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☆ Particle Vs Wave

Superposition of Wave :-

Interference of light waves

* During Initial development of interfer-ence the only source of coherence was to produce them from same source

* Two different light source are coherent only for 10^{-10} sec

• Lloyd, Fresnels double mirror

• Fresnel's Biprism

Division of Amplitude
Phase change on reflection

* Principle of optical reversibility :-

Reflection of light at an interface betn two media

In absence of any absorption a light ray that is reflected or refracted will retrace its original path if its dirn is reversed.

Consider a light ray incident at two media of refractive indices n_1 & n_2.

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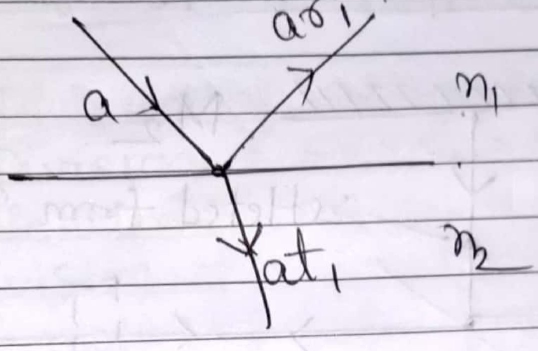


Fig.1

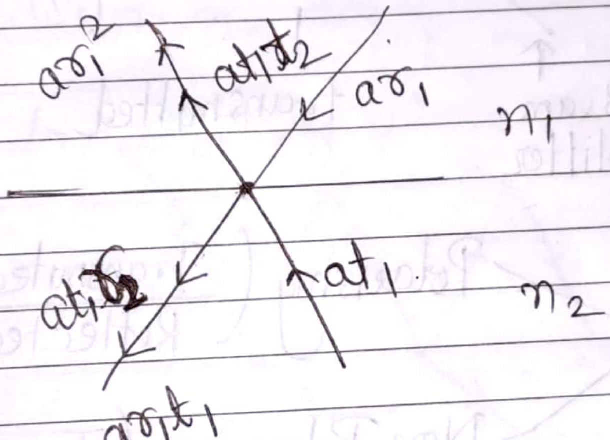


Fig.2

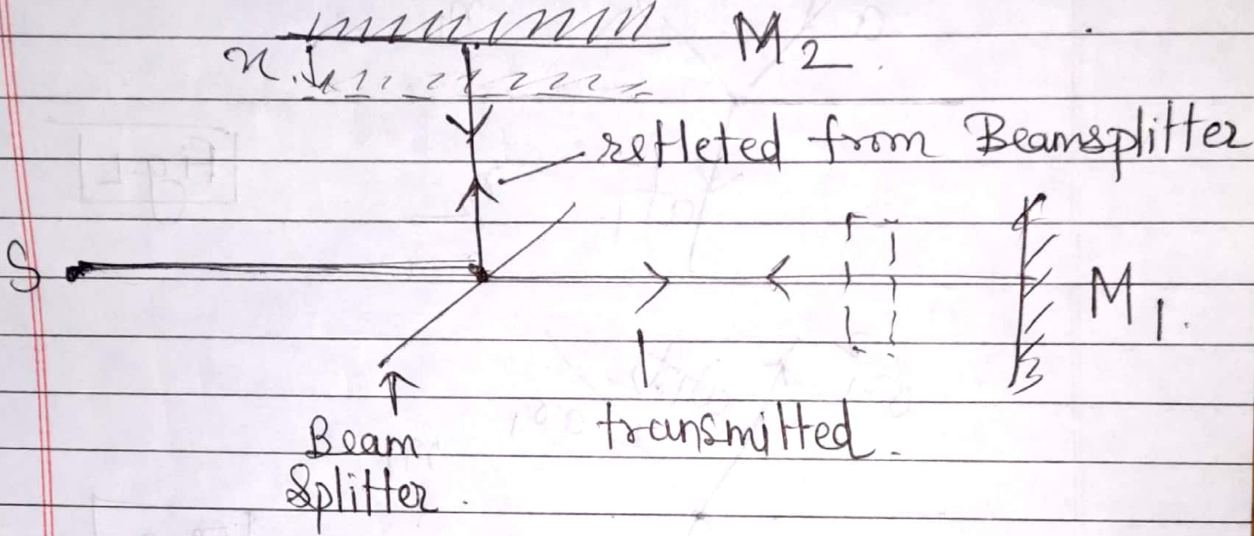
$$ar_1^2 + at_1t_2 = a$$

$$\boxed{t_1t_2 = 1 - r_1^2} \Rightarrow \text{Stoke's relation.}$$

$$at_1r_2 + ar_1t_1 = 0$$

$$\Rightarrow \boxed{r_2 = -r_1} \Rightarrow \text{Stoke's relation.}$$

★ Micheleon Interferometer :-



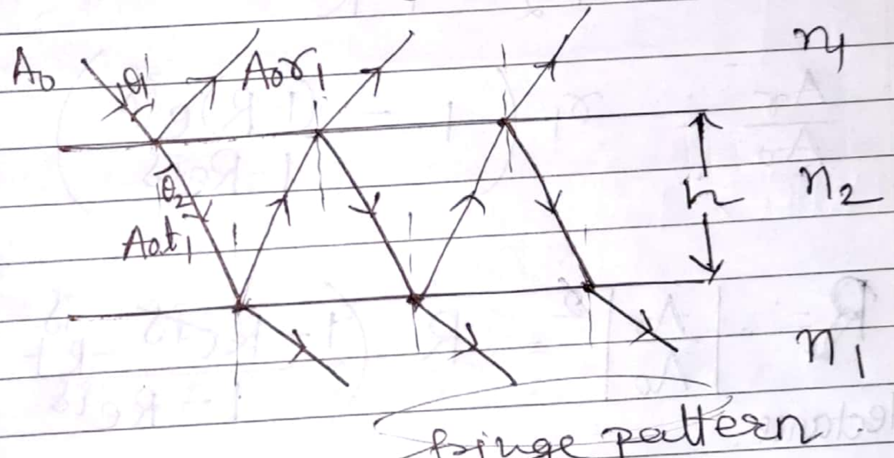
Beam Splitter
- Polarising $\left(\frac{\text{Transmited}}{\text{Reflected}} = 1\right)$
- Non-Polarising $\left(\frac{\text{Transmitted}}{\text{Reflected}} \neq 1\right)$

- If any object is placed in any path the intensity of light is further reduced
- So we prefer more light in the path where object is placed
- This experiment is used to calculate λ of light

- When any mirror is shifted some fringe pattern collapses.
- We try to calculate no. of fringes collapse.

* Two-Beam Interference

→ Division of Amplitude.
→ Division of Wavefront
→ Micheleon Interferrometer comes under division of Amplitude.
→ If more no. of ~~Beames~~ are used then resolution of fringes increase.



Application :- Fabry-Perot Interferrometer.

Path difference betn two wavefront $\Delta = 2n_2 h\cos\theta_2$

Phase difference $= \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} \times (2n_2 h\cos\theta_2)$

$$= \frac{4\pi n_2 h\cos\theta_2}{\lambda}$$

Amplitude of successive reflected waves

