## RATIO, PROPORTION AND PARTNERSHIP

### RATIO

Ratio is strictly a mathematical term to compare two similar quantities expressed in the same units.

The ratio of two terms 'x' and 'y' is denoted by x:y.

In general, the ratio of a number x to a number y is defined as the quotient of the numbers x and y.

The numerator of the ratio is called the antecedent (x) and the denominator is called consequent (y) of the ratio.

### COMPARISON OF TWO OR MORE RATIOS

Two or more ratios may be compared by reducing the equivalent fractions to a common denominator and then comparing the magnitudes of their numerator. Thus, suppose 2:5, 4:3 and 4:5 are three ratios to be compared then the fractions $\frac{2}{5}, \frac{4}{3}$ and $\frac{4}{5}$ are reduced to equivalent fractions with a common denominator. For this, the denominator of each is changed to 15 equal to the L.C.M. their denominators. Hence the given ratios are expressed $\frac{6}{15}, \frac{20}{15}$ and $\frac{12}{15}$ or 2:5, 4:3, 4:5 according to magnitude.

**Example 1:**

Which of the ratio 2:3 and 5:9 is greater?

**Solution:**

In the form of fractions, the given ratios are $\frac{2}{3}$ and $\frac{5}{9}$,

Reducing them to fractions with a common denominator they are written as $\frac{6}{9}$ and $\frac{5}{9}$

Hence the greater ratio is $\frac{6}{9}$ or 2:3.

**Example 2:**

Are the ratio 3 to 4 and 6:8 equal?

**Solution:**

The ratio are equal if 3/4 = 6/8.

These are equal if their cross products are equal; that is, if 3×8 = 4×6. Since both of these products equal 24, the answer is yes, the ratios are equal.

Remember to be care full order matters!

A ratio of 1:7 is not the same as a ratio of 7:1.

### ■ REMEMBER

- ❖ The two quantities must be of the same kind and in same unit.
- ❖ The ratio is a pure number, i.e. without any unit of measurement.
- ❖ The ratio would stay unaltered even if both the antecedent and the consequent are multiplied or divided by the same number.

**Compound ratio:** Ratios are compounded by multiplying together the antecedents for a new antecedent and the consequents for a new consequent.

The compound of a: b and c: d Is a*c/b*d, i.e.,

ac: bd.

**Example 3:**

Find the compound ratio of the four ratios: 4:5, 15:13, 26:3 and 6:17

**Solution:**

The required ratio $= \frac{4\times15\times26\times6}{5\times13\times3\times17} = \frac{48}{17}$

or 48: 17

The duplicate ratio of x: y is $x^2 : y^2$.

The triplicate ratio of x: y is $x^3 : y^3$.

The sub duplicate ratio x: y is $\sqrt{x} : \sqrt{y}$.

The sub triplicate ratio x: y is $\sqrt[3]{x} : \sqrt[3]{y}$.

Reciprocal ratio of a: b is $\frac{1}{a} : \frac{1}{b}$ or b: a.

**Inverse ratio**

Inverse ratio of x: y is y: x.

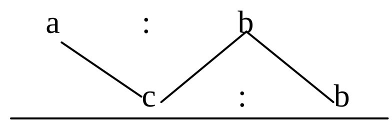
**PROPERTIES**

1. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$, i.e., the inverse ratios of two equal ratios are equal. The property is called Invertendo.
2. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$, i.e., the ratio of antecedents and consequents of two equal ratios are equal. This property is called Alternendo.
3. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$. This property is called Componendo.
4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$. This property is called Dividendo.
5. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. This property is called Componendo-Dividendo.
6. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} =$........ Then,
   Each ratio $= \frac{sum\ \ of\ Numerato\ rs}{sum\ \ of\ denominators}$
   i.e. $\frac{a}{b} = \frac{c}{d} = \frac{a+c+e+\cdots}{b+d+f+\cdots}$
7. If we have two equations containing three unknown as
   $a_1x + b_1y + c_1z = 0\ and\ \ldots$ (i)
   $a_2x + b_2y + c_2z = 0\ \ldots$ (ii)
   then, the values of x, y and z cannot be resolved without having a third equation.
   However, in the absence of a third equation, we can find the proportion x : y : z.
   This will be given by
   $$b_1c_2 - b_2c_1 : c_1a_2 - c_2a_1 : a_1b_2 - a_2b_1$$
8. To find the ratio of the two variables of a homogeneous equation of second degree.
   Fort this all the terms of the homogeneous equation are taken on one side and factorized into linear equation is formed from each of the factors and the ratio of the variables obtained.
9. A number which when subtracted from the terms of the ratio a:b make it equal to the ratio c:d is $\frac{bc-ad}{c-d}$



