Syllabus involves statics of rigid & deformable bodies.

Chapter - 1
Basic Mechanics

- Force ⟶ Body ⟶ Reaction

The above three are main components of mechanics. We can study each of them in detail.

→ BODY

- It may be of 2 types

a) Rigid (Pre - midsem)
b) Deformable (Post- midsem)

→ REACTION

- The reaction of our body may be of 2 types

- Statics (no motion)
- Dynamics (motion involved)
  - Kinematics (Study of s, v, a etc)
  - Kinetics (Study of F, E)

* → TERMINOLOGIES

1. **PARTICLE** : when we consider a body as a point mass, ie neglecting its shape and size, then the point mass is called particle

2. **FORCE** : an entity capable of moving or deforming a body. In engineering mechanics, a force is completely described by 3 parameters:

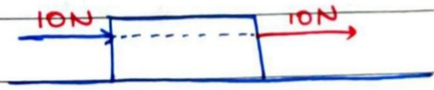
a) magnitude   b) direction   c) point of application

NOTE: Point of application is not needed for a particle.

• FORCE
- CONTACT
- NON- CONTACT

NOTE: POINT LOAD : when forces distributed over a body is replaced by a single equivalent force with an equivalent direction, it is called point load. It is applied on centre of gravity.

OTE:



If blue force is replaced by red one, effect on the body remains same.

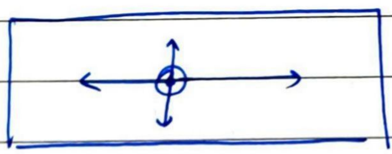
- Force may also be categorised based on the plane in which mechanics is involved

FORCE

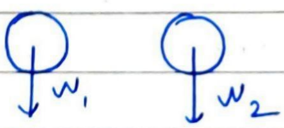
- PLANAR (2D)
- NON-PLANAR (3D)

· PLANAR FORCES: which act in a plane

a) Concurrent: which intersect at a point



b) Parallel:



(weight of 2 bodies)

c) General: neither // nor concurrent.

- We also have the same 3 categories ~~of~~ for 3D force.

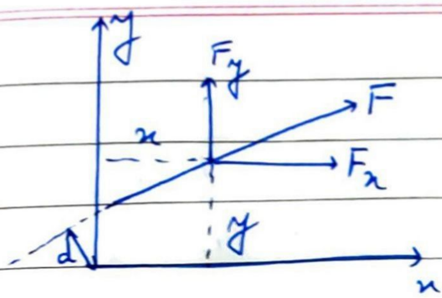
- Resolution of a force: If multiple forces act on a body then their vector sum is called their resultant force & they ~~are~~ are ~~xxx~~ called components of the resultant.

3. → WEIGHT

- weight is a force acting on body due to gravity. It is always vertically downwards.

4. NORMAL : A force (reaction force) which acts ⊥ to the surface on which body is placed.

5 MOMENT : The magnitude of moment of any force about a point is the ~~cross~~ product of the ~~lever~~ force and the ⊥ distance of the line of action of force from the point. moment is a vector quantity and vectorically, it equals the cross product ~~of~~ of position vector and the force.



$|M| = Fd$

$|M| = -F_x y + F_y x$

In vectorical form

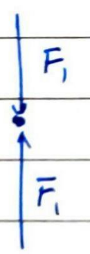
$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$; $\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$

$M = \vec{r} \times \vec{F} = -\vec{F} \times \vec{r}$

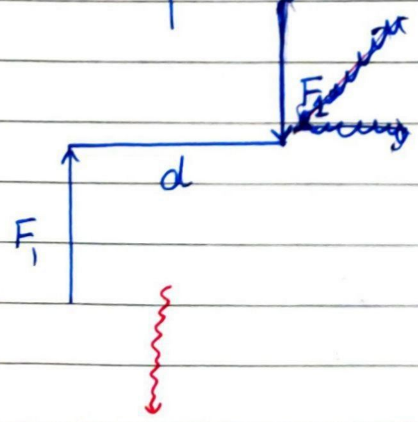
~~M=~~ $\boxed{M = \vec{r} \times \vec{F} = -\vec{F} \times \vec{r}}$

$$M = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ F_x & F_y & F_z \\ r_x & r_y & r_z \end{vmatrix}$$

eg- consider



moment = 0



moment = dF_1 (no matter wherever the body's COM is)

Such a case where a body is acted upon by equal and opposite forces is called couple

→ EQUILIBRIUM

- If a body experiences zero net force, and zero moment, it is said to be in equilibrium.

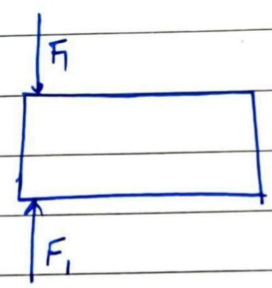
- In 2D, for equilibrium,

a) $\sum F = 0 \Rightarrow \sum F_x = 0 \; ; \; \sum F_y = 0$

&

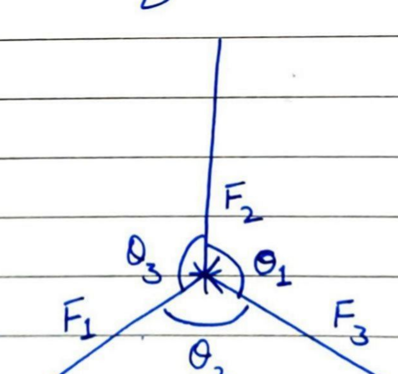
b) $\sum M = 0 \Rightarrow \sum M_z = 0$

$\left.\right\}$ 3 eqⁿs ∴ we can determine max^m of 3 variables.



⇝ equilibrium case.

- Consider a force arrangement as



If given that body is in eq^m, then we can directly write

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

- In 3D, for equilibrium

a) $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$

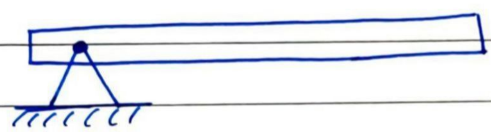
b) $\Sigma M_x = 0$; $\Sigma M_y = 0$; $\Sigma M_z = 0$

} 6 eq^{n}s $\therefore$ we can determine max^m of 6 variables. Any more and the problem is called statically indeterminant

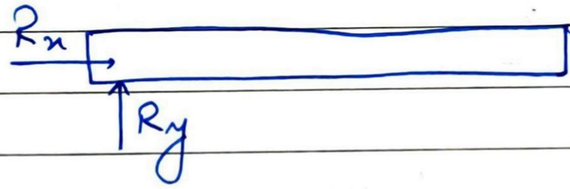
SOME SPECIAL CASES OF MOTION RESTRICTIONS →

2D JOINTS

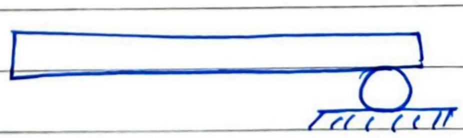
1. PIN JOINT :



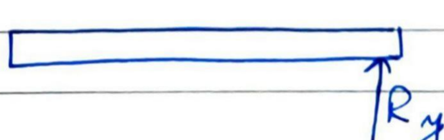
In FBD, we draw



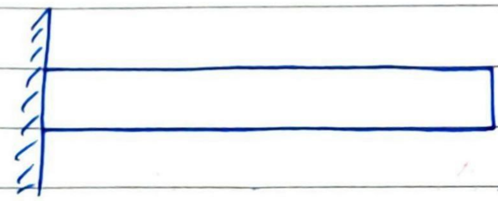
2. ROLLER JOINT



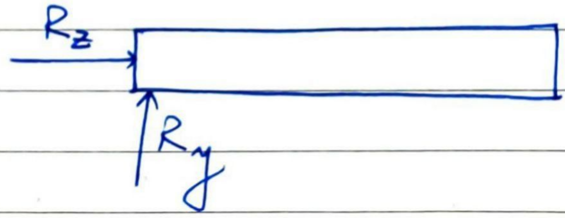
In FBD, we draw



3. Fixed Joint



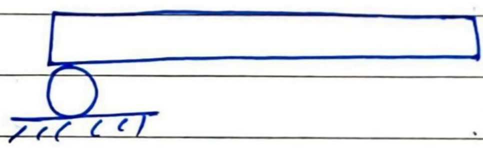
In FBD, we draw



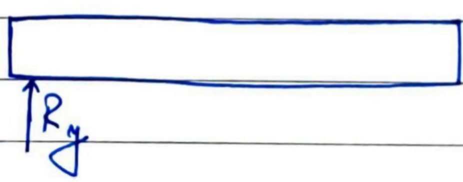
$\Sigma M_z = 0$

3D Joints

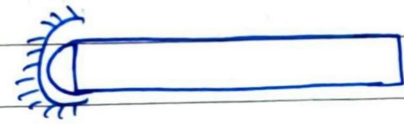
1. Roller Joint



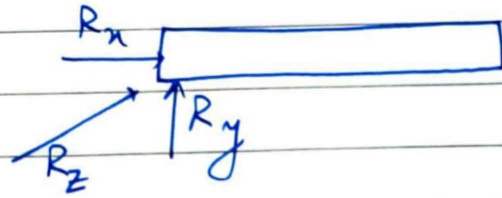
In FBD, we draw



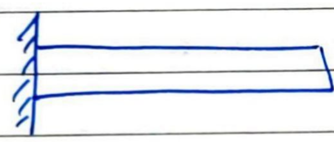
2. Ball and Socket Joint



In FBD we draw.

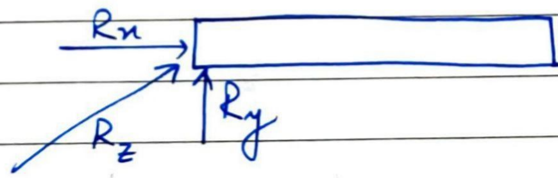


3. Fixed Joint

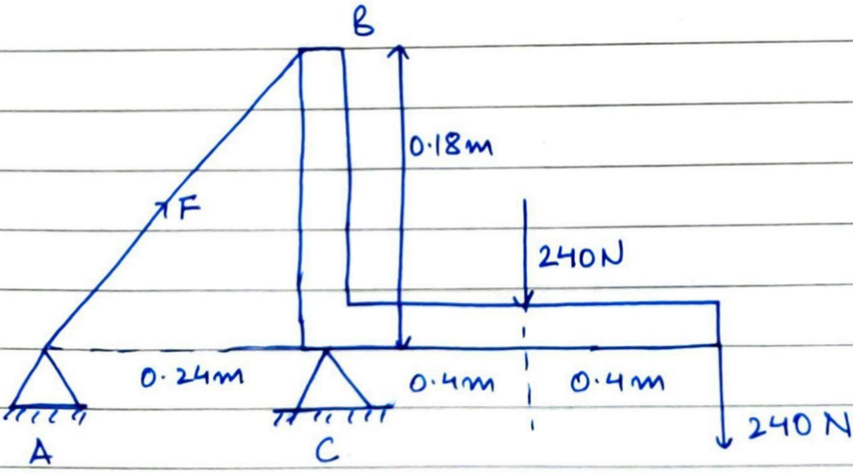


In FBD, we draw

$\Sigma M = 0$

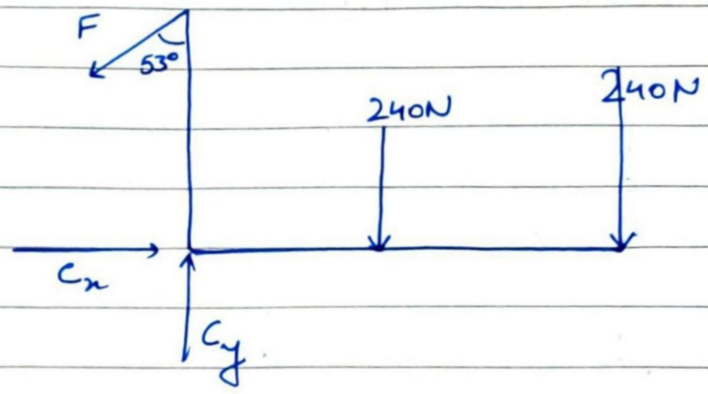


Q→



Find F & R_c.

Ans:



$\Sigma M = 0$

$\Rightarrow \cancel{240}^{60} \times 0.4 + \cancel{240}^{60} \times 0.8 = F \times \frac{4}{5} \times 0.18$

$\Rightarrow \cancel{72}^{4} = \frac{F}{5} \times \cancel{0.18}^{0.01}$

$\Rightarrow F = 2000N$

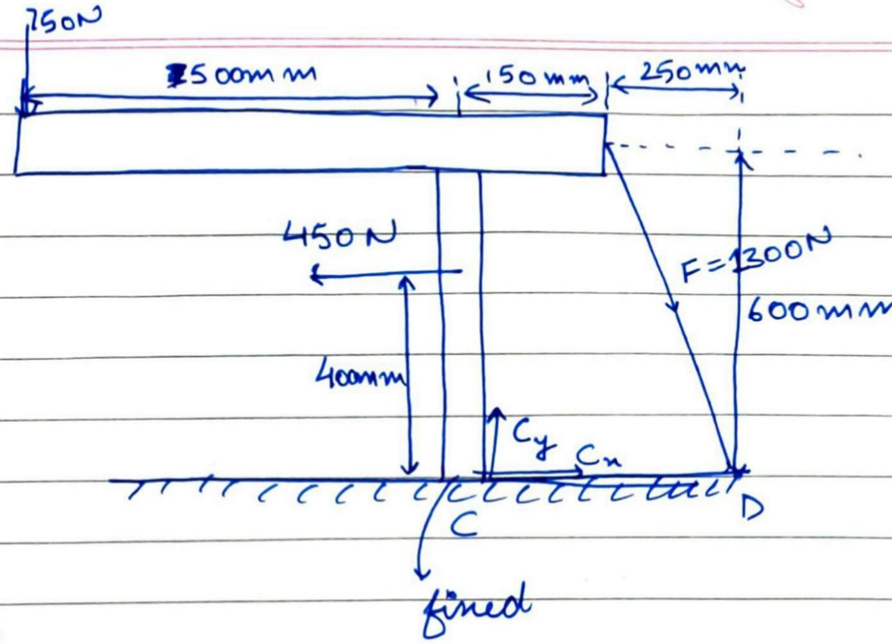
$\Sigma F_x = 0 \Rightarrow C_x = \cancel{2000}^{400} \times \frac{4}{\cancel{5}}$

$\Rightarrow C_x = 1600N$

$\Sigma F_y = 0 \Rightarrow 480 + F \times \frac{3}{5} = C_y$

$\Rightarrow C_y = 1680 N$

Q→

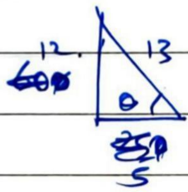


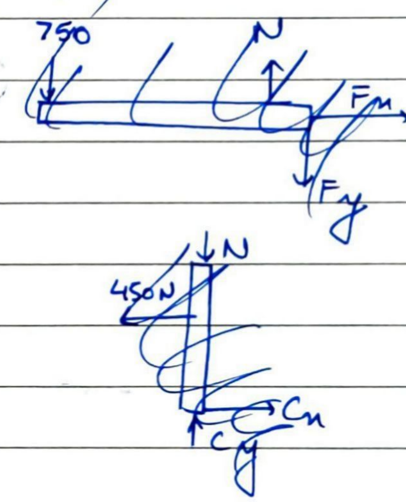
Find R_c

Ans: $\cancel{\Sigma M = 0 \Rightarrow 750 \times 500 \quad 450 \times 400 = M_c}$

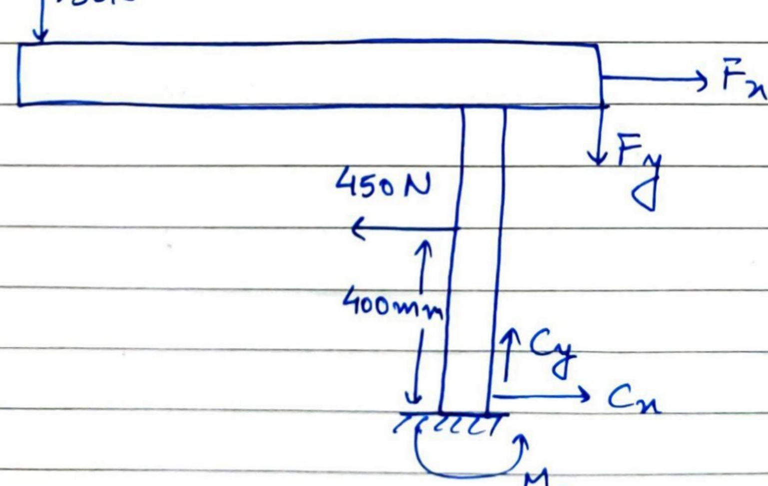
$\cancel{\Sigma F_x = 0 \Rightarrow C_x = 450N}$

$\cancel{\Sigma F_y = 0 \Rightarrow C_y = N}$





$\cancel{\Sigma M = 0 = 750 \times 500 = F_y \times 150}$



$\Sigma M = 0$

$\Rightarrow 600 \times F_x + 150 \times F_y = 750 \times 500 + 400 \times 450 + M_c$

$\Rightarrow 600 \times \frac{500}{\cancel{13}} + 150 \times \frac{1200}{\cancel{13}} = 750 \times 500 + 400 \times 450 + M_c$

$\Rightarrow 300000 + 180000 = 375000 + 180000 + M_c \Rightarrow M_c = 75000$ CW

$\sum F_x = 0 \Rightarrow 450 = -C_x + 500$

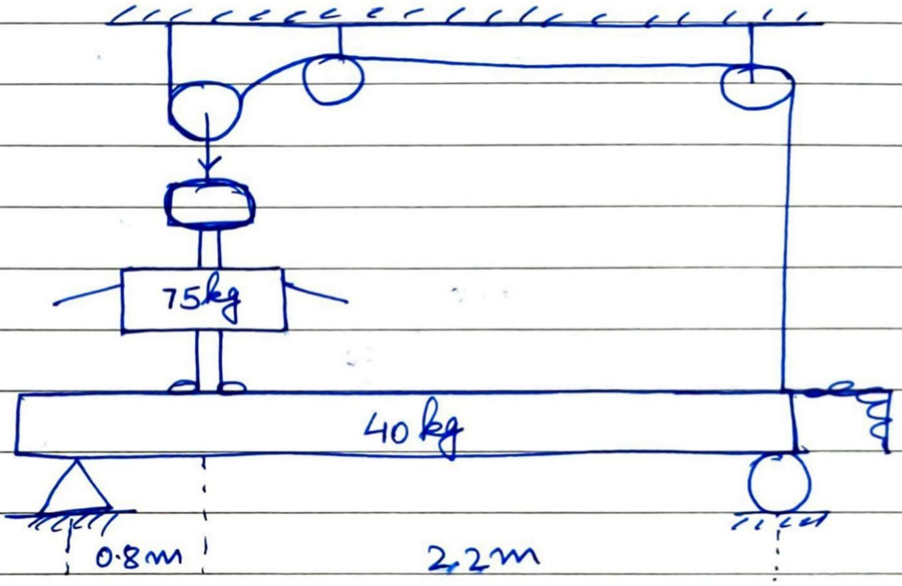
$\Rightarrow C_x = -50\,N$

- $F_y = 0 \Rightarrow 750 + 1200 = C_y$

$\Rightarrow C_y = 1950\,N$

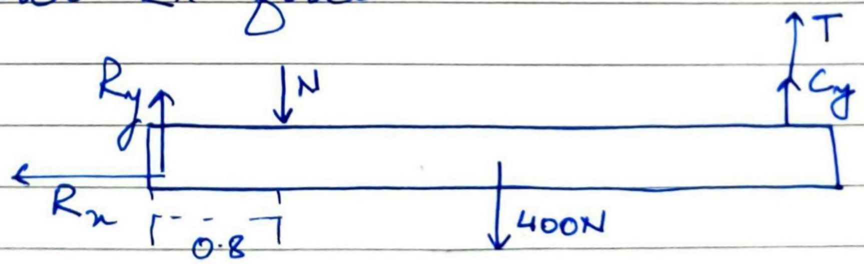
$M_c = 75\,Nm$

Q:



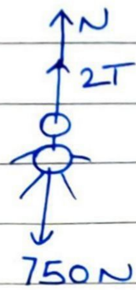
man pulls the pulley. Find cable tension and R^{xn} force

Ans:



$R_x = 0$

man



~~In total~~

Body

$0.8 \times N + 1.5 \times 40g = 3(T + C_y)$

$\cancel{R_y + 3T + C_y = 1150}$

$0.8N + 60g = 3T + 3C_y$ (i)

&

$75g = N + 2T$ (ii)

$\cancel{R_y + T + C_y = 400 + N}$

$\cancel{R_y + 3T}$

$\cancel{\Rightarrow 4T = 750 - N}$

$R_y + T + C_y = 40g + N$ (iii)

~~Moment abt. roller = 0~~

$\Rightarrow \cancel{60g + 2.2N = 3R_y}$ (iv)

$\cancel{\frac{60g}{3}^{20} + \frac{2.2}{3}N = R_y}$

$\therefore \cancel{20g + \frac{2.2N}{3} + T + C_y = 40g + N}$

$\cancel{T + C_y = 20g + \frac{0.8N}{3}}$

we have 3 equations and 4 variables.
∴ the problem is statically indeterminant.

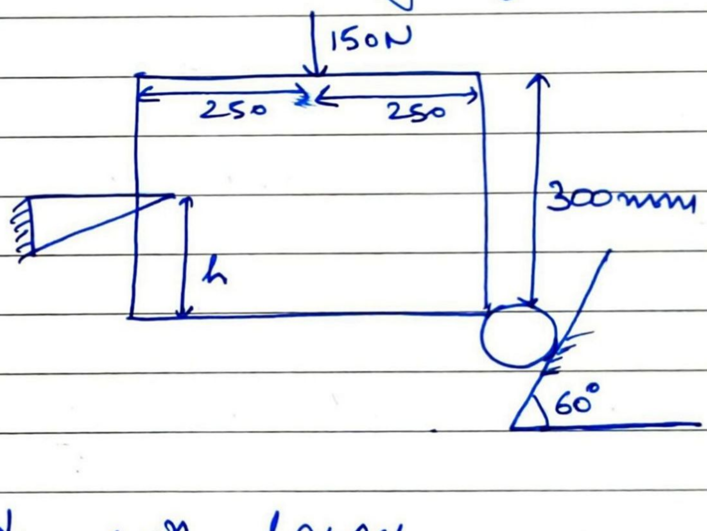
consider if a case is given where $C_y = 0$
ie the rod is almost about to lift up, then,

$$\therefore \; 0.8N + 60g = 3T \quad \text{(i)}$$
$$75g = N + 2T \quad \text{(ii)}$$

$$R_y + T = 40g + N \quad \text{(iii)}$$

∴ solve them to get your required ans

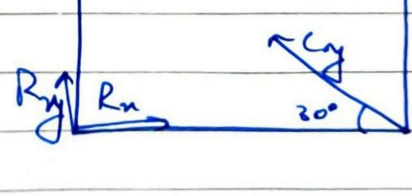
Q:



i) $h = 0$
ii) $h = 200$ mm

Find $r\times^n$ forces

Ans
i)



$$\Sigma F_y = 0 \Rightarrow R_y + C_y/2 = 150$$

$$\Sigma F_x = 0 \Rightarrow R_x = C_y \frac{\sqrt{3}}{2}$$

$\Sigma M_2 = 0$ (abt pin)

$\Rightarrow \frac{C_y}{2} \times 5\cancel{00} = 150 \times \cancel{250}$

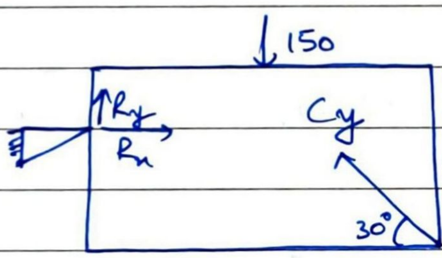
$\Rightarrow C_y \cancel{/2} = 150$

$\Rightarrow C_y = 150$ N

$R_x = 75\sqrt{3}$ N

$R_y + \frac{\cancel{150}}{\cancel{2}}^{75} = 150$

$\Rightarrow R_y = 150 - 75 = 75$ N



$\Sigma F_y = 0 \Rightarrow R_y + \frac{C_y}{2} = 150$

$\Sigma F_x = 0 \Rightarrow R_x = C_y \sqrt{3}/2$

$\Sigma M_2 = 0 \Rightarrow 1\cancel{00} \times C_y\sqrt{3} + 15\cancel{0} \times 25\cancel{0} = \frac{C_y}{2} \times 5\cancel{00}$

$\Rightarrow C_y\left(\sqrt{3} - 5/2\right) + 15 \times 25 = 0$

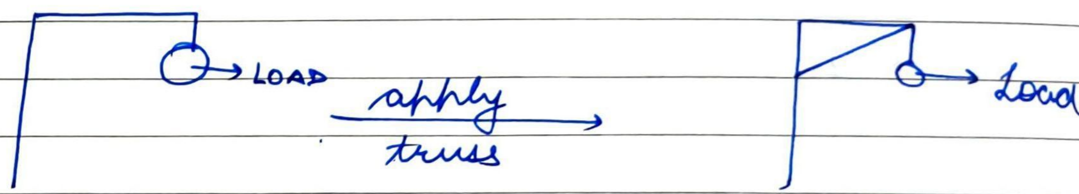
$\Rightarrow C_y = \frac{-375}{(\sqrt{3} - 5/2)}$

Chapter -2
Mechanical Structures

TRUSS

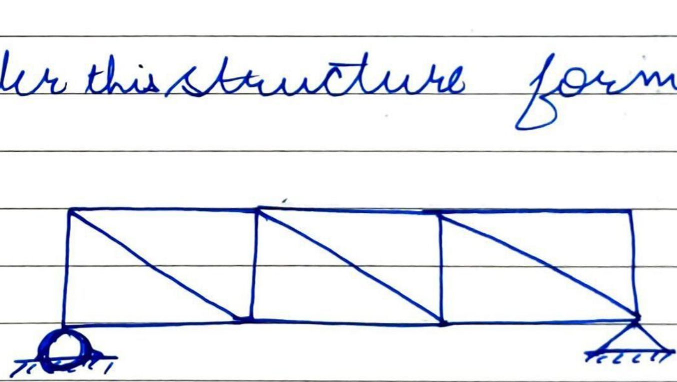
- Truss is a triangular support structure which is used to support mechanical load on a structure.

- eg-



load is supported in better way

- consider this structure formed by truss



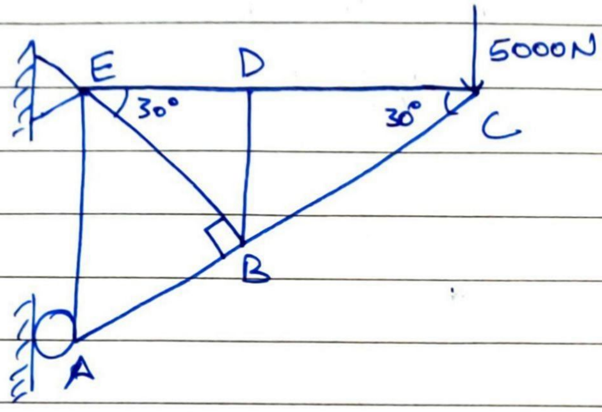
- Each side of triangle counts as a single member and there is a specific relation b/w number of members & joints.

imp.

$$\boxed{m = 2j - 3}$$

- In Trusses, one end is taken to be pin joint and other to be a roller joint. This is because if the other is also taken to be a pin joint then the length would get fixed due to which stress may break the ~~st~~ structure.

Q→

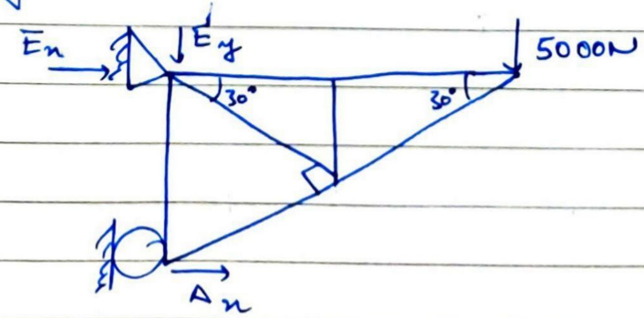


Find $r x^n$ forces at all points.

Ans: Method of joints

- We draw F.B.D of joints and use the equations
$$\Sigma F_x = 0 \quad \& \quad \Sigma F_y = 0$$
(since force on massless body $\underline{\underline{=}}\, 0$)

- Firstly we will determine $r x^n$ forces



$5000 + E_y = 0$

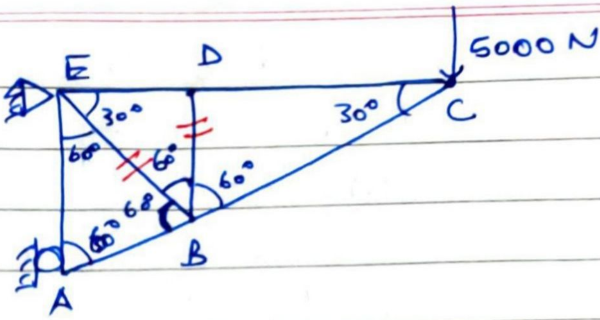
$\Rightarrow E_y = -5000N$

& $A_x + E_x = 0$

~~& $\Sigma M_E = 0$~~

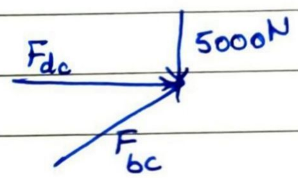
~~$\Rightarrow 2 \times 5000 = 1 \times A_x$~~

~~$\Rightarrow A_x = 10000N \therefore E_x = -10000N$~~



- now from the place where force was applied, move to adjacents points where we can apply $\Sigma F_x = 0$ & $\Sigma F_y = 0$

$\therefore$ at C



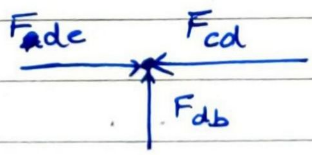
$\therefore F_{bc} \sin 30^\circ = 5000N$

$\Rightarrow F_{bc} = 10000N$

$F_{dc} = - F_{bc} \cos 30^\circ$

$\Rightarrow F_{dc} = - 5000\sqrt{3}\ N$

now at D



$F_{de} = F_{cd}$

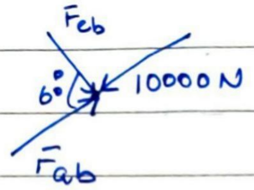
$\therefore F_{cd} = - 5000\sqrt{3}\ N = F_{de}$

$F_{db} = 0$

$\therefore E_x = 5000\sqrt{3}\ N$

$\therefore A_x = -5000\sqrt{3} N$

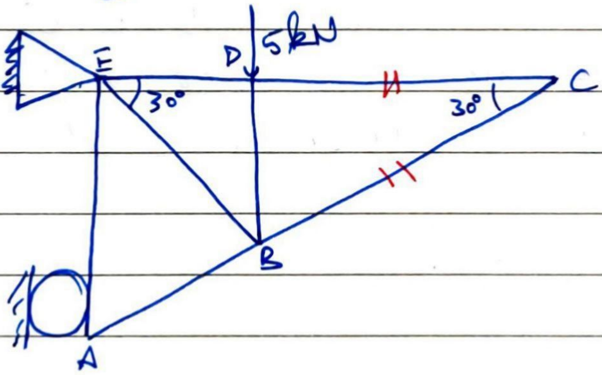
At B



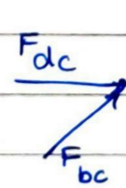
$F_{eb} = 0$ & $F_{ab} = 10000 N$

In given structure EB and DB are zero force members. They are very important since they take up the load if any other member breaks.

Q→ Find zero force member



Ans. At C



$\Sigma F_y = 0 \Rightarrow \quad F_{bc} \sin 30^\circ = 0$

$\Rightarrow F_{bc} = 0$

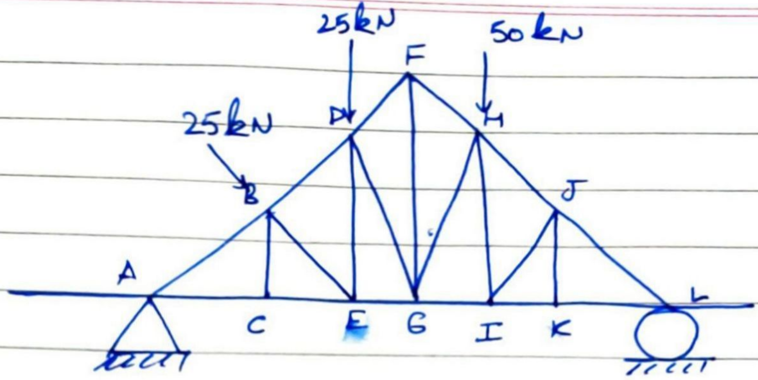
$\Sigma F_x = 0 \Rightarrow \quad F_{dc} + F_{bc} \cos 30^\circ = 0$

$\Rightarrow F_{dc} = 0$

$\therefore$ DC and BC are zero force members

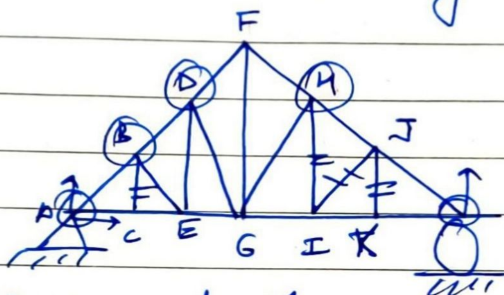
imp
read note on next page first

Q→



Determine zero force members without writing equations.

Ans: First we mark joints with force



We wont consider these joints.

