Tutorial - 7

CS - 303

M. Nitesh Reddy

1801CS32

1A:- We will prove this by using contradiction.
So, basically we will take two Languages which are context free grammar and we will try to prove that their intersection is not context free.

So,

$L_1 = \{a^n b^n c^m \mid n \geq 0 \text{ and } m \geq 0\}$

and

$L_2 = \{a^m b^n c^n \mid n \geq 0 \text{ and } m \geq 0\}$

So, both L_1 & L_2 are context free Languages.
now we will try to find $L_1 \cap L_2$

So, $L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$

In push down automata we can only compare two variables at time. But here we want to make sure that ~~a~~ no. of a is equal to no. of b's and that is equal to no. of c. so, we cannot compare all three at once. so, it cannot be accepted by push down automata.
Hence it is not context free.

$\therefore$ $L_1 \cap L_2$ is not context free.

$\therefore$ By contradiction if L_1, L_2 are context free then

$L_1 \cap L_2$ may not be context free.

2A:- Given

$L_1 = \{a^n b^n c^m\}$

$L_2 = \{a^m b^n c^n\}$

are two CFL.

Then we have comment about $L_1 \cap L_2$.

$L_1 \cap L_2 = ?$

L_1 says number of a's should be equal to number of b's and L_2 says number of b's equal to number of c's. So, their intersection says both conditions need to be true, but push down automata can compare only two. automata so, it cannot be accepted by push down

· Hence, $L_1 \cap L_2$ is not a context free grammar.

(Detailed explaination was done in 1 Ans).

3Ans:- CFLs are not closed under complement.

If L_1 is a CFL, then $\overline{L_1}$ may not be a CFL.

proof:

we know that CFL are closed under union & they are are not closed under intersection.

1. Assume the complement of every CFL is a CFL.
2. Let L_1 & L_2 are 2 CFLs
3. Since CFLs are closed under union, and we are assuming they are closed under complement,

$\overline{L_1 \cup L_2} = L_1 \cap L_2$ is a CFL

4. However, we know there are CFLs whose intersection is not a CFL.

5. Therefore, our assumption that CFLs are closed under complement is false.

Example

$L_1 = \{ a^i b^j c^k \mid i \neq j \}$

$L_2 = \{ a^i b^j c^k \mid j \neq k \}$

$L_3 = \{ a^i b^j c^k \mid k \neq i \}$

$L_4 = \overline{a^* b^* c^*}$

$L_5 = L_1 \cup L_2 \cup L_3 \cup L_4$

$\overline{L_5} = \{ a^i b^j c^k \mid i = j \,\&\, j = k \}$

L_1, L_2, L_3, L_4 all are context free

$\therefore$ L_5 which is union of 4 context free grammars is also context free

$\therefore$ L_5 is a context free grammar.

$\overline{L_5}$ is not a context free grammar because we cannot compare all three values in push down automata.

$\therefore$ L_5 is CFL but $\overline{L_5}$ is not a CFL.