$A_0 r_1,\ A_0 t_1 r_2 t_2 e^{i\delta},\ A_0 t_1 r_2^3 e^{2i\delta},\ \ldots$

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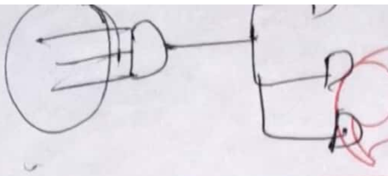

$$A_r = A_0\left[r_1 + t_1t_2r_2e^{i\delta}\left(1 + r_2^2e^{i\delta} + r_2^4e^{2i\delta} \ldots\right)\right]$$

$$= A_0\left(r_1 + \frac{t_1t_2r_2e^{i\delta}}{1-r_2^2e^{i\delta}}\right)$$

$$R = r_1^2 = r_2^2 \quad (\because r_2 = -r_1)$$

$$T = t_1t_2 = 1-R$$

$$\frac{A_r}{A_0} = r_1\left(1 - \frac{(1-R)e^{i\delta}}{1-Re^{i\delta}}\right)$$

$$\mathcal{R} = \left|\frac{A_r}{A_0}\right|^2 = R\left(\frac{1-\cancel{Re^{i\delta}} - e^{i\delta} + \cancel{Re^{i\delta}}}{1-Re^{i\delta}}\right)^2$$

Reflectance

$$= R\left|\frac{1-e^{i\delta}}{1-Re^{i\delta}}\right|^2$$

$$\mathcal{R} = \frac{4R\sin^2\delta/2}{(1-R)^2 + 4R\sin^2\delta/2}$$

let $F = \frac{4R}{(1-R)^2}$

$$\boxed{\mathcal{R} = \frac{F\sin^2\delta/2}{1 + F\sin^2\delta/2}}$$

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Here, $F \equiv$ Coefficient of Finnese

measure of interference fringe sharpness & contrast.

* If F <<< then multiple beam interference reduce to two beam interference.

r_1 & r_2 should be very high for multiple beam interference.

→ Amplitude of successive transmitted waves

$A_0 t_1 t_2,\ A_0 t_1 t_2 r_2^2 e^{i\delta},\ A_0 t_1 t_2 r_2^4 e^{2i\delta}$.

$$A_t = A_0 t_1 t_2 + A_0 t_1 t_2 r_2^2 e^{i\delta} + A_0 t_1 t_2 r_2^4 e^{2i\delta} + \_\_\_$$

$$= A_0 t_1 t_2 \left(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \_\_\_\right)$$

$$= A_0 \left(\frac{1-R}{1-Re^{i\delta}}\right)$$

$$T = \left|\frac{A_t}{A_0}\right|^2 = \left|\frac{1-R}{1-Re^{i\delta}}\right|^2$$

$$\boxed{T = \frac{1}{1+F\sin^2\delta/2}}$$

If there is no absorbtion,

$$\boxed{R + T = 1}$$

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FWHM : Full width at half maximum

Sharpness increases with decreasing width and increasing F

$n_2 = 1$; $h = 1.0\,cm$, $F = 400$

$\lambda_0 = \lambda_1 = 5000\,A^\circ$ $m = 1, 2,$ ———

$$\delta = \frac{4\pi n_2 h \cos\theta_2}{\lambda} = 2m\pi$$

$$\cos\theta_2 = \frac{m\lambda_0}{2n_2 h} \Rightarrow \theta_2 = \cos^{-1}\left(\frac{m}{40000}\right)$$