Now, at K, 2 members collinear ∴ 3rd member JK is 0 force member
∴ JI also is zero force and IH also zero force.
BC also zero force

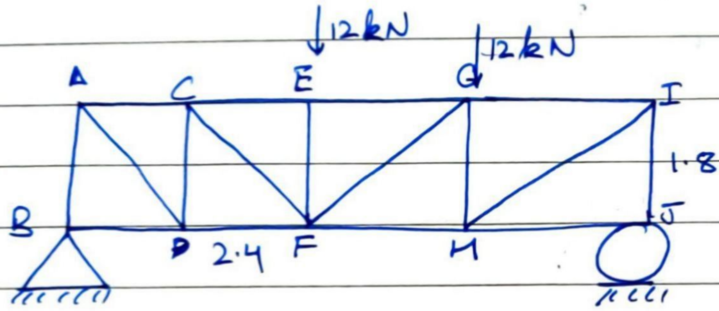
PTO

NOTE: To check for zero force members, conditions

i) no external force on joint (Don't use this method in exam)

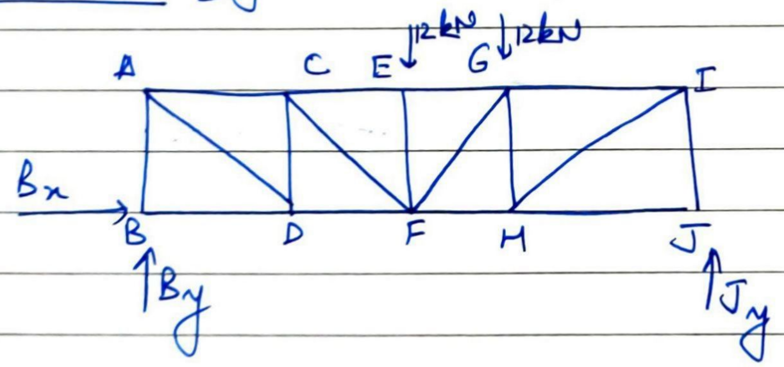
ii) 2 members are collinear and 3rd member (the one under observation) is at an angle on the joint (under observation. This joint must not ~~be~~ have any external force (valid for joint with 3 ~~forc~~ members)

Q→



Find ~~forces~~ forces for GF & FH

Ans: method of joints



$\sum F_x = 0$

$\Rightarrow B_x = 0$

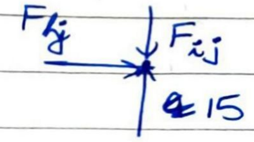
$\sum F_y = 0 \quad \Rightarrow \quad B_y + J_y = 24\,kN$

$\sum M_B = 0$

$\Rightarrow \quad J_y \times 9.6 - 12 \times 4.8 - 12 \times 7.2 = 0$

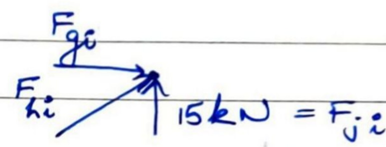
$\Rightarrow J_y = 15\,kN \quad \Rightarrow B_y = 9\,kN$

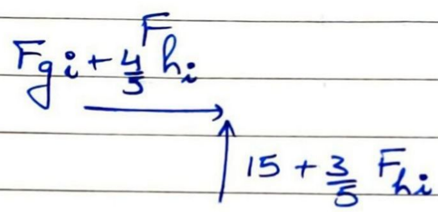
We start from joint J



$\therefore F_{hj} = 0$ & $F_{iJ} = 15\,kN$

At I





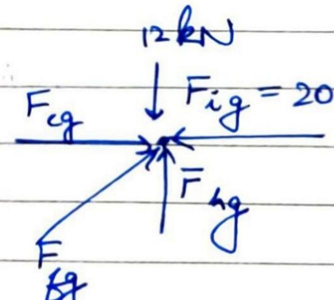
$15 + \frac{3}{5} F_{hi} = 0$

$\Rightarrow F_{hi} = -\cancel{\frac{75}{5}}\,25 = -25\,kN$

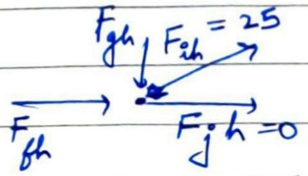
$F_{gi} - 20 = 0$

$\Rightarrow F_{gi} = 20\,kN$

At G



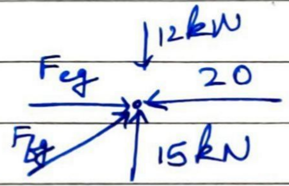
At H



$$F_{gh} = \cancel{25}^{5} \times \frac{3}{\cancel{5}} = 15 \text{ kN}$$

$$F_{fh} = -20 \text{ kN} \qquad \therefore \boxed{|F_{fh}| = 20 \text{ kN}}$$

at G



$$F_{fg} \times \frac{3}{5} = -3$$

$$\boxed{|F_{fg}| = 5 \text{ kN}}$$

imp

- **Sections Method**

- In this method we break the ~~s~~truss into sections and then solve.

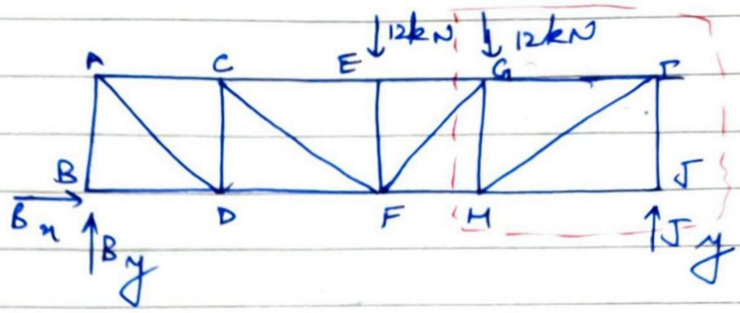
- Steps

i) Determine rxn forces

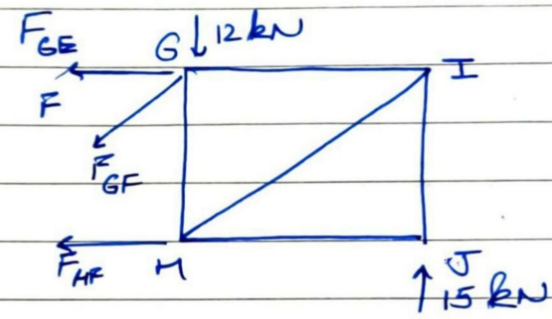
ii) Take a section

iii) Draw FBD.

consider previous question



We take section GHJI



$\Sigma F_y = 0$

$15 = 12 + F_{GF} \times 3/5$

$\Rightarrow \boxed{F_{GF} = 5kN}$

$\Sigma F_x = 0$

$\Rightarrow \quad F_{GE} + F_{GF} \times \frac{4}{5} + F_{HF} = 0 \qquad (i)$

We now write $\Sigma M_G = 0$

$1.8 \times F_{HF} = 15 \times 2.4$

$\boxed{F_{HF} = 20kN}$

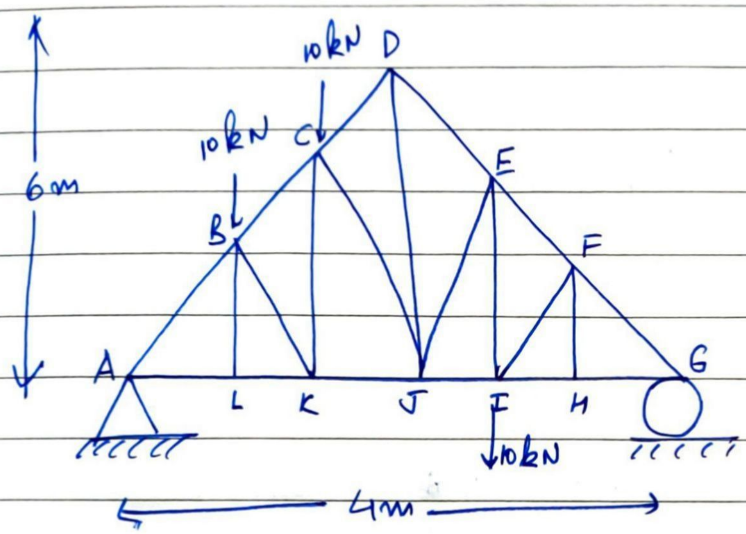
f. (~~from (i)~~

~~F_{GE}~~

V. Imp NOTE: you have to cut a section in such a way that maximum no. of unknowns is 3 and it passes through our region of interest

NOTE: Method of section is useful when force on few members are asked (say 2 or 3). But if forces on all members are asked, blindly apply method of joints.

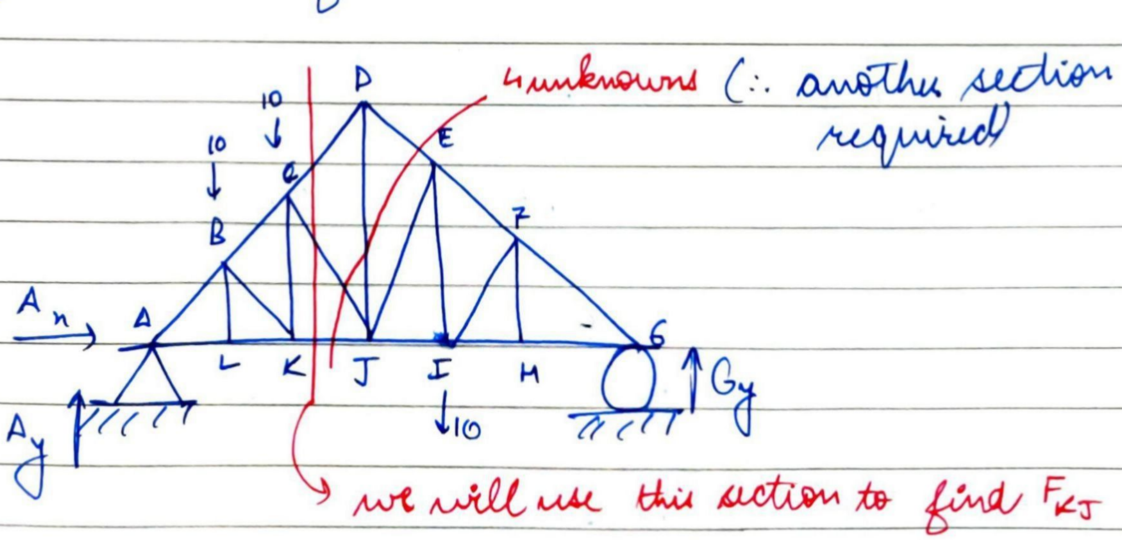
imp Q→

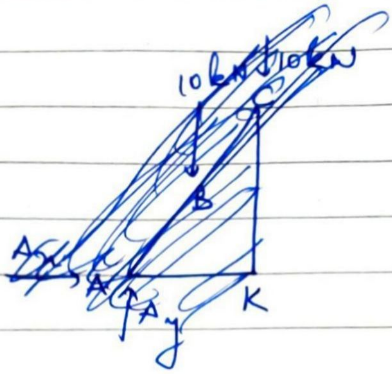


Find F_{DJ}

NOTE: method of sections can determine 3 unknowns at maximum ∴ If in cutting a section you get more than 3 unknowns, you need to cut another section to find rest of the unknowns.

Ans:





$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0 \Rightarrow A_y + G_y = 30$

$\Sigma M_A = 0$

$\Rightarrow 10 \times \frac{2}{3} + 10 \times \frac{4}{3} + 10 \times \frac{8}{3} = G_y \times 4$

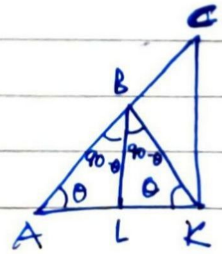
$\Rightarrow \frac{\overset{70\ 35}{\cancel{140}}}{3} = G_y \times 4$

$\Rightarrow G_y = 35/3 \text{ kN}$

$A_y + \frac{35}{3} = 30$

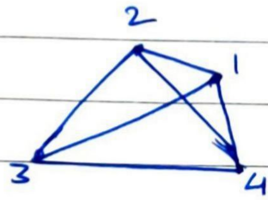
$\Rightarrow A_y = 55/3 \text{ kN}$

Now we operate on sections



→ 3-D TRUSS

- Basic element of a 3-D truss is a tetrahedron element

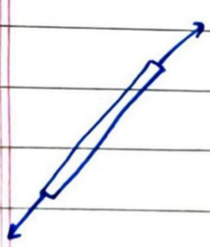


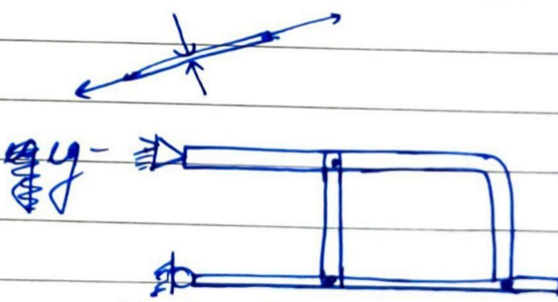
- The members and joints are related by relation

$$\boxed{m = 3j - 6}$$

* → MECHANICAL STRUCTURES →

- TRUSS | FRAME | MACHINE





eg-

eg- nutcracker

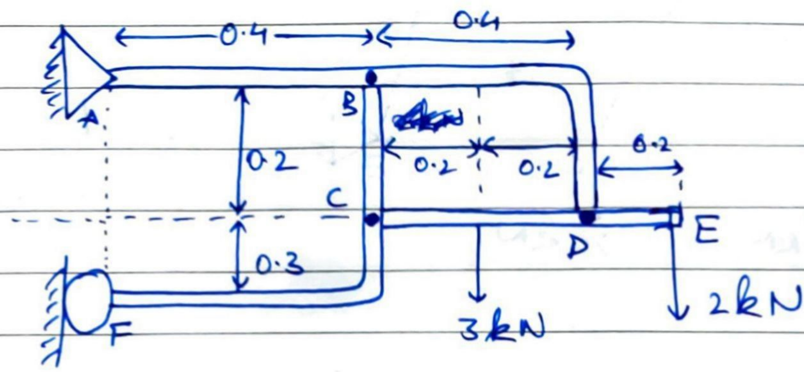
- 2 force member
- stationary

- At least one multiforce member
- stationary

- At least one multiforce mem[ber] & one moving member

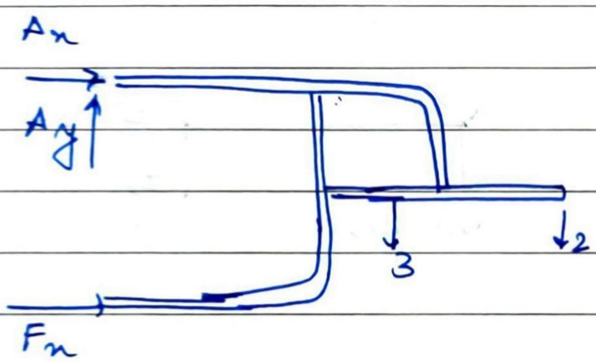
→ FRAME →

Q→



Find Forces on AD

Ans:



$\Sigma F_y = 0$

$3 + 2 = A_y$

$\Rightarrow A_y = 5$

$\Sigma F_x = 0 \Rightarrow A_x + F_x = 0$

$\Sigma M_A = 0$

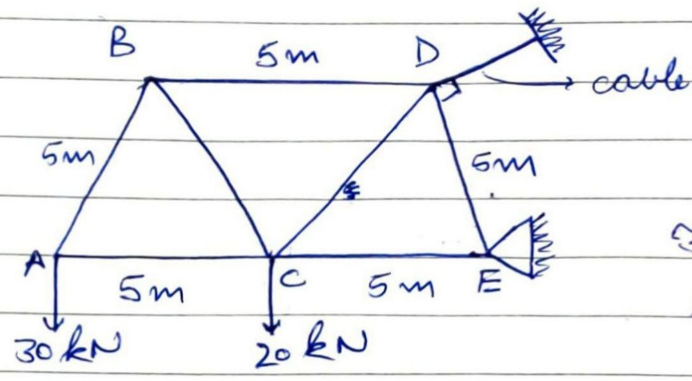
$\Rightarrow 0.6 \times 3 + 2 \times 1 = 0.5 F_x$

$\Rightarrow 1.8 + 2 = 0.5 F_x$

$\Rightarrow 3.8 = F_x/2 \Rightarrow F_x = 7.6$ kN

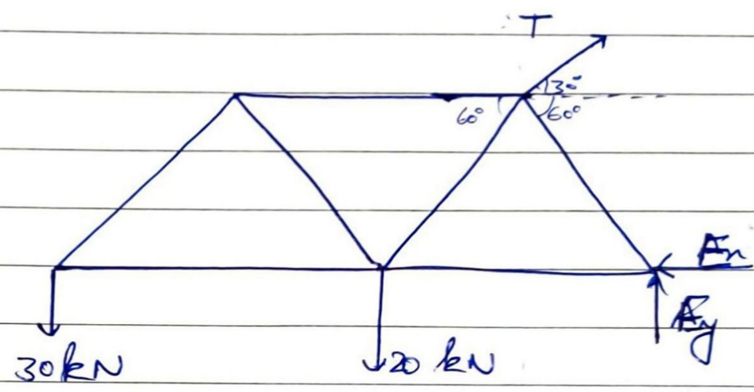
TUTORIAL

Q→



Find forces on each joint.

Ans.



ΣF_x

$T\cos 30° = E_x$

$\Sigma F_y = 0$

$\Rightarrow E_y = 50\,kN - T\sin 30°$

$\Sigma M_A = 0$

$\Rightarrow 20 \times 5 + 300 = 5T$

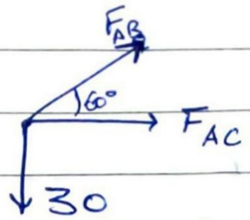
$\Rightarrow 400 = 5T$

$\Rightarrow T = 80\,N$

$E_x = 80 \dfrac{\sqrt{3}}{2} = 40\sqrt{3}\,kN$

$E_y = 10\,kN$

At A



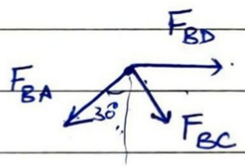
$F_{AB}\frac{\sqrt{3}}{2} = 30$

$\Rightarrow F_{AB} = \frac{60}{\sqrt{3}} = 20\sqrt{3}\ kN$

$F_{AB}\frac{1}{2} = -F_{AC}$

$\Rightarrow F_{AC} = \frac{-30}{\sqrt{3}} = -10\sqrt{3}\ kN$

At B



$\Rightarrow \frac{20\sqrt{3}}{\cancel{2}} = F_{BD} + \frac{F_{BC}}{2}$

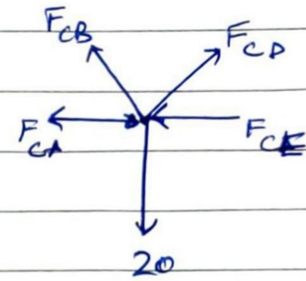
$\cancel{20}^{10}\sqrt{3}\times\frac{\sqrt{3}}{\cancel{2}} = -F_{BC}\frac{\sqrt{3}}{2}$

$F_{BC} = -20\sqrt{3}$

$10\sqrt{3} = F_{BD} - 10\sqrt{3}$

$20\sqrt{3} = F_{BD}$

At C



$$F_{CB} = -20\sqrt{3}$$

$$F_{CA} = -10\sqrt{3}$$

$$(F_{CB} + F_{CD})\frac{\sqrt{3}}{2} = 20$$

$$\Rightarrow (F_{CD} - 20\sqrt{3})\frac{\sqrt{3}}{2} = 20$$

$$\Rightarrow F_{CD}\frac{\sqrt{3}}{2} - 30 = 20$$

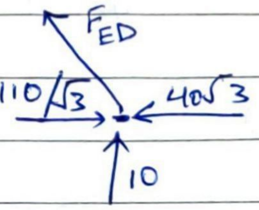
$$\Rightarrow F_{CD} = \frac{100}{\sqrt{3}}$$

$$\frac{F_{CB}}{2} + F_{CA} = \frac{F_{CD}}{2} - F_{CE}$$

$$\Rightarrow -10\sqrt{3} - 10\sqrt{3} = \frac{50}{\sqrt{3}} - F_{CE}$$

$$\Rightarrow F_{CE} = \frac{50}{\sqrt{3}} + 20\sqrt{3} = \frac{110}{\sqrt{3}} \text{ kN}$$

At E

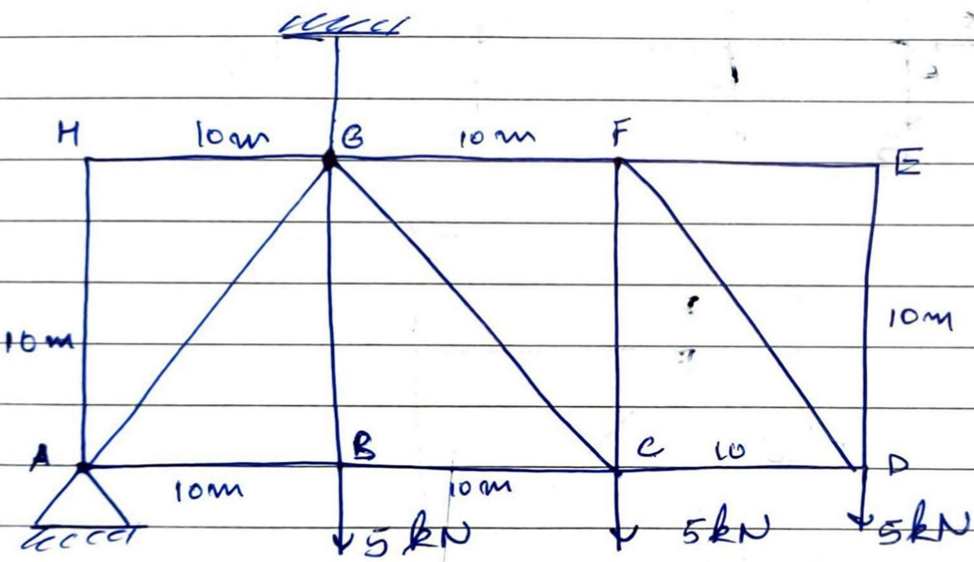


$$-F_{ED}\frac{\sqrt{3}}{2} = 10$$

$$\Rightarrow F_{ED} = \frac{-20}{\sqrt{3}}$$

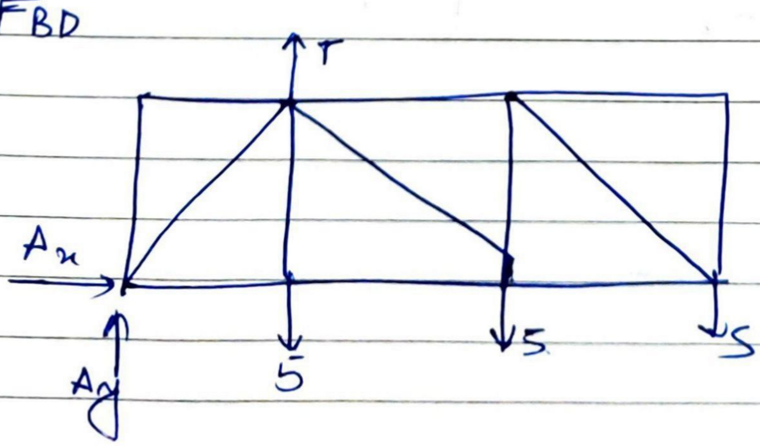
At D we have all forces.

Q→



Find F_{CG} using method of section

Ans. FBD



$$\Sigma F_x = 0 \Rightarrow A_x = 0$$

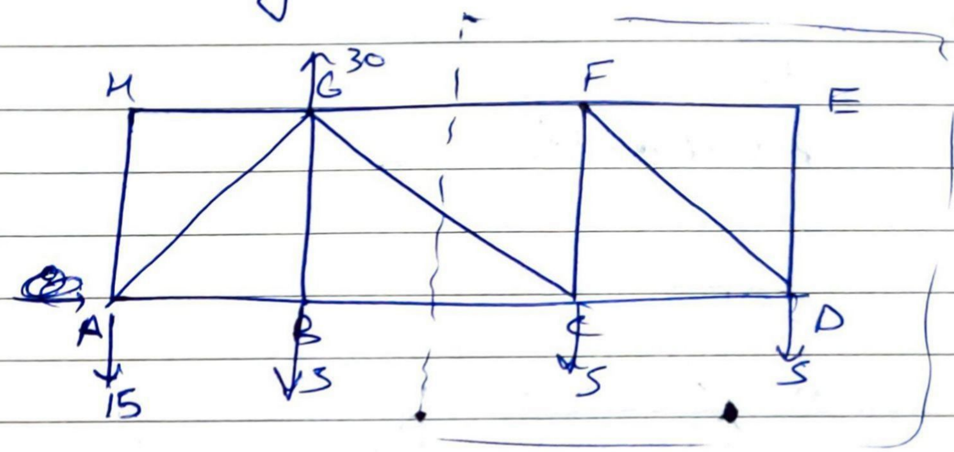
$$\Sigma F_y = 0 \Rightarrow 15 = T + A_y$$

$\sum M_A = 0 \Rightarrow 6\cancel{0}(5) = 1\cancel{0} \times T$

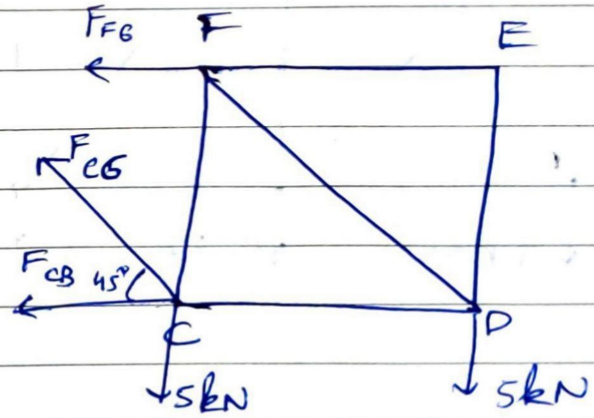
$\Rightarrow T = 30kN$

$\Rightarrow A_y + 30 = 15$

$\Rightarrow A_y = -15 kN$



section



$\sum F_y = 0$

$\Rightarrow \frac{F_{CG}}{\sqrt{2}} = 10$

$\Rightarrow F_{CG} = 10\sqrt{2} \ kN$

classmate

Date

Page

$\sum M_c = 0$

$\Rightarrow 5 \times \cancel{10} = F_{FG} \times \cancel{10}$

$\Rightarrow F_{FG} = 5\,kN$

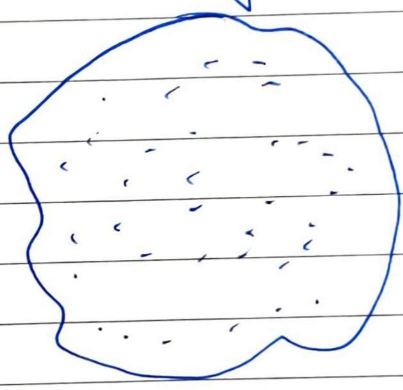
Chapter - 3
Geometric Properties ~~&~~ of masses

- Our areas of interest

a) Centroid
b) Centre of mass (COM)
c) Centre of gravity (COG)

→ **CENTRE OF GRAVITY**

- Consider a body



Say every small infinitesimal particle has weight acting on it δw & say we have n particles

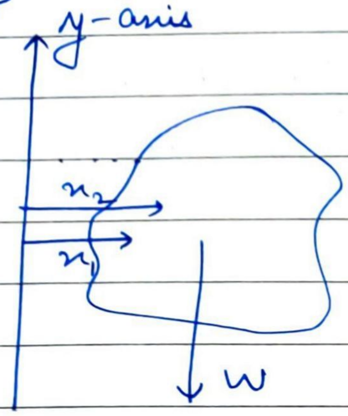
Then total weight $= n\delta w$

$= W$

$= \sum dw$

- Centre of gravity is a point at which we can assume all weight of body to be concentrated.

- To find COG



$$\therefore \Sigma M_y = \Sigma x_i \, dW$$

$$\boxed{x_{COG} = \frac{\Sigma M_y}{W} = \frac{\Sigma x_i \, dW}{W}}$$

- To find COG of an irregular body,

  Suspend it by 2 different pts and draw vertical lines through them. The pt. of intersection is COG.

Centre of Mass

- $W = mg$
  $= \rho V g$
- Centre of mass is a point where the mass of a body can be assumed to be concentrated.
- $m = \sum \delta m$
- $$\boxed{x_{COM} = \frac{\sum x\,\delta m}{\sum \delta m} = \frac{\sum x\,dm}{m}}$$

imp
- now COG is defined as

$$x_{COG} = \frac{\sum x\,dW}{\sum dW} = \frac{\sum x\,\cancel{g}(mg)}{\sum \cancel{g}(mg)}$$

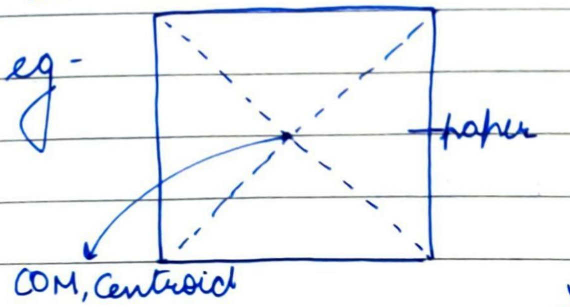
If g is constant, we can cancel it out & we get

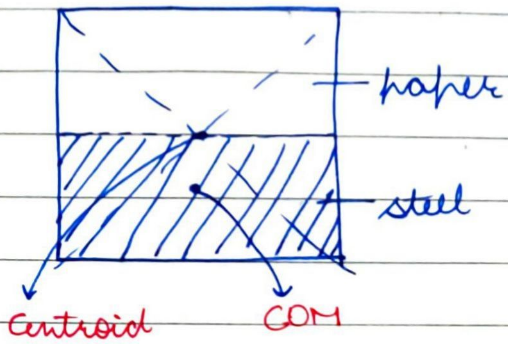
$$\boxed{x_{COG} = \frac{\sum x\,dm}{\sum dm} = x_{COM}}$$

ie COM and COG are same for constant g. ie COM is COG when g is constant.

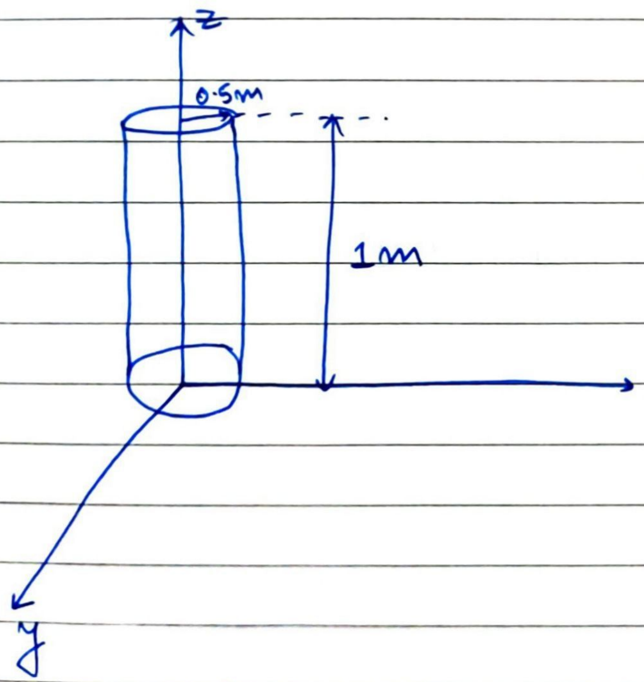
CENTROID

- It is the geometrical centre of a body
- $$\boxed{x_{centroid} = \frac{\sum x\, dV}{\sum dV}}$$
- eg-





Q→



$\rho = 200z\ kg/m^3$

Find COM

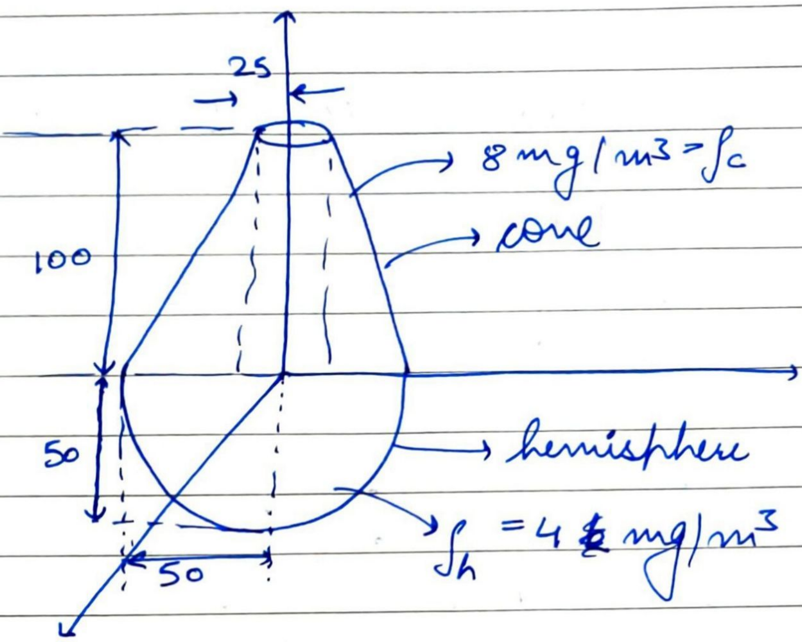
Ans:
$$z_{COM} = \frac{\int \cancel{\pi \times (0.5)^2}\, dz\, (\cancel{200}\, z^2)}{\sum \int \cancel{\pi (0.5)^2}\, \cancel{200}\, z\, dz}$$

$$= \frac{\frac{z^3}{3}\Big|_0^1}{\frac{z^2}{2}\Big|_0^1} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}\ m$$

$COM \equiv (0, 0, 2/3)\ m$

NOTE You can directly use COM formulas for known bodies like sphere, cone, etc.

