PH301 Assignment - I

Interference, Diffraction and Polarization

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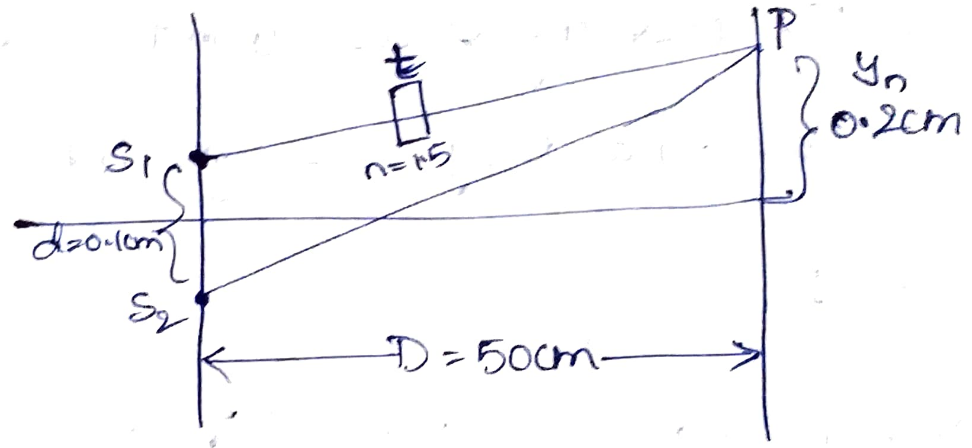
Answers

1. set-up looks like this



let thickness of mica sheet is t.

Path difference $= S_2P - [S_1P + (n-1)t] = 0$

$\Rightarrow S_2P - S_1P - (n-1)t = 0$

$\Rightarrow S_2P - S_1P = (n-1)t$

$\Rightarrow (n-1)t = \dfrac{y_n d}{D}$

$\Rightarrow (1.5-1)t = \dfrac{0.2 \times 0.1}{50}\,cm \Rightarrow 0.5t = 4\times10^{-4}$

$\Rightarrow t = 8 \times 10^{-4}$ cm

$\Rightarrow \boxed{t = 8\mu m}$

$\therefore$ $\boxed{\text{Thickness of mica sheet is } 8\mu m.}$

2.

The fringe pattern will disappear when destructive interference occurs between the two wavelengths.

Since, the wavelengths are almost equal, It is safe to assume they undergo interference.

For a fringe of order m,

We must have, $\left(m+\frac{1}{2}\right)\lambda_1 = m\lambda_2$ = Path diff.

where $\lambda_1 = 5890 A^\circ$ & $\lambda_2 = 5896 A^\circ$, $\lambda_1 < \lambda_2$

$\Rightarrow \left(m+\frac{1}{2}\right) 5890 = m \times 5896$

$\Rightarrow 5890m + 2945 = 5896m$

$\Rightarrow 6m = 2945 \Rightarrow m \approx 490.8$

Path diff. $= 490.8 \times 5896 A^\circ = \frac{y \cdot d}{D}$

where y is distance where fringe pattern disappears, from central fringe.

$d = 0.05cm \qquad D = 100cm$

$$\Rightarrow \quad y \times \frac{0.05}{100} = 490.8 \times 5896 \times 10^{-10}$$

$$\Rightarrow \quad \boxed{y = 0.578m = 57.8cm}$$

$\therefore$ The fringe pattern disappears at 57.8cm distance from central fringe. (either side)

3.

Fringe width is given by,

$$\boxed{\beta = \frac{\lambda D}{d}}$$

where λ is wavelength of light used in Young's experiment

D is distance between the slits and screen

d is distance between the slits

Given $\lambda = 3cm$

$D = 100cm$

<u>Case(i):</u> $d = 0.1cm$

$$\therefore \beta = \frac{\lambda D}{d} = \frac{3 \times 100}{0.1} cm = 3000 cm$$

$$\boxed{\beta = 30m}$$

<u>Case(ii):</u> $d = 1 cm$

$$\beta = \frac{\lambda D}{d} = \frac{3 \times 100}{1} cm = 300cm$$

$$\boxed{\beta = 3m}$$

<u>Case (iii):</u> $d = 4cm$

$$\beta = \frac{\lambda D}{d} = \frac{3 \times 100}{4} = 75cm$$

$$\boxed{\beta = 0.75m}$$

The fringe width of the pattern is

30m when $d = 0.1 cm$

3m when $d = 1 cm$

0.75m when $d = 4cm$

4.

