =17xy According to question, 17xy-714

$\Rightarrow xy=42=6 \times 7$

$\therefore x=6$ and $y = 7$

or $x = 7$ and $y=6$

First numbers = $17x = 17 \times 6 = 102$

Second numbers = $17y = 17\times7 = 119$

Required sum=102 + 119=221

Example 10:

Find the greatest number of six digits which on being divided by 6, 7, 8, 9 and 10 leaves 4, 5, 6, 7 and 8 as remainders respectively.

Solution:

The L.C.M. of 6, 7, 8, 9 and 10 = 2520

The greatest number of 6 digits = 999999 Dividing 999999 by 2520, we get 2079 as remainder. Hence the 6 digit number divisible by 2520 is 999999 - 2079 = 997920

Since 6 – 4=2, 7 – 5 = 2, 8 – 6 = 2, 9 – 7 = 2,

10 - 8 = 2, the remainder in each case is less than the divisor by 2.

$\therefore$Required number = 997920 – 2 = 997918

Example 11:

What least number must be subtracted from 1936 so that the remainder when divided by 9, 10,15 will leave in each case the same remainder 7?

Solution:

The L.C.M. of 9,10 and 15 is 90.

On dividing 1936 by 90, the remainder =46

But 7 is also a part of this remainder.

$\therefore$Required number = 46 - 7 = 39.

Example 12:

Find the greatest number which will divide 410, 751 and 1030 leaving a remainder 7 in each case.

Solution:

Required number

= H.C.F. of (410 - 7), (751 - 7) and (1030 - 7)

= 31

Example 13:

Find the H.C.F and L.C.M of 1.75,5.6 and 7.

Solution:

Making the same number of decimal places, the numbers may be written as 1.75, 5.60 and 7.00.

Without decimal point, these numbers are 175,560 and 700.

Now, H.C.F of 175,560 and 700 is 35.

∴H.C.F of 1.75,5.6 and 7 is 0.35.

L.C.M of175,560 and 700 is 2800.

∴L.C.M of 1.75,5.6 and 7 is 28.00 i.e. 28.

COMPARISON OF FRACTIONS:

Rule: Convert each one of the given fractions in the decimal form. Now, arrange them in ascending or descending order, as per requirement

Example 14:

Arrange the fractions $\frac{3}{8}, \frac{7}{12}, \frac{2}{3}, \frac{14}{19}, \frac{16}{25}$ and $\frac{1}{2}$ in ascending order of magnitude.

Solution:

Converting each of the given fractions into decimal form, we get: $\frac{3}{8} = 0.375, \frac{7}{12} = 0.583, \frac{2}{3} = 0.666, \frac{14}{19} = 0.736, \frac{16}{25} = 0.64$ and $\frac{1}{2} = 0.5$.

Clearly, 0.375 <0.5< 0.583<0.64 <0.666 < 0.736

$\therefore \frac{3}{8} < \frac{1}{2} < \frac{7}{12} < \frac{16}{25} < \frac{2}{3} < \frac{14}{19}$

Example 15:

The H.C.F. of two polynomials is $x^2 - 1$ and their L.C.M is $x^4 - 10x^2 + 9$. If one of die polynomials is $x^3 - 3x^2 - x + 3$, find the other.

Solution:

Given that H.C.F of p (x) and q (x) = $x^2 - 1 = (x + 1)(x - 1)$

Also, LCM of p(x)and q(x) = $x^4 - 10x^2 + 9 = x^4 - 9x^2 - x^2 + 9$

$= x^2 (x^2 - 9) - (x^2 - 9 = (x^2 - 9)(x^2 - 1)$

$= (x+3)(x-3)(x+1)(x-1)$

And $p(x) = x^3 - 3x^2 - x + 3 = x^2 (x-3) - (x-3)$

$= (x - 3)(x^2 - 1) = (x - 3)(x + 1)(x - 1)$

p(x) . q(x) = (H.C.F).(L.C.M)

$\therefore q(x) = \frac{\text{(H.C.F).(L.C.M)}}{p(x)} = \frac{(x+1)(x-1)(x+3)(x-3)(x+1)(x-1)}{(x-3)(x+1)(x-1)}$

$= (x+3)(x+1)(x-1)$

$= (x+3)(x^2 - 1) = x^2 + 3x^2 - x - 3$

Example 16:

Find the H.C.F and L.C.M of 6,72 and 120, using the prime factorisation method.