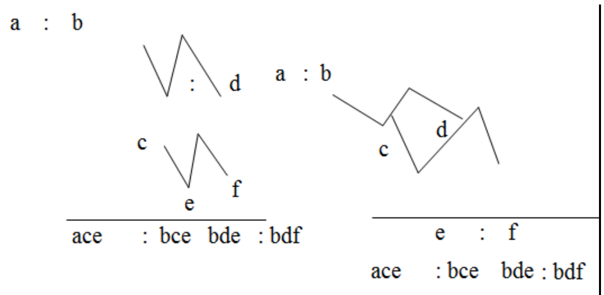
❖ If the ratio between the first and the second quantities is a : b and the ratio between the second and third quantities is c : d, then the ratio among first, second and third quantities is given by ac : bc : bd

❖ The above ratio can be represented diagrammatically as




ac : bc : bd

❖ If the ratio between the first and the second quantities is a:b and the ratio between the second and third quantities is c : d and the ratio between the third and fourth quantities is e : f, then the ratio among first, second, third and fourth quantities is given by ace : bce : bde : bdf




❖ To divide a given quantity into a given ratio.

Suppose any given quantity a, is to be divided in the ratio m:n

Let one part of the given quantity be a then the other part will be a-x.

$$\therefore \frac{x}{a-x} = \frac{m}{n} \ or$$

$$nx = ma - mx \ or \ (m+n)x = ma$$

$\therefore$ One part is $\frac{ma}{m+n}$ and the other part will be

$a - \frac{ma}{m+n} = \frac{na}{m+n}$

**Example 4:**

Divide 70 in the ratio 3:7

**Solution:**

Let one part be x

Then the other part = 70-x

$\therefore \frac{x}{70-x} = \frac{3}{7} \ or \ 7x = 210 - 3x$

or x=21 and 70-x=49

Hence the two required parts of 70 are 21 and 49.

Short cut method:

First part $= \frac{3}{10} \times 70 = 21$

Second part $= \frac{7}{10} \times 70 = 49$

**Example 5:**

What is the least integer which when subtracted from both the numerator and denominator of $\frac{60}{70}$ will give a ratio equal to $\frac{16}{21}$?

**Solution:**

Let x be the required integer. Then,

$\frac{60-x}{70-x} = \frac{16}{21}$

$\Rightarrow 1260 - 21x = 1120 - 16x$

$\Rightarrow 5x = 140 \Rightarrow x = 28$

**Example 6:**

If $\frac{x}{y} = \frac{4}{5}$, find the value of $\frac{3x+4y}{4x+3y}$.

**Solution:**

$$\frac{3x+4y}{4x+3y} = \frac{\frac{3x}{y}+4}{\frac{4x}{y}+3} = \frac{3\times\frac{4}{5}+4}{4\times\frac{4}{5}+3} = \frac{32}{31}$$



**Example 7:**

Find the value of $\frac{x+a}{x-a} + \frac{x+b}{x-b}, if \ x = \frac{2ab}{a+b}$.

**Solution:**

$x = \frac{2ab}{a+b} \Rightarrow \frac{x}{a} = \frac{2b}{a+b}$

By componendo – dividendo,

$$\frac{x+a}{x-a} = \frac{3b+a}{b-a}$$

Similarly, $\frac{x}{b} = \frac{2a}{a+b}$

$$\Rightarrow \frac{x+b}{x-b} = \frac{3a+b}{a-b}$$

$$\therefore \frac{x+a}{x-a} + \frac{x+b}{x-b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

$$= \frac{-3b+a}{a-b} + \frac{3a+b}{a-b} = \frac{2a-2b}{a-b} = 2$$

**Example 8:**

Divide `581 among A, B and C such that four time A's share is equal to 5 times B's share which is equal to seven times C's share.

**Solution:**

4 times A's share = 5 times B's share = 7 times C's share

$$\frac{A's\ share}{35} = \frac{B's\ share}{28} = \frac{C's\ share}{20}$$

[Dividing by L.C.M. of 4, 5 and 7 i.e. 140]

$$\therefore A:B:C = 35:28:20$$

$\therefore Share\ of\ A = \frac{35}{35+28+20} \times 581 =$ ` 245

Share of B $= \frac{28}{83} \times 581 =$ ` 196

Share of C $= \frac{20}{83} \times 581 =$ ` 140

- In any 2-dimensional figures, if the corresponding sides are in the ratio x:y, then their areas are in the ratio $x^2:y^2$.