When 2 light waves of intensities I_1, I_2 interfere at a point, then the resultant intensity is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$$

where θ is phase difference.

In Young's experiment, $I_1 = I_2 = I_0$ (say)

$$\Rightarrow \quad I = 2I_0 + 2I_0\cos\theta$$

$$= 2I_0(1+\cos\theta)$$

$$= 2I_0 \times 2\cos^2\frac{\theta}{2}$$

$$\Rightarrow \quad \boxed{I = 4I_0\cos^2\frac{\theta}{2}}$$

I_{max} occurs when the 2 waves are in-phase, i.e., $\theta = 2n\pi$, where n is an integer

$$\Rightarrow \quad \boxed{I_{max} = 4I_0}$$

~~For pat~~

$$\Rightarrow \quad \boxed{\frac{I}{I_{max}} = \cos^2\frac{\theta}{2}}$$

For path difference $= \frac{\lambda}{5}$, phase difference

is $\frac{2\pi}{\lambda} \cdot \frac{\lambda}{5}$

$$\Rightarrow \boxed{\theta = \frac{2\pi}{5}}$$

$$\Rightarrow \frac{I}{I_{max}} = \cos^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \frac{I}{I_{max}} = \cos^2\left(\frac{\frac{2\pi}{5}}{2}\right)$$

$$\Rightarrow \boxed{\frac{I}{I_{max}} = \cos^2\left(\frac{\pi}{5}\right) = \frac{3+\sqrt{5}}{8} = 0.65}$$

5.

Grating Equation is given by

$$d \sin\theta = m\lambda$$

where d is separation between 2 slits

θ is angle of diffraction

m is the order of spectrum

λ is wavelength of light

Given,

$$d = \frac{1}{1000} \text{ cm} = \underline{10^{-5} m}$$

$m = 2$

$\lambda = 650 nm$

$$\Rightarrow \sin\theta = \frac{m\lambda}{d} = \frac{2 \times 650 \times 10^{-9}}{10^{-5}}$$

$$= 1300 \times 10^{-4}$$

$$= 0.13$$

$$\Rightarrow \theta = \sin^{-1}(0.13)$$

$$\Rightarrow \boxed{\theta \approx 7.47^\circ}$$

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6. Grating Equation is $\boxed{d\sin\theta = m\lambda}$

$$\lambda = \frac{c}{\underbrace{v}_{\text{frequency of light}}} = \frac{3 \times 10^{8}}{4 \times 10^{14}} = \boxed{7.5 \times 10^{-7} m = \lambda}$$

$$d = \frac{1}{10,000} \text{ cm} = \boxed{10^{-6} m = d}$$

For highest order spectrum to be obtained $\sin\theta$ must have ~~highest~~ a value as large as possible.

$\therefore$ Consider $\sin\theta = 1$ for highest order spectrum.

$$\Rightarrow d \cdot 1 = n\lambda$$

$$\Rightarrow n = \frac{d}{\lambda}$$

$$= \frac{\cancel{10^{-6}}/10}{7.5 \times \cancel{10^{-7}}} = \frac{10 \times 10}{75}$$

$$\Rightarrow \boxed{n = \frac{4}{3} = 1.333}$$

As n can take only integer values, $n = 1$

$\therefore$ **Only First Order Spectrum is visible**

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7. Resolving power of a grating is given as,

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

where, λ is the wavelength of the spectrum

$\Delta\lambda$ is the seperation of the two wavelengths the grating can just resolve

m is the order of the spectrum

N is the total no. of lines in the grating

Given

$$\Delta\lambda = (5896 - 5890)\,A^\circ = 6A^\circ$$

$$\lambda = 5893A^\circ$$

$$\Rightarrow R = \frac{\lambda}{\Delta\lambda} = \frac{5893}{6} = 982.167$$

$$m = 3$$

$$\therefore 982.167 = 3N$$

$$\Rightarrow \boxed{N = 327.389}$$

Hence the grating must have 327 lines in order just seperate the sodium doublet in third order.

8.

The intensity distribution for Fraunhoffer diffraction pattern is given by

$$\boxed{I = I_0 \frac{\sin^2\beta}{\beta^2}}$$

where I_0 is the intensity at $\theta = 0^\circ$

θ is the angle of diffraction

and $\beta = \frac{\pi b \sin\theta}{\lambda}$

b is the width of the slit

λ is wavelength of light.