Solution:

We have: $6 = 2 \times 3$, $72 = 2^3 \times 3^2$, $120 = 2^3 \times 3 \times 5$

Here, 2^1 and 3^1 are the smallest powers of the common factors 2 and 3 respectively.

So, H.C.F(6,72,120) = $2^1 \times 3^1 = 2 \times 3 = 6$

$2^3, 3^2$ and 5^1 are the greatest powers of the prime factors 2,

3 and 5 respectively involved in the three numbers. So, L.C.M(6,72,120)=$2^3 \times 3^2 \times 5^1$ =360.

Example 17:

Find the GCD of: $14x^3+14$, $42(x^2 +4x+3)$

$(x^2 -x+1)$

Solution:

L.C.M = $2 \times 7 \times 11 \times 13 \times x^3 \times y^4 = 2002x^3y^4$.

$p(x)=14x^3+14 = 14(x^3+1) = 2 \times 7(x+1)(x^2 - x+1)$

$q(x)=42(x^2+4x+3)(x^2-x+1)$

$= 42(x^2+3x+x+3)(x^2 - x+1)$

$= 42[x(x+3) + (x+3)](x^2 - x+1)$

$=2\times3\times7(x+3)(x+1)(x^2 - x+1)$

$\therefore$GCD of p(x) and q(x) = $14(x+1)(x^2-x+1) = 14(x^3+1)$

Example 18:

Two bills of Rs 6075 and Rs 8505 respectively are to be paid separately by cheques of same amount. Find the largest possible amount of each cheque.

Solution:

Largest possible amount of cheque will be the H.C.F (6075, 8505).

We can write $8505=6075 \times 1 +2430$

Since, remainder $2430 \neq 0$ again applying division concept we can write

$6075 = 2430 \times 2+1215$

Again remainder $1215 \neq 0$

So, again applying the division concept we can write $2430=1215\times2+0$

Here the remainder is zero

So, H.C.F = 1215

Therefore, the largest possible amount of each cheque will be 1215.

Example 19:

A garden consists of 135 rose plants planted in certain number of columns. There are another set of 225 marigold plants which is to be planted in the same number of columns. What is the maximum number of columns in which they can be planted?

Solution:

To find the maximum number of columns we need to find the H.C.F(135,225)

We can write, $225 = 135 \times 1 + 90$

Since, remainder $90 \neq 0$

So, again applying division concept, we can write, $135 = 90 \times 1+45$

Remainder $45 \neq 0$ again using division concept, we have, $90 = 45 \times 2 + 0$

Since, remainder is 0 So, H.C.F=45

Therefore, 45 is the maximum number of columns in which the plants can be planted.

Example 20:

A watch ticks 90 times in 95 seconds and another watch ticks 315 times in 323 seconds. If both the watches are started together, how many times will they tick together in the first hour?

Solution:

The first watch ticks every $\frac{95}{90}$ seconds.

They will tick together after (L.C.M of $\frac{95}{90}$ & $\frac{323}{315}$)seconds.

Now L.C.M of $\frac{95}{90}$ and $\frac{323}{315}$

$= \frac{\text{L.C.M.of } 95,323}{\text{H.C.F.of } 90,315} = \frac{19\times5\times17}{45}$

The number of times, they will tick in the first 3600 seconds

$= 3600 \div \frac{19\times5\times17}{45} = \frac{3600\times45}{19\times5\times17} = 100\frac{100}{323}$

Once they have already ticked in the beginning; so in 1 hour they will tick 100+1 = 101 times.

Example 21:

Find the H.C.F. and L.C.M of $14xy^3$, $22x^2y$ and $26x^3y^4$.

Solution:

$14xy^3 = 2\times7\times x\times y^3$

$\Rightarrow 22x^2y = 2 \times 11 \times x^2 \times y$

$26x^3y^4 = 2 \times 13 \times x^3 \times y^4$

H.C.F $=2 \times x \times y=2xy$

L.C.M = $2 \times 7 \times 11 \times 13 \times x^3 \times y^4 = 2002x^3y^4$.