Bright rings $\Rightarrow$ 0° , 0.41° , 0.57° , 0.70° ,

$m \Rightarrow 40000, 39999, 39998, 39997$

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If $\lambda_1 = 4999.98\ A^\circ$

$$\theta_2 = \cos^{-1}\left[\frac{m}{40000.16}\right]$$

$\theta_2 \rightarrow 0.167^\circ, 0.436^\circ, 0.595^\circ, \text{———}$
$m \rightarrow 40000, 39999, 39998, \text{———}$

$$\boxed{\text{visibility} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}}$$

H.W. for $T = 1$ & $T = 1/2$; find δ.

$T = 1$; $T = \frac{1}{1 + F\sin^2\delta/2}$; $\delta = 2m\pi$; $m = 1, 2, \ldots n$.

$T = 1/2$ $\quad \frac{1}{2} = \frac{1}{1 + F\sin^2\delta/2}$

$\Rightarrow F\sin^2\delta/2 = 1$

$\Rightarrow \sin^2\frac{\delta}{2} = \frac{1}{F}$

$\Rightarrow \sin\frac{\delta}{2} = \sqrt{\frac{1}{F}}$

$\Rightarrow \delta = 2.\sin^{-1}\sqrt{\frac{1}{F}}$

$$\delta = 2m\pi \pm \frac{\Delta\delta}{2}$$

$$\sin^2 \frac{\Delta\delta}{4} = \frac{1}{F}$$ ($\sin\theta \simeq \theta$ for small θ)

$$\rightarrow \boxed{\Delta\delta = \frac{4}{\sqrt{F}} = \frac{2(1-R)}{\sqrt{R}}}$$

* Optical Flatness :- measured in terms of wavelength

Correlation length $\frac{\lambda}{10}$; $\frac{\lambda}{20}$; $\frac{\lambda}{50}$; ----

$n_2 = 1$

$h' = 1cm + \frac{\lambda}{20} = 1.0000025$ cm

$\lambda_0 = 5000$ A° $[\lambda_1 = 4999.98$ A°$]$

h $\theta_2 = \cos^{-1} \frac{m}{40000}$ $\theta_2 = \cos^{-1} \frac{m}{40000.16}$

h' $\theta_2 = \cos^{-1} \frac{m}{40000.1}$ $0.126°, 0.45°, 0.587°, ---$

$0, 0.41°, 0.57°, --$ $0.162°, 0.436°, 0.59°$

40000, 39999, 39998, --

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So, for change of 250 nm due to optical flatness (change) in h; we get well separated fringes.

This principle is used in filters for specific wavelengths.

★ **Resolving Power of Interferrometer:-**

$RP = \left|\frac{\lambda}{\Delta\lambda}\right|$

$= \left|\frac{\nu}{\Delta\nu}\right|$

For two frequencies ν_1 & ν_2 ~~$(\nu+\Delta\nu)$~~ to be just resolved, we assume that the half intensity points of ν_1 falls on the half intensity of ν_2

$$\delta = \frac{\delta_1}{2} = 2m\pi \pm \frac{\Delta\delta}{2} \qquad \Delta\delta = \frac{4}{\sqrt{F}}$$

$$\delta = \frac{4\pi n_2 h\cos\theta_2}{\lambda}$$

for $\cos\theta_2 = 1$; $n_2 = 1$

$$\delta = \frac{4\pi h}{\lambda} = \frac{4\pi h\nu}{c}$$

ν_1 $\quad \delta_1 = \frac{4\pi h_1\nu_1}{c} = 2m\pi$ ——— (1)

ν_2 $\quad \delta_2 = \frac{4\pi(h_1+\Delta h_1)(\nu_1+\Delta\nu_1)}{c} = 2m\pi$ — (2)

$$\frac{\cancel{4\pi} h_1 \nu_1}{\cancel{c}} = \frac{\cancel{4\pi}(h_1+\Delta h_1)(\nu_1+\Delta\nu_1)}{\cancel{c}}$$

$$\cancel{h_1\nu_1} = \cancel{h_1\nu_1} + h_1\Delta\nu_1 + \Delta h_1\nu_1 + \Delta h_1\Delta\nu_1 \text{ (neglected)}.$$

$$\Rightarrow \boxed{\Delta h_1 = -\frac{h_1}{\nu_1}\cdot\Delta\nu_1}$$

* For Δh_1 to be positive $\Delta\nu_1$ should be $-ve$.