**Example 9:**

The ratio of the radius of two circles is 2:5. Find the ratio of their areas.

**Solution:**

Ratio of their area $= 2^2:5^2 = 4:25$

❖ In any two 3-dimensional figures, if the corresponding sides are in the ratio x:y, then their volumes are in the ration $x^3:y^3$.

❖ If the ratio between two numbers is a:b and if each number is increased by x, the ratio become c:d.

Then, Sum of the two numbers $= \frac{x(a+b)(c-d}{ad-bc}$

Difference of the two numbers

$$= \frac{x(a-b)(c-d)}{ad-bc}$$

Two numbers are given as $\frac{xa(c-d}{ad-bc}$ $and$ $\frac{xb(c-d}{ad-bc}$

**Example 10:**

The ratio between two numbers is 3:4. If each number be increased by 2, the ratio becomes 7:9. Find the numbers.

**Solution:**

Numbers are $\frac{2\times3(7-9}{3\times9-4\times7}$ $and$ $\frac{2\times4(7-9}{3\times9-4\times7}$ or 12 and 16.

❖ If the sum of two numbers is A and their difference is a, then the ratio of numbers is given by A + a: A - a.

**Example 11:**

The sum of two numbers is 60 and their difference is 6. What is the ratio of the two numbers?

**Solution:**

The required ratio of the numbers

$$= \frac{60+6}{60-6} = \frac{66}{54} = \frac{11}{9}\ or\ 11:9$$

**Example 12:**

Three persons A, B, C whose salaries together amount of ` 14400, spend 80, 85 and 75 per cent of their salaries respectively. If their savings are in the ratio 8:9:20, find their respective salaries.

**Solution:**

A, B and C spend 80%, 85% and 75% respectively of their salaries

A,B&C save 20%, 15% and 25% respectively of their salaries.

So, 20% of A's salary: 15% of B's salary: 25% of C's salary = 8:9:20

$$\Rightarrow \frac{1}{5} of\ A's\ Salary: \frac{3}{20}\ of\ B's\ Salary: \frac{1}{4}\ of\ C's\ Salary = 8:9:20 \ldots \text{(i)}$$

Now $\frac{\frac{1}{5}\ of\ A's\ salary}{\frac{3}{20}\ of\ B's\ salary} = \frac{8}{9}$

$$\frac{A's\ salary}{B's\ salary} = \frac{3}{20} \times 8 \times \frac{5}{9} = \frac{2}{3}$$

A's salary: B's salary = 2:3 … (ii)

Similarly, B's salary: C's salary = 3:4 … (iii)

From (ii) and (iii)

A's salary: B's salary: C's salary = 2:3:4.

$\therefore A's\ salary = \frac{2}{2+3+4} \times 14400 =$ ` 3200

B's salary $= \frac{3}{2+3+4} \times 14400 =$ ` 4800

C's salary $= \frac{4}{2+3+4} \times 14400 =$ ` 6400

(viii) If the ratio $\frac{a}{b} > 1$ and k is a positive number, then $\frac{a+k}{b+k} < \frac{a}{b}$ and $\frac{a-k}{b-k} > \frac{a}{b}$

Similarly, if$\frac{a}{b} < 1$

$$\frac{a+k}{b+k} > \frac{a}{b} \; and \; \frac{a-k}{b-k} < \frac{a}{b}$$

(ix) If $\frac{c}{d} > \frac{a}{b}$, then $\frac{a+c}{b-k} > \frac{a}{b}$

and if $\frac{c}{d} < \frac{a}{b}$, then $\frac{a+c}{b+d} < \frac{a}{b}$

**PROPORTION**

When two ratios are equal, the four quantities composing them are said to be in proportion.

If $\frac{a}{b} = \frac{c}{d}$, then a, b, c, d are in proportion.

This is expressed by saying that 'a' is to 'b' as 'c' is to 'd' and the proportion is written as

a:b::c:d or a:b=c:d

The terms a and d are called the extremes while the terms b and c are called means.

**■ REMEMBER**

❖ If four quantities are in proportion, the product of the extremes is equal to the product of the means.
Let a, b, c, d be in proportion, then
$\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$

❖ If three quantities a, b and c are in continued proportion, then a:b=b:c
$\therefore ac = b^2$
b is called mean proportional.