Q→



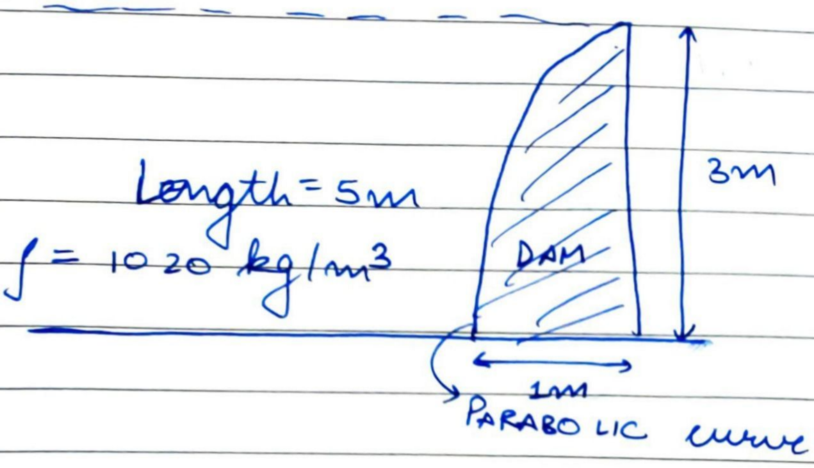
Ans: The body is symmetric about z axis

ie $x_{COM} = y_{COM} = 0$

We need z_{COM}

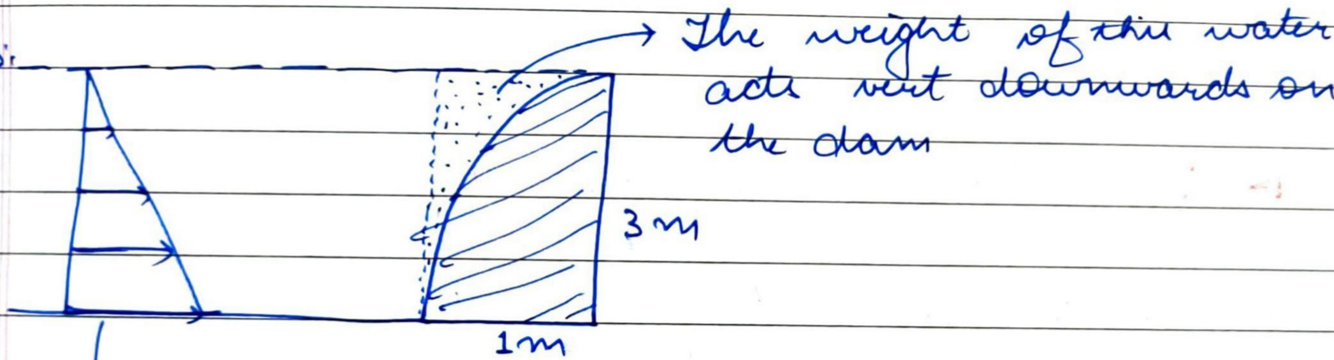
For that consider 2 bodies

Q: Given a dam



Determine equivalent force in the wall

Ans:

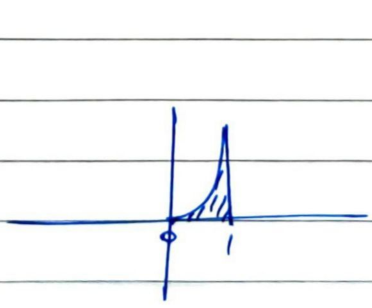


pressure increases downwards & increasing is acts as horizontal force on the wall

$y = ax^2$

$\Rightarrow 3 = a$

$\Rightarrow a = 3$



$\therefore \int_0^1 ax^2\,dx$ = Area

$\Rightarrow 3 \left.\frac{x^3}{3}\right|_0^1$

$\Rightarrow = \frac{1}{3} \times 3 = 1$

$\therefore$ Volume $= 1 \times 5 = 5\ m^3$

NOTE: Double or second moment involves a power of 2. eg → $\Sigma x\,dA$ is moment & $\Sigma x^2 dA$ is second moment.

$\therefore W = 5 \times 1020 \times g \downarrow$

For horizontal force

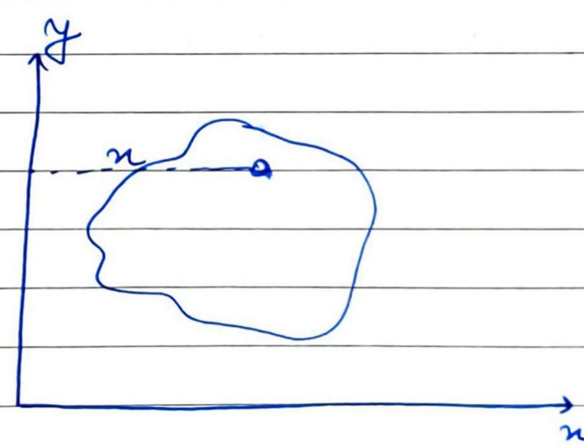
$$F = \int_0^3 \rho g h \, dh \times 5$$

$$\Rightarrow F = 5\rho g \left.\frac{h^2}{2}\right|_0^3$$

$$\Rightarrow F = 5 \times 1020 g \times \frac{9}{2}$$

AREA MOMENT OF INERTIA

- Area moment of inertia is the double (or second moment) moment about an ~~body~~ axis.

- 

$$\boxed{I_y = \sum x^2 dA}$$

$$\boxed{I_x = \sum y^2 dA}$$

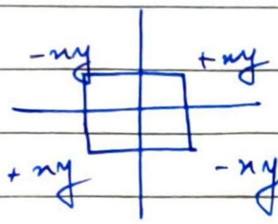
$$\boxed{I_{xy} = \sum xy \, dA}$$

- It signifies the direction with maximum stiffness.

NOTE: For a circle

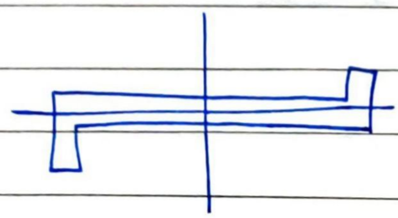
$I_x = \frac{\pi r^4}{4}$

- I_{xy} is zero for symmetric bodies eg-



even if rotated,
$I_{xy} = 0$.

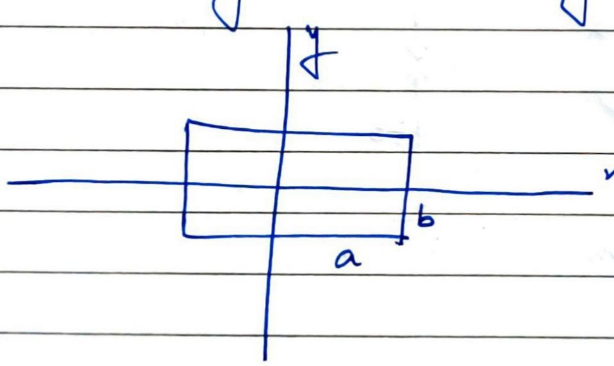
eg-



I_{xy} may not be zero for all orientatio

- Unit - m^4

- For a rectangular body

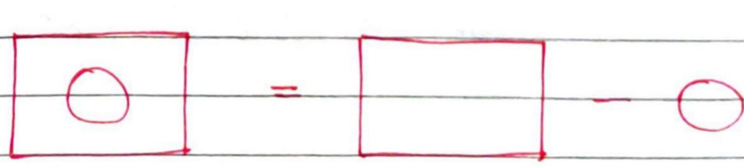


$\boxed{I_x = \frac{1}{12} ab^3}$

$\boxed{I_y = \frac{1}{12} ba^3}$

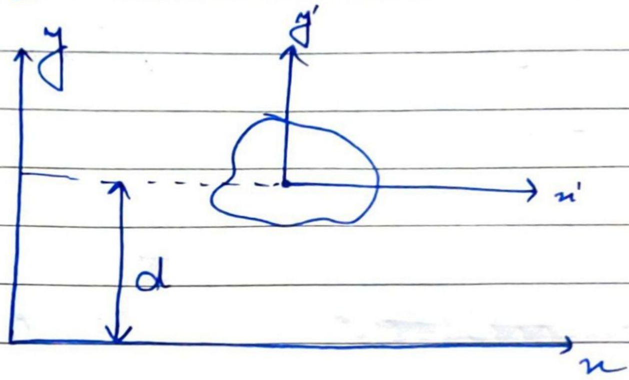
$\boxed{I_{xy} = 0}$

- For a body with a hole, we can calcula in the following manner



→ PARALLEL AXIS THEOREM →

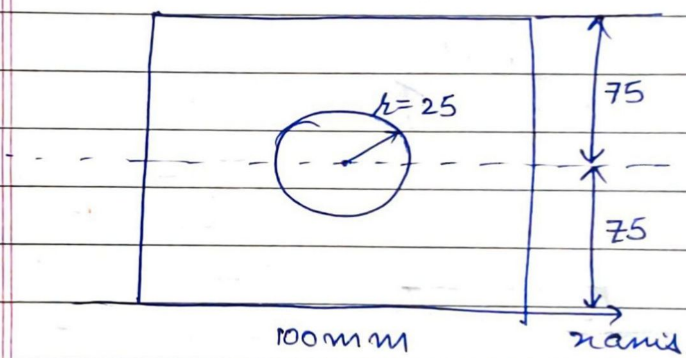
- consider



we can write,

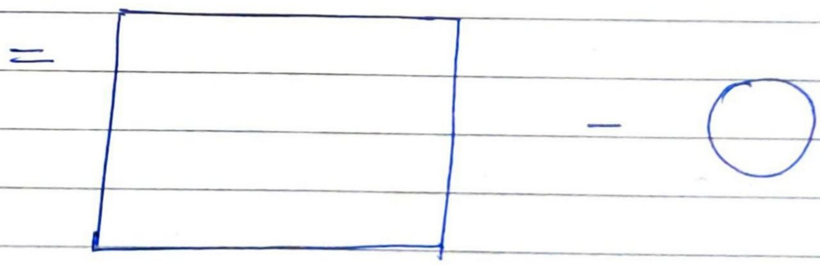
$$\boxed{I_x = I_x' + Ad^2}$$

Q→



Find I_x.

Ans:



Abt axis through centre of circle & // to x axis

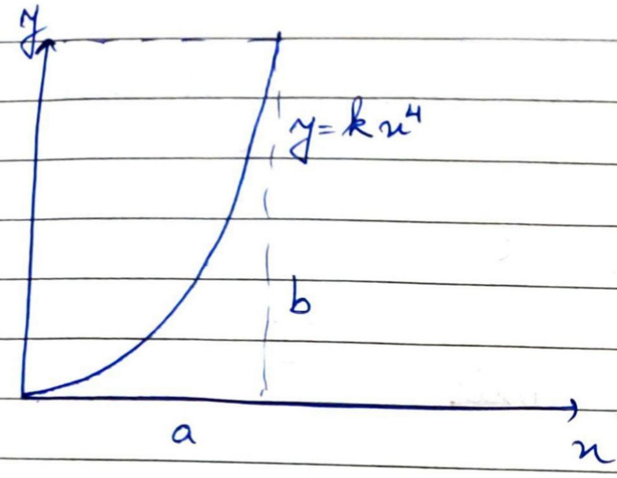
$$I = \frac{1}{12} bh^3 - \frac{\pi r^4}{4}$$

$$\Rightarrow I = \frac{1}{12} \times 50 \times (75)^3 - \frac{\pi (25)^4}{4}$$

$$\therefore I_x = \frac{1}{6}(25)^4\left[3^3 - \frac{\pi}{4}\right] + \left[150\times100 - \pi(25)^2\right](75)^2$$

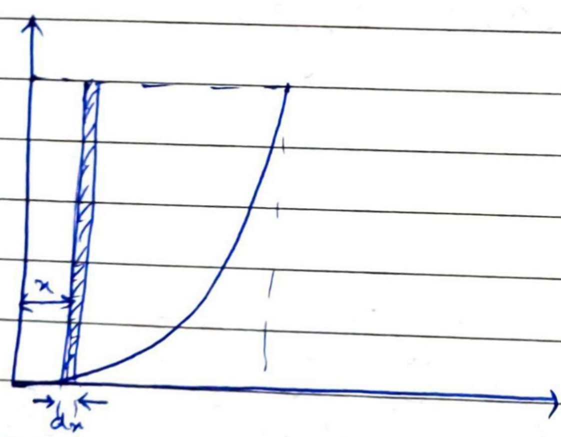
$$= \frac{1}{6}(25)^4\left[27 - \pi/4\right] + \frac{1}{6}(25)^4\left[24 - \pi\right]9$$

Q



$I_y = ?$

Ans



$$dA = (b-y)\,dx$$

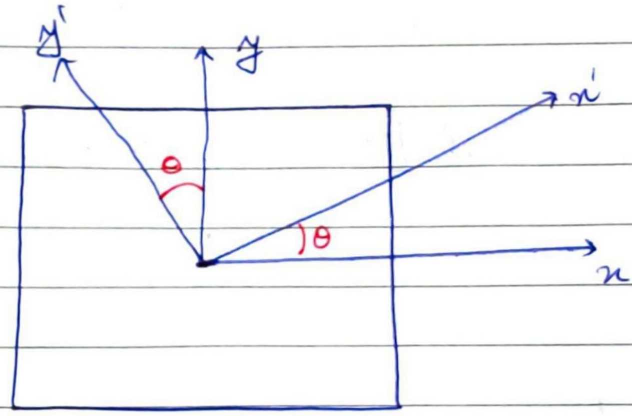
$$\therefore I_y = \int_0^a x^2 (b-y)\,dx$$

$$\Rightarrow I_y = \int_0^a x^2 (b - kx^4)\,dx$$

$$\Rightarrow I_y = \left(\frac{bx^3}{3} - \frac{kx^7}{7}\right)\Bigg|_0^a$$

$$\Rightarrow I_y = \frac{ba^3}{3} - \frac{ka^7}{7} = \frac{ba^3}{3} - \frac{ba^3}{7} = \frac{4ba^3}{21}$$

- Consider



We rotated our axis and we get new values of

$I_{x'}$ $I_{y'}$ $I_{x'y'}$

When $I_{x'y'} = 0$ then that axis is called principal axis

<u>NOTE:</u> Principal axis means $I_{xy} = 0$

*→ MOHR'S CIRCLE

- Mohr's circle is a structure (trick) used to determine the principal and to interrelate I_x, I_y & I_{xy}

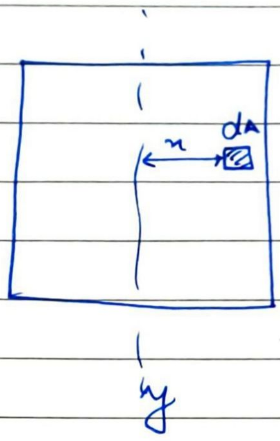
PTO

MOHR'S CIRCLE NOT WRITTEN. READ IT

- Consider we know I_x, I_y & I_{xy}

➔ RADIUS OF GYRATION →

- In calculation of Inertia, ~~xxx~~ if we assume inertia to be concentrated at a point then the coordinate (distance) of that point is called radius of gyration.

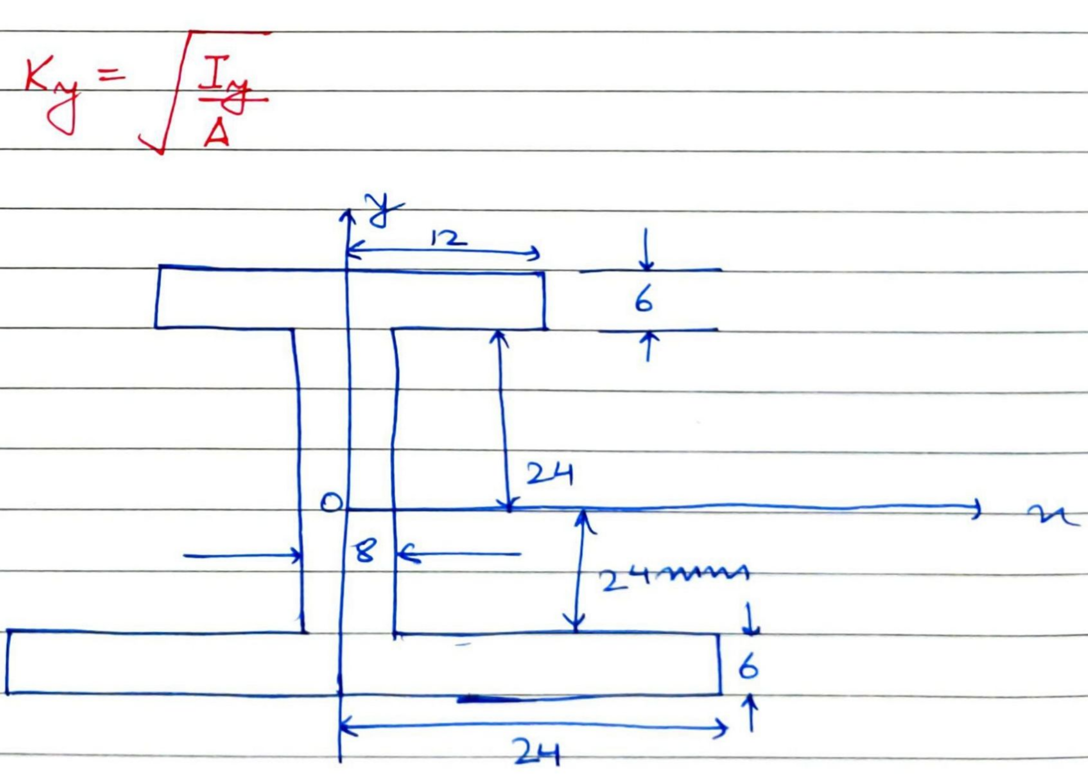


$$I_y = \sum x^2 dA$$

$$\Rightarrow I_y = \underbrace{k_y}_{\text{radius of gyration.}}{}^2 A$$

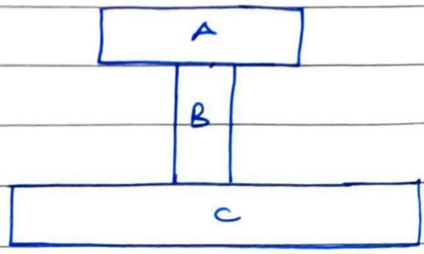
$$\therefore K_y = \sqrt{\frac{I_y}{A}}$$

Q→



$I_y = ?$; $K_y = ?$;

Ans:



$I_y = I_{yA} + I_{yB} + I_{yC}$

$\Rightarrow I_y = \frac{1}{2}\left(24 \times 4^3 + 3 \times 12^3 + 3 \times 24^3\right)$

$I_y = 64.6 \times 10^3 \, mm^3$

$K_y = 8.87 \, mm$

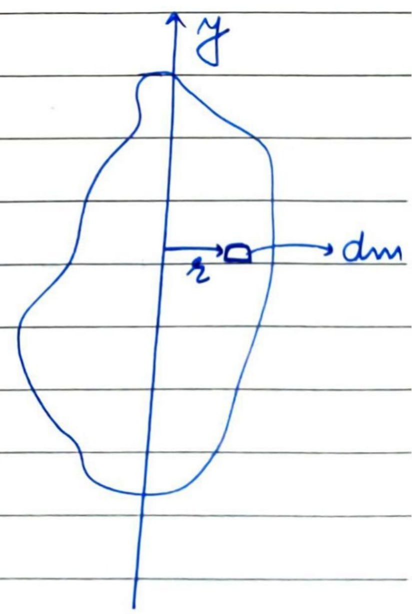
➔ MASS MOMENT OF INERTIA

- Area moment of inertia was

$$I_y = \sum y^2 dA$$

- Similarly we define a term mass moment of inertia as

$$\boxed{I_y = \sum r^2 dm}$$



$$\boxed{I_y = \sum r^2 \rho dV}$$

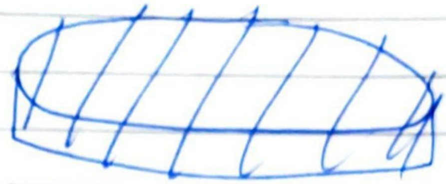
If ρ is constant,

$$\boxed{I_y = \rho \sum r^2 dV}$$

- Parallel axis theorem and radius of gyration is applicable in this case also.

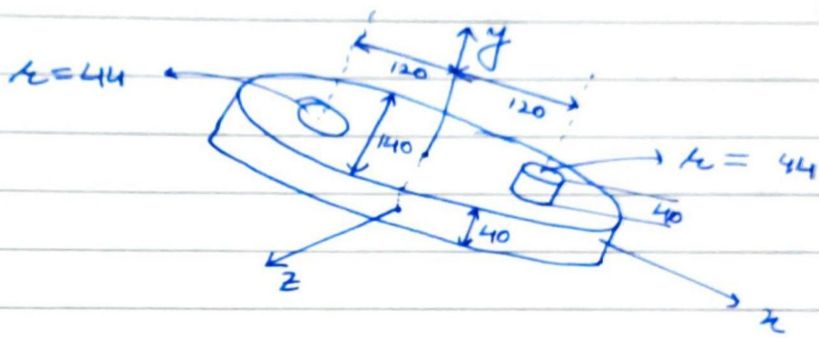
$\boxed{I = K^2 m}$ $\boxed{I_{y'} = I_y + d^2 m}$

Q→

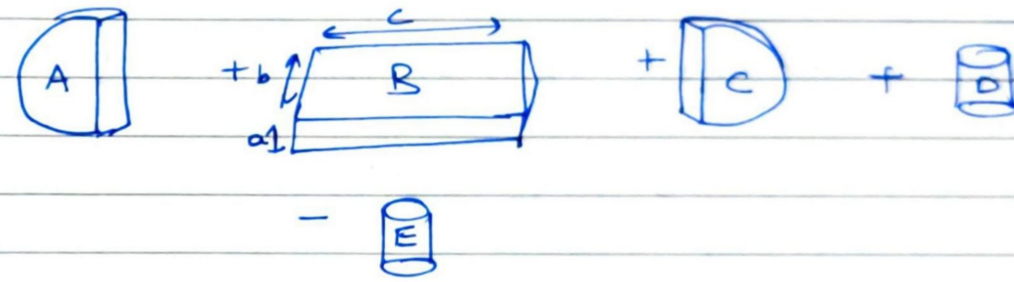


$\rho = 7850\, kg/m^3$

I_x, I_y, k_x, k_y?



Ans:



$m_a = m_c = \frac{\pi \times 70^2}{2} \times 40 \times \rho$

$m_d = m_e = \pi \times 44^2 \times 40 \times \rho$

$m_b = 140 \times 240 \times 40 \times 10^{-9} \times 7850$

$$I_y = 2\left(\frac{m_a r_a^2}{2} + m_a (120)^2\right) + m_b\left(\frac{b^2+c^2}{12}\right) + 0$$

cancellation for D & E.

Important : remember

For

I_x

$\cancel{I_{xA}}$ $\boxed{I_{xA} = \frac{m_a}{12}(3r^2 + h^2)} = I_{xC}$

$I_{xB} = \frac{m_b (a^2 + b^2)}{12}$

$I_{xD} = \frac{m_d}{12}(3r^2 + h^2) + m_d\, 40^2$

$I_{xE} = \frac{m_e}{12}(3r^2 + h^2)$

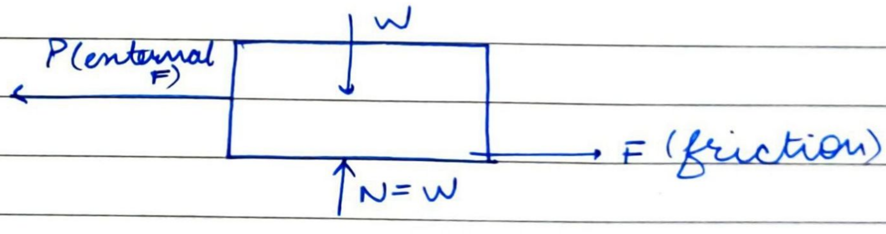
$I_x = 2I_{xA} + I_{xB} + I_{xD} - I_{xE}$

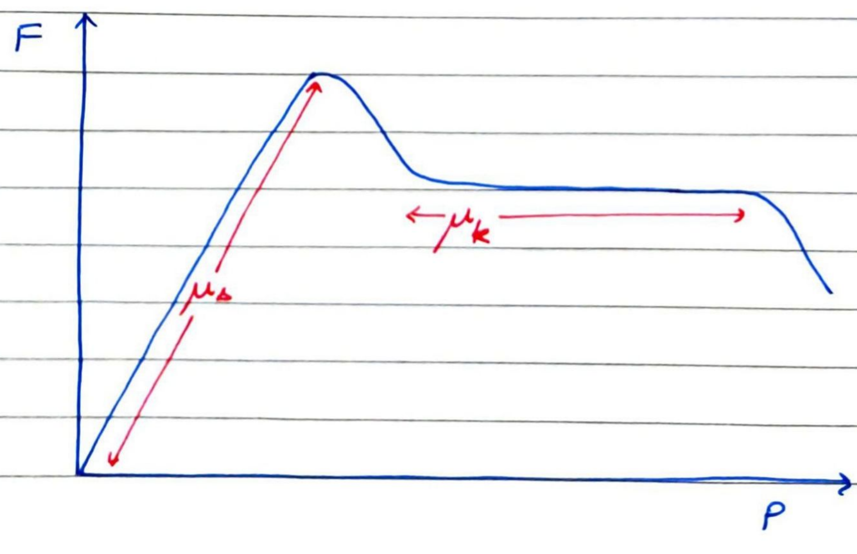
Chapter-4
Friction

→ REASON FOR FRICTION

- All surfaces whatsoever are not smooth at all.
- Uneven surfaces exert force on each other.

→ APPLIED FORCE AND FRICTION

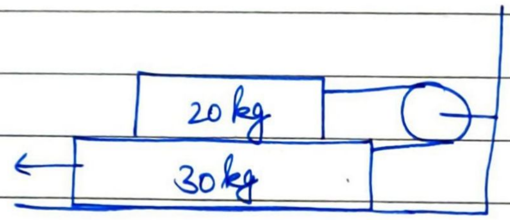
- Friction is a reaction force. It only appears as a result of some external force.
- 

- 

- μ_s (static F) is always greater than μ_k (kinetic F) since the uneven surface fitting is better in the static case.

- On increasing force to a huge extent, the uneven surface might face wear and tear, reducing friction.

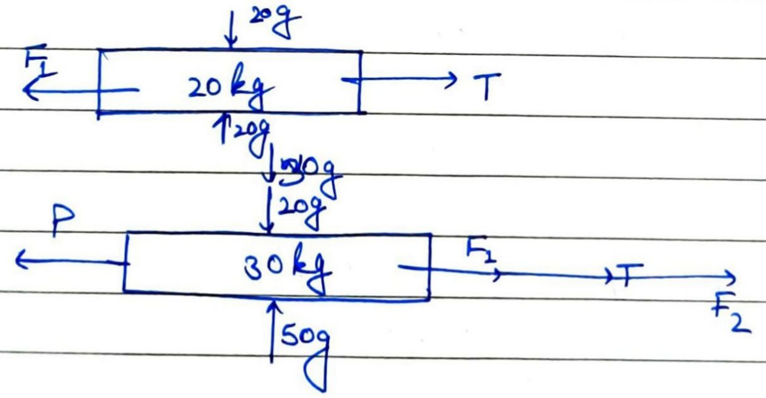
Q→



$\mu_s = 0.4$
$\mu_k = 0.3$

Find smallest force for which the system starts moving

Ans.



~~0.4×20g + T = ...~~ ~~→ 30 kg~~

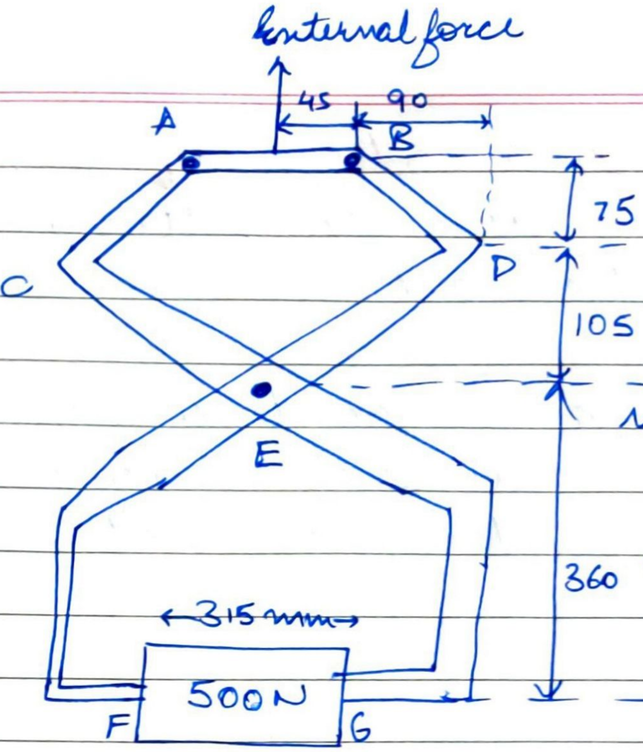
& $0.4 \times 20g = T$ → 20 kg

~~P = 0.4 × 70g ...~~

$F_1 + F_2 + T = P \Rightarrow 20g \times 0.4 + 50g \times 0.4 + 20g \times 0.4 = P$

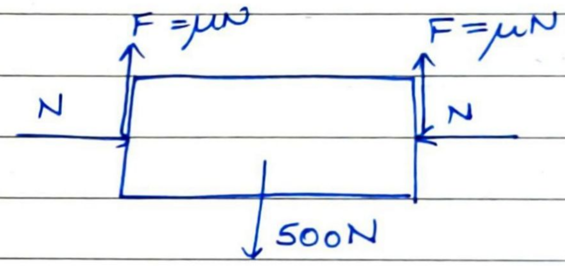
$\Rightarrow P = 35316\,N$

Q→



Find smallest value of μ so that the industrial tong can lift the body.

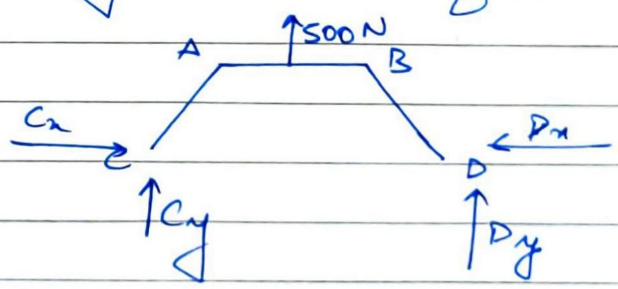
Ans:



$$\therefore\ 2\mu N = 500$$

$$\Rightarrow \mu N = 250$$

Obviously external force = 500N



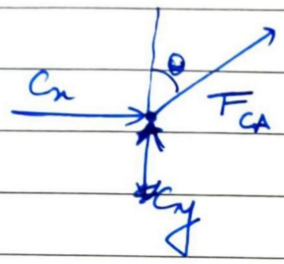
$\Sigma F_x = 0 \Rightarrow C_x = D_x$

$\Sigma F_y = 0 \Rightarrow C_y + D_y = -500N$

Due to symmetry

$C_y = D_y = -250N$

At point c



$F_{CA} \cos\theta = -C_y$

$\Rightarrow F_{CA} \cos\theta = 250N$

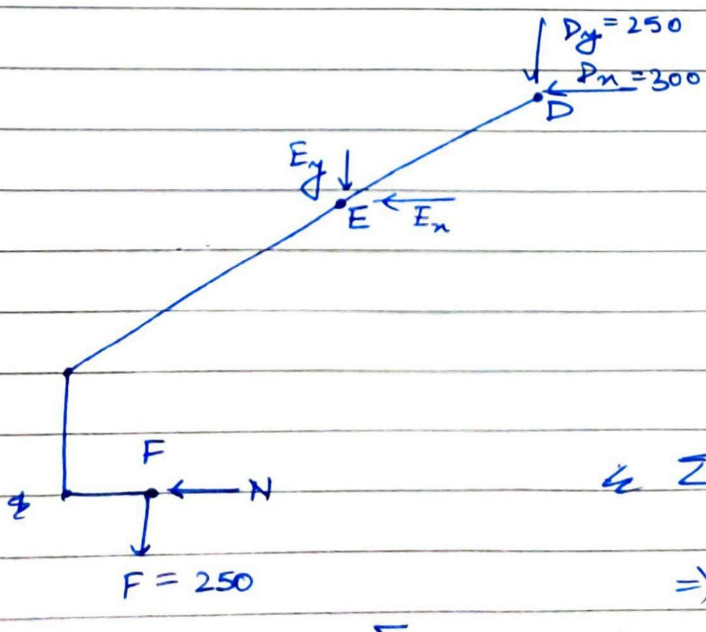
$\Rightarrow F_{CA} \frac{75}{\sqrt{90^2+75^2}} = 250N$

$F_{CA} \sin\theta = -C_x$

$\Rightarrow C_x = \frac{\cancel{250}^{50} \cancel{\sqrt{90^2+75^2}}}{\cancel{75}\,\cancel{5}} \frac{\cancel{90}^{6}}{\cancel{\sqrt{90^2+75^2}}}$

$\Rightarrow C_x = 300N = D_x$

now



& $\Sigma F_y = 0$

$\Rightarrow E_y = -500$

$\Sigma F_x = 0 \Rightarrow 300 + E_x + N = 0$

$\sum M_F = 0$

$\Rightarrow \frac{315}{2} \times E_y - \frac{315}{2} \times E_x - \left(\frac{315}{2} + 90\right) \times D_x + \left(\frac{315}{2} + 90\right) D_y$
$= 0$

$\Rightarrow 315(-250) - \frac{315 E_x}{2} + \left(\frac{495}{2}\right)(-50) = 0$

$\Rightarrow E_x \frac{315}{2} = -78750 - 12375$

$\Rightarrow E_x = -\frac{2}{315}(91125)$

$\Rightarrow E_x = -578.57 \text{ N}$

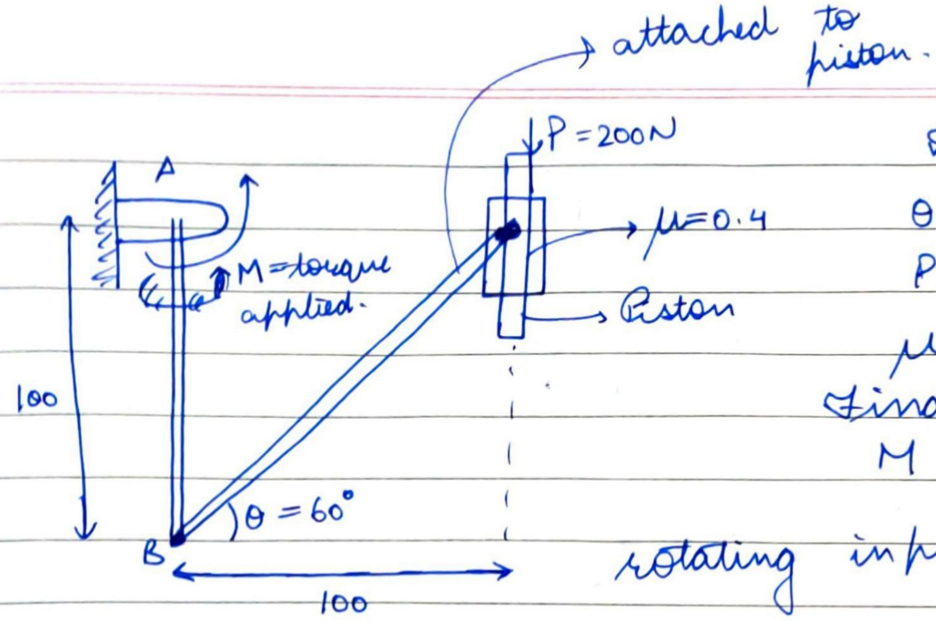
$300 - 578.57 + N = 0$

$\Rightarrow N = 278.57 \text{ N}$

Now
$\mu N = 250$

$\Rightarrow \mu = \frac{250}{278.57} = 0.8974$

Q.