* For freq. ν_1 let the half intensity point occur at

$$h = h_1 + \delta h_1 \quad \left(\text{corresponding } 2m\pi + \frac{\Delta\delta}{2}\right).$$

$$\frac{4\pi\nu_1\delta h_1}{c} = \frac{1}{2}\Delta\delta = \frac{2}{\sqrt{F}}.$$

$$\delta h_1 = \frac{c}{2\pi\nu_1\sqrt{F}}.$$

$$\Delta h_1 = 2\delta h_1 = \frac{c}{\pi\nu_1\sqrt{F}}.$$

$$R.P. = \left|\frac{\nu_1}{\Delta\nu_1}\right| = \frac{\pi h_1\nu_1\sqrt{F}}{c}.$$

$$\boxed{RP = \frac{\pi h\nu\sqrt{F}}{c} = \frac{\pi h\sqrt{F}}{\lambda}.}$$

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* $\lambda = 488$ nm
$n_2 = 1.35$ normal incidence $\Rightarrow \theta_2 = 0°$
$h = ?$

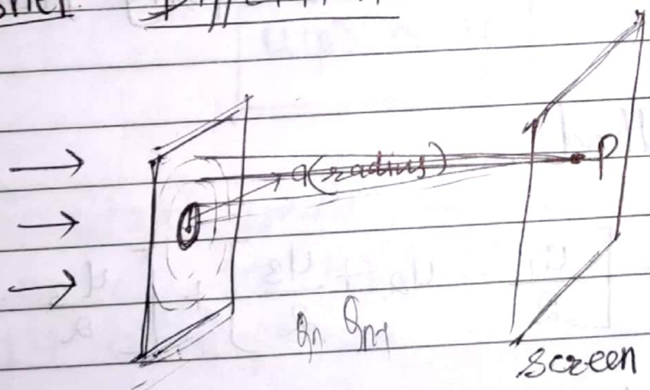
$$2n_2 h \cos\theta_2 = m\lambda$$

$$\Rightarrow 2(1.35)(h)(1) = m(488 \times 10^{-9})$$

$$\Rightarrow h = \frac{(488 \times 10^{-9})(m)}{2 \times 1.35}$$

for $m=1$; $\boxed{h = 180.74 \text{ nm}}$

Fresnel Diffraction



$a <<< d$.

$d + \frac{\lambda}{2}$, $d + \frac{2\lambda}{2}$, ...

$\leftarrow$ finite $\rightarrow$

Half period zones

$$r_n = \left[\left(d + \frac{n\lambda}{2}\right)^2 - d^2\right]^{1/2}$$

$$= \cancel{d^2} + \frac{n^2\lambda^2}{4} + nd\lambda - \cancel{d^2}$$

$$= \left[\frac{n^2\lambda^2}{4} + nd\lambda\right]^{1/2} = \sqrt{nd\lambda}\left[1 + \frac{n\lambda}{4d}\right]^{1/2}$$

$$r_n = \sqrt{nd\lambda}\left(1 + \frac{n\lambda}{8d}\right) \simeq \sqrt{nd\lambda}.$$

$$Q_nP - Q_{n-1}P = \frac{\lambda}{2} = \pi \text{ (phase diff)}$$

Angular region between Q_n & Q_{n-1} zones.

$$A = \pi r_n^2 - \pi r_{n-1}^2$$

$$= \pi n d\lambda - \pi(n-1)d\lambda$$

$$\boxed{A = \pi d\lambda}$$

★ $u(P) = u_1 - u_2 + u_3 - u_4 + \ldots + (-1)^{m+1} u_m + \ldots$

u (Amplitude)

$$u_1 > u_2 > u_3 \ldots$$

$$u_n(p) = \text{Constant} \frac{A_n}{Q_nP} \cdot \frac{1+\cos\delta}{2}$$

optical field due to n^{th} zone. obliquity factor.

★ Schuster Method.

$$u(p) = \frac{u_1}{2} + \left[\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right] + \left[\frac{u_3}{2} - u_4 + \frac{u_5}{2}\right]$$

$$+ \ldots$$

last term $\Rightarrow \frac{1}{2} u_m$ or $\frac{1}{2} u_{m-1} - u_m$ $\quad m =$ odd or even.

★ If obliquity factor is such that

$$u_n > \frac{1}{2}(u_{n-1} + u_{n+1}).$$

Then $u(p) < \frac{1}{2} u_1 + \frac{1}{2} u_m \quad$ (m odd).

$$u(p) < \frac{1}{2} u_1 + \frac{1}{2} u_{m-1} - u_m \approx \frac{u_1}{2} - \frac{u_m}{2} \quad \text{(m even)}$$

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To obtain upper limits,

$$u(p) = u_1 - \frac{u_2}{2} - \left[\frac{u_2}{2} - u_3 + \frac{u_4}{2}\right] - \left[\frac{u_4}{2} - u_5 + \frac{u_6}{2}\right]$$

$$u(p) > u_1 - \frac{u_2}{2} - \frac{u_{m-1}}{2} + u_m \approx \frac{u_1}{2} + \frac{u_m}{2} \quad (m \text{ odd})$$

$$u(p) > u_1 - \frac{u_2}{2} - \frac{u_m}{2} \approx \frac{u_1}{2} - \frac{u_m}{2} \quad (m \text{ even}).$$

Approximating $u(p) \approx \frac{u_1}{2} + \frac{u_m}{2}$ (m odd).

$$u(p) \approx \frac{u_1}{2} - \frac{u_m}{2} \quad (m \text{ even}).$$

If we neglect u_m in comparision to u_1 then

$$\boxed{u(p) \approx \frac{u_1}{2}} \Rightarrow \boxed{I \approx \frac{I_0}{4}}$$

Resultant amplitude produced by entire wavefront is only one half of amplitude produced by 1st half-period zone.

