23.09.2020

CS303- Tutorial 4

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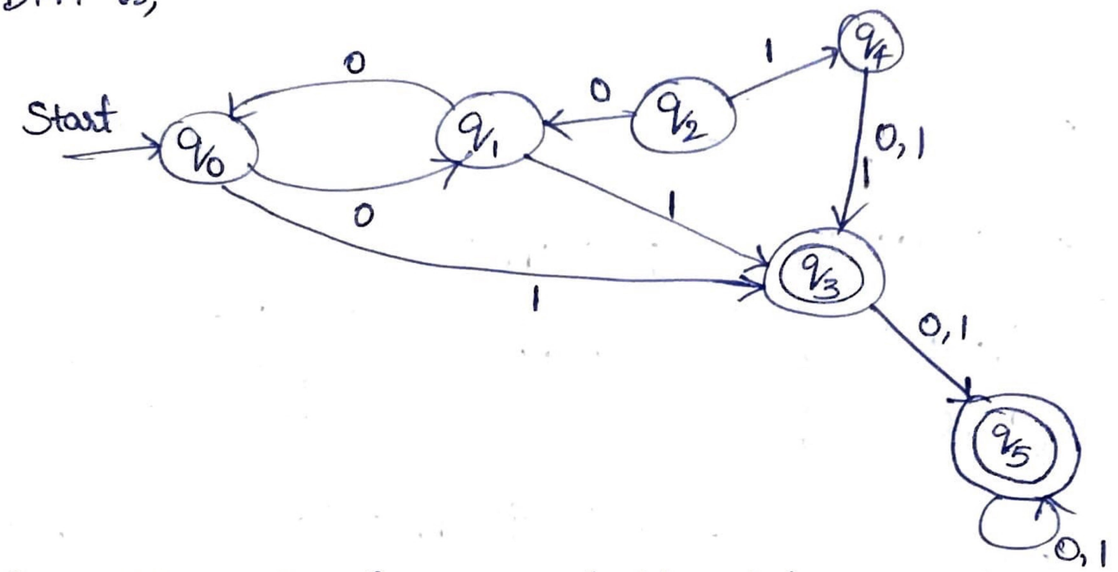
Ans 1:

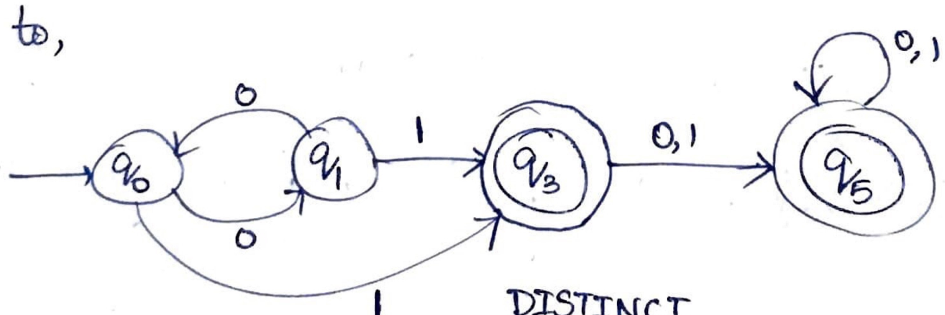
Given DFA is,



Step 0:

In this DFA, q_2 is unreachable state and q_4 can be reached only from q_2. Hence q_2 and q_4 can be removed.

DFA reduces to,



Step 1:

Construct the lower-triangular table DISTINCT, initially blank. For every pair of states (p, q), if p is final and q is not or vice-versa, then DISTINCT$(p,q) = \epsilon$

DISTINCT
after Step 1

q_0				
q_1				
q_3	e	ϵ		
q_5	ϵ	ϵ		
	q_0	q_1	q_3	q_5

Step 2: For every pair of states (p,q) and each symbol $\alpha \in \{0,1\}$,

If DISTINCT(p,q) is blank and DISTINCT$(\delta(p,\alpha), \delta(q,\alpha))$ is not blank

then, DISTINCT$(p,q) = \alpha$

for $(q_1, q_0) \rightarrow$ DISTINCT(q_1, q_0) is blank

$\left.\begin{array}{l}\delta(q_1,0) = q_0 \\ \delta(q_0,0) = q_1\end{array}\right\} \rightarrow$ DISTINCT is blank