At an instant.

$\theta = 60^\circ$

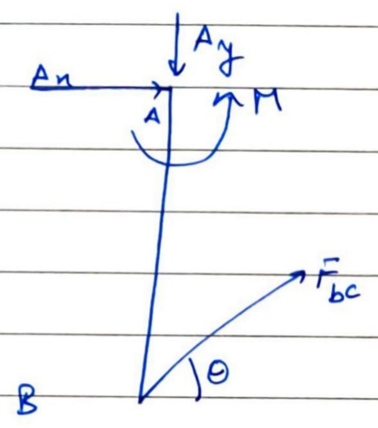
$P = 200N$

$\mu = 0.4$

Find range of M

rotating in plane.

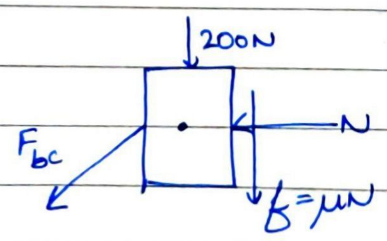
Ans. Consider AB



$\Sigma M_A = 0$

$\Rightarrow F_{bc} \cos 60^\circ \times \frac{100}{1000} + M = 0$

Consider upward motion In that case for piston



$\Sigma F_y = 0 \Rightarrow F_{bc} \frac{\sqrt{3}}{2} + \mu N + 200 = 0$

$\Sigma F_x = 0 \Rightarrow \frac{F_{bc}}{2} + N = 0$

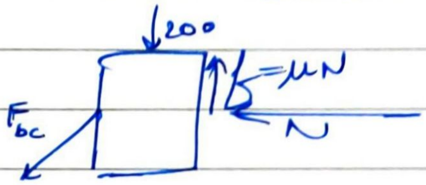
$\therefore \quad \frac{F_{bc}\sqrt{3}}{\cancel{2}} - \frac{\mu F_{bc}}{\cancel{2}} + 400 = 0$

$$\Rightarrow F_{bc} = \frac{-400}{\sqrt{3}-0.4}$$

$$\therefore M = \frac{+400}{\sqrt{3}-0.4} \frac{0.1}{.2}$$

$$\Rightarrow M = \cancel{40e} \; \frac{20}{\sqrt{3}-0.4}$$

Downward motion.



$$\mu N = 200 + \frac{F_{bc}\sqrt{3}}{2}$$

$$\& \quad \frac{F_{bc}}{2} + N = 0$$

$$\Rightarrow \frac{\mu F_{bc}}{\cancel{2}} + 200 + \frac{F_{bc}\sqrt{3}}{\cancel{2}} = 0$$

$$\Rightarrow F_{bc} = \frac{-400}{(\sqrt{3}+0.4)}$$

$$\therefore M = \frac{\overset{20}{\cancel{400}}}{\sqrt{3}+0.4} \times \frac{\cancel{0.1}}{\cancel{2}}$$

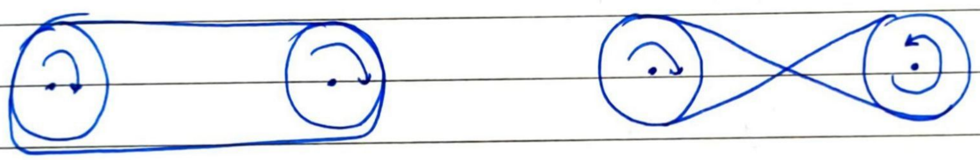
$$\Rightarrow M = \frac{20}{\sqrt{3}+0.4}$$

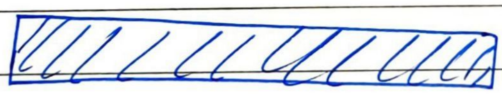
∴ M ranges as

$$\frac{20}{\sqrt{3}+0.4} \leq M \leq \frac{20}{\sqrt{3}-0.4}$$

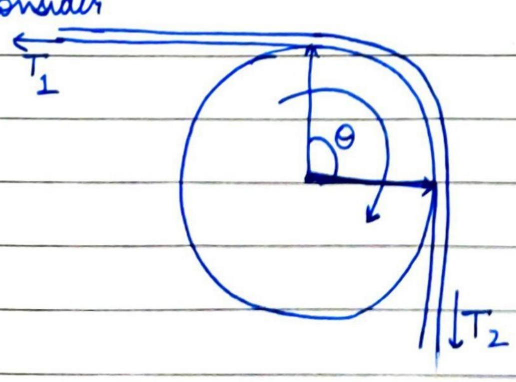
* → SPECIALIZED CASES OF FRICTION

→ 1. FLAT BELT →

- moved with the help of rollers and friction
- 
- Cross section of flat belt is ~~as~~ rectangular



- consider



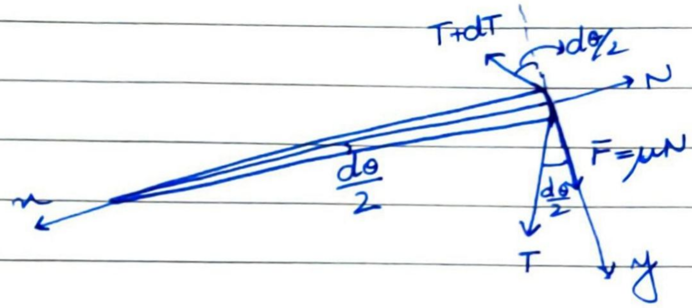
Observe the direction of rotation and then we can say that $T_1 > T_2$ since horizontal bed T_1 is being pulled.

PTO

classmate

Date ______
Page ______

Take a small element $(d\theta)$



$\sum F_x = 0$

$\Rightarrow \quad (T+dT+T)\sin\left(\frac{d\theta}{2}\right) = N$

$\Rightarrow \quad (2T+dt)\frac{d\theta}{2} = N$

$\Rightarrow \quad \cancel{2}T\,d\theta = N \qquad -(i)$

$\sum F_y = 0$

$\Rightarrow (T+dT)\cos\left(\frac{d\theta}{2}\right) = \mu N + T\cos\left(\frac{d\theta}{2}\right)$

$\Rightarrow \quad dT + \cancel{T} = \mu T\,d\theta + \cancel{T}$

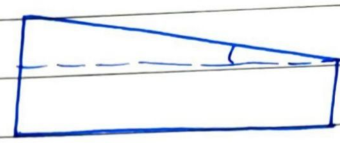
$\Rightarrow \quad \frac{dT}{T} = \mu\,d\theta$

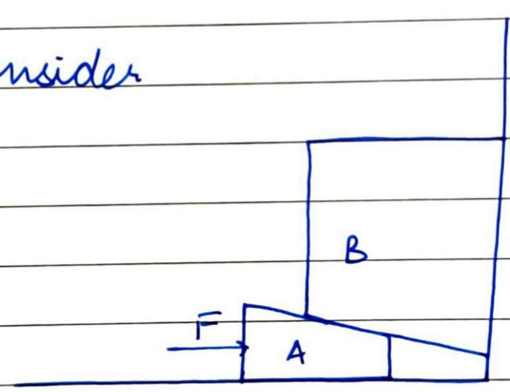
$\Rightarrow \quad \int_{T_2}^{T_1} \frac{dT}{T} = \mu\int_0^{\theta} d\theta$

$\Rightarrow \quad \boxed{\ln\left(\frac{T_1}{T_2}\right) = \mu\theta}$ → check once

$\Rightarrow$

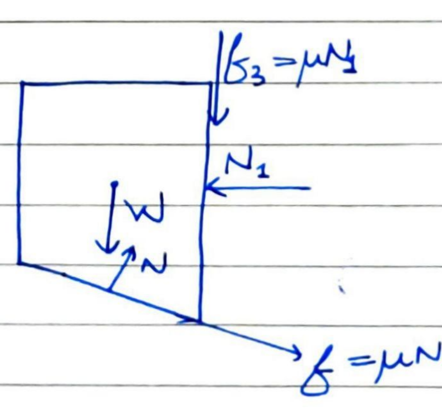
3. WEDGE

- Wedge is a trapezoidal structure
- 
- may be used to make structures stable (eg- when we put a wedge under our desk)
- may also be used to lift objects.
- Consider

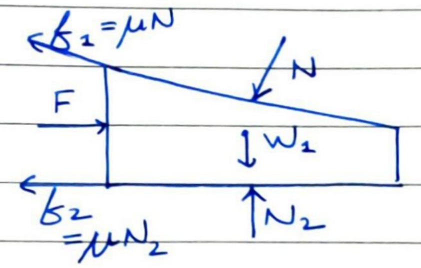


In a condition when it is just about to move

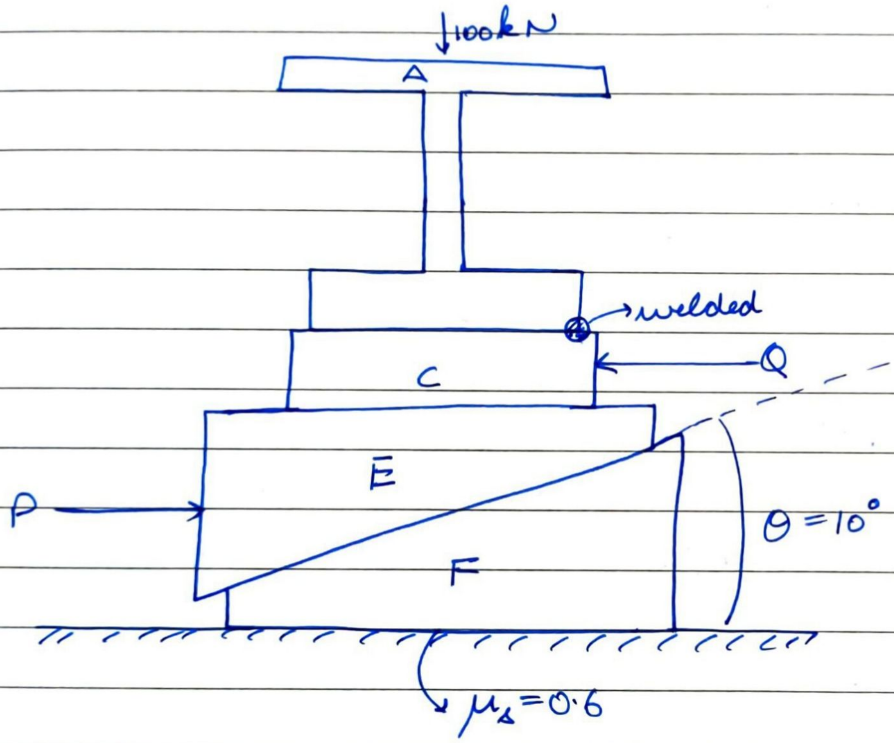
For B we draw FBD



For A, we draw



Q→



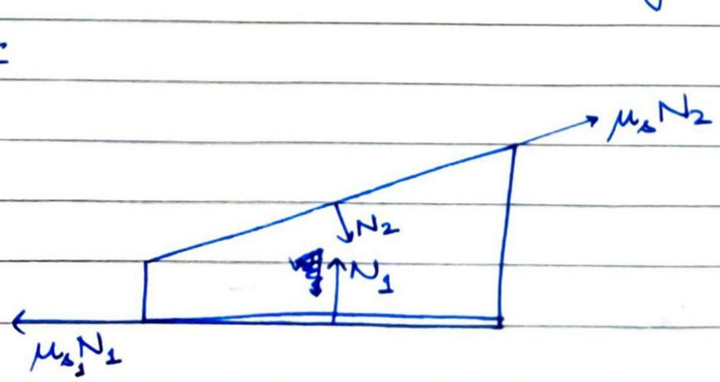
$\mu = 0.3$

consider massless bodies.

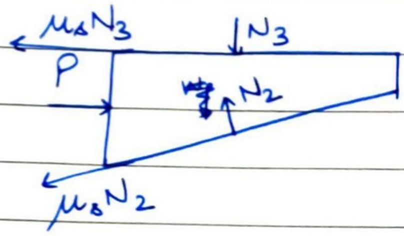
F remains stationery & A & C don't move horizontally

Find P & Q when beam is just about to move

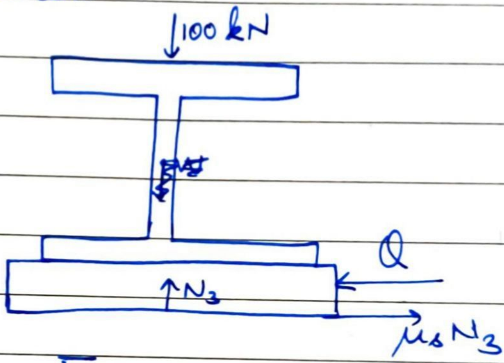
Ans: For F



For E



For C & A



$\sum F_x = 0 \qquad \sum F_y = 0$

$N_3 = 100\,kN$

$\mu_s N_3 = 30\,kN$

$\therefore Q = 30\,kN$

~~For F~~

~~$0.6 \times N_1 = N_2 \sin 10° + N_2\, 0.3 \cos 10°$~~

For E

$\sum F_x = 0 \Rightarrow 30 + N_2 \sin 10° + 0.3 N_2 \cos 10° = P$

$\sum F_y = 0 \rightarrow 100 = N_2 \cos 10° - 0.3 N_2 \sin 10°$

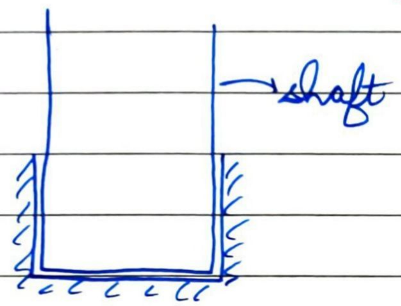
$\Rightarrow N_2 = \dfrac{100}{\cos 10° - 0.3 \sin 10°}$

$\therefore P = 30 + \dfrac{100(\sin 10° + 0.3 \cos 10°)}{\cos 10° - 0.3 \sin 10°} = 30 + \dfrac{46.91}{0.9327}$

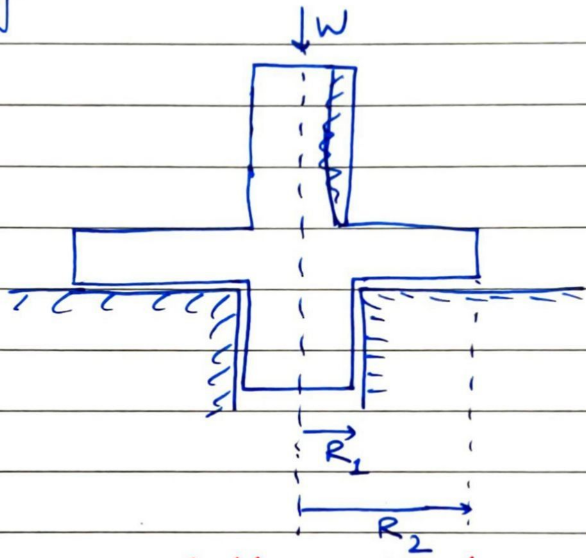
$\Rightarrow P = 80.29$ N

4. BEARING

- Bearings are support structures for shafts generally
  ↳ always rotating



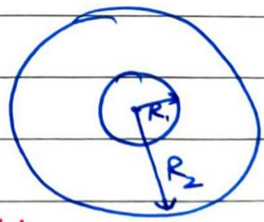
Pivot Shaft



Collar Shaft

- For collar shaft consider it is almost about to rotate.

$$P = \frac{W}{\pi(R_2^2 - R_1^2)}$$



Scalar integral of frictional force:

$dA = r\,d\theta\,dr$

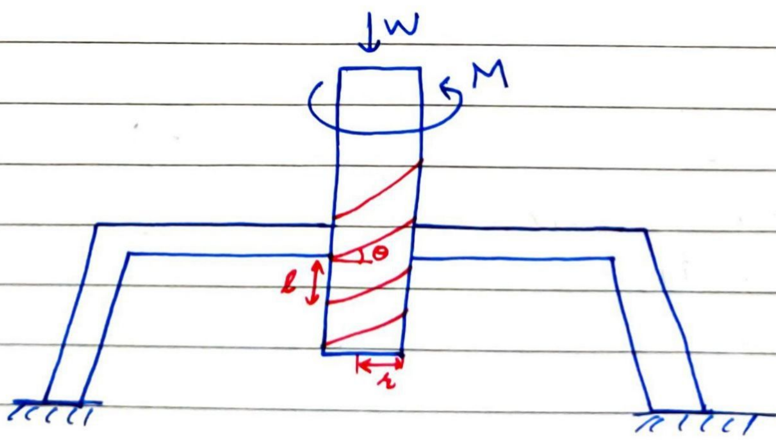
$dF = \mu P\,dA$

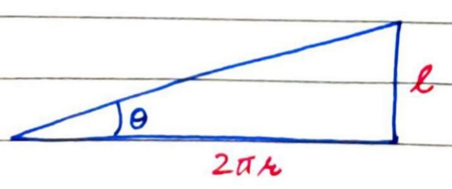
$$F = \int_{R_1}^{R_2}\int_{0}^{2\pi} \frac{\mu W}{\pi(R_2^2 - R_1^2)}\, r\,d\theta\,dr$$

$$\Rightarrow F = \int_{R_1}^{R_2} \frac{2\pi \mu W}{\pi(R_2^2 - R_1^2)}\, r\,dr \Rightarrow F = \mu W$$

- similarly we find torque using integral.

5. <u>SCREW</u> :

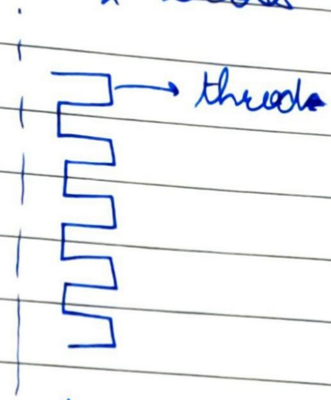
- 

- 

$$\boxed{\tan\theta = \frac{l}{2\pi r}}$$

On every rotation (ie $2\pi r$ rotation), we get a rise or fall in height of length l

- Consider ~~screw~~ square cross section of threads

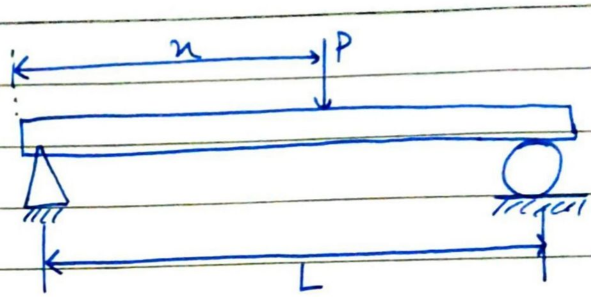


vertical cross section of

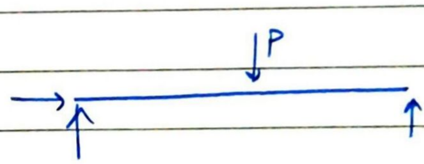
Chapter -5

→ SHEARING FORCE AND BENDING MOMENT

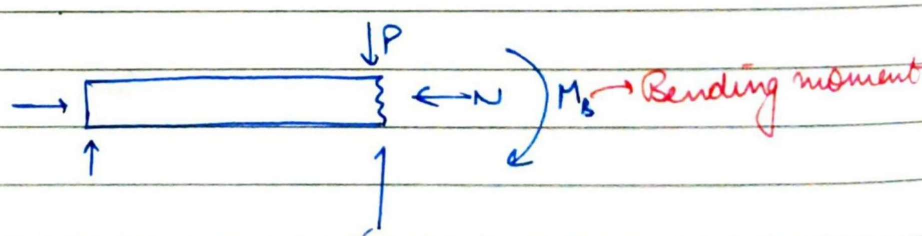
- Consider



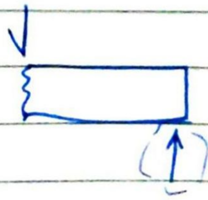
We draw FBD as



- But if we need the FBD for a section, we draw



For other section



→ This force must be balanced, ~~this~~

The resistive force responsible for balancing is called sheer force

~~NOTE: Bending moment ... ie SF × x = BM~~

→ +ve bending

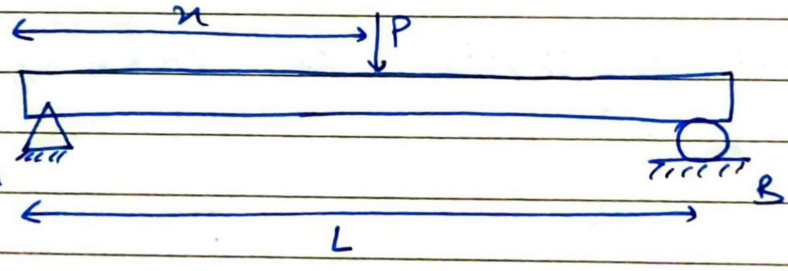
→ -ve bending

- ↑ → +ve shear force (if on left side)

↓ → -ve shear force (if on right side)

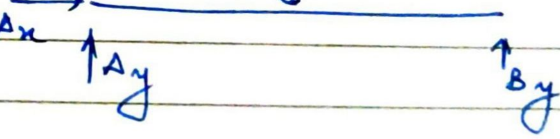
→ SHEER FORCE & BENDING MOMENT DIAGRAMS

Q→



Draw graphs for Sheer force & Bending moment

Ans. $\Sigma F_x = 0 \Rightarrow A_x = 0$

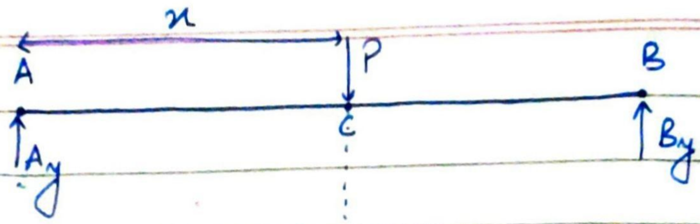


$\Sigma F_y = 0 \Rightarrow A_y + B_y = 0$

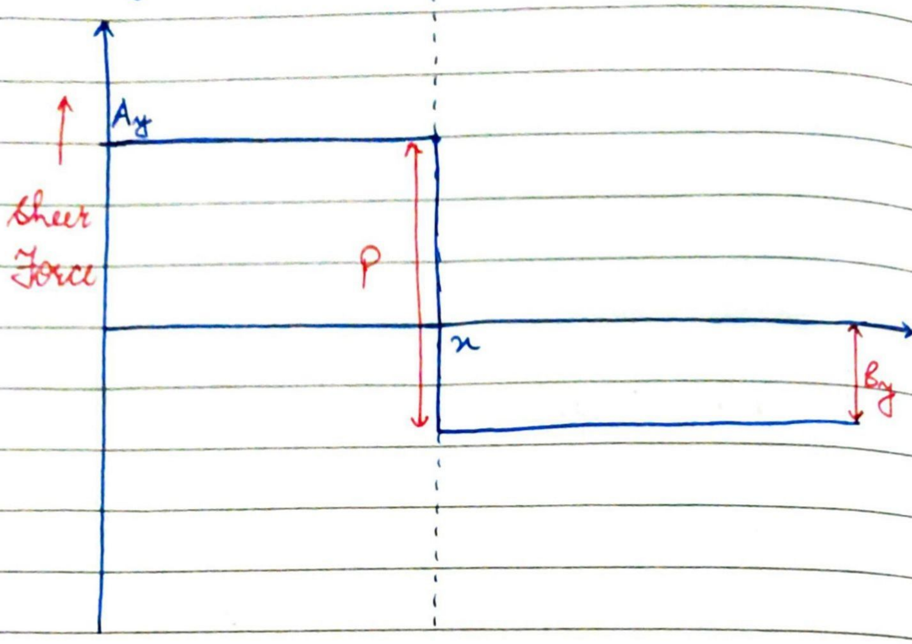
$\Sigma M_B = 0 \quad A_y L = P(L-x)$

$\Rightarrow A_y = P \frac{(L-x)}{L}$

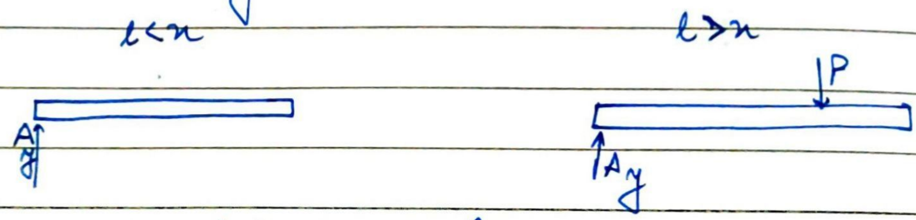
now for graphs,



SHEER FORCE DIAGRAM

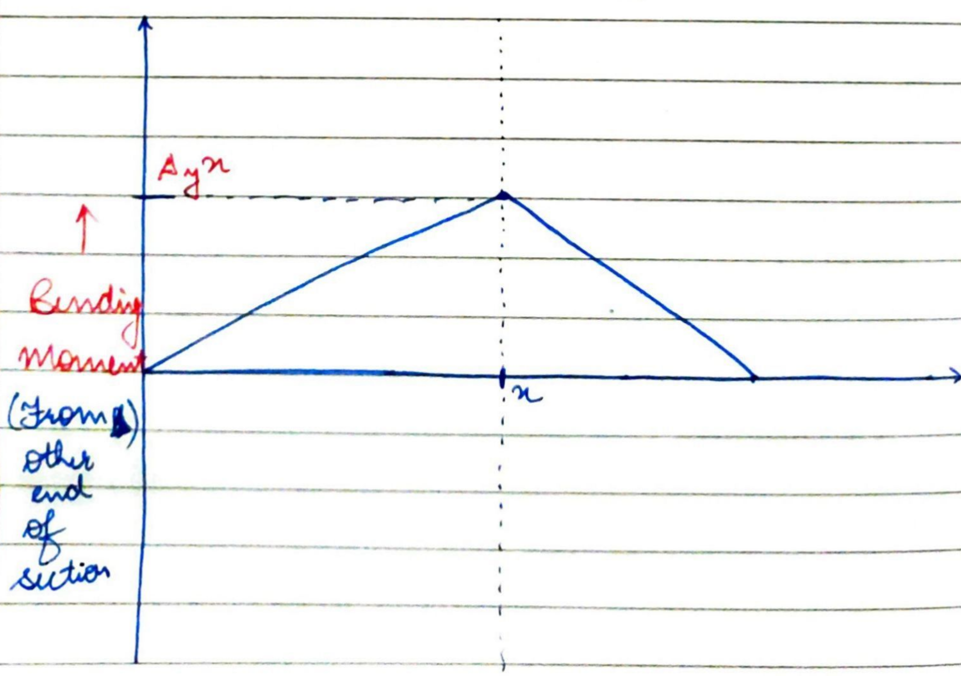


For finding sheer force, take sections like



Calculate sheer force as such.

BENDING MOMENT DIAGRAMS

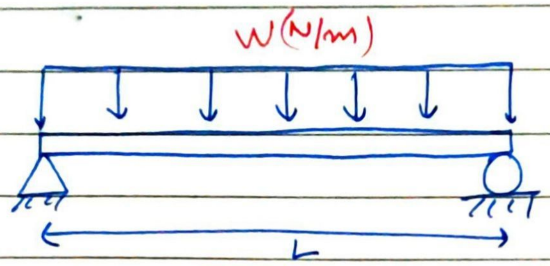


NOTE: Bending moment is maximum where shear force is zero.

you can find moments ~~using~~ by drawing diagrams of sections similarly. (moment to be taken from other end of section acc. to graph)

→ DISTRIBUTED LOAD

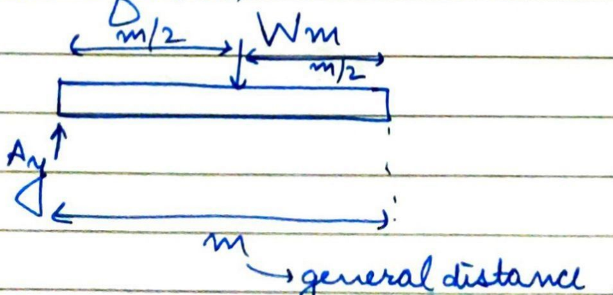
- Consider a load on a rod as force per unit length.



- In such a case, we consider the force to be concentrated at the centroid of its distribution.

- Consider the given case, we draw SFD & BMD for it.

