Tutorial-4

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1) From the given DFA, we can obtain the following tables.

	0	1
q_0	q_1	q_3
q_1	q_0	q_3
q_2	q_1	q_4
q_3	q_5	q_5
q_4	q_3	q_3
q_5	q_5	q_5

q_1					
q_2					
q_3	ε	ε	ε		
q_4				ε	
q_5	~~ε~~	~~ε~~	~~ε~~		ε
	q_0	q_1	q_2	q_3	q_4

First Iteration :

(i) Distinct (q_1, q_0) = blank

$\delta(q_1, 0) = q_0$; $\delta(q_0, 0) = q_1$

$\delta(q_0, 1) = q_3$, $\delta(q_1, 1) = q_3$

$\Rightarrow$ Distinct (q_0, q_1) = blank

$\Rightarrow$ Distinct (q_0, q_1) = blank

(ii) Distinct (q_2, q_0) = blank

$\delta(q_2, 0) = q_1$, $\delta(q_0, 0) = q_1$

$\delta(q_2, 1) = q_4$, $\delta(q_0, 1) = q_3$

$\Rightarrow$ Distinct $(q_2, q_0) = 1$

(iii) Distinct (q_4, q_0) = blank

$\delta(q_4, 0) = q_3$; $\delta(q_0, 0) = q_1$

$\Rightarrow$ Distinct $(q_4, q_0) = 0$

(iv) Distinct (q_2, q_1) = blank

$\delta(q_2, 0) = q_1$, $\delta(q_1, 0) = q_0$

$\delta(q_2, 1) = q_4$, $\delta(q_1, 1) = q_3$

$\Rightarrow$ Distinct $(q_2, q_1) = 1$

(v) Distinct (q_4, q_1) = blank

$\delta(q_4, 0) = q_3$, $\delta(q_1, 0) = q_0$

$\Rightarrow$ Distinct $(q_4, q_1) = 0$

(vi) Distinct (q_4, q_2) = blank

$\delta(q_4, 0) = q_3$, $\delta(q_2, 0) = q_1$

$\Rightarrow$ Distinct $(q_4, q_2) = 0$

(vii) Distinct (q_5, q_3) = blank

$\delta(q_5, 0) = q_5$, $\delta(q_3, 0) = q_5$

$\delta(q_5, 1) = q_5$, $\delta(q_3, 1) = q_5$

No update

q_1					
q_2	1	1			
q_3	ε	ε	ε		
q_4	0	0	0	ε	
q_5	ε	ε	ε		ε
	q_0	q_1	q_2	q_3	q_4

Second Iteration:

i) Distinct (q_1, q_0) = blank

$\delta(q_1, 0) = q_0$, $\delta(q_0, 0) = q_1$

$\delta(q_1, 1) = q_3$, $\delta(q_0, 1) = q_3$

$\Rightarrow$ No update

ii) Distinct (q_5, q_3) = blank

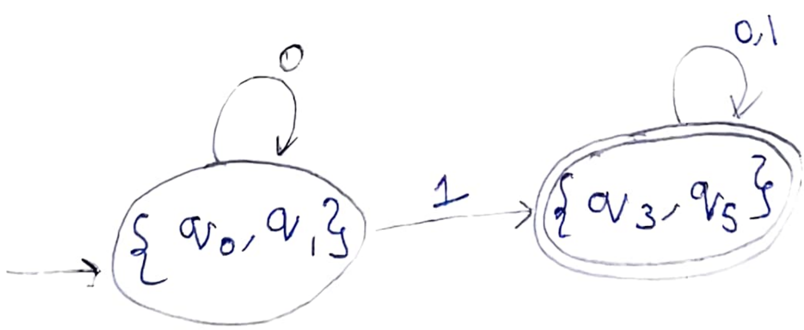
$\delta(q_5, 0) = q_5$, $\delta(q_3, 0) = q_5$

$\delta(q_5, 1) = q_5$, $\delta(q_3, 1) = q_3$

$\Rightarrow$ No Update

$\therefore$ Resulting Table is same.

Minimal DFA:



2) RTP: $L = \{a^k \mid k \text{ is a prime number}\}$ is not regular using pumping lemma.

Proof: Proof by contradiction

We first assume L is regular. And therefore has to satisfy the lemma.

Let the pumping length be "m" and consider $s \in L$ such that $|s| \geq m$.

i.e) Let $s = a^n$ $\quad n \geq m$

$$s = \overbrace{\underbrace{a \; a}_{a^k} \; \underbrace{a \; \cdots}_{a^{m-k}}}^{m} \underbrace{\cdots\cdots a}_{a^{n-m}}$$

$x \quad y \quad z$

i.e) $s = xyz$

$= a^k a^{m-k} a^{n-m}$

According to the lemma $s = xy^iz \quad \forall\, i \geq 0$ should belong to L aswell.

i.e) $s = (a^k)^i (a^{m-k}) (a^{n-m})$

$$= a^{n-\cancel{m}+\cancel{m}-k+ik} = a^{n+k(i-1)}$$

i.e) $n+k(i-1)$ should be prime $\forall\, i \geq 0$

but we can see that for $i = n+1$

$n + k(n+1-1) = n + kn$

$= n(1+k)$ which is clearly not prime. (as 2 factors $n, k+1$)

$\therefore$ Hence proved.