

Tut 10 (S 303)

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Q1) RTP: A CFL is not closed under complementation.

Assume that $\overline{L_1}$ is a context free language.

Context free languages are closed under union.

$\Rightarrow L_1 \cup \overline{L_1}$ is also context free.

$$\text{But, } \overline{L_1 \cup \overline{L_1}} = \overline{L_1} \cap L_1$$

As we have assumed, complementation is also closed.

$\Rightarrow \overline{L_1 \cup \overline{L_1}}$ is also context free.

$\Rightarrow \overline{L_1} \cap L_1$ is also context free.

i.e) Intersection of L_1 & $\overline{L_1}$ (two CFL) is also context free.

(But we know that intersection of two CFL is not always CFL)

\therefore Contradiction

\therefore Initial assumption that $\overline{L_1}$ is CFL is wrong.

Q2) Given, L is a CFL

R is a Regular Language

$\Rightarrow R$ is also a CFL (According to Chomsky Hierarchy).

We know, UNION of any two CFL languages is also context free.

{ In general: For context free languages L_1, L_2 with context free grammars G_1, G_2 and start variables S_1, S_2

The grammar of the union $L_1 \cup L_2$ has a new start variable S and additional product
on $S \rightarrow S_1 \mid S_2$

So, L and R both are CFLs

$\therefore L \cup R$ is also a CFL, Hence proved.

3) (a) All strings over $\{0, 1\}$ with the substring '0101'

Alphabet: $\Sigma = \{0, 1\}$

Reg. Exp: $(0+1)^* 0101 (0+1)^*$

- (b) All strings beginning with '1' and ending with 'ab'.

Alphabet : $\Sigma = \{0, 1, a, b\}$

Reg-Exp : $1(0+1+a+b)^*ab$

- (c) Set of all strings over $\{a, b\}$ with 3 consecutive b's.

Alphabet : $\Sigma = \{a, b\}$

Reg-Exp : $(a+b)^* bbb (a+b)^*$

- (d) Set of all strings that end with '1' and has no substring '00'.

Alphabet : ~~$\Sigma = \{0, 1, a, b\}$~~ $\Sigma = \{0, 1\}$

Reg-Exp : ~~$(0+1)^*$~~ $(01+1)(01+1)^*$

Q4)

Substring length	Count	Eg: abcde
0	1	$\{\lambda\}$
1	n	$\{a, b, c, d, e\}$
2	n-1	$\{ab, bc, cd, de\}$
3	n-2	$\{abc, bcd, cde\}$
\vdots	\vdots	
n	1	$\{abcde\}$

$$\text{Total count} = (1+2+3+\dots+n) + 1 = \frac{(n)(n+1)}{2} + 1$$

∴ the total no. of substrings can be formed.