Now, $\lambda = 6000\,A^\circ$

$b = 0.02\,cm$

For minima to occur,

$$\beta = m\pi \quad , \quad m \neq 0$$

$$\Rightarrow \frac{\cancel{\pi} b \sin\theta}{\lambda} = m\cancel{\pi}$$

$$\Rightarrow \boxed{\sin\theta = \frac{m\lambda}{b},} \quad m \neq 0$$

$\therefore$ For first minima, $m = 1$

$$\Rightarrow \theta = \pm \sin^{-1}\left(\frac{\lambda}{b}\right)$$

$$\boxed{\theta = \pm 0.17^\circ}$$

For second minima, $m = 2$

$$\Rightarrow \theta = \pm \sin^{-1}\left(\frac{2\lambda}{b}\right)$$

$$= \pm \sin^{-1}\left(\frac{\cancel{2} \times 6 \times \cancel{10^{-7}}^{10^{-3}}}{\cancel{2} \times \cancel{10^{-4}}}\right)$$

$$\Rightarrow \boxed{\theta = \pm 0.34^\circ}$$

For maxima to occur,

$$\tan\beta = \beta,$$

first two solutions for this equation are,

$$\beta = 1.43\pi, \quad \beta = 2.46\pi$$

For first maxima, $\frac{\cancel{\pi} b \sin\theta}{\lambda} = 1.43\cancel{\pi}$

$$\Rightarrow \theta = \pm \sin^{-1}\left(\frac{1.43\cancel{\pi} \times 6 \times 10^{-7}}{\cancel{\pi} \times 2 \times 10^{-4}}\right)$$

$$\Rightarrow \boxed{\theta = \pm 0.24^\circ}$$

For second maxima,

$$\frac{\cancel{\pi} b \sin\theta}{\lambda} = 2.46\cancel{\pi}$$

$$\Rightarrow \theta = \pm \sin^{-1}\left(\frac{2.46 \times 6 \times 10^{-7}}{2 \times 10^{-4}}\right)$$

$$\Rightarrow \boxed{\theta = \pm 0.42^\circ}$$

∴ First Minima occurs at $\theta = 0.17^\circ$

Second Minima occurs at $\theta = 0.34^\circ$

First Maxima occurs at $\theta = 0.24^\circ$

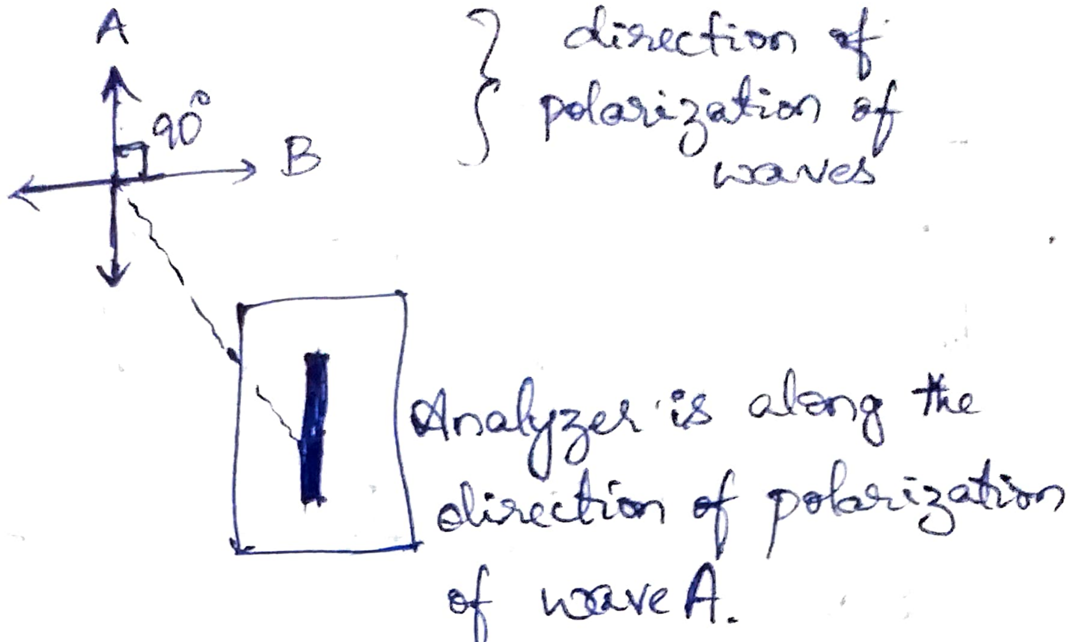
Second Maxima occurs at $\theta = 0.42^\circ$

9.