❖ If three quantities are proportional, then the first is to be third is the duplicates ratio of the first is to the second.
If $a : b :: b : c \; then \; a : c = a^2 : b^2$

**TO FIND THE MEAN PROPORTIONAL**

**Example 13:**

Find the mean proportional between 3 and 75.

**Solution:**

Let x be the required mean proportional. Then,

$3 : x :: x : 75$

$\therefore x = \sqrt{3 \times 75} = 15$

**Example 14:**

A courier charge to a place is proportional to the square root of the weight of the consignment. It costs ` 54 to courier a consignment weighing 25 kilos. How much more will it cost (in rupees) to courier the same consignment as two parcels weighing 16 kilos and 9 kilos respectively?

**Solution:**

Courier charges

$\propto \sqrt{weight \; of \; the \; consignment}$

or Courier Charges

=k $\sqrt{weight \; of \; the \; consignment.}$

For weight = 25 kilos, courier charges is given to ` 54.

$\therefore 54 = K\sqrt{25} \; or \; k = 10.80$

Cost of courier for 16 kilos,

$C_{16} = k\sqrt{16} = 10.8\sqrt{16}$

$\therefore C_{16}$ =` 43.20

Cost of courier for 9 kilos,

$C_9 = k\sqrt{9} = 10.8\sqrt{9}$ = `32.4

Total cost of courier for two parcels

$C_{16} + C_9 = 43.20 + 32.40 = 75.60$

Difference to be paid = 75.60 – 54 = `21.60.

**TO FIND THE VALUES OF AN UNKNOWN WHEN FOUR NUMBERS ARE IN PROPORTION**

**Example 15:**

What must be added to each of the four numbers 10, 18, 22, 38 so that they become in proportion?

**Solution:**

Let the number to be added to each of the four numbers be x.

By the given condition, we get

$(10+x):(18+x):(22+x):(38+x)$

$\Rightarrow\ 10+x)(38+x)=(18+x)(22+x$

$\Rightarrow 380+48x+x^2=396+40x+x^2$

Cancelling $x^2$ from both sides, we get

$380+48x=396+40x$

$\Rightarrow 48x-40x=396-380$

$\Rightarrow 8x-16x=\frac{16}{8}=2$

Therefore, 2 should be added to each of the four given numbers.

**TO FIND THE FOURTH PROPORTIONAL**

**Example16:**

Find the fourth proportional to $p^2-pq+q^2, p^3+q^3, p-q$

**Solution:**

Let x be the fourth proportional

$\therefore (p^2-pq+q^2):(p^3+q^3)=(p-q):x$

$\Rightarrow\ p^2-pq+q^2)\times x=(p^3+q^3)(p-q$

$$\Rightarrow x=\frac{(p^3+q^3)(p-q)}{(p^2-pq-q^2)}$$

$\therefore x=\frac{(p+q\ \ (p^2-pq+q^2)(p-q)}{(p^2-pq-q^2)}$

$\therefore$ The required fourth proportional is $p^2-q^2$

**TO FIND THE THIRD PROPORTIONAL**

**Example 17:**

Find third proportional to $a^2-b^2$ and a + b

**Solution:**

Let x be the required third proportional

Then$a^2-b^2:a+b=a+b:x$

$\therefore (a^2-b^2)x=(a+b)(a+b)$

$\therefore x=\frac{(a+b\ \ (a+b)}{a^2-b^2}=\frac{a+b}{a-b}$

**USING THEORM ON EQUAL PROPORTION**

**Example 18:**

If$\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$, prove that each is equal to $\frac{1}{2}$ or -1.

**Solution:**

We have$\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$

Each ratio $=\frac{sum\ of\ antecedents}{sum\ of\ consequents}$

[By theorem on equal ratios]

$$=\frac{a+b+c}{b+c+c+a+a+b}=\frac{a+b+c}{2(a+b+c)}$$

$$=\frac{1}{2}\ if\ a+b+c\neq 0$$

If a + b + c = 0, then b + c = -a

$$\therefore \frac{a}{b+c}=\frac{a}{-a}=-1$$

Similarly,$\frac{b}{c+a}=\frac{b}{-b}=-1,\frac{c}{a+b}=\frac{c}{-c}=-1$

Hence each ratio $=\frac{1}{2}\ if\ a+b+c\neq 0$

$=-1\ if\ a+b+c=0$

**DIRECT PROPORTION**

If on the increase of one quantity, the other quantity increases to the same extent or on the decrease of one, the other decrease to the same extent, then we say that the given two quantities are directly proportional. If A and B are directly proportional then we denote it by $A\propto B$.