$\left.\begin{array}{l}\delta(q_1,1) = q_3 \\ \delta(q_0,1) = q_3\end{array}\right\} \rightarrow$ DISTINCT is blank

Also, for $(q_5, q_3) \longrightarrow$ DISTINCT(q_5, q_3) is blank,

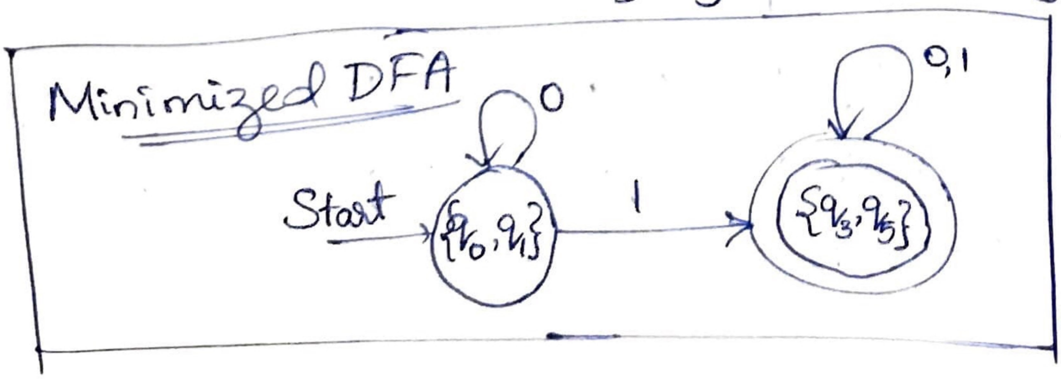
$\left.\begin{array}{l}\delta(q_5,0) = q_5 \\ \delta(q_3,0) = q_5\end{array}\right\} \rightarrow$ DISTINCT is blank

$\left.\begin{array}{l}\delta(q_5,1) = q_5 \\ \delta(q_3,1) = q_5\end{array}\right\} \rightarrow$ DISTINCT is blank.

So, DISTINCT table doesn't change from Step 1 to Step 2.

q_1, q_0 can be merged and

q_5, q_3 can be merged



Ans 2:

Given: Language $L = \{a^k \mid k \text{ is a prime number}\}$

To prove: L is not regular

Proof: Assume L is regular language

Then $\exists\, p > 0$ such that for any string $s \in A$ with $|s| \geq p$,

the string can be divided into 3 pieces: $s = xyz$, such that

i) for each $i \geq 0$, $xy^iz \in A$

ii) $|y| > 0$

iii) $|xy| \leq p$

Consider string $s = a^q$ such that $|s| = q \geq p$ and q is prime.

As there is no number called as largest prime number, we can always choose a prime number greater than the given prime number.

Now, split $s = a^q$ into 3 pieces: x, y, z such that

$y = a^m \Rightarrow |y| = m > 0$ and $|xy| \leq p$

Since, L is regular, there exists another string $w = xy^iz$ such that $|w|$ is prime.

Now, $w = xy^iz \Rightarrow |w| = |xy^iz| = |xyz| + |y^{i-1}|$

$\Rightarrow |w| = |s| + m(i-1) = q + m(i-1)$

For $i = q+1$, $\boxed{|w| = q + m(q+\cancel{1}-\cancel{1}) = m(1+q)}$

For $i = q+1$, $|w|$ has turned out to be $m(1+q)$

clearly, $m > 0$ and $(1+q) > 0$

$\therefore$ $|w|$ is composite.

But as per our initial assumption that L is regular language, $|w|$ should be prime.

We obtained a contradiction. So, the assumption is wrong.

$\therefore$ $\boxed{L = \{a^k \mid k \text{ is prime number}\} \text{ is not a regular language}}$

Hence Proved.