For sheer force,

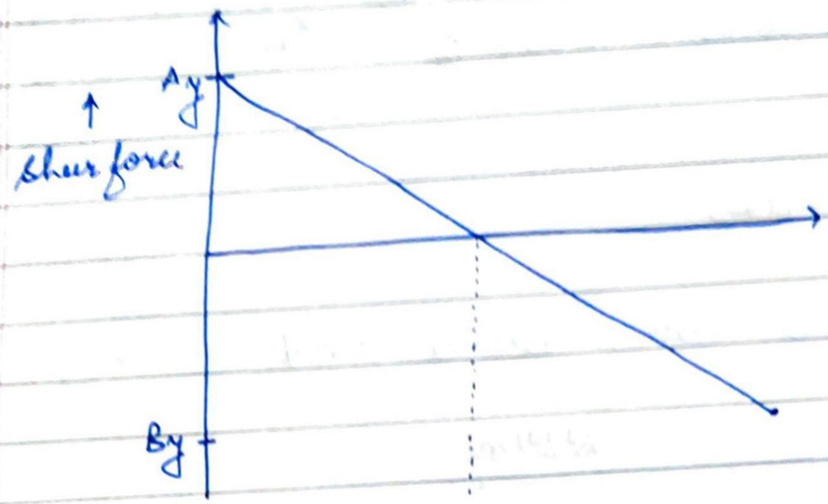


$$\therefore SF = A_y - Wm$$

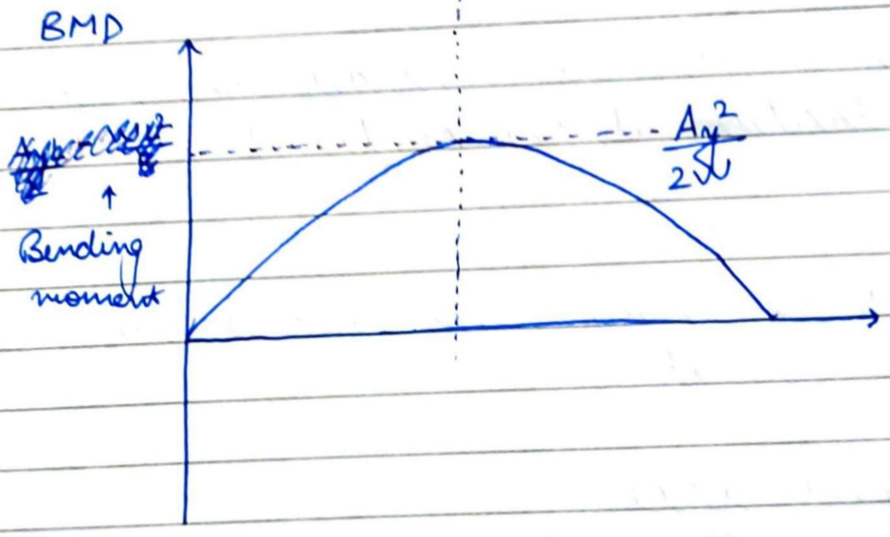
For BM

$$BM = A_y m - \frac{Wm^2}{2}$$

∴ SFD

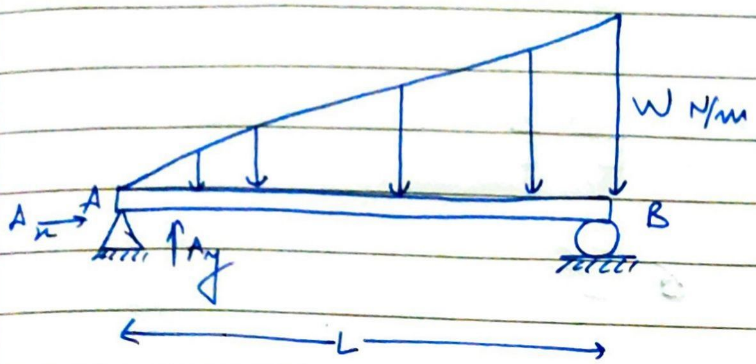


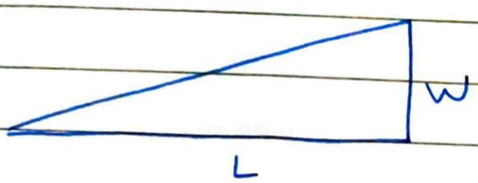
BMD



- ~~Non~~ uniformly distributed load (as a f^n)

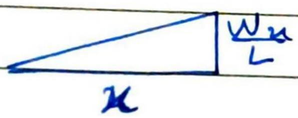
- Consider a case with force distribution as





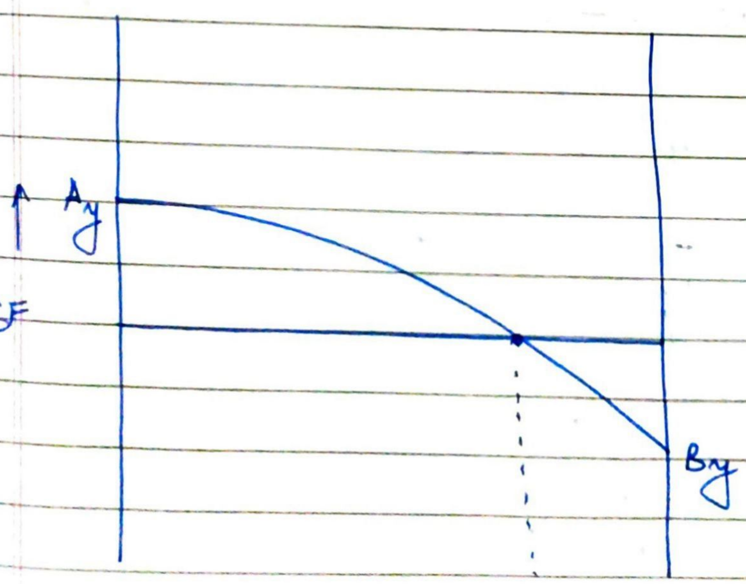
Total load $= \frac{LW}{2}$

- For shear force

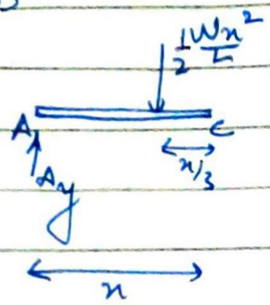


$\therefore SF = A_y - \frac{1}{2}\frac{Wx^2}{L}$

$\therefore$ SFD



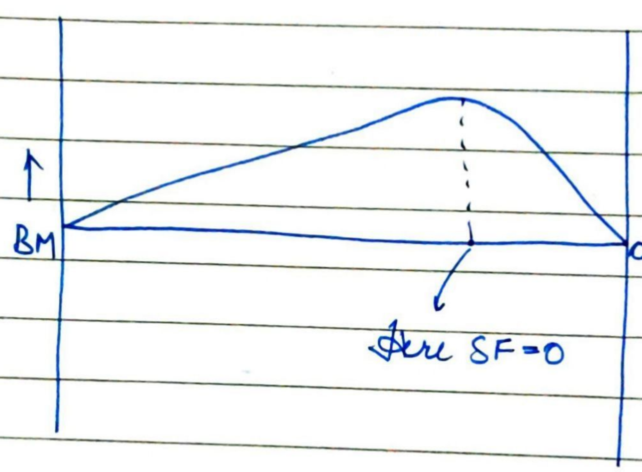
- For BMD



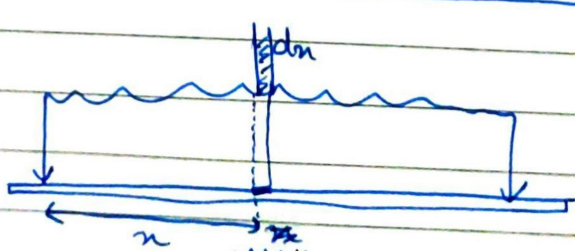
$$\Sigma M_c = A_y x - \frac{1}{2}\frac{Wx^2}{L}\frac{x}{3}$$

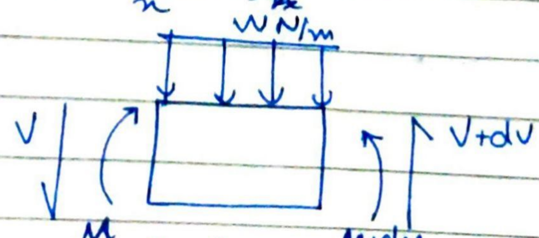
$$\therefore M_c = A_y x - \frac{1}{6}\frac{Wx^3}{L}$$

$\therefore$ BMD



→ FORCE RELATION WITH SHEER FORCE →





In element dx

$\sum F_y = 0$

$\Rightarrow V + w\,dx - (V + dV) = 0$

$$\boxed{w = \frac{dV}{dx}} \quad -(i)$$

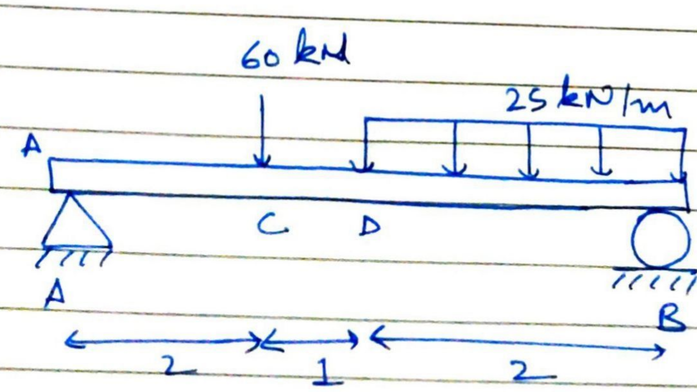
& $\sum M = 0$

$-\cancel{M} + (\cancel{M} + dM) + dx(V + dV) - w\,dx \cdot \frac{dx}{2} = 0$

$\Rightarrow dM + (dx)V = 0$

$$\Rightarrow \quad \boxed{V = -\frac{dM}{dx}} \quad -(ii)$$

Q. Find SFD, BMD and max SF & BM for



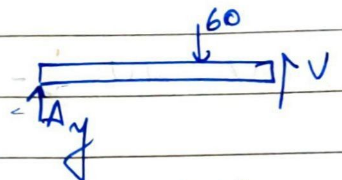
Ans. For SFD

$B_y = 64N \quad A_y = 46N$

From A to C

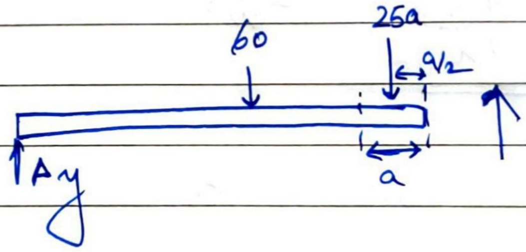
$SF = +46N$ & $BM = A_y x$

From C to D



$SF = -14N$ & $BM = A_y x - 60(x-2)$

From D to B

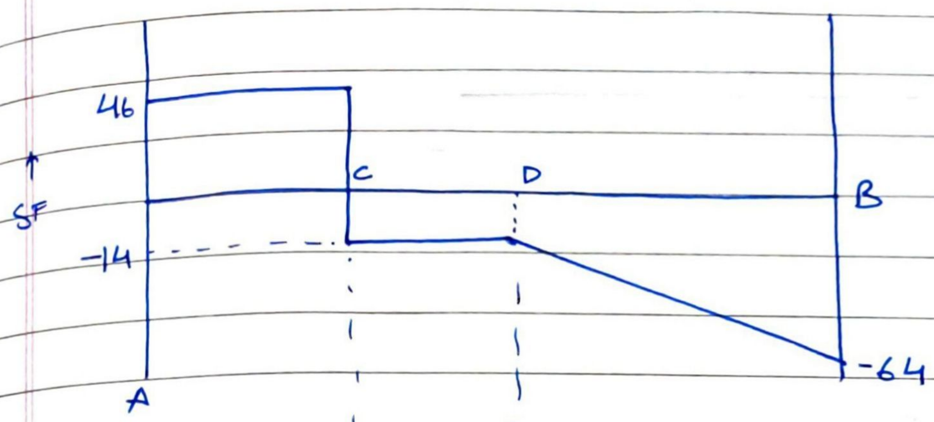


$\therefore SF = -14 - 25a$

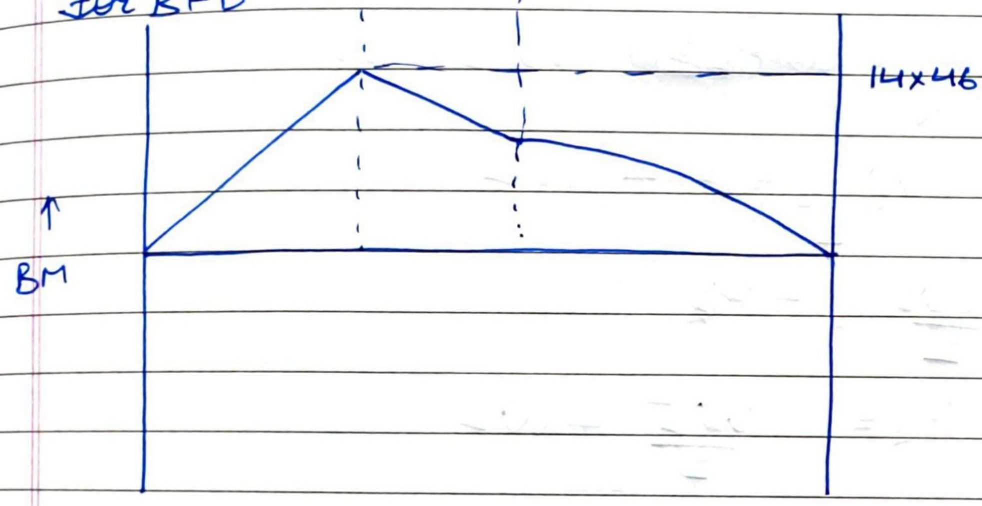
& $BM = A_y x - 60(x-2) - \frac{25(x-3)^2}{2}$

PTO

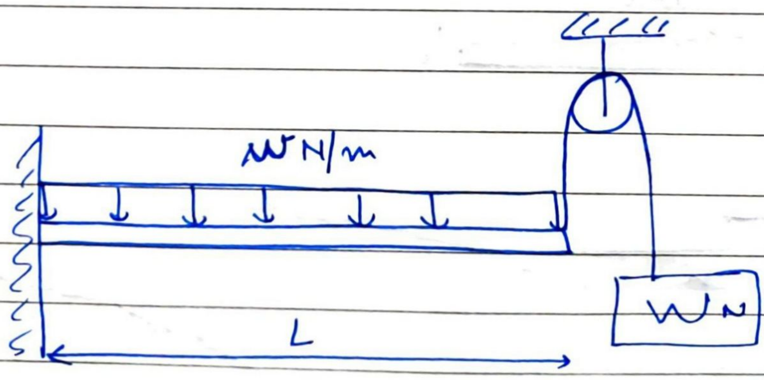
∴ SFD



For BFD

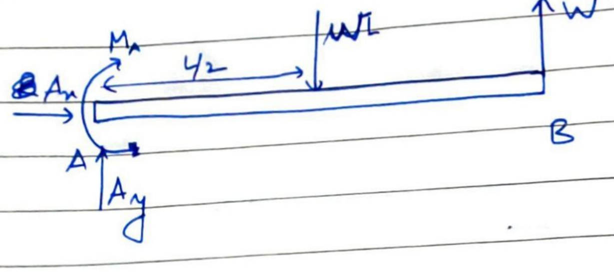


Q→



Find SF, BM and M_{max}. Also find these if W is removed (mass)

Ans:



$\therefore A_x = 0$

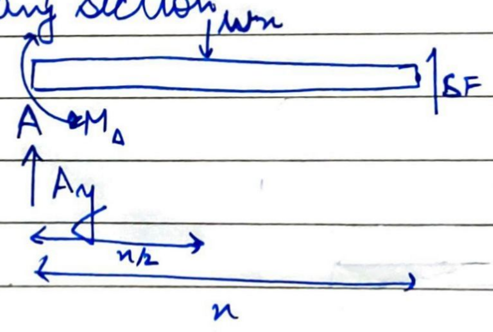
$-A_y + wL - W = 0$

$\Rightarrow A_y = wL -$

$M_A + \frac{w \times L^2}{2} - WL =$

$\Rightarrow M_A = -\frac{wL^2}{2} + WL$

For any section



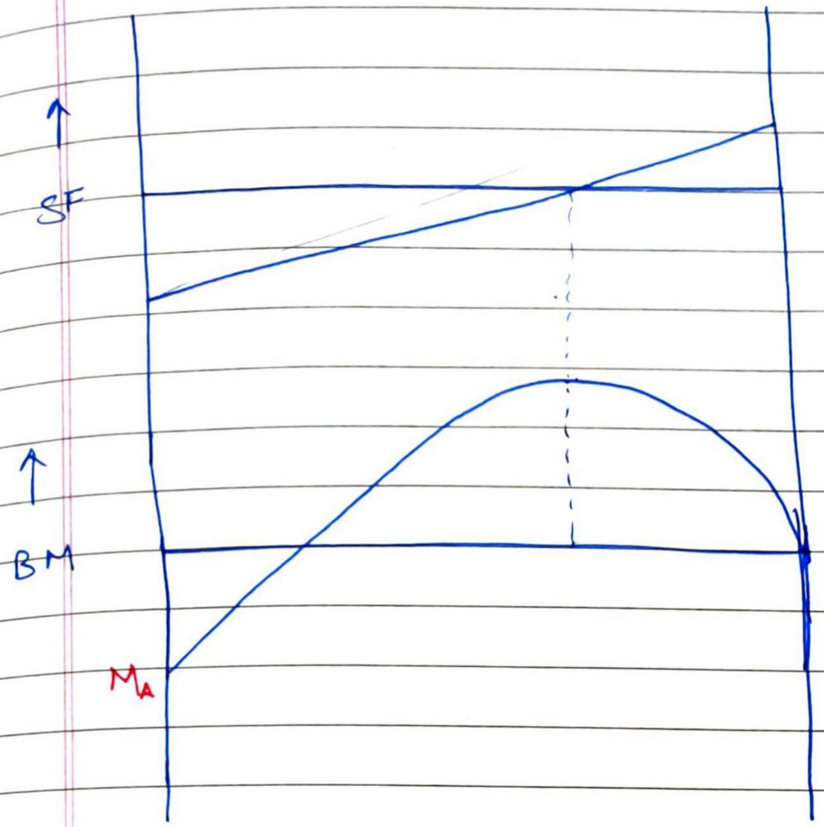
$SF + A_y = wx$

$\Rightarrow SF = wx - A_y$

&

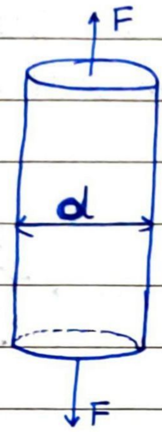
$A_y x + M_A - \frac{wx^2}{2} = B$

$$\Rightarrow BM = \cancel{A}\,(wL - W)x + WL - \frac{wL^2}{2} - \frac{wx^2}{2}$$



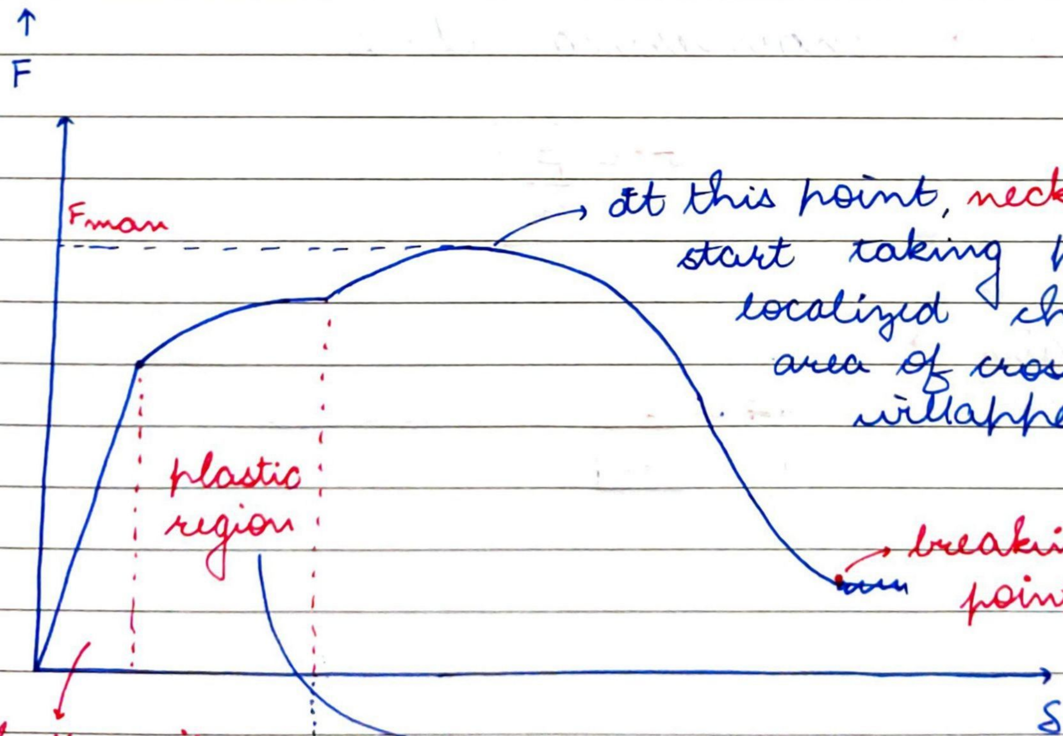
Chapter - 6
Deformable bodies (Forces & effects)

- consider a deformable body



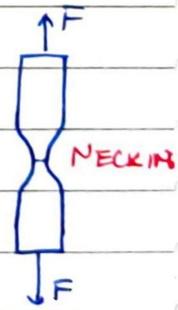
$\sigma = F/A$

$A = \frac{\pi}{4} d^2$

- 

at this point, necking will start taking place, ie localized change in area of cross section willappear

breaking point

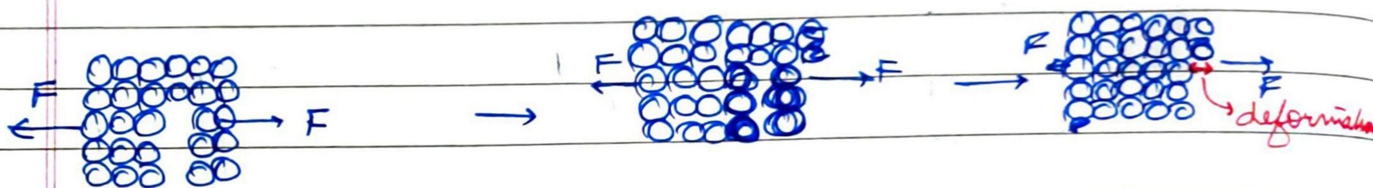


elastic region (object will deform but will come back to its original size in this region)

(It will deform but will have permanent deformation in this region)

NOTE: no solid has a perfect stack of atoms & has some defects. These defects are the reason for deformable properties like necking.

- Dislocation → · irreversible · cause of deformation · due to movements in imperfect stack



- Stress (axial) → Engineering Stress:

$$\boxed{\sigma = \frac{F}{A_0}}$$

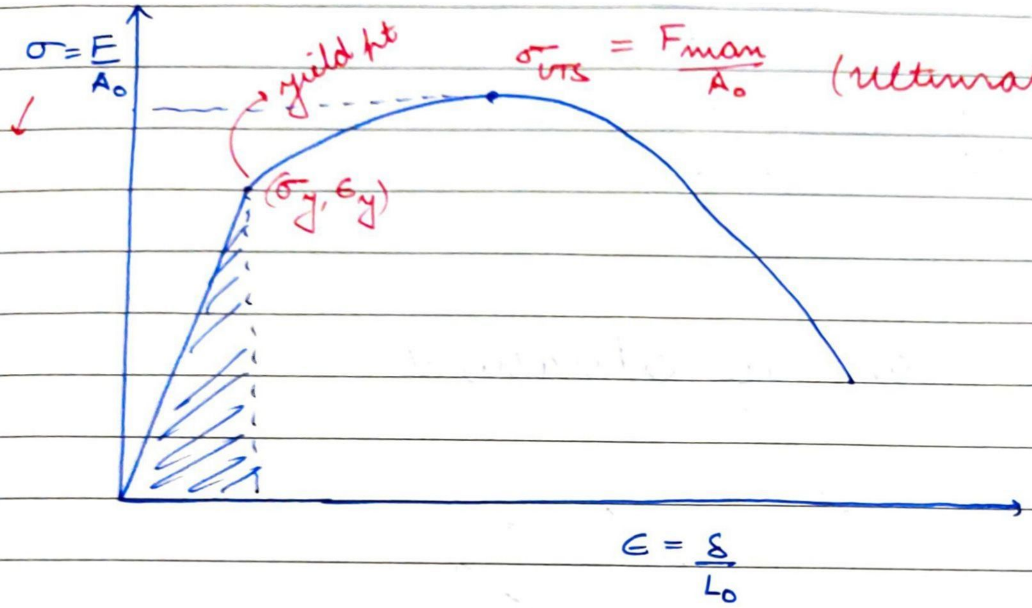
- Strain (axial) →

$$\boxed{\epsilon = \frac{\delta}{L_0}}$$

PTO

- Stress vs Strain graph

similar to F vs δ graph



when we design substances we make the σ accordingly to yield point.

$$\boxed{\sigma_{allowed} = \frac{\sigma_y}{F.S.}}$$

factor of safety

where F.S. is some factor (given as per problem)

- Consider the highlighted

Area under the curve = workdone/unit volume
reversible.

$$\therefore \boxed{\frac{W_e}{V} = \frac{1}{2}(\sigma_y \epsilon_y)}$$

now

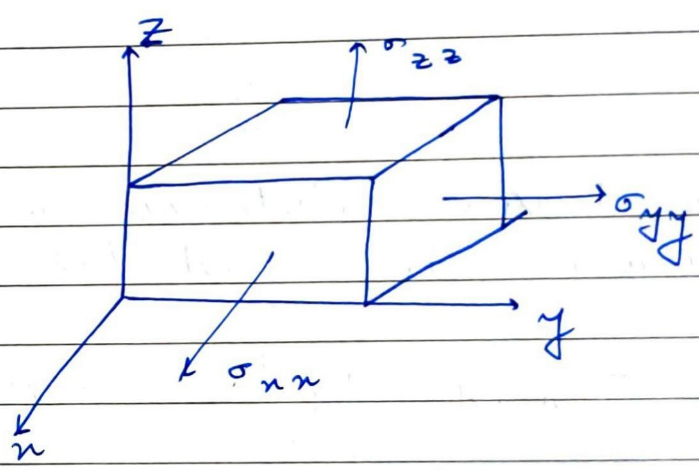
$$\sigma \propto \epsilon$$

$$\Rightarrow \boxed{\sigma = E\epsilon}$$ modulus of elasticity

$$\epsilon = \frac{\sigma}{E}$$

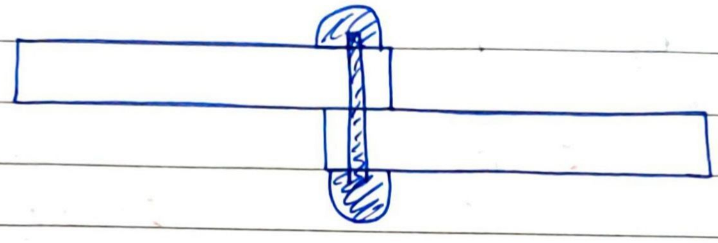
$$\therefore \boxed{\frac{W_e}{V} = \frac{1}{2}\frac{\sigma^2}{E}}$$

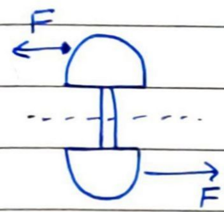
- In real life situations



we may have stress along all 3 directions
'cuz life is not so simple

→ SHEAR STRESS

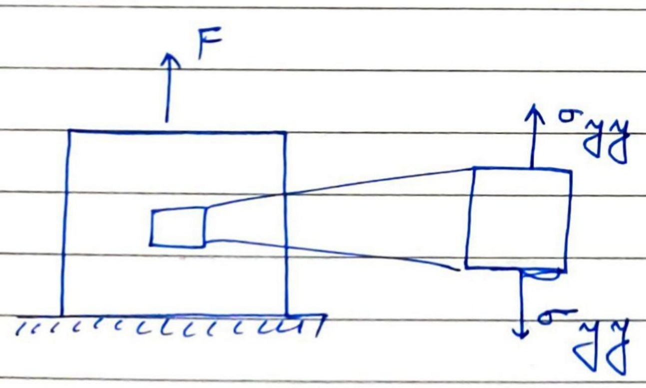




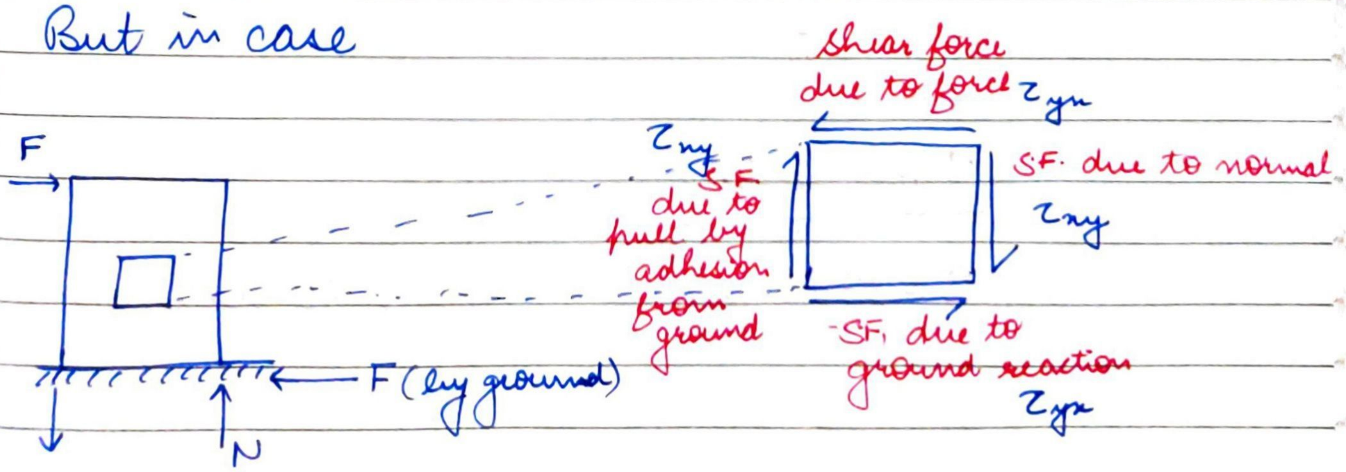
→ Shear Stress

$$\tau = \frac{F}{A_0}$$

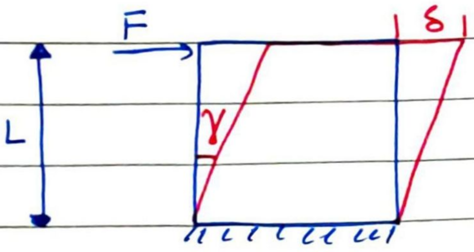
• consider a case



But in case



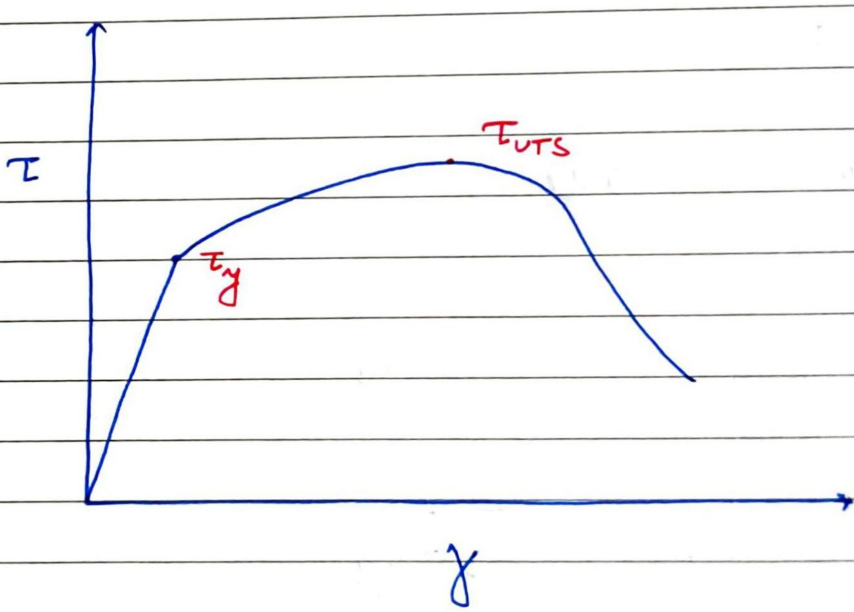
- In case of shear stress, our deformation is like



$$\tau = \frac{F}{A_0}$$

$$\gamma = \frac{\delta}{L}$$

$\underline{\tau = G\gamma}$ → shear modulus (G); valid upto yield point

- 

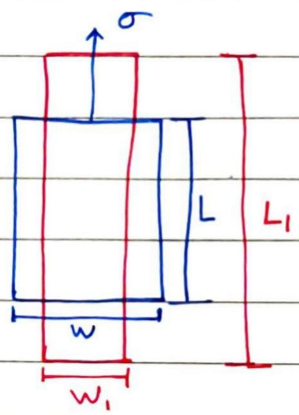
$$\boxed{\sigma_y = 2\tau_y}$$

$$\boxed{\sigma_{UTS} = 2\tau_{UTS}}$$

$$\boxed{E = 2G(1+\mu)}$$; G is shear modulus

→ POISSON'S RATIO →

- consider a body



When we stretch a body, we consider V to be constant (but it is not const) due to which we also get another component of ~~stress~~ strain due to decrease in width. Because of this, we have changes in original strain values. The correction is provided by Poisson's ratio.