Also, A = kB, k is constant

$$\Rightarrow \frac{A}{B}=k$$

If $b_1$ and $b_2$ are the values of B corresponding to the values $a_1, a_2$ of A respectively, then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

**Some Examples:**

1. Work done ∝ number of men
2. Cost ∝ number of men
3. Work ∝ wages
4. Working hour of a machine ∝ fuel consumed
5. Speed ∝ distance to be covered

**INDIRECT PROPORTION (OR INVERSE PROPORTION)**

If on the increase of one quantity, the other quantity decreases to the same extent or vice versa, then we say that the given two quantities are indirectly proportional. If A and B are indirectly proportional then we denote it by $A \propto \frac{1}{B}$.

Also, $A = \frac{k}{B}$ (k is a constant)

$$\Rightarrow AB = k$$

If $b_1, b_2$ are the values of B corresponding to the values $a_1, a_2$ of A respectively, then

$a_1 b_1 = a_2 b_2$

Some Examples:

1. More men, less time
2. Less men, more hours
3. More speed, less time
4. More speed, less taken time to be covered distance

**Example 19:**

A garrison of 3300 men had provision for 32 days, when given at the rate of 850 gm per head. At the end of 7 days, a reinforcement arrived and it was found that the provision would last 17 days more, when given at the rate of 825 gm per head. What was the strength of the reinforcement?

**Solution:**

There is a provision for 2805 x 32 kg for 3300 men for 32 days @ 850 gm per head per day.

In 7 days, 3300 men consumed

$$\frac{2805 \times 32}{32} \times 7 = 2805 \times 7\ kg$$

Let the strength of the reinforcement arrived after 7 days be x.

∴ $(3300 + x)$ men had provision of 2805 × 25 kg for 17 days @ 825 gm per head per day, i.e.

$$\therefore \frac{(3300 + x) \times 825 \times 17}{1000} = 2805 \times 25$$

$$\Rightarrow\ 3300 + x) = \frac{1000 \times 2805 \times 25}{825 \times 17}$$

$$= 5000$$

$$\Rightarrow x = 1700$$

∴ Strength of the reinforcement arrived after 7 days = 1700

On k-method

**Example 20:**

If a, b, c be in continued, prove that $\frac{(a+b)^2}{(b+c)^2} = \frac{a}{c}$

**Solution:**

Let $\frac{a}{b} = \frac{b}{c} = k$, then a = bk and b = ck

Hence $a = bk = ck.k = ck^2$ and b = ck

Substituting these values of a and b, we get

L.H.S. $= \frac{(ck^2+ck}{(ck+c)^2}^{2} = \frac{[ck(k+1)^2}{[c(k+1)]^2} = \frac{c^2k^2(k+1)^2}{c^2(k+1)^2}$

and R.H.S. $= \frac{ck^2}{c} = k^2$

$$\therefore L.H.S. = R.H.S.. Hence \frac{(a+b)^2}{(b+c)^2} = \frac{a}{c}$$

**RULE OF THREE**

In a problem on simple proportion, usually three terms are given and we have to find the fourth term, which we can solve by using Rule of three. In such problems, two of given terms are of same kind and the third term is of same kind as the required fourth term. First of all we have to find whether given problem is a case of direct proportion or indirect proportion. For this, write the given quantities under their respective headings and then mark the arrow in increasing direction. If both arrows are in some direction then the relation between them is direct otherwise it is indirect or inverse proportion. Proportion will be made by either head to tail or tail to head.

The complete procedure can be understand by the examples.

**Example 21:**

A man completes 5/8 of a job in 10 days. At this rate, how many more days will it take him to finish the job?

**Solution:**

Work done $= \frac{5}{8}$. Balance work $= 1 - \frac{5}{8} = \frac{3}{8}$

Less work, Less days (Direct Proportion)

Let the required number of days be x. Then,

$$\begin{array}{ccc} & Work & days \\ \uparrow & 5/8 & 10 \uparrow \\ & 3/8 & x \end{array}$$

Then, $\frac{5}{8} : \frac{3}{8} :: 10 : x \Rightarrow \frac{5}{8} \times x = \frac{3}{8} \times 10$

$\Rightarrow x = \frac{3}{8} \times 10 \times \frac{8}{5} = 6$

**Example 22:**

A fort had provision of food for 150 men for 45 days. After 10 days, 25 men left the fort. The number of days for which the remaining food will last, is:

**Solution:**

After 10 days:150 men had food for 35 days.