1. Circular aperture
2. Rectangular aperture.
3. Opaque disc.

Fresnel Approximation.

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Tut 1

1) $\beta = \dfrac{D\lambda}{d} = \dfrac{(50\times10^{-2})\times(5\times10^{-5}\times10^{-2})}{(0.5\times10^{-3})}$

$\boxed{\beta = 0.5 \text{ mm}}$

2) $n = 1.5$, $d = 0.1$ cm, $D = 50$ cm, $t = ?$
0.2 cm

$\text{shift} = \dfrac{(n-1)tD}{d}$

$\Rightarrow 0.2\times10^{-2} = \dfrac{(0.5)\times t\times 50\times10^{-2}}{0.1\times10^{-2}}$

$\Rightarrow \boxed{t = 0.008 \text{ mm}}$

3) $\lambda_1 = 5890\,A^\circ$; $\lambda_2 = 5896\,A^\circ$
$d = 0.5$ mm ; $D = 100$ cm

$\beta_1 = \dfrac{D\lambda_1}{d}$ $\qquad$ $\beta_2 = \dfrac{D\lambda_2}{d}$

$\boxed{\beta_1 = 1.178 \text{ mm}}$ $\qquad$ $\boxed{\beta_2 = 1.1192 \text{ mm}}$

4) $\lambda = 3$ cm ; $d = (0.1, 1, 4)$ cm ; $D = 100$ cm

$\beta = \dfrac{\lambda D}{d}$

$\beta_1 = \dfrac{3\times10^{-2}\times100\times10^{-2}}{0.1\times10^{-2}} = 30$ m

$\beta_2 = \dfrac{3\times10^{-2}\times100\times10^{-2}}{1\times10^{-2}}$

$\beta_2 = 3$ m

$\beta_3 = \dfrac{3}{4}$ m

Visible : $4000 A^\circ - 7000 A^\circ$

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(5) $\Delta_1 = 5000 A^\circ$ $\quad \Delta_2 = 40000 A^\circ$. $\lambda = ?$

$\Delta_1 = S_2P - S_1P = n\lambda \rightarrow$ const

$\Delta_2 = S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda \rightarrow$ Dist.

let $n=1$. $\quad \Delta_1 = 5000 A^\circ = 1 \cdot \lambda$.

$\Delta_2 = 40000 A^\circ = \left(1 + \frac{1}{2}\right)\lambda$.

(6). $\frac{I}{I_{max}}$ $\qquad \Delta = \frac{\lambda}{5}$

$I = 4I_0 \cos^2 \frac{\delta}{2}$ $\qquad I_{max} = 4I_0$

$\frac{I}{I_{max}} = \frac{\cancel{4I_0} \cos^2 \delta/2}{\cancel{4I_0}} = \cos^2 \frac{\delta}{2}$

Bright fringe $\qquad$ Dark fringe.

$\Delta = n\lambda$ $\qquad \Delta = \left(n + \frac{1}{2}\right)\lambda$.

$\delta = \frac{2\pi}{\lambda} \Delta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{5} = 0.4\pi$.

$\frac{I}{I_{max}} = \cos^2(0.2\pi) =$

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(7) $RP = \left|\frac{\lambda}{\Delta\lambda}\right| \Rightarrow$ $R = 0.85$ $h = 1\ mm$

$\lambda = 4880 A^\circ$ $F = \frac{4R}{(1-R)^2}$

$= \frac{\pi h \sqrt{F}}{\lambda}$

(8) $\Delta\lambda = 0.1 A^\circ$ $\lambda = 6000 A^\circ$ $R = 0.8$

$\boxed{\text{Ans } h = 1.28\ mm}$.

$F = \frac{4R}{(1-R)^2}$

(9) $\lambda = 6000 A^\circ$. $F = 200$.

$n_2 = 1$

$h = 1\ cm$ for $T = 1$.

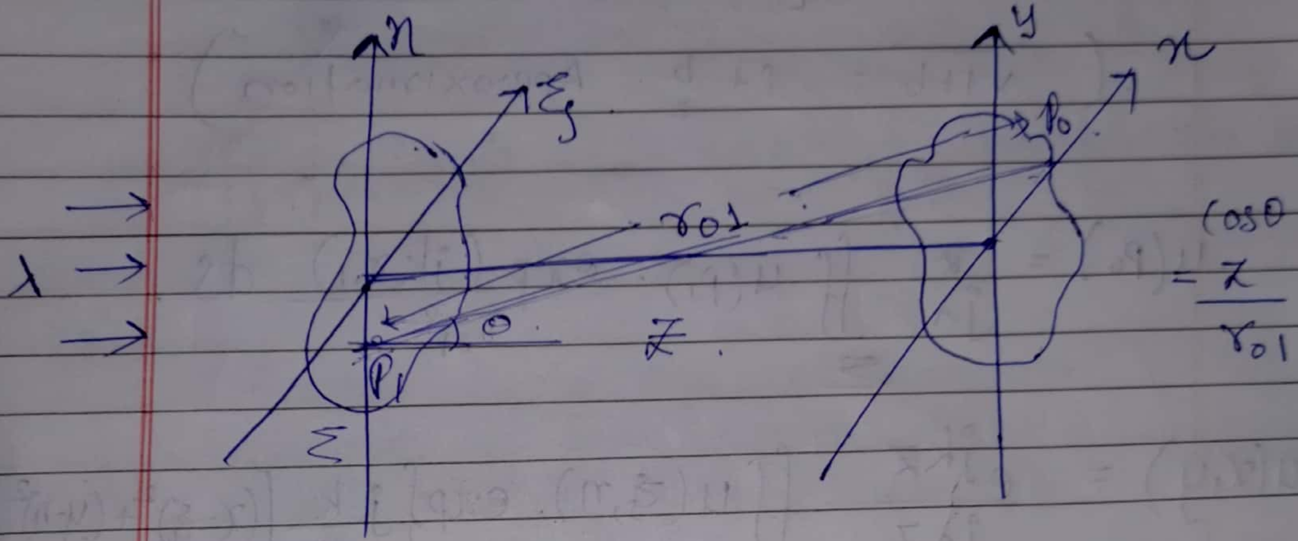
$\delta = \frac{4\pi n_2 h \cos\theta_2}{\lambda}$ $\delta = 2m\pi$.