$\Delta L = L_1 - L$ (+ve)

$\Delta W = W_1 - W$ (−ve)

$\epsilon_L = \dfrac{\Delta L}{L}$ $\qquad \epsilon_W = \dfrac{\Delta W}{W}$

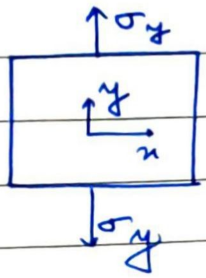
we define a ratio

$$\boxed{\mu = \frac{-\epsilon_W}{\epsilon_L}}$$ Poisson's ratio

- For steel upto yield point $\mu = 0.3$
- For all metals after yield point $\mu = 0.5$

- We can use Poisson's Ratio to find strain along different axis as in →

PTO



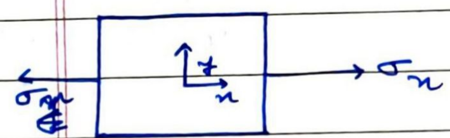
$$\epsilon_y = \frac{\sigma_y}{E}$$

$$\mu = -\frac{\epsilon_x}{\epsilon_y}$$

$$\epsilon_x = -\mu \epsilon_y$$

$$\Rightarrow \boxed{\epsilon_x = -\mu \frac{\sigma_y}{E}}$$

~~Sx~~ for case



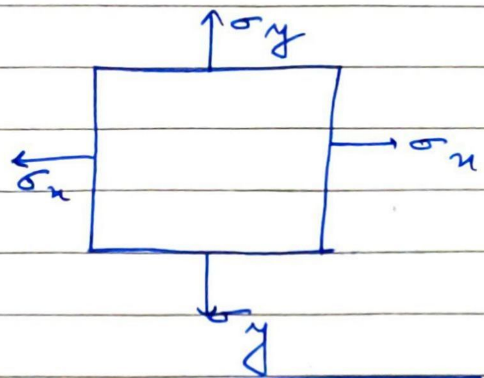
$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\mu = -\frac{\epsilon_y}{\epsilon_x}$$

$$\epsilon_y = -\mu \epsilon_x$$

$$\Rightarrow \boxed{\epsilon_y = -\mu \frac{\sigma_x}{E}}$$

- now if



$$\boxed{\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}}$$

$$\boxed{\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}}$$

- In 3-D

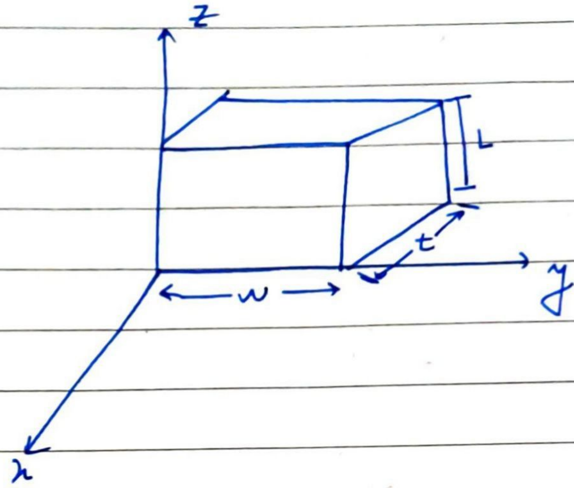
$$\boxed{\epsilon_y = \frac{\sigma_y}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_z}{E}\right)}$$

$$\boxed{\epsilon_z = \frac{\sigma_z}{E} - \mu\left(\frac{\sigma_y}{E} + \frac{\sigma_x}{E}\right)}$$

$$\boxed{\epsilon_x = \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)}$$

→ VOLUME CHANGES →

•



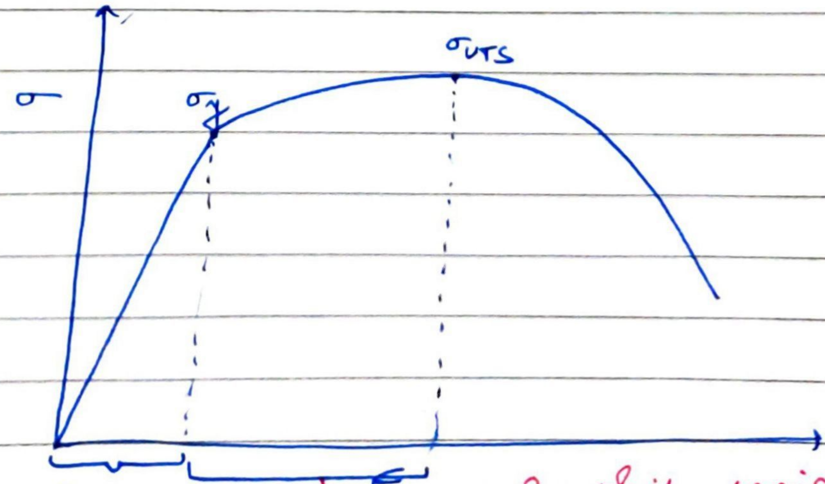
$V = LWt$

$dV = LWdt + LtdW + tWdL$

$\Rightarrow \frac{dV}{V} = \frac{dt}{t} + \frac{dW}{W} + \frac{dL}{L}$

$\Rightarrow e_v = \epsilon_x + \epsilon_y + \epsilon_z$

imp
• In graph



In elastic region V may change

In this region V remains const.

classmate

Date

Page

- $\therefore$ In elastic region

$$\boxed{\epsilon_x + \epsilon_y + \epsilon_z = 0}$$

- Between σ_y & σ_{UTS}

$$e_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \underbrace{\frac{\sigma_x + \sigma_y + \sigma_z}{E} - 2\mu\left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)}$$

from earlier derivation

$$\Rightarrow e_v = \frac{(1-2\mu)}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$\boxed{e_v = \left(\frac{1-2\mu}{E}\right)(\sigma_x + \sigma_y + \sigma_z)}$$

$$\Rightarrow \boxed{e_v = \left(\frac{1-2\mu}{E}\right)(-3P)}$$

NOTE: $\sigma_x + \sigma_y + \sigma_z = -3P$ ⟶ learn this (explaination not required)

- Bulk modulus $\boxed{K = \frac{-P}{e_v} = \frac{E}{3(1-2\mu)}}$

$$\boxed{E = 3K(1-2\mu)}$$ ⟶ Relation b/w Young's modulus & Bulk's modulus

NOTE:

$$\sigma_x$$

$$\frac{\sigma_x}{3} \qquad\qquad \frac{2\sigma_x}{3}$$

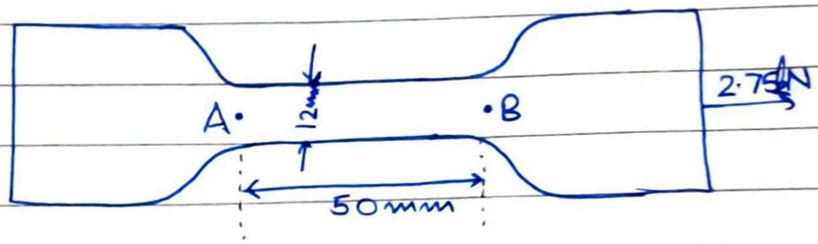
$\frac{\sigma_x}{3}$: This part of stress is used for volume change

$\frac{2\sigma_x}{3}$: This part of stress is used for plastic deformation

Q→

$E = 200\,GPa$

$\mu = 0.3$



Determine $\Delta L, \Delta W, \Delta t, \Delta A, \Delta V$

Ans: $A_0 = tw = 1.6 \times 12$

$$\sigma_x = \frac{F}{A_0} = \frac{2.75 \times 10^3}{1.6 \times 12} \; N/mm^2$$

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{\Delta L}{L}$$

$$\Delta L = \frac{\sigma_x}{E} L$$

$$\epsilon_y = \epsilon_z = -\mu \frac{\sigma_x}{E} = \frac{\Delta t}{t} = \frac{\Delta W}{W}$$

$A_0 = tW$

$$\Rightarrow A + \Delta A = (t + \Delta t)(W + \Delta W)$$

& similarly

$$V + \Delta V = (L + \Delta L)(t + \Delta t)(w + \Delta W)$$

or

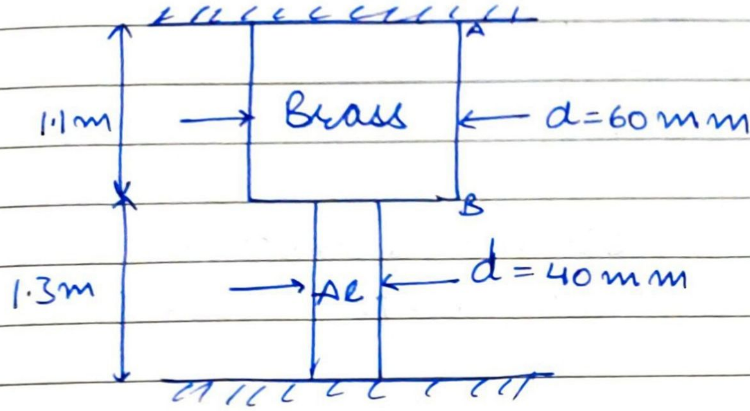
$$e_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{\sigma_x}{E} - \frac{2\mu\sigma_x}{E}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\sigma_x}{E}(1 - 2\mu)$$

NOTE: μ: Poisson's ratio may vary for different directions. This happens in case of anisotropic materials.

Ques:



$E_b = 105\,GPa$

$\alpha_b = 20.9 \times 10^{-6}/°C$

$E_a = 72\,GPa$

$\alpha_a = 23.9 \times 10^{-6}/°C$

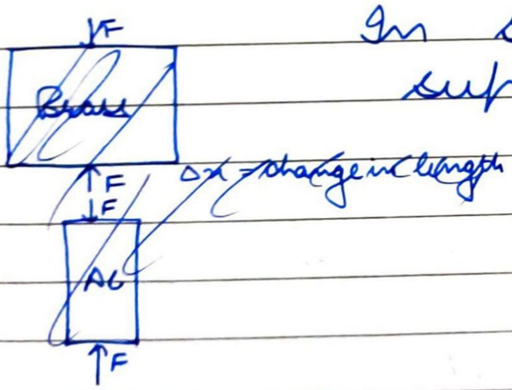
$\sigma_{AB} = ?$

$\sigma_{BC} = ?$

$\Delta T = +42°C$

Find deflection of point B.

Ans.



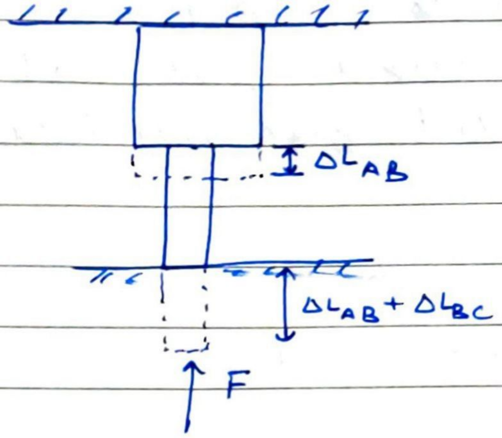
In such questions, we use superposition theorem. First we find thermal elongation.

$$\left.\Delta L_{AB}\right|_T = \alpha_b \Delta T L_b$$

$$\left.\Delta L_{BC}\right|_T = \alpha_a \Delta T L_a$$

elongation due to thermal expansion

$$\delta_T = \Delta T(\alpha_b L_b + \alpha_a L_a)$$



Now we assume a force F such that ~~to~~ the bodies come back to their position

Due to 'F'

$$\Delta L_{AB} = \frac{F \cdot L_{AB}}{A_{AB} E_{AB}}$$

$$\Delta L_{AB}|_F = \frac{1.1 \times F}{\frac{\pi}{4}(0.06)^2 \times E_b}$$

similarly

$$\Delta L_{BC}|_F = \frac{1.3 \times F}{\frac{\pi}{4}(0.04)^2 \times E_a}$$

elongation due to force

Now we can write

$$\Delta L_{AB}|_T + \Delta L_{BC}|_T = \Delta L_{AB}|_F + \Delta L_{BC}|_F$$

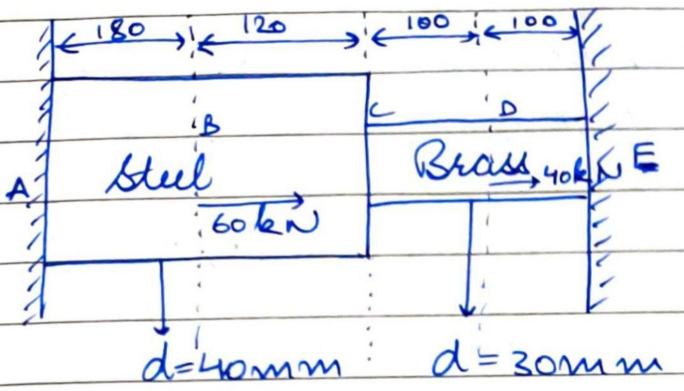
From this eq^n, we can find force F

$F = 125.62 \times 10^3$ N

& $\Delta L_{AB} = 0.5$ mm

$(\Delta L_{AB}|_T + \Delta L_{AB}|_F)$

Que:



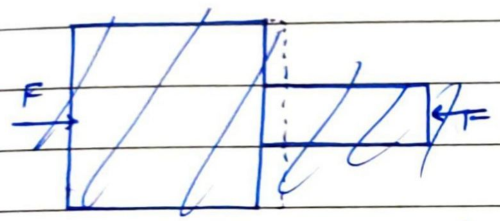
$E_s = 200\,GPa$

$E_b = 105\,GPa$

Deflection in 'c' = ?

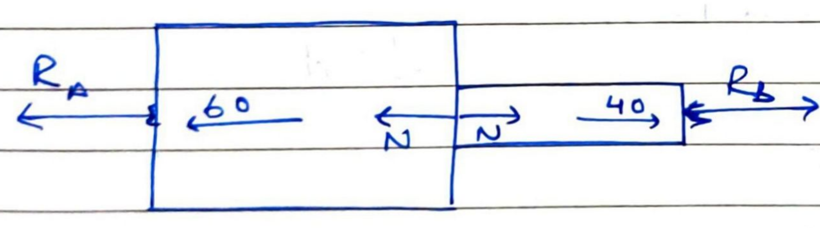
Reaction at A & E = ?

Ans:



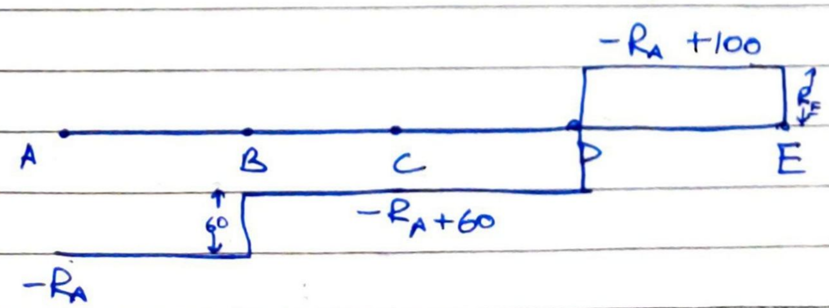
$\cancel{\Delta L_{steel} = \frac{FL_s}{A_s}} = \cancel{\frac{F \times 300}{A_s}}$

$\cancel{\Delta L_{brass} = \frac{FL_b}{A_b}} = \cancel{\frac{F \times 200}{A_b}}$



$R_A = N + 60$; $R_B = N + 40$

If we draw a force diagram,



$\delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 0$

$\Rightarrow \dfrac{-R_A L_{AB}}{A_s E_s} + \dfrac{(-R_A + 60) L_{BC}}{A_s E_s} + \dfrac{(-R_A + 60) L_{CD}}{A_b E_b}$

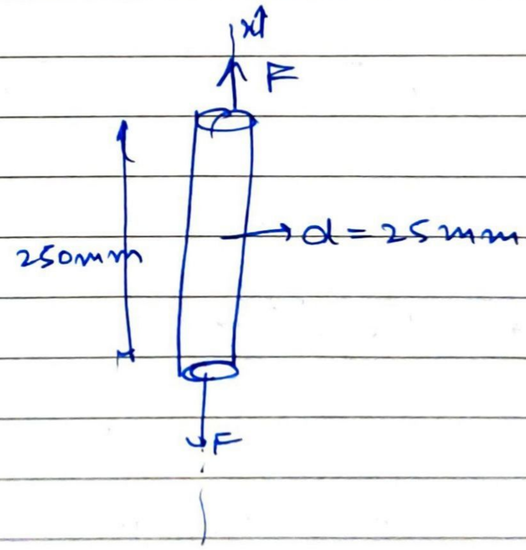
$+ \dfrac{(-R_A + 100) L_{DE}}{A_b E_b} = 0$

we can find R_A from here

& $-R_A + 60 + 40 + R_E = 0$
Find R_E from this.

Def. in C = $\delta_{AB} + \delta_{BC}$

Q →



$F = 165\ kN$
$\delta = 1.2\ mm$
Find $E = ?$
$\delta_d = ?$

$G = 26\ GPa$ → shear modulus
$\sigma_y = 440\ MPa$

Ans: $\delta = \dfrac{FL}{AE}$

$\Rightarrow 1.2 \times 10^{-3} E = \dfrac{\cancel{165}^{33} \times 10^{3} \times \cancel{250}^{10^2} \times 10^{-3}}{\frac{\pi}{4}\, 25^{2} \times 10^{-6}} = \dfrac{66 \times 10^{9}}{1.2\, \pi/4}$

$= 70\ GPa$

$$E = 2G(1+\mu)$$

$$\Rightarrow 70 = 2 \times 26 (1+\mu)$$

$$\Rightarrow \frac{35}{26} - 1 = \mu$$

$$\Rightarrow \frac{9}{26} = \mu = -\frac{\epsilon_w}{\epsilon_L}$$

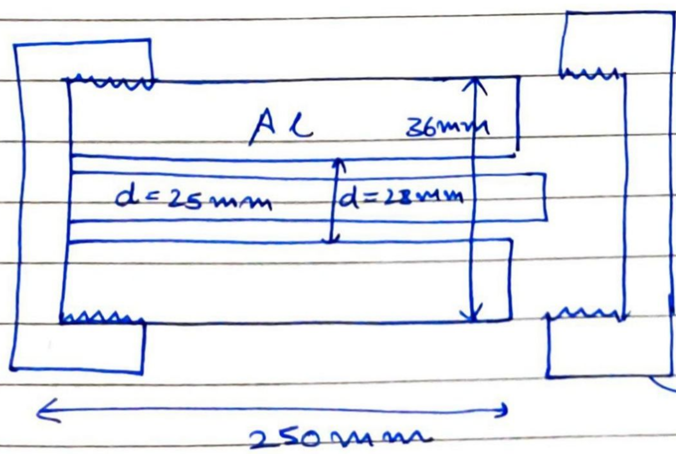
$$\epsilon_y = -\mu \frac{\sigma_x}{E}$$

$$\Rightarrow \epsilon_y = \frac{-9}{26} \quad \frac{165 \times \cancel{10^3}}{\frac{\pi}{4} \times 25^2 \times \cancel{10^{-6}}} \quad \times \frac{1}{70 \times \cancel{10^9}\ 10^3}$$

$$\Rightarrow \frac{\delta_d}{25} = \frac{-9}{26} \times \frac{165}{\frac{\pi}{4} \times 25^2} \times \frac{1}{70}$$

$$\Rightarrow \delta_d = -0.04155 \text{ mm}$$

Q.



→ Pitch of cap = 1.5 mm

If we turn by 1/4 pitch, the cap will close.

$$\delta_b = \frac{1}{4} \times pitch$$

Brass → $E_b = 105\ GPa$

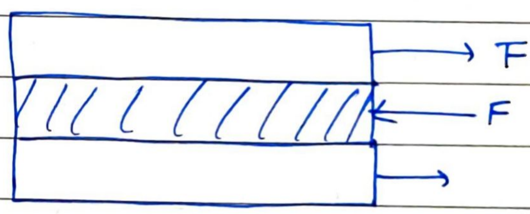
$\alpha_b = 20.9 \times 10^{-6} /^\circ C$

Al → $E_a = 70\ GPa$

$\alpha_a = 23.6 \times 10^{-6} /^\circ C$

$\Delta T = 40^\circ C$ Find stress in both

Ans: On closing, tube will extend, and rod will be compressed & forces on them will be equal



For brass

$$\{\delta_b\} = -\frac{FL}{AE}$$

$$= -\frac{F \times (251.5)}{A_b \times 105 \times 10^9}$$ → we could have taken $L = 250$ mm (very low error)

& for Al

$$\{\delta_a\} = \frac{F \times 250}{A_a \times 70 \times 10^9}$$

& we can write an eqn

$$\{\delta_a\} = \{\delta_b\} + \frac{1}{4} \times 1.5$$

If we also consider ΔT, then in a combined manner, we write

$$\delta_b = \frac{-F \times 251.5}{A_b \times 105 \times 10^9} + \alpha_b \Delta T \times (251.5)$$

$$\delta_a = \frac{F \times 250}{A_a \times 70 \times 10^9} + \alpha_a \Delta T (250)$$

& eq^n

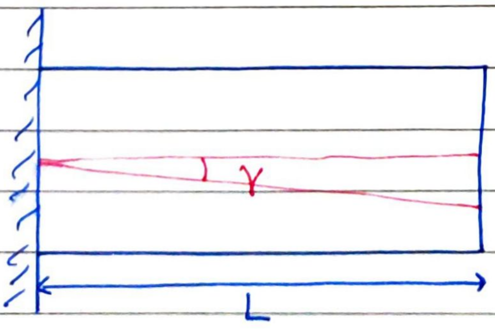
$$\delta_a = \delta_b + \frac{1}{4} \times 1.5$$

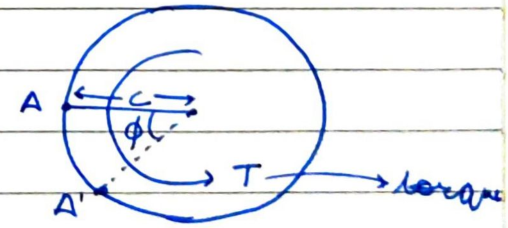
$$\therefore F\left(\frac{250}{A_a \times 70 \times 10^9} + \frac{251.5}{A_b \times 105 \times 10^9}\right) + 40\left(20.9 \times 10^{-6} \times 251.5 + 23.6 \times 10^{-6} \times 250\right)$$

$$= \frac{1}{4} \times 1.5$$

→ MORE ON SHEAR STRESS AND STRAIN

- consider





$AA' = c\phi$

shear strain $\boxed{\gamma = \frac{c\phi}{L}}$

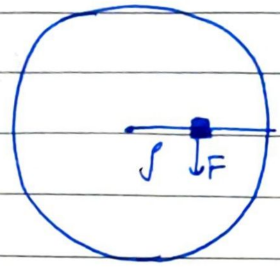
shear modulus

$\boxed{\tau = G\gamma}$

shear stress

$\left[G = 2E[1+\mu]\right]$

- To find torque consider an element at dist (radial) $= \rho$



$$dF = \tau \, dA$$

$$dT = \tau \rho \, dA$$

$$\therefore \cancel{dT} \; T = \int \tau \rho \, dA = \int_0^{2\pi} dT \quad -(i)$$

now if we check, we can write that τ is maximum at the periphery of the disc and thus we can write

$$\frac{\tau_{max}}{c} = \frac{\tau}{\rho}$$

$$\Rightarrow \frac{\tau_{max}}{c} \rho = \tau$$

$\therefore$ in (i)

$$T = \int \tau_{max} \frac{\rho^2}{c} \, dA$$

$$\Rightarrow \boxed{T = \frac{\tau_{max}}{c} J}$$

J for circular shaft (solid) $= \frac{\pi}{32} d^4$

(hollow) $= \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$

imp

$$\therefore \boxed{\frac{T}{J} = \frac{\tau}{\rho} = \frac{G\phi}{L}}$$

$\boxed{W = F.d}$ $\boxed{P = F.v}$

$\boxed{P = T.\omega}$

Q. given $P = 150\ kW$ (360 rpm) $G = 77.2\ GPa$

$\tau_{max} = 50\ MPa$, $L = 2.5\ m$ $\phi_{max} = 3^\circ$

$d = ?$

Ans.

$P = T\omega$

$\Rightarrow \frac{150 \times 10^3}{360 \times 2\pi} = T$

$\therefore \frac{T}{J} = \frac{\tau}{c}$

$c = \frac{\tau J}{T}$

$= \frac{\tau_{max} \times \frac{\pi}{32}(2c)^4}{T}$

classmate

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Q→ $L = 2.5\,m$; $d = 30\,mm$; $f = 30\,Hz$; $P_{max} = ?$
$G = 77.2\,GPa$ $\tau_{allowed} = 50\,MPa$; $\phi_{max} = 7.5^\circ$

For a shaft

Ans. $\frac{T}{J} = \frac{\tau}{\rho} = \frac{G\phi}{L}$

$J = \frac{\pi}{32} d^4$

For $\frac{T}{J} = \frac{\tau}{\rho}$

$T = \frac{2\tau}{d} \times \frac{\pi}{32} d^{3}$

$\Rightarrow T = 50 \times 10^6 \times \frac{\pi}{16} d^3$

$= 265.07\ N\text{-}m$

For

$\frac{T}{J} = \frac{G\phi}{L}$

$\Rightarrow T = \frac{77.2 \times \cancel{7.5}^{3}}{\cancel{2.5}} \times \frac{\pi}{180} \times \frac{\pi}{32} d^4$

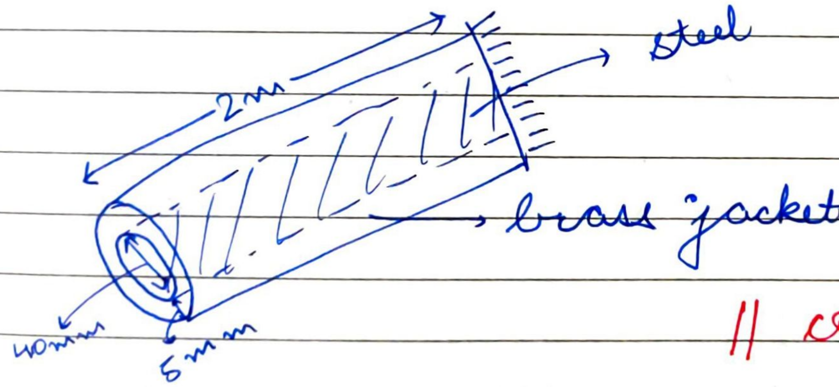
$= 803.60\ N\text{-}m$

we get 2 values of T from the τ allowed and ϕ_{max}.

But since both of them must be satisfied, $\therefore$ we take minimum of 2 values ie 265.07 N-m = T since if we exceed this T all will be exceeded.

NOTE: In case we are given values of ~~τ_{max} T_{max} J G ϕ~~ T, τ, J, ϕ, L and we have to find ρ, then we use highest value of ρ obtained from 2 eqns. (Since neither of the conditions must be exceeded).

Q →



composite shaft

// connection

$G_B = 39$ GPa ; $G_s = 77.2$ GPa ; $T = 600$ N-m ;

$\tau_B = ?$ $\tau_S = ?$ $\phi = ?$

Ans: we can write initially

$\phi_s = \phi_B$ & $T = T_s + T_B$

Ans: $\tau_B = 17.45\ MPa$

$\tau_S = 27.6\ MPa$

Starting from brass

$$\frac{T_B}{J_B} = \frac{G_B\phi}{L}$$

$$\frac{T_S}{J_S} = \frac{G_S\phi}{L}$$

$$\therefore \frac{T_B}{T_S} = \frac{G_B J_B}{G_S J_S}$$

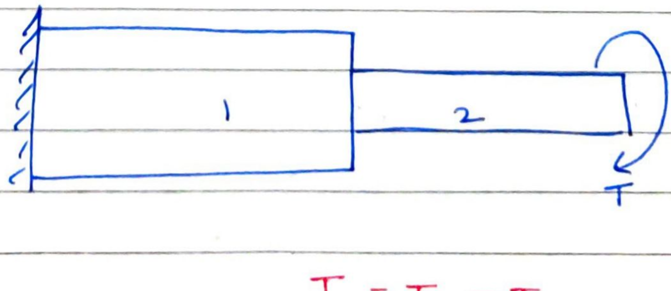
$$600 = T_B + T_S$$

$$\Rightarrow T_S + \frac{G_B J_B}{G_S J_S} T_S = 600$$

$$\therefore T_S = \frac{600}{\left(1 + \frac{G_B J_B}{G_S J_S}\right)} \quad \& \quad T_B = \frac{600}{\left(\frac{G_S J_S}{G_B J_B} + 1\right)}$$

solve it yourself now (Khali eqⁿs me daalna hai)

NOTE:



series connection

$$T_1 = T_2 = T$$

$$\phi = \phi_1 + \phi_2$$

classmate

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Q: In previous que,

$G_B = 39\ GPa$
$G_S = 77.2\ GPa$
$\tau_B = 20\ MPa$
$\tau_S = 45\ MPa$] — max allowable
$T_{max} = ?$
$\phi = ?$

Ans: For steel

$$\frac{45 \times 10^6}{20 \times 10^{-3}} = \frac{77.2 \times 10^9}{2}\phi$$

$$\frac{4.5}{77.2} = \phi$$

$\Rightarrow \phi = 0.05829$

For brass

$$\frac{20 \times 10^6}{25 \times 10^{-3}} = \frac{39 \times 10^9}{2}\phi$$

$$\frac{40}{39 \times 25} = \phi$$

$\phi = 0.041025$

$\phi = 2.3505°$

we take min. ϕ

$\therefore \phi = 2.35°$

now

$$\frac{T_B}{J_B} = \frac{G\phi}{L} \qquad \frac{T_S}{J_S} = \frac{\phi G_S}{L}$$

$$\Rightarrow T_B = \frac{39 \times 10^9 \times 0.041025}{2} \times \frac{\pi}{32} \times (40)^4 \times 10^{-12}$$

$T_B = 201.0587\ N \cdot m$

$$\cancel{T_S = \frac{G_S \phi}{L} \times J_S}$$

$$\cancel{= \frac{77.2 \times 10^9 \times 0.041025}{2} \times \frac{\pi}{32} \times \left((50)^4 - (40^4)\right) \times 10^{-12}}$$

$$\cancel{= 573.67002 \text{ N-m}}$$

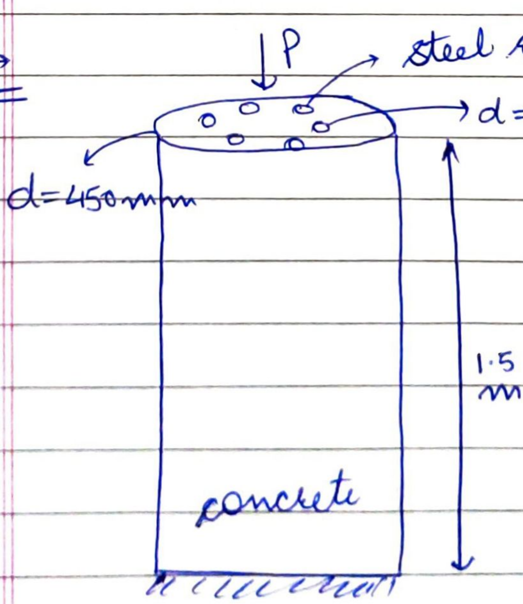
$$T = T_S + T_B$$

$$= \frac{\phi}{L}\left(G_B J_B + G_S J_S\right)$$

$$= \frac{0.041025}{2}\left(39 \times \frac{\pi}{32} \times (40)^4 \times 10^{-3} + 77.2 \times \frac{\pi}{32} \times \left((50)^4 - 40^4\right) \times 10^{-3}\right)$$

$$= \frac{0.041025}{2} \times \frac{\pi}{32} \times 10^{-3}\left(39 \times 40^4 + 77.2 \times \left((50)^4 - (40)^4\right)\right)$$

~~imp~~ **Q→**



$E_s = 200$ GPa

$E_c = 25$ GPa

$P = 1550$ kN

Find σ_c & σ_s

~~Plus reinforcement force~~

Ans: $P = 6P_s + P_c$

$\delta_c = \delta_s = \delta$ $\epsilon_s = \epsilon_c = \epsilon$

$$E_s = \frac{\sigma_s}{\epsilon_s}$$

$$\Rightarrow E_s = \frac{P_s}{A_s \epsilon}$$

$$\& \quad E_c = \frac{P_c}{A_c \epsilon}$$

$$6E_sA_s\epsilon + E_cA_c\epsilon = P$$

$$\Rightarrow \epsilon = \frac{P}{6E_sA_s + E_cA_c}$$

$$= \frac{1550 \times \cancel{10^3}}{\cancel{10^3} \times \left(6 \times 200 \times \pi \times 14^2 + 25 \times \pi (225^2 - 14^2 \times 6)\right)}$$

$$\Rightarrow \epsilon = 3.353 \times 10^{-4}$$

$$\therefore \sigma_s = 200 \times 10^9 \times 3.353 \times 10^{-4}$$

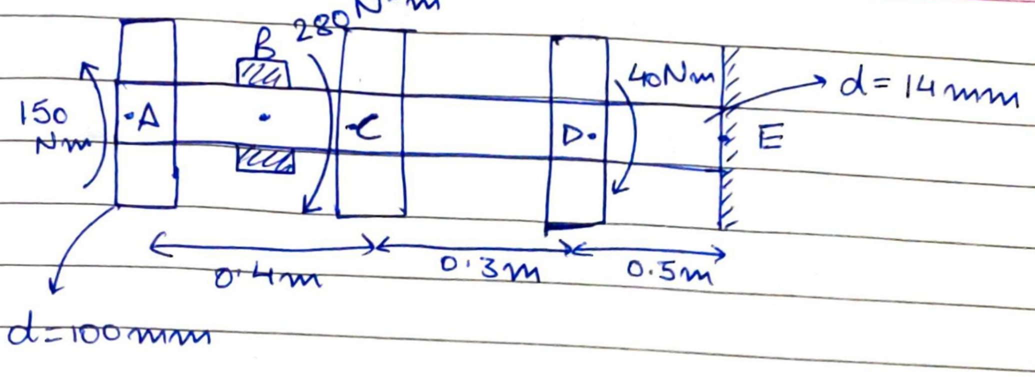
$$= 670.6 \times 10^5$$

$$= 67.06 \text{ MPa}$$

$$\sigma_c = 25 \times 10^9 \times 3.353 \times 10^{-4}$$

$$= 8.3825 \text{ MPa}$$

Ques:

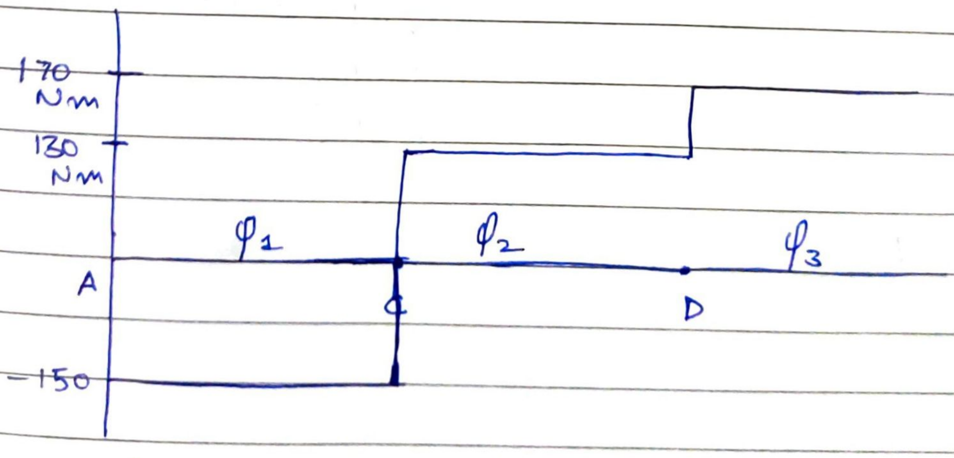


$G_s = 80$ MPa

Find displacement of a point on A gear.

Ans: It is a series connection. To find displacement, we need to find ϕ and ~~ϕ~~ ~~ϕ~~ ~~ϕ~~ $\phi \times$ radius is our required displacement.

Here there is an issue that the torque is not same ~~for~~ throughout the shaft. ∴ firstly we will need a torque diagram.