Suppose 125 men had food for x days. Now,

Less men, More days (Indirect Proportion)

Then,

$$\begin{array}{ccc} & men & days \\ \downarrow & 150 & 35 \downarrow \\ & 125 & x \end{array}$$

$\therefore 125 : 150 :: 35 : x \Rightarrow 125 \times x = 150 \times 35$

$\Rightarrow x = \frac{150 \times 35}{125} \Rightarrow x = 42$

Hence, the remaining food will last for 42 days.

**Compound Proportion or Double Rule of Three**

In the compound proportion, number of ratios are more than two.

**Example 23:**

If the cost of printing a book of 320 leaves with 21 lines on each page and on an average 11 words in each line is ` 19, find the cost of printing a book with 297 leaves, 28 lines on each page and 10 words in each line.

**Solution:**

Less leaves, less cost (Direct Proportion)

More lines, more cost (Direct Proportion)

Less words, less cost (Direct Proportion)

$$\left.\begin{array}{r} leaves\ 320:297 \\ lines\ 21:28 \\ words\ 11:10 \end{array}\right\} :: 19 : x$$

$\therefore 320 \times 21 \times 11 \times x = 297 \times 28 \times 10 \times 19$

$\Rightarrow x = \frac{171}{8} = 21\frac{3}{8}$

**Example 24:**

If 80 lamps can be lighted, 5 hours per day for 10 days for ` 21.25, then the number of lamps, which can be lighted 4 hours daily for 30 days, for ` 76.50, is:

**Solution:**

Let the required number of lamps be x.

Less hours per day, More lamps (Indirect Proportion)

More money, More lamps (Direct Proportion)

More days, Less lamps (Indirect Proportion)

$$\left.\begin{array}{r} Hours\ per\ day\ 4:5 \\ Money\ 21.25:76.50 \\ Number\ of\ days\ 30:10 \end{array}\right\} :: 80:x$$

$$\therefore 4 \times 21.25 \times 30 \times x$$

$$= 5 \times 76.50 \times 10 \times 80$$

$$\Rightarrow x = \frac{5 \times 76.50 \times 10 \times 80}{4 \times 21.25 \times 30} \Rightarrow x = 120$$

**PARTNERSHIP**

A partnership is an association of two or more persons who lives invest their money in order to carry on a certain business.

A partner who manages the business is called the working partner and the one who simply invests the money is called the sleeping partner.

Partnership is of two kinds:

(i) Simple

(ii) Compound

**Simple partnership**: If the capitals is of the partners are invested for the same period, the partnership is called simple.

**Compoundpartnership:** If the capitals of the partners are invested for different lengths of time, the partnership is called compound.

❖ If the period of investment is the same for each partner, then the profit or loss is divided in the ratio of their investments.

If A and B are partners in a business, then

$$\frac{Investment\ \ of\ A}{Investment\ \ of\ B} = \frac{Profit\ \ of\ A}{Profit\ \ of\ B}\ or = \frac{Loss\ of\ A}{Loss\ of\ B}$$

If A, B and C are partners in a business, then

Investment of A:Investment of B:Investment of C

=Profit of A:Profit of B: Profit of C, or

=Loss of A: Loss of B: Loss of C

**Example 25:**

Three partner Rahul, Puneet and Chandan invest ` 1600, ` 1800 and ` 2300 respectively in a business. How should they divide of ` 399?

**Solution:**

Profit is to be divided in the ratio 16:18:23

Rahul's share of profit $= \frac{16}{16+18+23} \times 399$

$= \frac{16}{57} \times 399 =$`112

Puneet's share of profit $= \frac{18}{57} \times 399=$`126

Chandan's share of Profit$=\frac{23}{57} \times 399=$`161

**Example 26:**

A, B and C enter into a partnership by investing 1500, 2500 and 3000 rupees, respectively. A as manager gets one-tenth of the total profit and the remaining profit is divided among the three in the ratio of their investment. If A's total share is `369, find the shares of B and C.