$\Rightarrow \frac{\cancel{4}^{2}\cancel{\pi}}{\lambda} n_2 h \cos\theta_2 = \cancel{2} m \cancel{\pi}$

$\Rightarrow \cos\theta_2 = \left(\frac{m\lambda}{2h_2\lambda}\right)$

$\theta_2 = \cos^{-1}\left(\frac{3m}{100000}\right)$

Huygen's Fresnel Diffraction :-



$$u(P_0) = \frac{1}{j\lambda} \iint_{\Sigma} u(P_1) \frac{\exp(jkr_{01})}{r_{01}} \cos\theta \, ds$$

$$k = \frac{2\pi}{\lambda}$$

$$u(P_0) = \frac{Z}{j\lambda} \iint_{\Sigma} u(P_1) \frac{\exp(jkr_{01})}{r_{01}^2} ds$$

$$r_{01} = \sqrt{Z^2 + (x-\xi)^2 + (y-\eta)^2}$$

$$= Z \sqrt{1 + \frac{(x-\xi)^2}{Z^2} + \frac{(y-\eta)^2}{Z^2}}$$

$$r_{01} = Z \sqrt{1 + \left(\frac{x-\xi}{Z}\right)^2 + \left(\frac{y-\eta}{Z}\right)^2}$$

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$$r_{01} = z\left[1 + \frac{1}{2}\left(\frac{x-\xi}{z}\right)^2 + \frac{1}{2}\left(\frac{y-\eta}{z}\right)^2\right]$$

$\left(\sqrt{1+b} = 1 + \frac{b}{2} \text{ Approximation}\right)$.

$$u(P_0) = \frac{z}{j\lambda} \iint_{\Sigma} u(P_1) \cdot \frac{\exp(jkr_{01})}{r_{01}^2} \, ds.$$

$$u(x,y) = \frac{e^{jkz}}{j\lambda z} \iint u(\xi,\eta) \cdot \exp\left\{j \frac{k}{2z}\left[(x-\xi)^2 + (y-\eta)^2\right]\right\} d\xi \, d\eta.$$

$$u(x,y) = \iint_{-\infty}^{\infty} u(\xi,\eta) \cdot h(x-\xi, y-\eta) \, d\xi \, d\eta.$$

Convolution term $h(x,y) = \frac{e^{jkz}}{j\lambda z} \exp\left[\frac{jk}{2z}(x^2+y^2)\right]$

{Convulution function, Correlation function}

Fourier Transform

$$F(w) = \int_{-\infty}^{\infty} f(x) \cdot \exp(-j2\pi x w) \cdot dx.$$

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Fresnel Diffraction Integral.

$$U(x,y) = \frac{e^{jkz}}{j\lambda z} \cdot e^{j\frac{k}{2z}(x^2+y^2)} \cdot \iint_{-\infty}^{\infty} \left\{ u(\xi,\eta) \cdot e^{j\frac{k}{2z}(\xi^2+\eta^2)} \right\} e^{-j\frac{2\pi}{\lambda z}(x\xi+y\eta)} \, d\xi \, d\eta$$

(HW)

1. Diffraction by a circular aperture.
2. Diffraction by an opaque disc.
3. Propagation of Guassian Mean.

Polarization.

1. Linearly Polarized → x-polarized (Horizontal), → y-polarized (Vertical).
2. Circularly Polarized → Left Circularly, → Right Circularly.
3. Elliptically Polarized → Left Elliptically, → Right Elliptically.

Human Eyes can not distinguish whether light is polarized or not.

$$E_1 = \hat{x} a_1 \cos(kz - \omega t + \theta_1)$$
$$E_2 = \hat{x} a_2 \cos(kz - \omega t + \theta_2)$$

linearly polarized & propagating in same dir^n.

xx

① $E = E_1 + E_2$

$= \hat{x} a_1 \cos(kz - \omega t + \theta_1) + \hat{x} a_2 \cos(kz - \omega t + \theta_1)$

$E = E_1 + E_2 = \hat{x} a \cos(kz - \omega t + \theta_2)$

$$a = \left[a_1^2 + a_2^2 + 2a_1 a_2 \cos(\theta_1 - \theta_2)\right]^{1/2}$$

② xy

$E_1 = \hat{x} a_1 \cos(kz - \omega t)$.