↓

reverse the graph (take anticlockwise torque to be +ve)

We know now

$T_1 = 150$
$T_2 = -130$
$T_3 = -170$

$\therefore$ we can now find ϕ

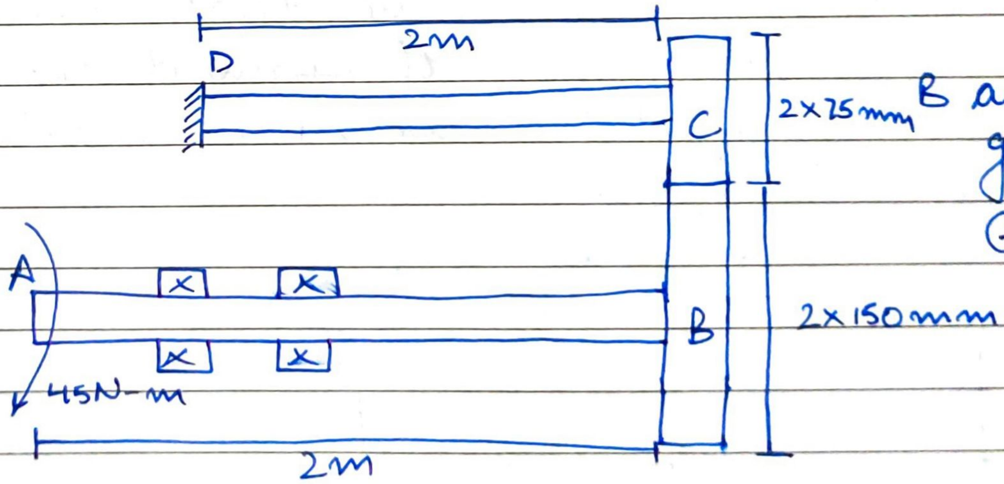
$$\frac{150}{J_1} = \frac{G\phi_1}{L}$$

$$\Rightarrow \frac{150}{\frac{\pi}{2} \times (0.04)^4} = \frac{80 \times 10^6 \times \phi_1}{0.4}$$

~~$\Rightarrow \phi_1 = 0.02689$~~

~~ϕ Displacement~~

Q→



B and C are gears

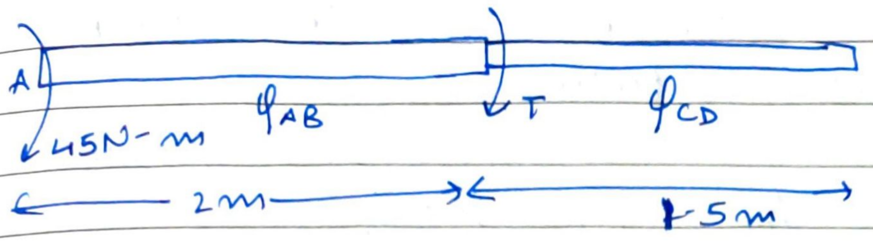
$G = 80$ GPa

$d = 20$ mm for both rods

Find ϕ_A

Ans:

If we carefully observe, we can substitute this system as a series connection



If we carefully observe, ~~the~~ & at equilibrium & the force on the periphery of B = force on periphery of C (third law)

& A is in eq^m ∴ torque on AB is balanced.

$$\therefore \quad 0.15 \times F = 45$$

$$\Rightarrow F = \frac{45}{0.15}$$

now for CD,

$$T = \frac{45}{\cancel{0.15}_2} \times \cancel{0.075}$$

$$\phi_A = \frac{TL}{JG} = \frac{45 \times 2}{\frac{\pi}{2} \times \left(\frac{10}{1000}\right)^4 \times 80 \times 10^9} = \frac{180}{\pi \times 800}$$

$$= 0.071619$$

$$\phi_B = \frac{(45/2) \times 1.5}{\frac{\pi}{2}(0.01)^4 \times 80 \times 10^9} = \frac{202.5}{800\pi \times 3}$$

$$= 0.0269$$

8/10

now the ϕ we have calculated ~~is~~ only ~~ϕ~~ counts for rotation due to torque. But, for AB, we also need to ~~[illegible]~~ add rotation due to gear B for ϕ_{AB} (total rotation)

factor dependent on gear

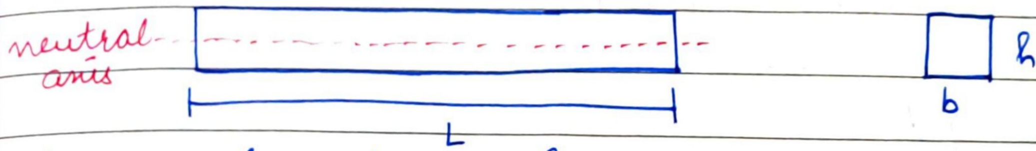
$$\therefore \phi_{AB} = \phi_A + x \cdot \phi_B$$

$$\Rightarrow \phi_{AB} = \phi_A + 0.5\,\phi_B$$

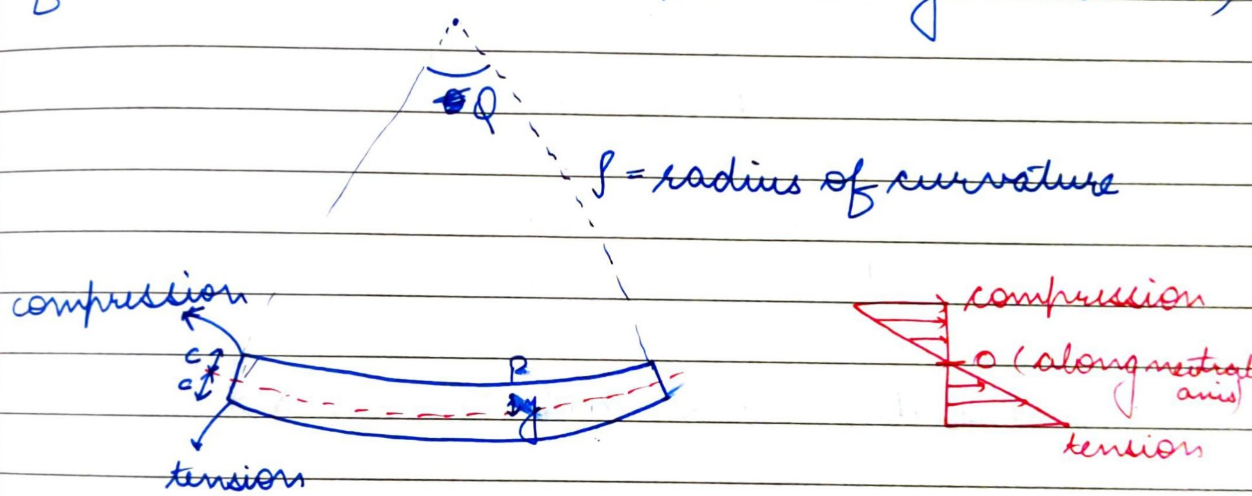
Chapter - 7

→ BENDING IN BEAMS

- consider a rod / beam



If we bend this rod (by a moment)



- Neutral axis is a ~~axis~~ plane along which stress always remains zero

- $L = \rho\phi$

We will find stress at any general point P

for P

$$L' = (\rho - y)\,\phi$$

$$\delta = (\rho - y)\,\phi - \rho\phi = L' - L$$

(since initially length of fibre was L but after bending it becomes L')

$$\epsilon = \frac{\delta}{L} = \frac{-y\cancel{\phi}}{\rho\cancel{\phi}} = \frac{-y}{\rho}$$

$$\therefore \boxed{\epsilon_{max} = \frac{c}{\rho}}$$

$$\sigma_{max} = E\epsilon_{max}$$

$$\Rightarrow \boxed{\sigma_{max} = E\frac{c}{\rho}}$$

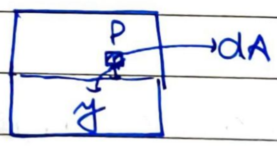
For pt. P

$$\sigma = E\frac{y}{\rho}$$

$$\Rightarrow \sigma = E\frac{c}{\rho}\frac{y}{c}$$

$$\Rightarrow \boxed{\sigma = \sigma_{max}\frac{y}{c}}$$

- Consider a small element



$$dM = dF \cdot y$$

$$= \sigma \, dA \, y$$

$$= \frac{y^2}{c}\sigma_{max} \, dA$$

$$\Rightarrow M = \frac{\sigma_{max}}{c}\int y^2 dA$$

$$\Rightarrow M = \frac{\sigma_{max}}{c} I$$

$$\therefore \boxed{\frac{M}{I} = \frac{\sigma_{max}}{c} = \frac{\sigma}{y}} \qquad (i)$$

$$\sigma_{max} = E\,\epsilon_{max}$$

$$\frac{Mc}{I} = \frac{Ec}{\rho}$$

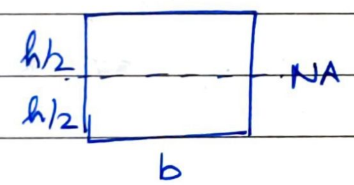
$$\Rightarrow \frac{M}{I} = \frac{E}{\rho} \qquad (ii)$$

$\therefore$ from (i) & (ii)

imp remember

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{\rho}}$$

• Section modulus



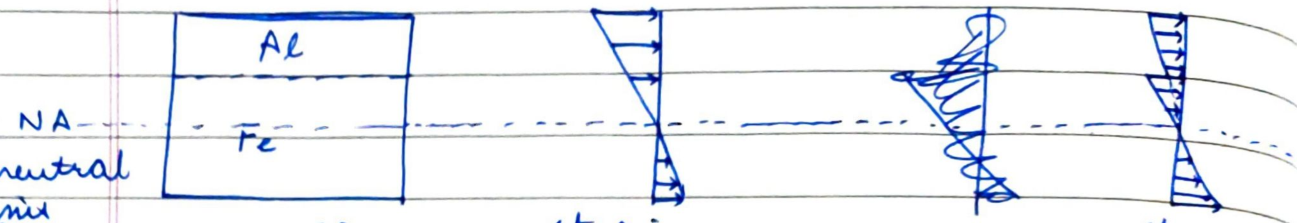
$$\frac{M}{I} = \frac{\sigma_{max}}{c}$$

$$\sigma_{max} = \frac{M}{(I/c)} \longrightarrow Z = \text{section modulus}$$

$$Z = \frac{I}{c} = \frac{\frac{1}{\cancel{12}\,6} b h^{\cancel{3}\,2}}{\cancel{h}/\cancel{2}}$$

$$\Rightarrow \boxed{Z = \frac{1}{6} b h^2}$$

• Consider a composite rod.



(neutral axis shifts a little)

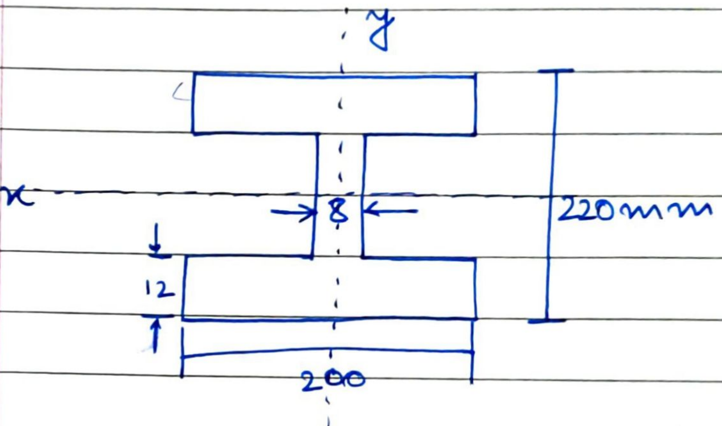
$E_a = 70 \text{ GPa}$

$E_i = 200 \text{ GPa}$

strain graph

stress graph

Q →



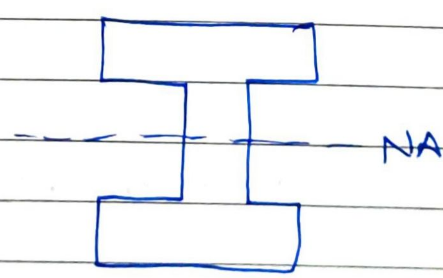
σ_{all} (allowable) = 155 M

$M_{max} = ?$

(abt x)

Ans: In general case, take moment to be applied ~~along~~ by forces along y axis.

∴



∴ we need to find I

$$\therefore I = \frac{1}{12} 8 \times \left(196\right)^3 + \frac{1}{\cancel{36}} 200 \times \left(12\right)^3 + 2 \times 2400 \times 10$$

$\Rightarrow$ ~~$I = 63157290.67 \text{ mm}^4$~~

$$\Rightarrow I = 56994090.666 \text{ mm}^4$$

$$\Rightarrow I = 56.9 \text{ m}^4 \times 10^{-6}$$

We know

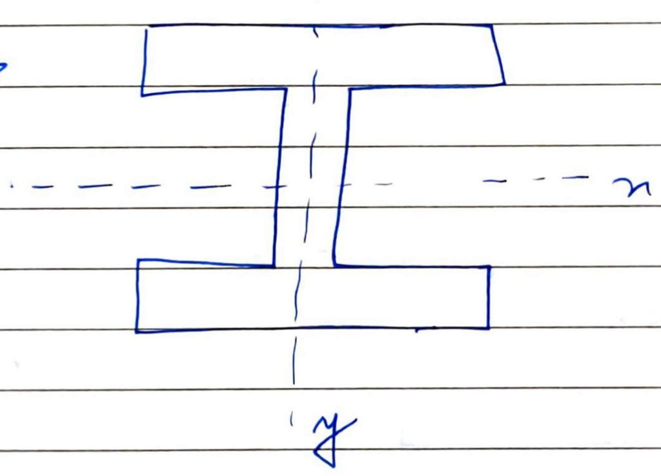
$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\Rightarrow \frac{155\times 10^{6}}{110\times 10^{-3}} = \frac{M}{56.9\times 10^{-6}}$$

$$\Rightarrow M = 80.177\times 10^{3}\ N\text{-}m$$

Q- In previous problem, find M_{max} (about y)

Ans:



$$I_y = \left(\frac{1}{\cancel{12}\ 6}\times \cancel{196}\ 98\times 8^{3} + \frac{1}{6}\times 12\times(200)^{3}\right)\ mm^{4}$$

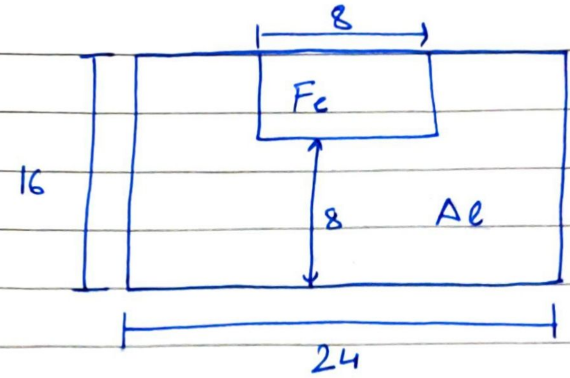
$$= 16.008\times 10^{-6}\ m^{4}$$

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\Rightarrow \frac{155\times 10^{6}}{100\times 10^{-3}} = \frac{M}{16.008\times 10^{-6}}$$

$$\Rightarrow M = 24.8124\times 10^{3}\ N\text{-}m$$

imp
Que:

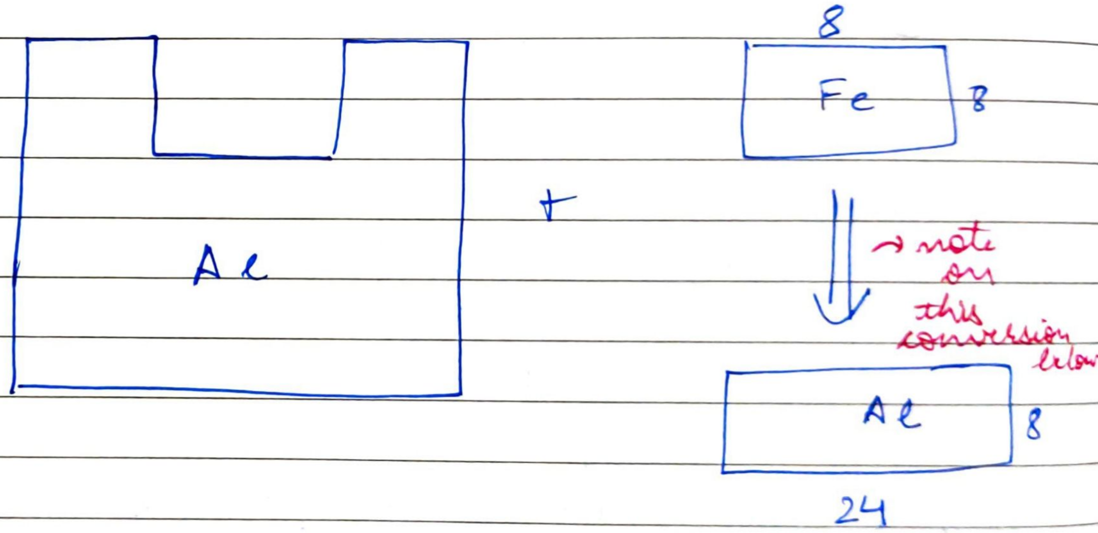


cross section of a beam.

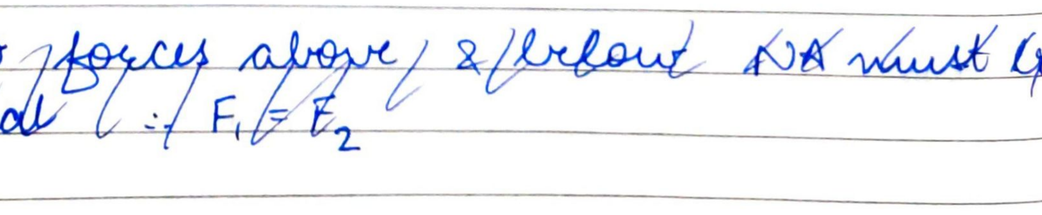
$E_s = 210$ GPa
$E_a = 70$ GPa
$M_x = 60$ Nm

$\sigma_a = ?$
$\sigma_s = ?$

Ans: In such problems, using the ratio of E, we transform one material to other



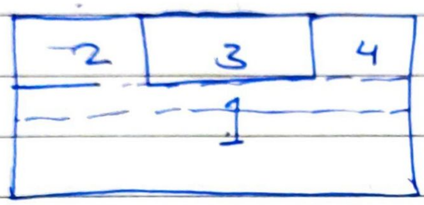
NOTE: Consider the beam in this problem



~~now forces above & below NA must be equal ∴ $F_1 = F_2$~~

now we will find NA. NA is at centroid.
$\therefore$ for centroid

$$y = \frac{\sum yA}{\sum A} = \frac{y_1A_1 + y_2A_2 + y_3A_3 + y_4A_4}{A_1 + A_2 + A_3 + A_4}$$

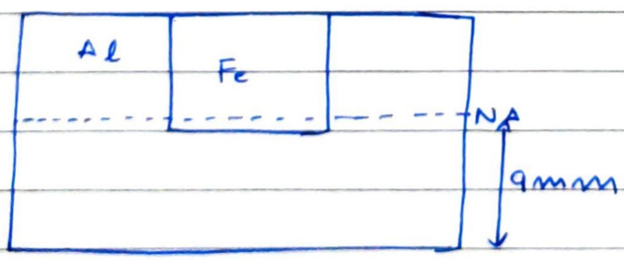


$$\therefore y = \frac{y_1A_3 + y_2A_2 + n y_3A_3 + y_4A_4}{A_1 + A_2 + nA_3 + A_4}$$

$$\Rightarrow y = \frac{4\times \cancel{8}\times \cancel{24}^{3} + 12\times\cancel{8}\times\cancel{8} + 12\times\cancel{8}\times\cancel{8} + 12\times\cancel{8}\times\cancel{24}^{3}}{\cancel{8}\times\cancel{24}_{3} + \cancel{8}\times\cancel{8}_{1} + \cancel{8}\times\cancel{8}_{1} + \cancel{8}\times\cancel{24}_{3}}$$

$$\Rightarrow y = \frac{12 + 12 + 12 + 36}{8} = \frac{72}{8} = 9\,mm$$

now for I_{NA}



$$I_{NA} = \cancel{\frac{1}{12}(24\times 8^3} \; \frac{1}{12}\times 24\times 16^3 + 24\times 16\times 1^2$$

$$+ \frac{1}{12}\times 16\times 8^3 + 16\times 8\times 5^2$$

$$\Rightarrow I_{NA} = 12714.6666\ mm^4$$

$$\Rightarrow I_{NA} = 0.012715\times 10^{-6}\ m^4$$

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$I_{NA} = \frac{1}{\cancel{12}} \times \overset{2}{\cancel{24}} \times 8^3 + \cancel{2}\, 24 \times 8 \times 5^2 + \frac{\cancel{2}}{\cancel{12}\,3} \times \overset{2}{\cancel{8}} \times 8^3 \quad \cancel{\tfrac{1}{12}} \times 8 \times 8 \times 3^2$

$+ \frac{1}{\cancel{12}} \times \overset{2}{\cancel{24}} \times 8^3 + 8 \times 24 \times 3^2$

$= 10410.67 \times 10^{-12}\ m^4$

$\therefore$ now

$\sigma_a = \frac{60 \times 9 \times 10^{-3}}{10.4106 \times 10^{-9}} = 51.870$ MPa

$\sigma_s = 3 \times \frac{60 \times 7 \times 10^{-3}}{10.4106 \times 10^{-9}} = 121.030$ MPa

Steps :→

a) $A_s \longrightarrow mA_s$

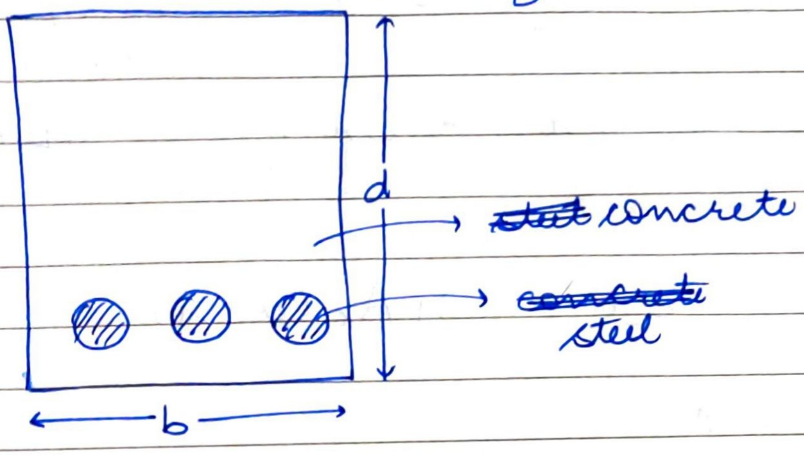
b) $\cancel{2}$ NA

c) I_{NA}

d) $\sigma_s \longrightarrow mM, \quad \sigma_s = \frac{mMy}{I}$

NOTE: concrete can only take compressive load not tensile load

Ques: cross section of a beam



$E_c = 25\ GPa$

$E_s = 200\ GPa$

$b = 200\ mm$

$d = 450\ mm$

$\sigma_{c\,all} = 12.5\ MPa$

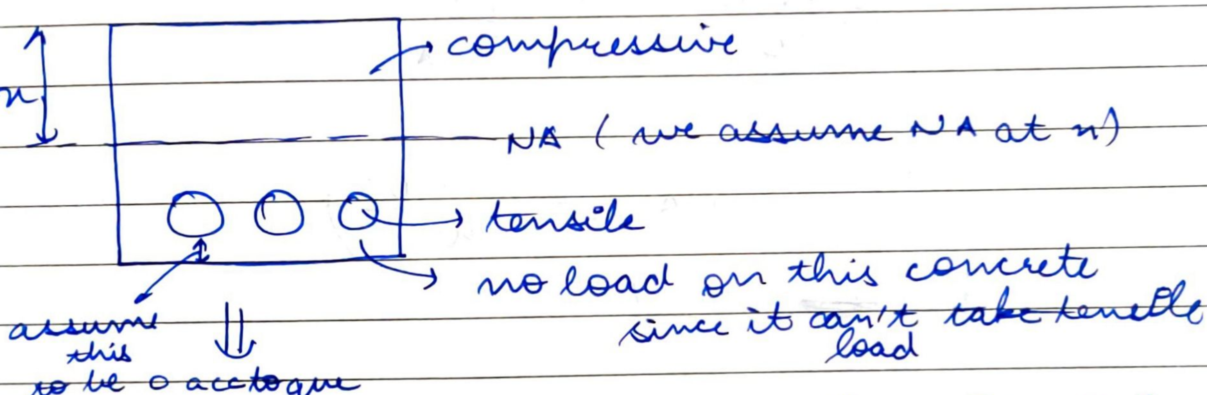
$\sigma_{s\,all} = 140\ MPa$

$A_s = ?$

$M = ?$

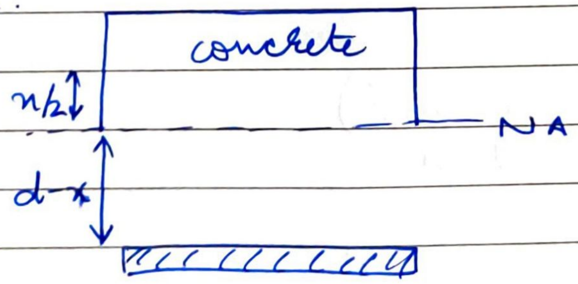
Ans: only steel will be able to take tensile load

∴



$n = \frac{200}{25} = 8$

∴



For centroid to be at x

$$d - x = \frac{\left(d - \frac{x}{2}\right) \times x \times b + nA_s \times 0}{nA_s + xb}$$

$$nA_s(d-x) + xb(\cancel{d}-x) = xb(\cancel{d}-x/2)$$

$$nA_s(d-x) = xb(x/2)$$

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$I_{NA} = I_c + I_s$ → for I_s, since area is very small, we can neglect its I about its centroid

$\Rightarrow I_{NA} = \frac{1}{12} \times b x^3 + b x \left(\frac{x}{2}\right)^2 + n A_s \times (d-x)^2$

$I_{NA} = \frac{b x^3}{3} + n A_s (d-x)^3$

For ~~steel~~ conc.

$\frac{\sigma}{y} = \frac{M}{I}$

$\Rightarrow \frac{12.5 \times 10^6}{x} = \frac{M}{I_{NA}}$

$\Rightarrow M = I_{NA} \left(\frac{12.5 \times 10^6}{x} \right)$ (i)

For ~~con~~ steel

$\frac{\sigma}{y} = \frac{M n}{I}$

$\Rightarrow M = \frac{I_{NA}}{n} \left(\frac{140 \times 10^6}{d-x} \right)$

$\Rightarrow M = \frac{I_{NA}}{8} \left(\frac{140 \times 10^6}{0.45 - x} \right)$ (ii)

moment along the rod will be the same ∴ from (i) & (ii)

$\frac{\cancel{140}^{35} \cancel{\times 10^6}}{2\cancel{8}(0.45-x)} = \frac{12.5 \cancel{\times 10^6}}{x}$

$\Rightarrow \frac{7}{0.45 - x} = \frac{5}{x} \Rightarrow x = 0.1875 \, m$

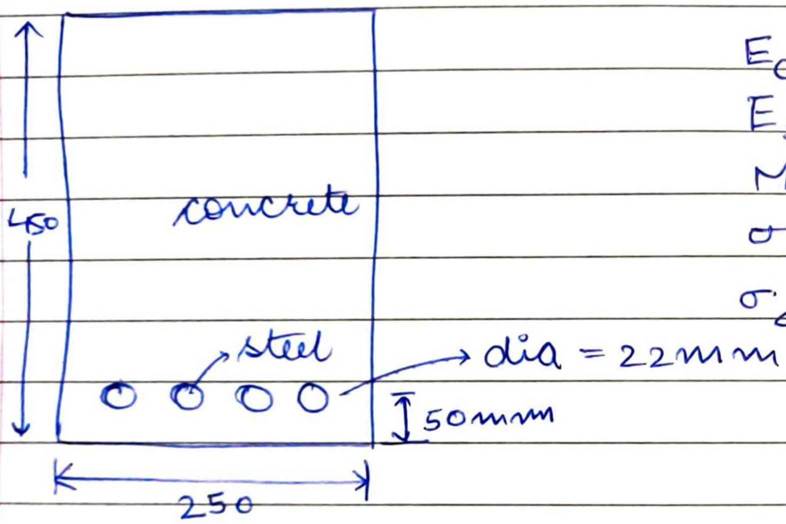
$\therefore\ 8A_s(0.2625) = 3.515625 \quad \Rightarrow A_s = 1.6741\ m^2 \times 10^{-2}$

$\therefore\ I_{NA} = 0.0028619\ m^4$

$$M = \frac{0.0028619 \times 12.5\times10^6}{0.1875}$$

$$\boxed{M = 90.8\ kN\text{-}m}$$

Que:



$E_c = 25\ GPa$

$E_s = 200\ GPa$

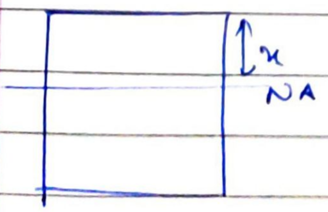
$M = 170\ kN\text{-}m$

$\sigma_s = ?$

$\sigma_c = ?$

Ans $n = \frac{200}{25} = 8$

$A_s = \cancel{4} \times \frac{\pi}{\cancel{4}} \times 22^2$



$\longrightarrow \quad \therefore\ x = \dfrac{\frac{x}{2}\times 250 \times x + (400-x) A_s n}{n A_s + 250x}$

$\Rightarrow x = \dfrac{125x^2 + (400-x)\,\pi \times 22^2 \times 8}{8\times\pi\times22^2 + 250x}$

$\Rightarrow x = 154.55\ mm$

$$\sigma_s = \frac{M y_n}{I} = \frac{170 \times 8 \times (400 - x) \times 10^3}{I}$$

$$\sigma_c = \frac{170 \times 10^3 \times x}{I}$$

where

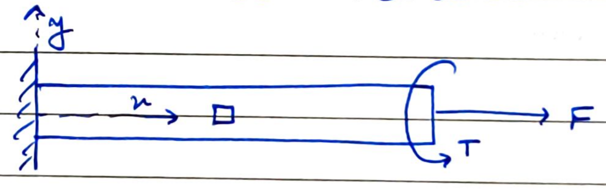
$$I = \frac{1}{12}(250)x^3 + x(250)\left(\frac{x}{2}\right)^2 + 8A_s(400-x)^2$$

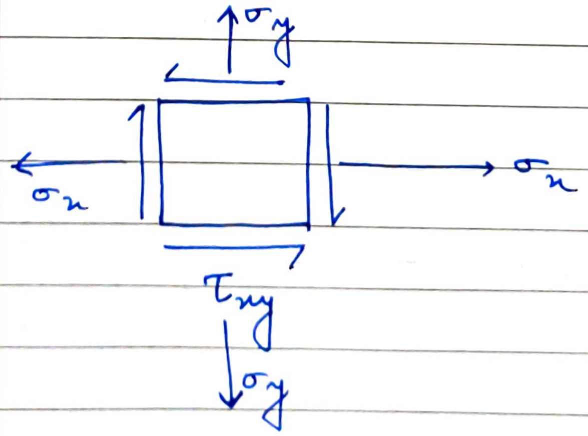
$\sigma_c = -26$ MPa

$\sigma_s = 330$ MPa

→

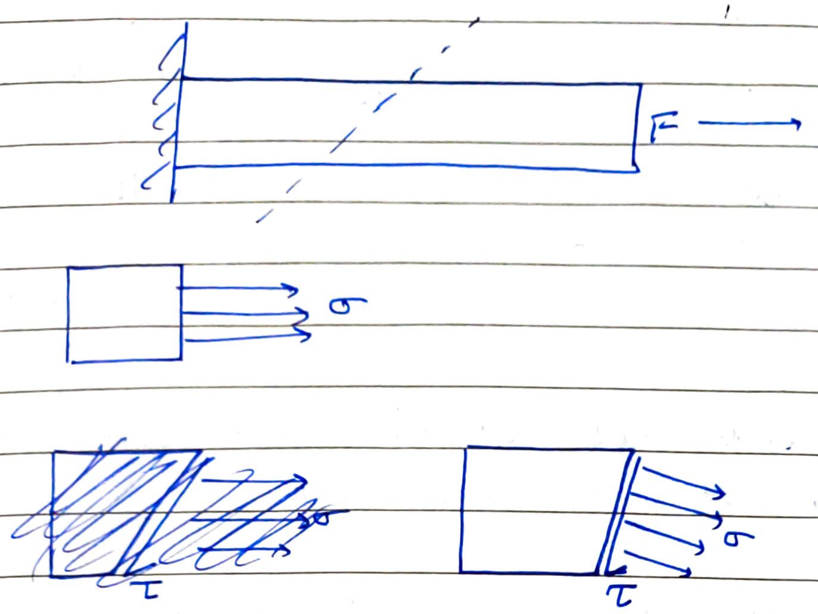
- consider a case where we provide both shear stress & axial stress





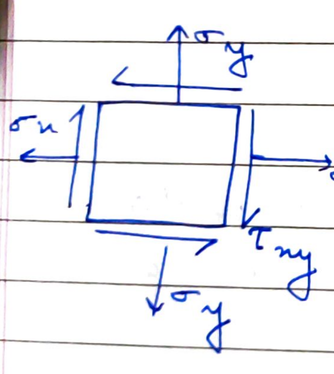
NOTE: Don't think about direction of τ in this diagram. Feel nahi ayegi. There will be some shear stress in the given manner but its hard to imagine so just assume this direction.

NOTE: ~~tate~~ If we take a section like

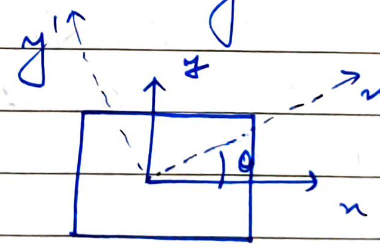


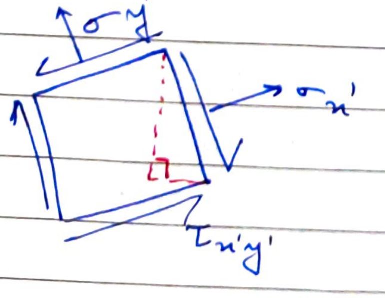
∴ σ is ⊥ to surface & τ is along it.

