DPDA

Deterministic PDA

Deterministic PDA: DPDA

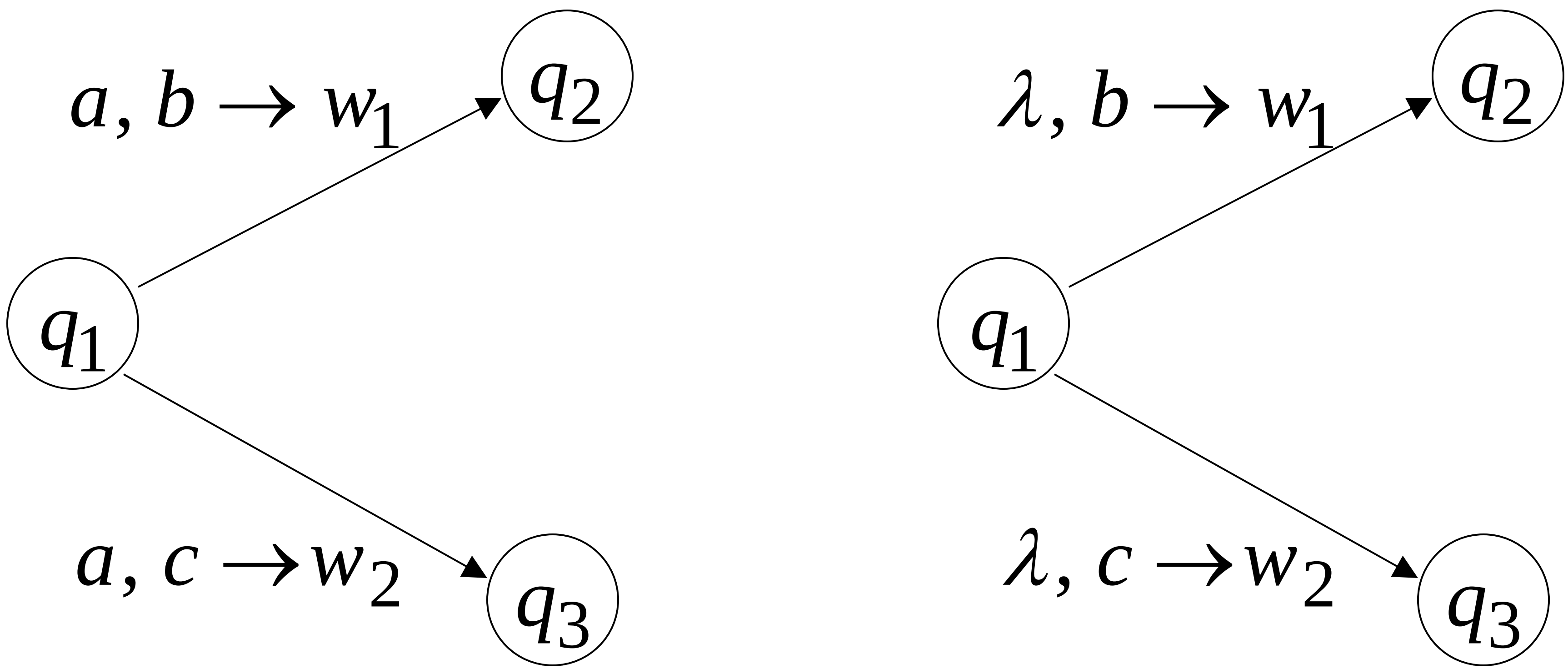
Allowed transitions:

$$q_1 \xrightarrow{a,\, b \to w} q_2$$

$$q_1 \xrightarrow{\lambda,\, b \to w} q_2$$

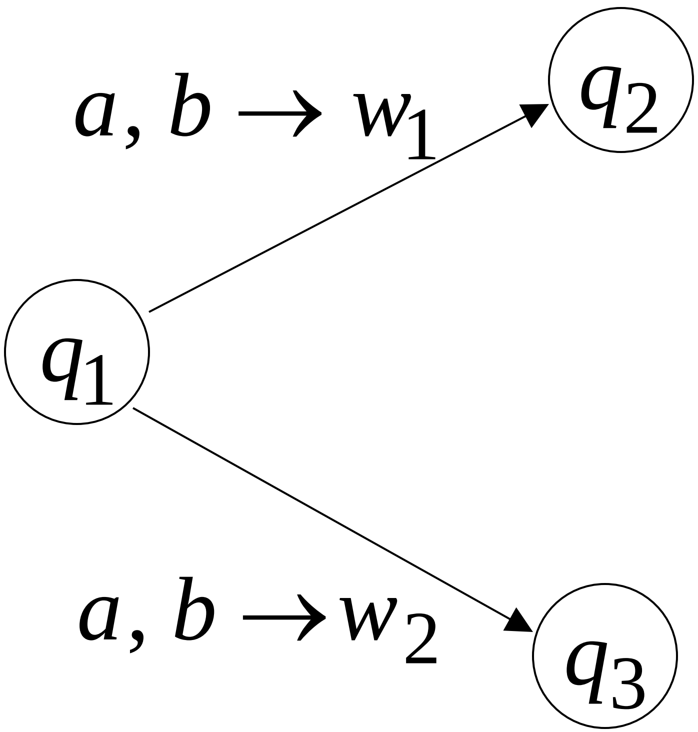
(deterministic choices)

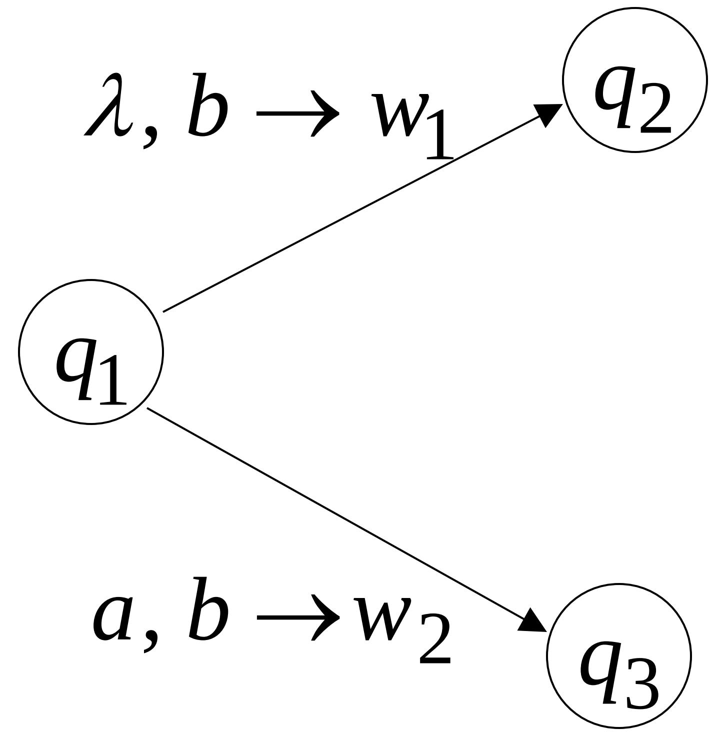
Allowed transitions:



(deterministic choices)

Not allowed:

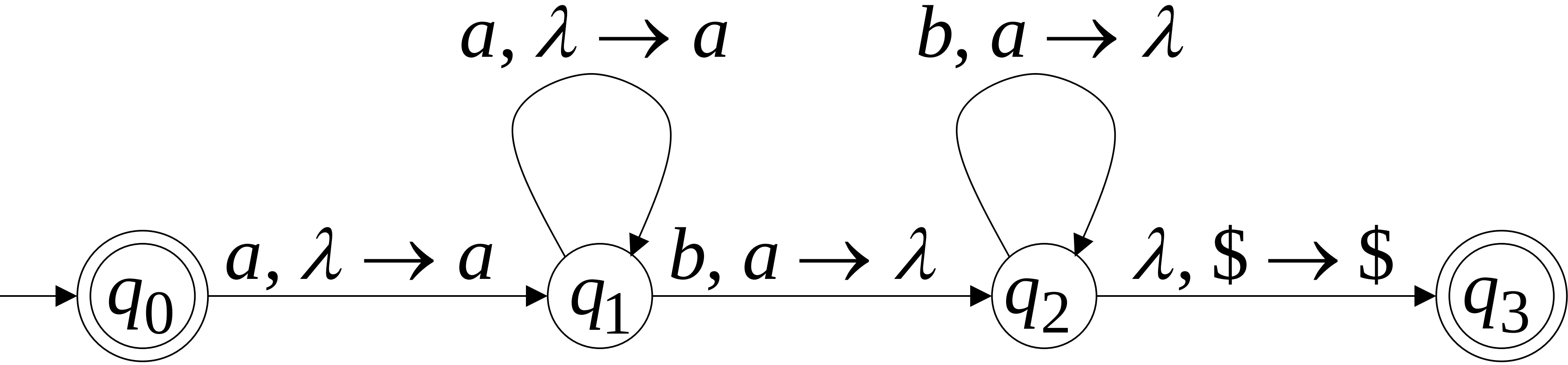




(non deterministic choices)