**Solution:**

If total profit is x, then

A's share $= \frac{1}{10}x + \frac{15}{15+25+30}$ of the balance$\frac{9}{10}x$

$$\Rightarrow \frac{1}{10}x + \frac{27x}{140} = 369$$

$$\Rightarrow 14x + 27x = 369 \times 140$$

$$\Rightarrow x = \frac{369 \times 140}{41} = 91 \times 140 = 1260$$

B's share $= \frac{5}{14} \times \frac{9}{10} \times 1260 =$`405

C's share $= \frac{6}{14} \times \frac{9}{10} \times 1260 =$ `486

**Example 27:**

A and B invested in the ratio 3:2 in a business. If 5% of the total profit goes to charity and A's share is ` 855, find the total profit.

**Solution:**

Let the total profit be ` 100

Then, ` 5 goes to charity

Now, ` 95 is divided in the ratio 3:2

$\therefore A'sshare = \frac{95}{3+2} \times 3$ =`57

But A's actual share is ` 855

$\therefore$ Actutal total profit $= 855\left(\frac{100}{57}\right)$=`1500

## MONTHLY EQUIVALENT INVESTMENT

It is the product of the capital invested and the period for which it is invested.

If the period of investment is different, then the profit or loss is divided in the ratio of their Monthly Equivalent Investment.

$$\frac{Monthly\ Equivalent\ Investment\ of\ A}{Monthly\ Equivalent\ Investment\ of\ B} = \frac{Profit\ of\ A}{Profit\ of\ B}\ or\ \frac{Loss\ of\ A}{Loss\ of\ B}$$

i.e.,$\frac{Investment\ \ of\ A \times Period\ \ of\ Investment\ \ of\ A}{Investment\ \ of\ B \times Period\ \ of\ Investment\ \ of\ B}$

$$= \frac{Profit\ of\ A}{Profit\ of\ B}\ or\ \frac{Loss\ of\ A}{Loss\ of\ B}$$

- If A, B and C are partners in a business, then

  Monthly Equivalent Investment of B: Monthly equivalent Investment of A: Monthly Equivalent Investment of C

  = Profit of A: Profit of B: Profit of C

  = Loss of A: Loss of B: Loss of C

**Example 28:**

A and B start a business. A invests ` 600 more than B for 4 months and B for 5 months. A's share is ` 48 more than that of B, out of a total profit of ` 528. Find the capital contributed by each.

**Solution:**

B's profit = $\frac{528-48}{2}$=`240

A's profit = 528 – 240 = `288

$$\frac{A's\ capital \times 4}{B's\ capital\ \times 5} = \frac{288}{240} = \frac{6}{5}$$

$$\frac{A's\ capital}{B's\ capital} = \frac{6}{5} \times \frac{5}{4}$$

$$\Rightarrow \frac{B's\ capital + 600}{B'scapital} = \frac{3}{2}$$

⇒ B's capital = `1200 and A's capital = `1800

**Example 29:**

Three persons A, B, C rent the grazing of a park for `570. A puts in 126 oxen in the park for 3months, B puts in 162 oxen for 5 months and C puts in 216 oxen for 4 months. What part of the rent should each person pay?

**Solution:** Monthly equivalent rent of A $= 126 \times 3 = 378$

Monthly equivalent rent of B $= 162 \times 5 = 810$

Monthly equivalent rent of C $= 226 \times 4 = 864$

$\therefore$ Rent is to be divided in the ratio 378:810:864, i.e. 7:15:16

$\therefore$ A would have to pay$\frac{7}{7+15+16}$ of the rent

$= \frac{7}{38}$ of the rent $= \frac{7}{38} \times 570 =$ □105

$\therefore$ B would have to pay$\frac{15}{38}$ of the rent $= \frac{15}{38} \times 570$

$= \frac{A'\ s\ capital\ \times 4}{B'\ s\ capital\ \times 5}$225

and C would have to pay$\frac{16}{38}$, i.e.$\frac{8}{19}$ of the rent

$= \frac{8}{19} \times 570$=`240

**Example 30:**

Shekhar started a business investing `25,000 in 2010. In 2011, he invested an additional amount of `10,000 and Rajeev joined him with an amount of `35,000. In 2013, Shekhar invested another additional amount of `10,000 and Jatin joined then with an amount of `35,000. What will be Rajeev's share in the profit of `1, 50,000 earned at the end of 3 years from the stsart of the business in 2010?