Now, consider



If we take an element at angle theta





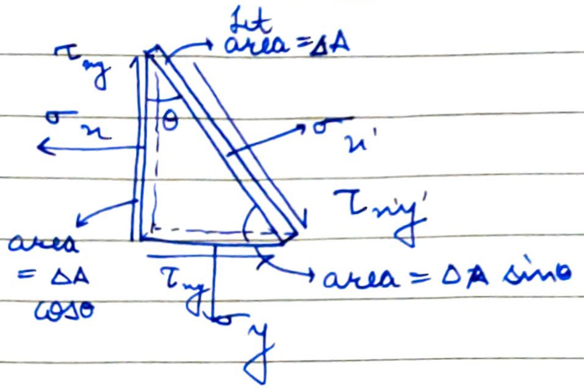
$\sigma_{x'} = f(\sigma_x, \sigma_y, \tau_{xy})$

$\sigma_{y'} = f(\sigma_x, \sigma_y, \tau_{xy})$

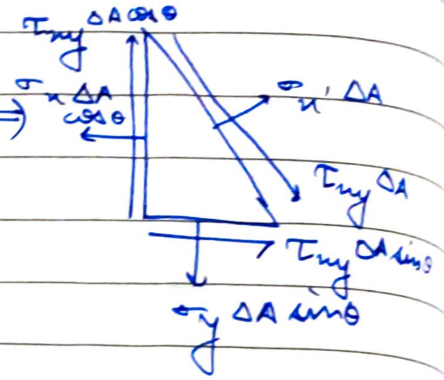
$\tau_{xy} = f(\sigma_x, \sigma_y, \tau_{xy})$

To find it, we take an element as shown above (in red). (We take such element because we know ~~tire~~ stresses along red lines & can use them to find $\sigma_{x'}$)

For our element



$\therefore$ FBD $\Rightarrow$



We ~~~~ use force balancing and using it we will get

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

- Principle stress: The stresses obtained when $\tau_{x'y'} = 0$ are known as principle stresses.

$\therefore$ From above

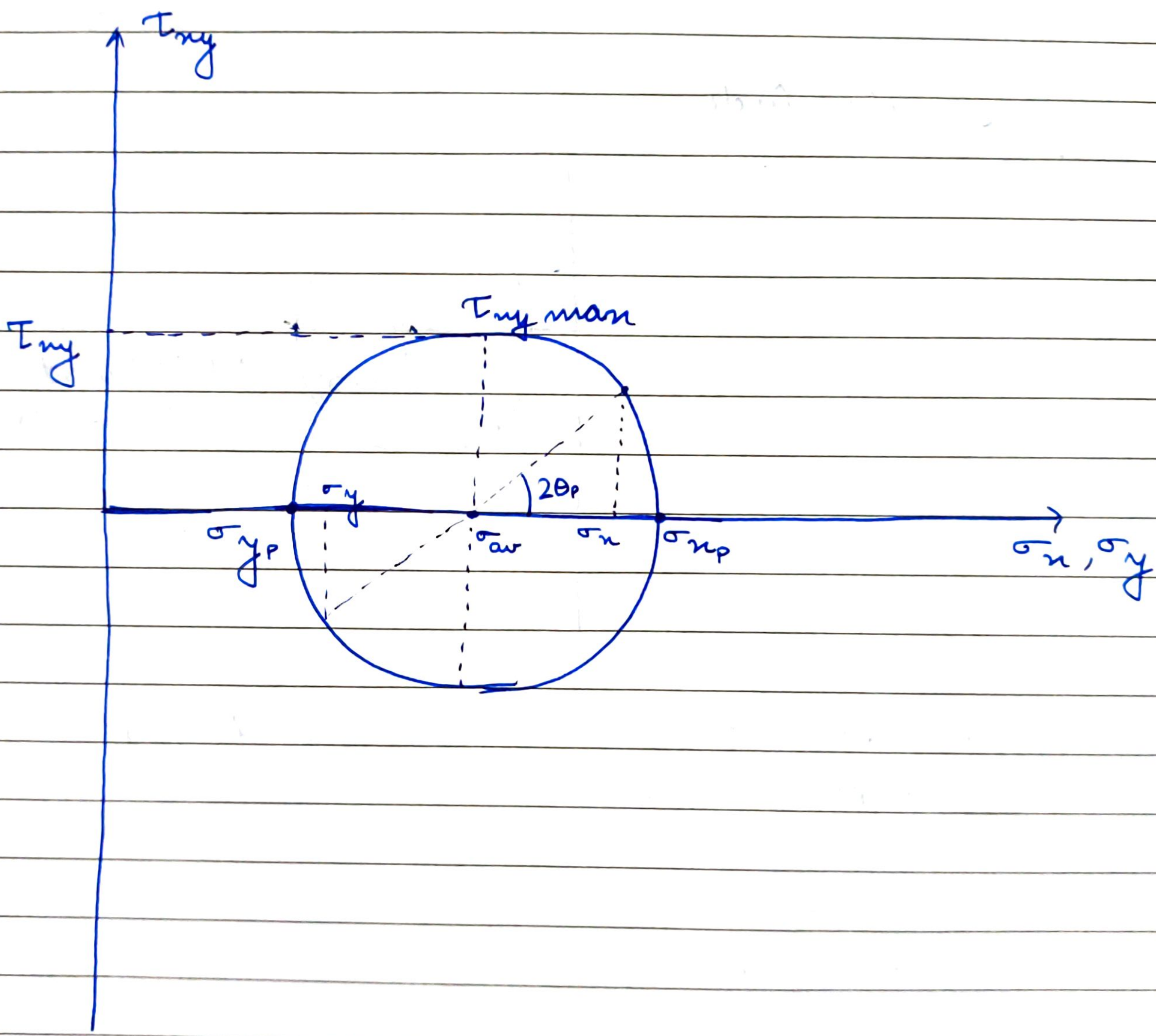
$$\Rightarrow \boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

using this 2θ, we get $\sigma_{x'}$ & $\sigma_{y'}$ ie principle stresses

$$\boxed{\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\boxed{\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

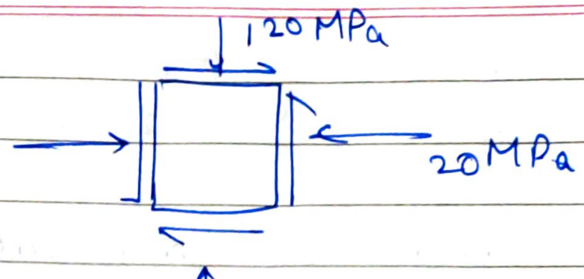
All of this can also be calculated using Mohr's circle



$\tau_{max} = R$ where $\sigma_1 = \sigma_2 = \sigma_{av}$

$$\& \; R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

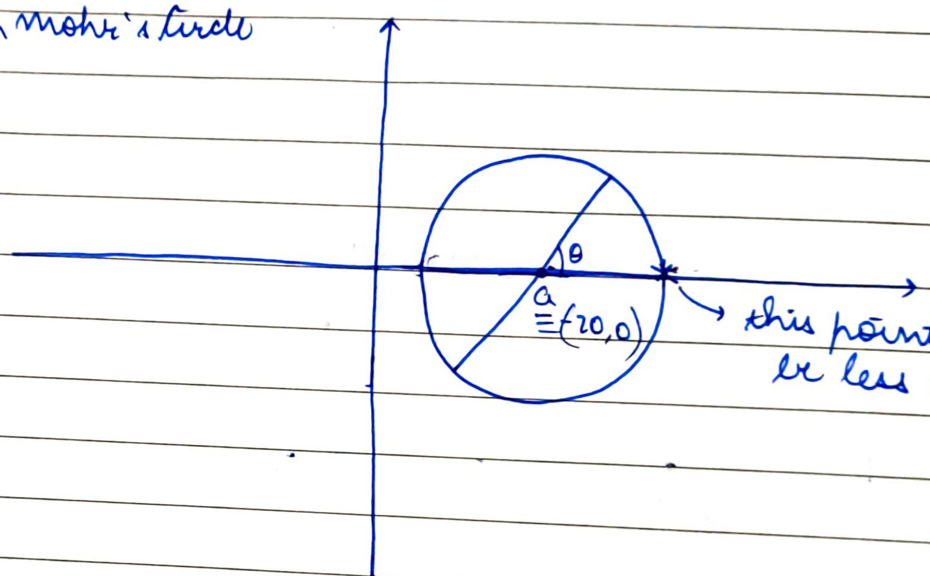
Ques:



$\sigma_{max} \leq 60$ MPa

give the range for τ_{xy} in the given situation.

Ans: In mohr's circle



$a + R\cos\theta = -20$

$a - R\cos\theta = -120$

$2a = -140$

$a = -70$

& $R\cos\theta = 50$

$\therefore R \leq 60 + 70$

$\Rightarrow R \leq 130$

$|\tau_{xy}| = R\sin\theta$

Ans: $[-120, 120]$

$$\sqrt{R^2\cos^2\theta + R^2\sin^2\theta} \leq 130$$

$$\Rightarrow 2500 + R^2\sin^2\theta \leq 16900$$

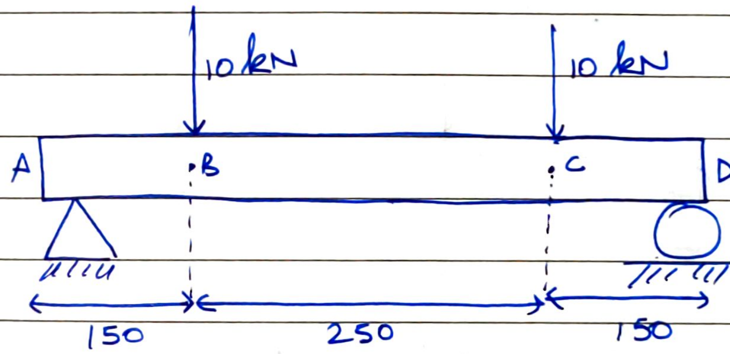
$$\Rightarrow R^2\sin^2\theta \leq 14400$$

$$\Rightarrow \{R\sin\theta\} \leq 120$$

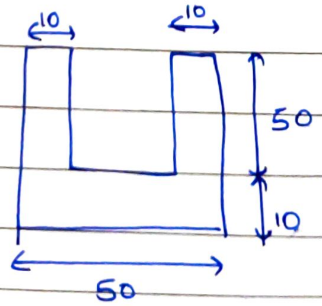
$$\therefore |\tau_{xy}| \leq 120$$

$$\therefore -120 \leq \tau_{xy} \leq 120$$

Que:



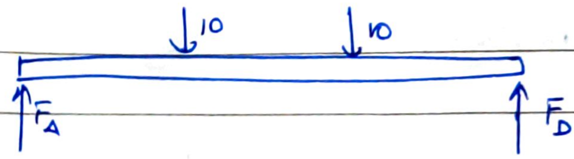
For region BC, find σ_c & σ_t given that the cross section of beam looks like



Ans:

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\Rightarrow \sigma = \frac{My}{I}$$



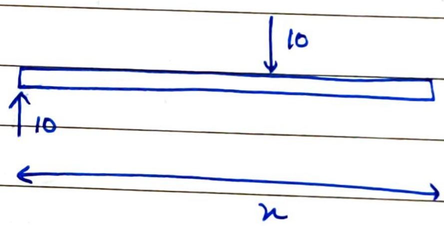
$\therefore F_A + F_D = 20$

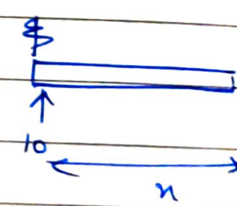
$10 \times 15\cancel{0} + 10 \times 40\cancel{0} = F_D \times 55\cancel{0}$

$F_D = \frac{650}{55}$

$\Rightarrow F_D = 10$

$\therefore F_A = 10$



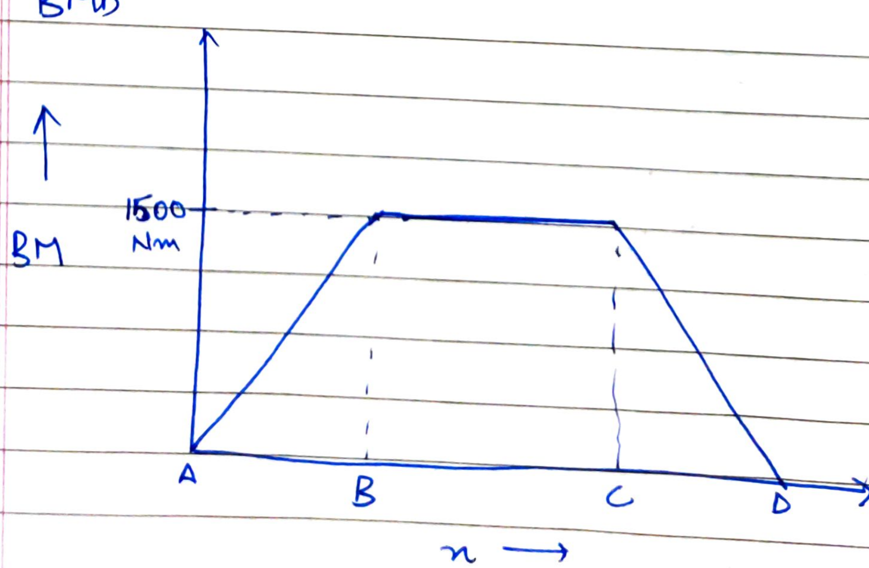


$M = 10x$

$10x - 10(x - 150) = M$

$\Rightarrow M = 1500$ Nm

BMD

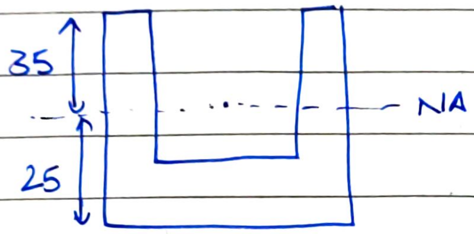


now we need I_{NA}

For NA we find centroid.

from bottom,

$$y_{NA} = \frac{5 \times 500 + 35 \times 500 + 25 \times 500}{1500} = \frac{375}{15} = 25 \text{ mm}$$



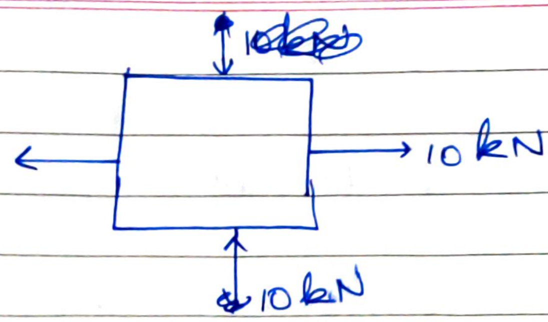
$$I_{NA} = \frac{1 \times 50 \times 1000}{12} + \frac{2}{12} \times 10 \times 50^3 + 500 \times 20^2 + 1000 \times 10^2$$

$$= 512500 \times 10^{-12} \text{ m}^4$$

$$\therefore \sigma_c = \frac{My}{I} = \frac{1.5 \times 35}{5.125 \times 10^{-7}} = 102.44 \text{ MPa}$$

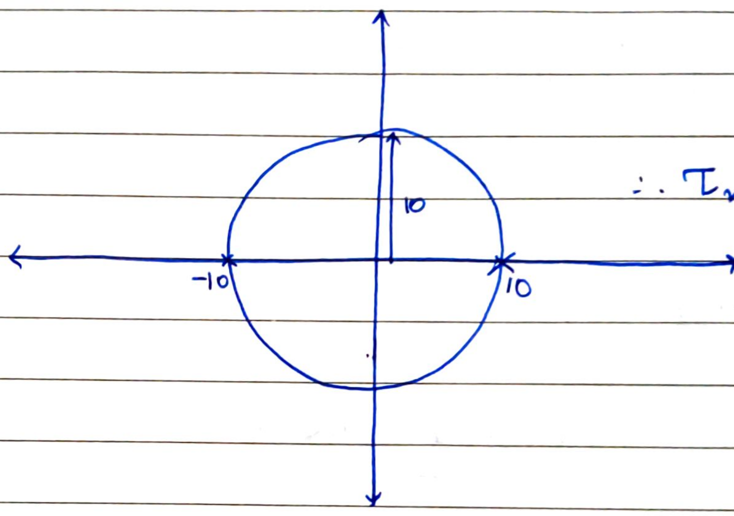
$$\sigma_t = \frac{1.5 \times 25}{5.125 \times 10^{-7}} \quad 73.17 \text{ MPa}$$

Que:



$\tau_{max} = ?$

Ans:



$\therefore \tau_{max} = 10$

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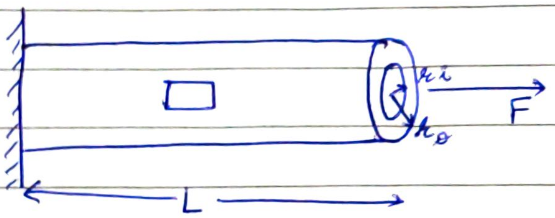
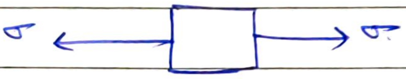
→ MOHR'S CIRCLE FOR DIFFERENT CASES

1.



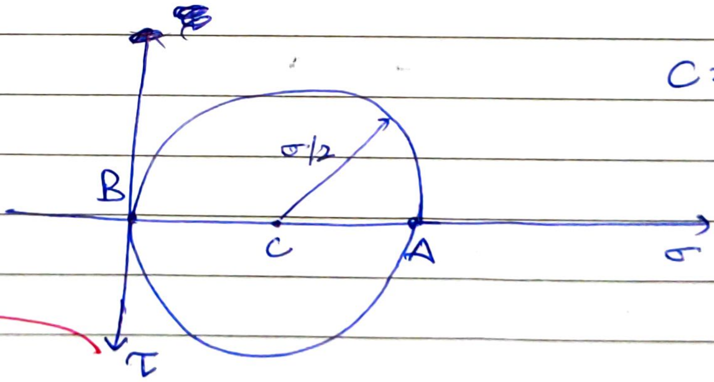
$$\sigma = \frac{F}{A} = \frac{F}{\pi(r_o^2 - r_i^2)}$$



Plotting points for Mohr's circle

$A = (\sigma, 0)$, $B = (0,0)$

For x direction   For y direction

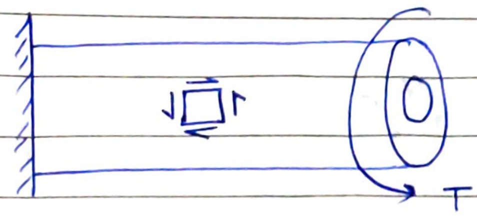


$C = \left(\frac{\sigma}{2}, 0\right)$

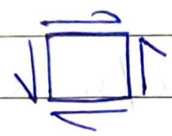
τ is +ve in downward direction

NOTE: τ in anticlockwise direction is positive and -ve for clockwise

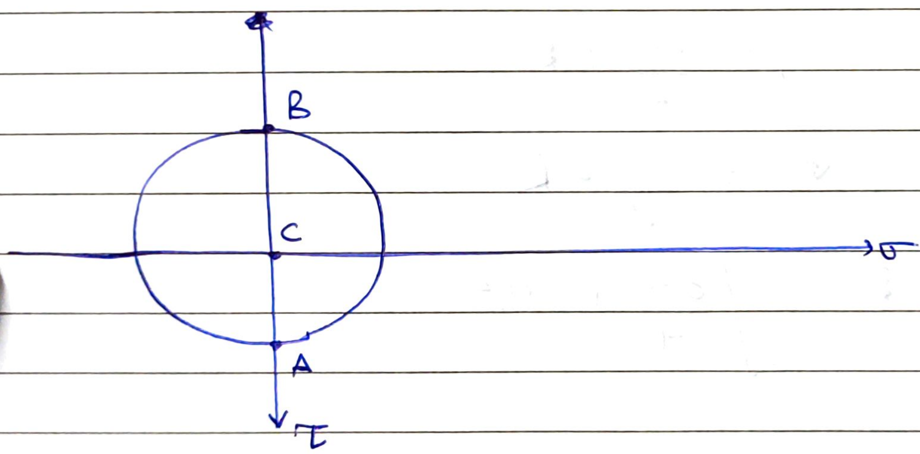
2.



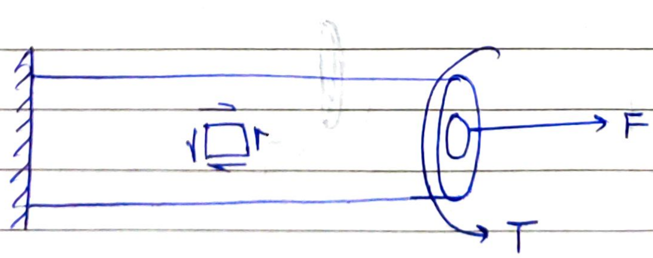
$$\frac{T}{J} = \frac{\tau}{\rho} \Rightarrow \tau = \frac{T\rho}{J} \quad (\rho = r_o)$$

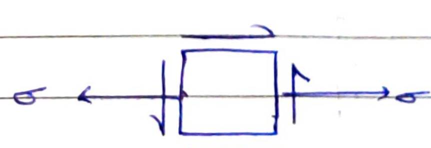


$A = (0, \tau)$, $B = (0, -\tau)$, $C = (0, 0)$

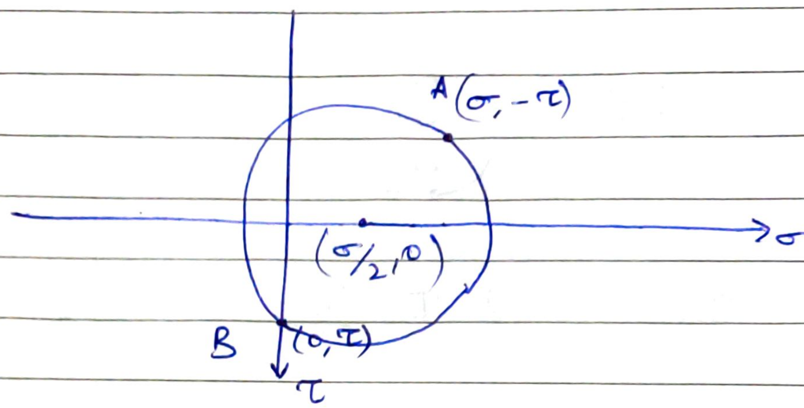


3.





$A = (\sigma, -\tau)$, $B = (0, +\tau)$, $C = \left(\frac{\sigma}{2}, 0\right)$



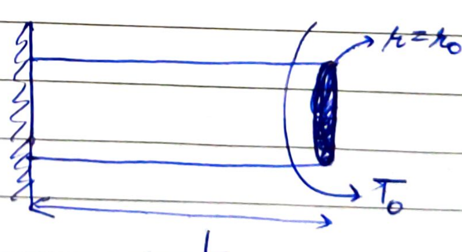
We can find principle stresses in this case by using eq^n of circle

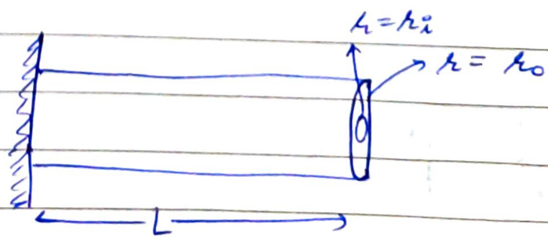
$$\sigma_1 = \frac{\sigma}{2} + R$$

$$\sigma_2 = \frac{\sigma}{2} - R$$

$$\& \ R = \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Que.





For both cases, compare values of τ_{max}, τ per unit mass.

Ans: case 1:

$\tau = \frac{T\rho}{J}$

τ in this case will be lower due to higher J

$J = \frac{\pi}{2} r_o^4$

$m = \text{density} \times \text{vol}$
$= d \times \pi r_o^2 L$

~~$\frac{\tau}{m}$ is lower for this~~

For $\frac{\tau}{m}$, we need τ_{av}

$\tau_{av} = \frac{\tau_{max} + \tau_{min}}{2}$

$= \frac{\tau_{max}}{2}$

solve

$\frac{\tau}{m}$ is lower

case 2:

$\tau = \frac{T\rho}{J}$

τ will be greater in this case

$J = \frac{\pi}{2}(r_o^4 - r_i^4)$

$m = \text{den} \times \text{vol}$
$= d \times \pi (r_o^2 - r_i^2) L$

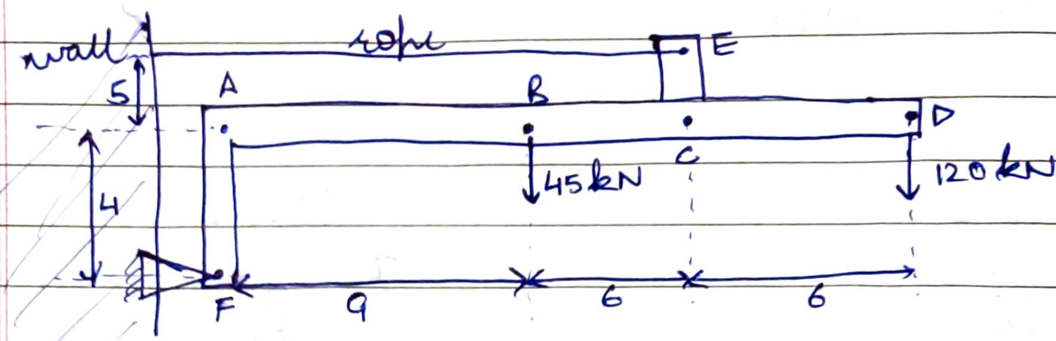
~~$\frac{\tau}{m}$ is higher for this~~

$\tau_{av} = \frac{\tau_{max} + \tau_{min}}{2}$

solve

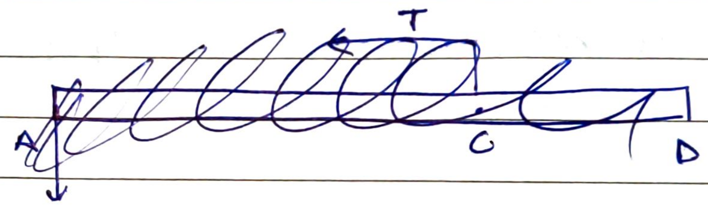
$\frac{\tau}{m}$ is higher.

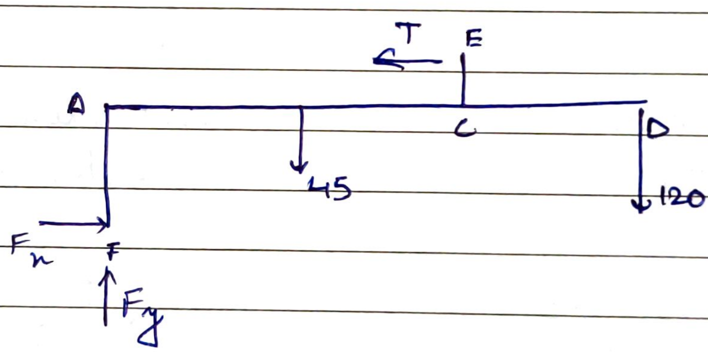
Que:



Draw SFD, BMD from A to D, Further find $\sigma_{max/allow}$

Ans:





$\Sigma F_x = 0 \Rightarrow F_x = T$

$\Sigma F_y = 0 \Rightarrow F_y = 45 + 120 = 165$ kN

$\Sigma M_A = 0 \Rightarrow 9 \times 45 - 5T + 120 \times 21 - F_x 4 = 0$

$\Rightarrow \cancel{9} \times 45 + \overset{40}{\cancel{120}} \times \overset{7}{\cancel{21}} = \cancel{9}T$

$\Rightarrow 45 + 280 = T$

$\Rightarrow T = 325$ kN

$F_x = 325$ kN

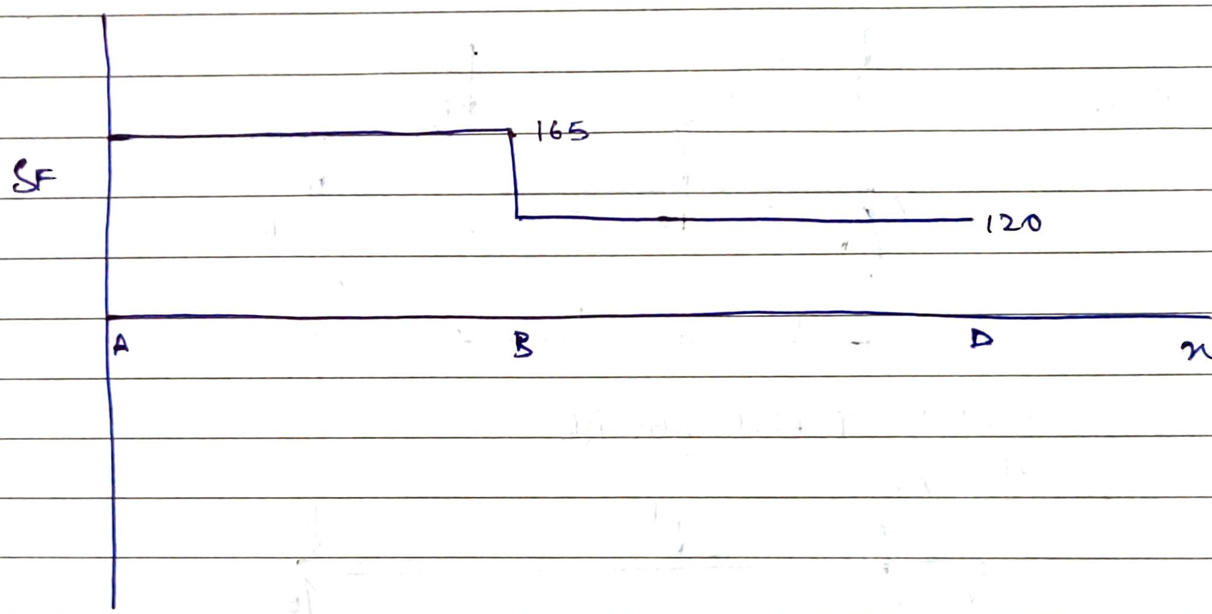
For SF

From A to B

SF = + 165 kN

From B to D

SF = + 120 kN



For BMD

From A to B

$BM = xF_y - 4F_x = 165x - 1300$

From B to C

$BM = F_y x - 4F_x - (x-9)45$

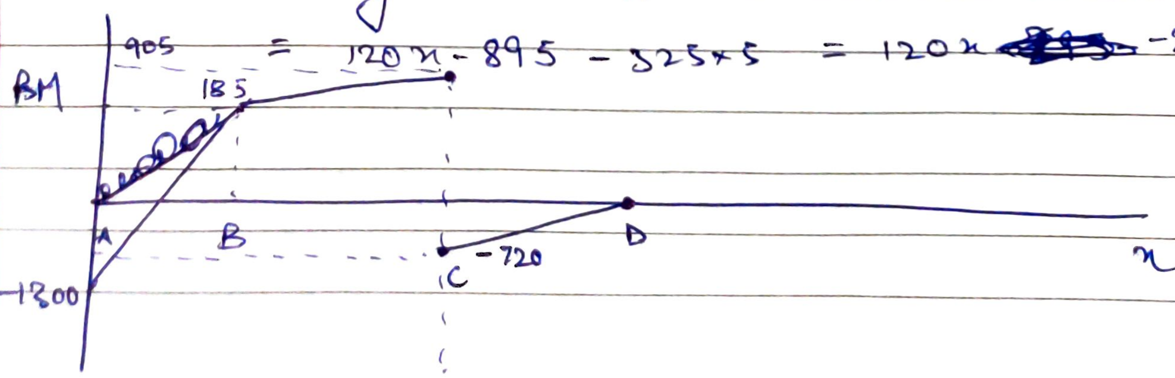
$= 165x - 1300 + 405 - 45x$

$= 120x - 895$

From C to D

$BM = F_y x - 4F_x - (x-9)45 - T\times 5$

$= 120x - 895 - 325\times 5 = 120x - 2520$

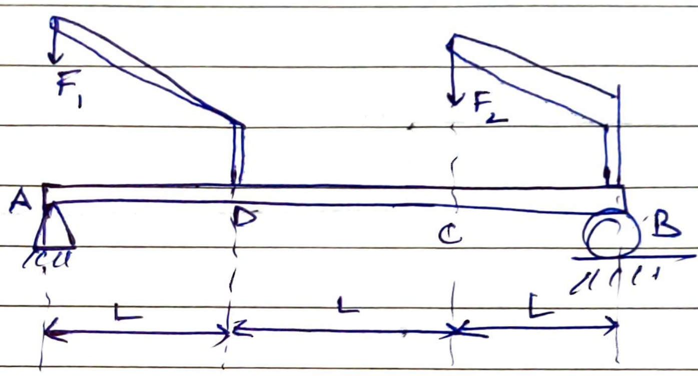


$$\sigma = \frac{My}{I}$$

$$\therefore \sigma_{max} = \frac{M_{max}\, y}{I}$$

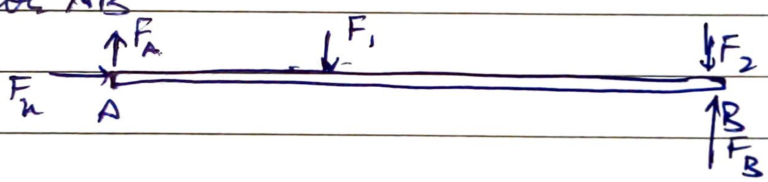
$$\sigma_{max} = \frac{1300 \times y}{I}$$ → for this problem.

Que:



Draw SFD, BMD for AB

Ans: For AB



$$\Sigma F_x = 0 \Rightarrow F_x = 0$$

$$\Sigma F_y = 0 \Rightarrow F_A + F_B = F_1 + F_2$$

$$\Sigma M_A = 0 \Rightarrow 2L F_2 = 3L F_B$$