$E_2 = \hat{y} a_2 \cos(kz - \omega t + \theta)$.

$E = E_1 + E_2$.

For $\theta = n\pi$ resultant will be linearly polarized wave.

if $z = 0$ $E_x = a_1 \cos\omega t$ $E_y = a_2 \cos(\omega t - \theta)$.

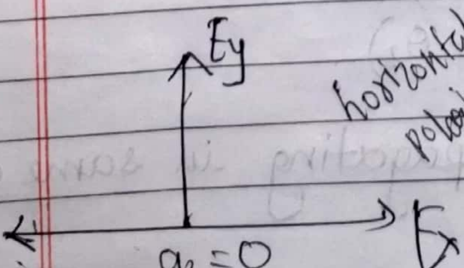
For $\theta = n\pi$; $E_x = a_1 \cos\omega t$ $E_y = (-1)^n a_2 \cos\omega t$

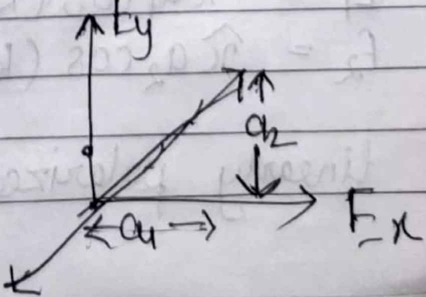
$\Rightarrow \dfrac{E_y}{E_x} = \pm \dfrac{a_2}{a_1} \Rightarrow$ Straight line.

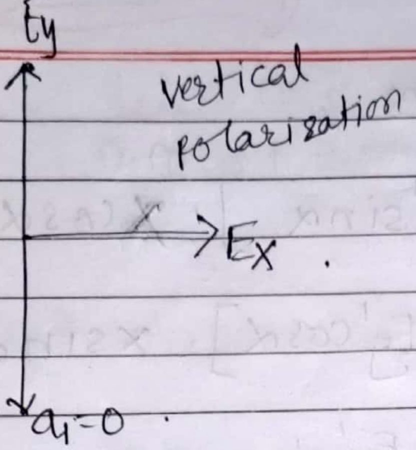
$$\phi = \tan^{-1}\left(\pm \frac{a_2}{a_1}\right).$$

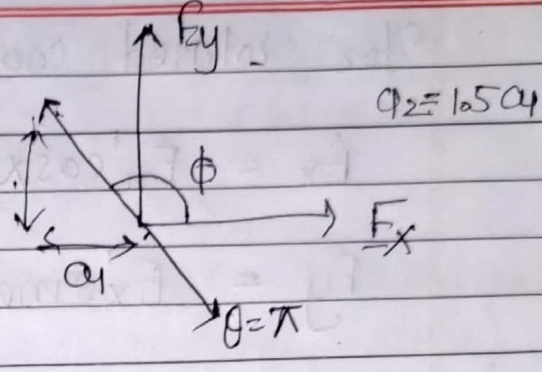
For $\theta = n\pi$; $n = 0, 2, 4,$ —— (In phase).

$n = 1, 3, 5,$ —— (Out of phase.







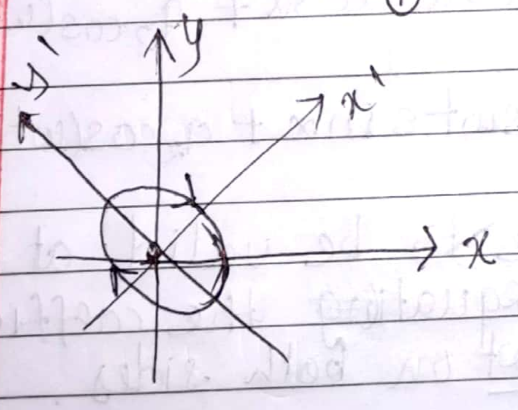


Consider $\theta = \pi/2$ $\quad a_1 = a_2 = a$

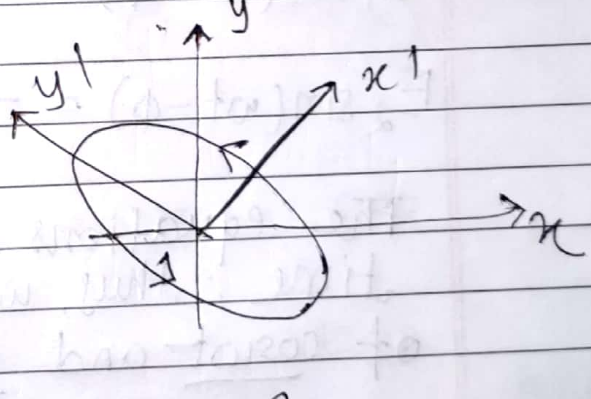
- $E_x = a\cos\omega t \;;\; E_y = a\sin\omega t$

$$E_x^2 + E_y^2 = a^2 \Rightarrow \text{circle}$$

- $E_x = a_1\cos\omega t$ —(1) $;\; E_y = a_2\cos(\omega t - \theta)$ —(2)



LEP



REP

$E_x' = E_1\cos(\omega t - \phi)$ —(3) $\qquad \left(\dfrac{E_x'}{E_1}\right)^2 + \left(\dfrac{E_y'}{E_2}\right)^2 = 1$