(a) `45,000 (b) `50,000

(c) `70,000 (d) `75,000

**Solution:**

(b) Ratio of Shekhar, Rajeev and Jatin's investments

$= 25000 \times 36 + 10000 \times 24 + 10000 \times 12 : 35000 \times 24 : 35000 \times 12,$

$= 25 \times 36 + 10 \times 24 + 10 \times 12 : 35 \times 24 : 35 \times 12$

$= 25 \times 3 + 10 \times 2 + 10 \times 1 : 35 \times 2 : 35 \times 1$

= 75+20+10:70:35

= 105:70:35, i.e. 3:2:1

∴ Rajeev's share in the profit $= \frac{2}{6} \times 150000 =$ □$50000$

**Example 31:**

A began a business with □ 4500 and was joined afterwards by B with □5400. If the profits at the end of year was divided in the ratio 2:1 then B joined the business after:

(a) 5 months (b) 4 months

(c) 6 months (d) 7 months

**Solution:**

(d) Let B joined after × months

Then, $4500 \times 12 : 5400 \times (12 - x) = 2:1$

or $\frac{4500 \times 12}{5400 \times (12-x)} = \frac{2}{1}$

$\Rightarrow \frac{45 \times 12}{54 \times (12 - x)} = \frac{2}{1}$

or $\frac{5}{12-x} = 1$

or x = 7 months

**Example 32:**

A sum of `3115 is divided among A, B and C so that if `25, `28 and `52 be diminished from their shares respectively, the remainders shall be in the ratio 8:15:20. Find the share of each.

**Solution:**

(A's share-25):(B's share-28):(C's share-52)=8:15:20

$$\therefore \frac{A's\ share - 25}{8} = \frac{B's\ share - 28}{15} = \frac{C's\ share - 52}{20} = k(say)$$

∴ A's share - 25 = 8k

∴ A's share = 8k + 25

Similarly, B's share = 15k+28 and C's share = 20k+52

∴ 8k+25+15k+28k+20k+52 = 3115

∴ k = 70

∴ A gets `585, b gets `1078 and C gets `1452.

**Example 33:**

A, B and C enter into a partnership. A advances `1200 for 4 months, B gives ` 1400 for 8 months and C `1000 for 10 months. They gain `585 altogether. Find the share of profit each.

**Solution:**

Monthly Equivalent Investment of A =1200×4=4800

Monthly Equivalent Investment of $B = 1400 \times 8 = 11200$

Monthly Equivalent Investment of $C = 1000 \times 10 = 1000$

Profit is divided in the ratio 48:112:100, i.e., 12:28:25

A's share of profit is$\frac{12}{65} \times 585$=`108

B's share of profit is$\frac{28}{65} \times 585$=`252

C's share of profit is$\frac{25}{65} \times 585$=`225

**Example 34:**

Three man A, B and C subscribe `4700 for a business. A subscribes `700 more than B and B `500 more than C. How much will each receive out of a profit of `423?

**Solution:**

If C subscribes `x, then,

B subscribes ` (x+500) and A subscribes ` (x+1200)

∴ 3x+1700=4700; ∴ $x$=`1000

∴ Ratio of profits of C, B and A = 1000:1500:2200

i.e. 10:15:22

∴ C's share of profit = $\frac{10}{47} \times 423$=`99

∴ B's share of profit = $\frac{15}{47} \times 423$=`135

∴ A's share of profit = $\frac{22}{47} \times 423$=`198

**Example 35:**

Two partners invested `1250 and `850 respectively in a business, they decided to distribute equally 60% of the profit, and the remaining as the interest on their capital. If one receives `320 more than the other, find the total profit.

**Solution:** If the total profit is 100x, each gets 30x as equal distribution Balance profit of 40x is divided in the ratio of capital =1250:850=25:17

∴ One partner gets$\frac{25}{42}$ ×40x and the other gets$\frac{17}{42} \times 40$

∴ This difference$\frac{25}{42}40x - \frac{17}{42} \times 40x = 320$

∴ $x = 42$ ∴ Total profit = `4200.

**Example 36:**

Divide `581 among A, B and C such that four times A share is equal to 5 times B's share which is equal to seve times C's share.

**Solution:**

4 time A's share = 5 times B's share = 7 times C's share

$\frac{A's\ share}{35} = \frac{B's\ share}{28} = \frac{C's\ share}{20}$ ….(dividing by LCM of 4, 5 and 7 i.e., 140)

∴ A:B:C=35:28:20

∴ Share of A = $\frac{35}{35+28+20} \times 581$=`245

Share of B = $\frac{28}{83} \times 581$=`196

Share of C = $\frac{20}{83} \times 581 = $ ⍰140