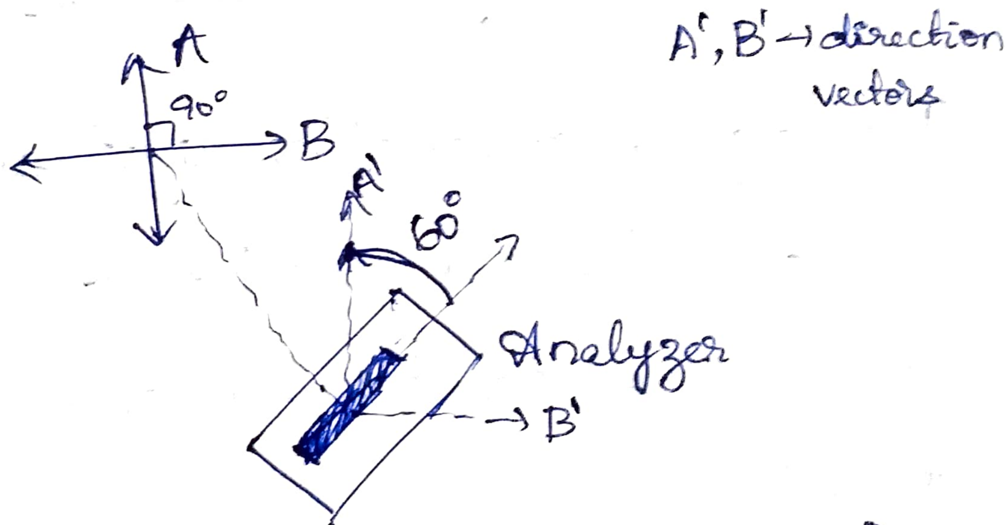
In the first orientation of the analyser, the intensity of wave B is zero.

∴ The direction of analyzer is perpendicular to direction of polarization of ~~plane~~ wave B.

So, the scenario can be visualized as.



Now, the analyzer is rotated through 60°.



Let intensities of waves A, B are I_A, I_B respectively.

Given that $I_A \cos^2 60° = I_B \cos^2(90° - 60°)$

$\Rightarrow I_A \times \frac{1}{4} = I_B \cos^2 30 = I_B \times \frac{3}{4}$

$\Rightarrow \boxed{\frac{I_A}{I_B} = 3}$

$\therefore$ $\boxed{\text{Intensity of wave A is \underline{thrice} that of wave B}}$

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10. If intensity of incident light is I_0, the emergent light from first polarizer has intensity $I_0 \cos^2\theta$ as per Malus' Law.

This light is polarized at an angle θ with the horizontal direction.

The angle between direction of polarization of this emergent beam and second polarizer is 2θ.

$\therefore$ The intensity of final emergent beam is found from Malus' Law as

$$\boxed{I_0 \cos^2\theta \cos^2 2\theta}$$

It's direction of polarization ~~m~~ makes angle of $-\theta$ with the horizontal direction.