$\dfrac{E_x'}{E_1} = \cos(\omega t - \phi)$

$E_y' = E_2\sin(\omega t - \phi)$ —(4) $\Rightarrow$ Ellipse

$\dfrac{E_y'}{E_2} = \sin(\omega t - \phi)$

For rotated coordinates;

$$E_x = E_x' \cos\alpha - E_y' \sin\alpha \quad] \cdot \cancel{x} \cos\alpha$$

$$E_y = E_x' \sin\alpha + E_y' \cos\alpha \quad] \times \sin\alpha$$

$$E_x \cos\alpha = E_x' \cos^2\alpha - E_y' \sin\alpha \cos\alpha$$
$$+ E_y \sin\alpha = E_x' \sin^2\alpha + E_y' \sin\alpha \cos\alpha .$$

$$E_x \cos\alpha + E_y \sin\alpha = E_x' \quad \text{——} \ (5)$$

$$-E_x \sin\alpha + E_y \cos\alpha = E_y' \quad \text{——} \ (6) .$$

Use (1), (2), (3), (4), (5) & (6) and solve;

$$E_1 \cos(\omega t - \phi) = a_1 \cos\omega t \cos\alpha + a_2 \cos(\omega t - \theta) \sin\alpha$$

$$E_2 \sin(\omega t - \phi) = -a_1 \cos\omega t \sin\alpha + a_2 \cos(\omega t - \theta) \cos\alpha$$

The equations have to be valid at all time, thus we equating the coefficients of $\cos\omega t$ and $\sin\omega t$ on both sides.

$$E_1 \cos\phi = a_1 \cos\alpha + a_2 \cos\theta \sin\alpha \quad \text{——} \ (A)$$
$$E_1 \sin\phi = a_2 \sin\theta \sin\alpha \quad \text{——} \ (B)$$
$$-E_2 \sin\phi = -a_1 \sin\alpha + a_2 \cos\theta \cos\alpha \quad \text{——} \ (C) .$$
$$E_2 \cos\phi = a_2 \sin\theta \cos\alpha \quad \text{——} \ (D) .$$

If we square the four eq^ns and add

$$E_1^2 + E_2^2 = a_1^2 + a_2^2 .$$

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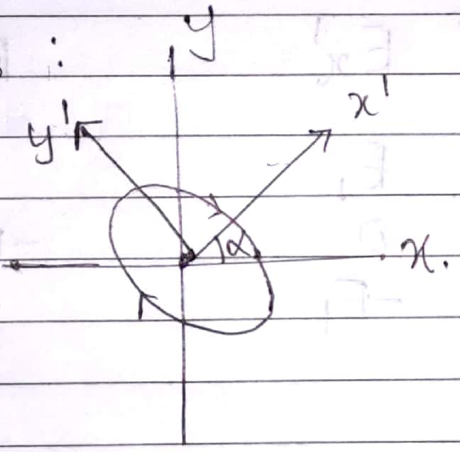
$$\frac{E_2}{E_1} = \frac{a_2 \sin\theta \cos\alpha}{a_1 \cos\alpha + a_2 \cos\theta \sin\alpha} = \frac{a_1 \sin\alpha - a_2 \cos\theta \cos\alpha}{a_2 \sin\theta \sin\alpha}.$$

$$a_2^2 \sin^2\theta \sin\alpha \cos\alpha = a_1^2 \sin\alpha \cos\alpha - a_2^2 \cos^2\theta \sin\alpha \cos\alpha - a_1 a_2 \cos\theta (\cos^2\alpha - \sin^2\alpha).$$

$$\boxed{\tan 2\alpha = \frac{2a_1 a_2 \cos\theta}{a_1^2 - a_2^2}}$$

Examples :



If $a_1 = a_2$

$\Rightarrow \alpha = \pi/4$.

$$\frac{E_2}{E_1} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$$

$\theta = \frac{\pi}{3}$; $\frac{E_2}{E_1} = \frac{1}{\sqrt{3}} = 0.577$. REP

$= \frac{\pi}{2}$; $\frac{E_2}{E_1} = 1$. RCP

$= \frac{2\pi}{3}$; $\frac{E_2}{E_1} = 1.732$. REP

$= \frac{4\pi}{3}$; $\frac{E_2}{E_1} = -1.732$. LEP

$= \frac{3\pi}{2}$; $\frac{E_2}{E_1} = -1$. LCP

$= \frac{5\pi}{3}$; $\frac{E_2}{E_1} = -0.577$. LEP.

P = Polarised
R = right
L = left
E = elliptically
C = circularly

for $\theta = \frac{4\pi}{3}$; $E_x' = E_1 \cos(\omega t - \phi)$

$E_y' = -1.732 \sin(\omega t - \phi)$

To determine the state of polarization

Choose $t = 0 \Rightarrow \phi = 0$

$E_x' = E_1 \cos \omega t$

$E_y' = -1.732 E_1 \sin \omega t$

Thus at:

	t	E_x'	E_y'
Hori. pol	0	E_1	0
Vertical pol.	$\pi/2\omega$	0	$-1.732 E_1$
	π/ω	$-E_1$	0