It is linearly polarized wave

The intensity of the emergent light from second polarizer becomes zero when,

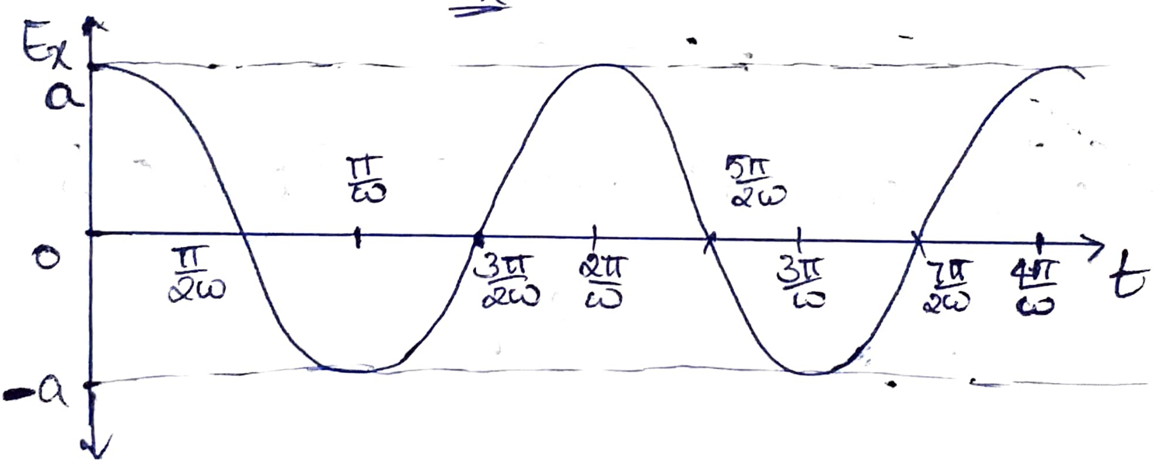
$$\cos^2\theta \cos^2 2\theta = 0$$

$$\Rightarrow \cos\theta \cos 2\theta = 0$$

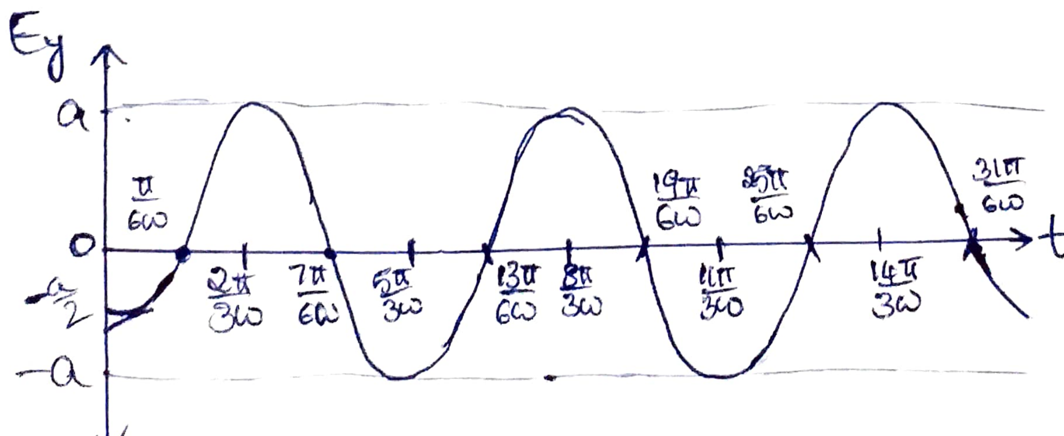
This happens when θ is an odd multiple of 45° or 90° or 135°

11.

E_x



E_y



State of Polarization

$$\frac{E_2}{E_1} = \tan\frac{\theta}{2} \quad , \quad \theta = \frac{2\pi}{3}$$

$$= \tan\frac{\pi}{3}$$

$$\Rightarrow \boxed{\frac{E_2}{E_1} = \sqrt{3} = 1.732}$$

$\therefore$ The resultant wave is Right Elliptically Polarized

12. Since, the incident beam is left circularly polarized,

the electric field for the incident beam at $x=0$ is

$$E_y = \frac{E_0}{\sqrt{2}} \sin(\omega t)$$

$$E_z = \frac{E_0}{\sqrt{2}} \cos(\omega t)$$

where E_0 is amplitude of electric field.

Phase difference between o-wave and e-wave emerging from crystal is

$$\theta = \frac{\omega}{c}(n_o - n_e)d$$

$$= \frac{2\pi}{\lambda}(n_o - n_e)d$$

$$= \frac{2\pi}{589.3\times10^{-9}} \times (1.65836 - 1.48641) \times 0.005141\times10^{-3}$$

$$\Rightarrow \theta \approx 2\pi \times 1.5 \Rightarrow \boxed{\theta \approx 3\pi}$$

$\therefore$ The emergent wave will be

$$E_y = \frac{E_0}{\sqrt{2}} \sin(\omega t - 3\pi)$$

$$\boxed{\begin{aligned} E_y &= -\frac{E_0}{\sqrt{2}} \sin(\omega t) \\ E_z &= \frac{E_0}{\sqrt{2}} \cos(\omega t) \end{aligned}}$$

$\therefore$ The emergent wave is right circularly polarized