DPDA example

$$L(M) = \{a^n b^n : n \geq 0\}$$



Definition:

A language L is **deterministic context-free** if there exists some DPDA that accepts it

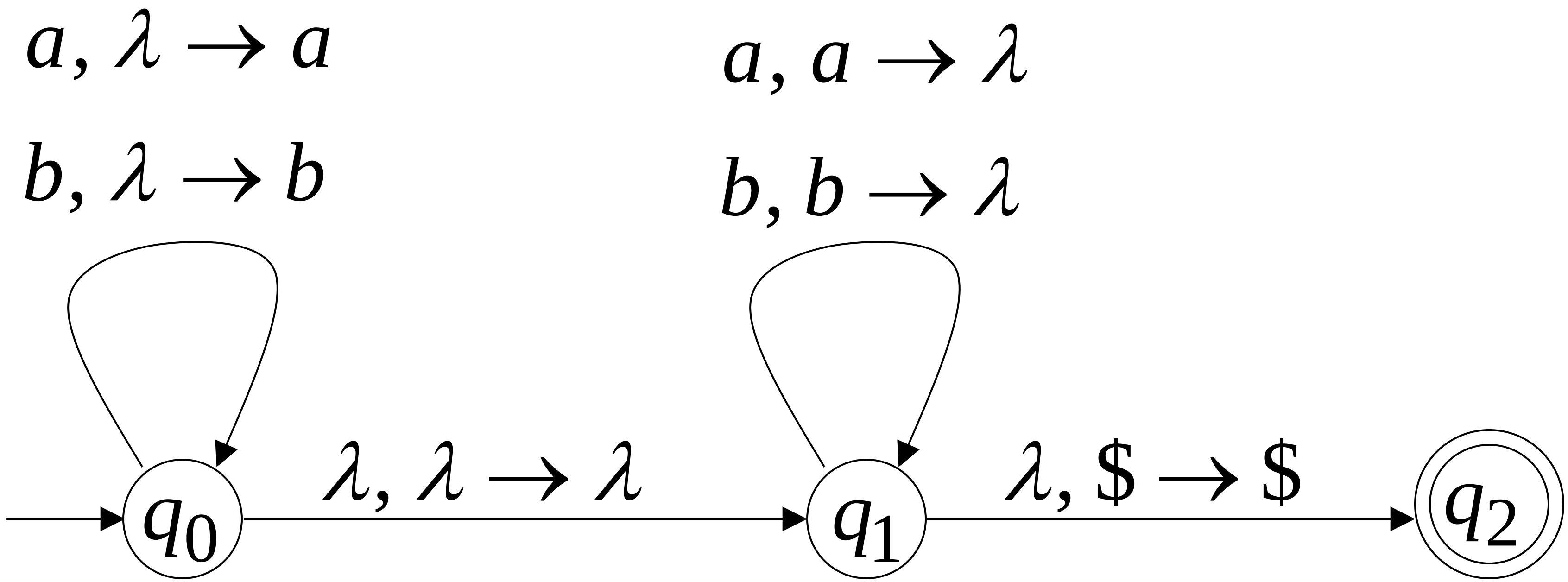
Example:

The language $L(M) = \{a^n b^n : n \geq 0\}$

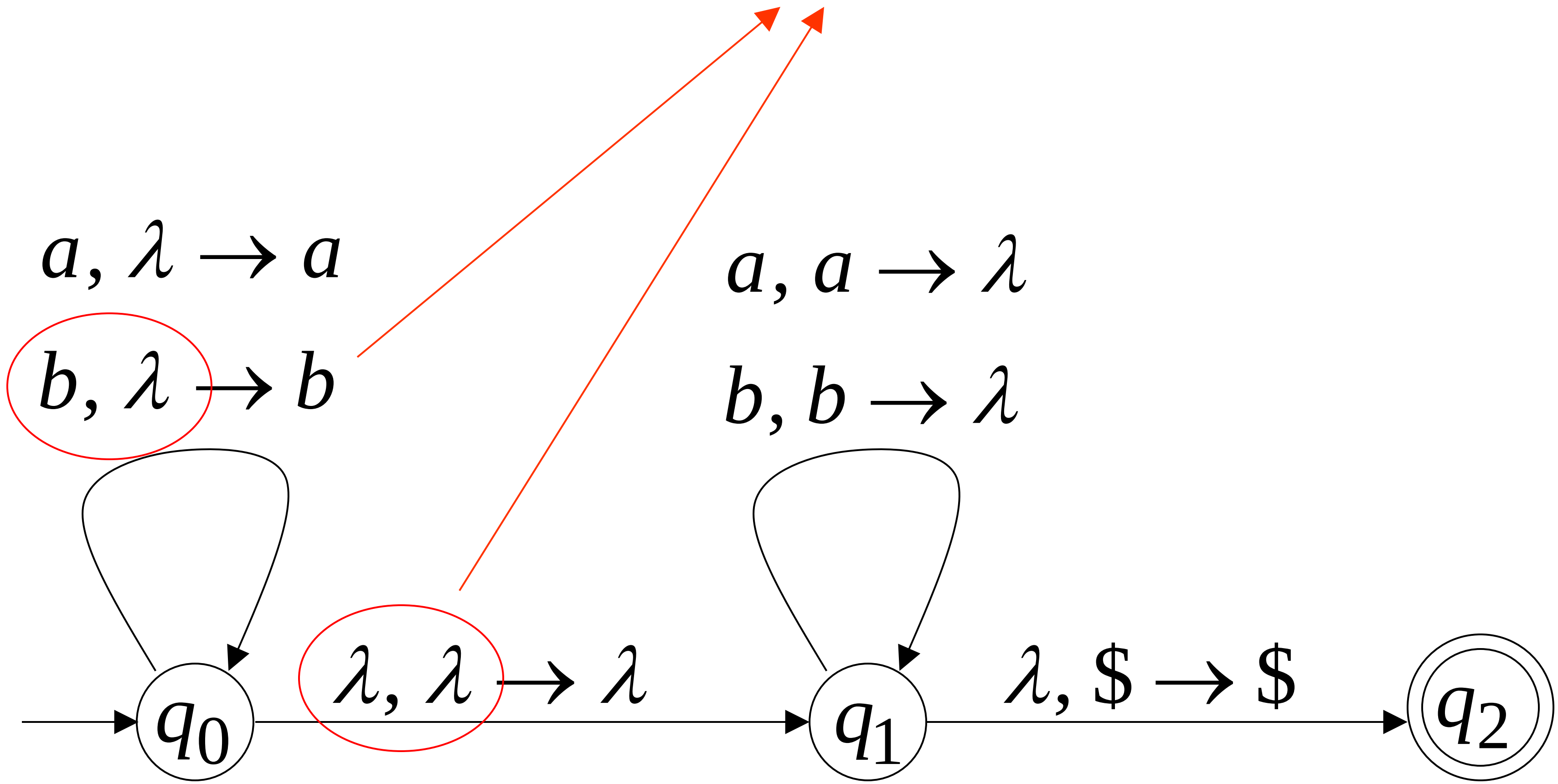
is **deterministic context-free**

Example of Non-DPDA (PDA)

$$L(M) = \{vv^R : v \in \{a,b\}^*\}$$



Not allowed in DPDAs



PDAs

Have More Power than

DPDAs

It holds that:

$$\left\{\begin{array}{c}\text{Deterministic}\\ \text{Context-Free}\\ \text{Languages}\\ \text{(DPDA)}\end{array}\right\} \subseteq \left\{\begin{array}{c}\text{Context-Free}\\ \text{Languages}\\ \text{PDAs}\end{array}\right\}$$

Since every DPDA is also a PDA

We will actually show:

$$\underset{L \notin}{\left\{ \begin{array}{c} \text{Deterministic} \\ \text{Context-Free} \\ \text{Languages} \\ \text{(DPDA)} \end{array} \right\}} \subset \underset{L \in}{\left\{ \begin{array}{c} \text{Context-Free} \\ \text{Languages} \\ \text{(PDA)} \end{array} \right\}}$$

We will show that there exists a context-free language L which is not accepted by any DPDA

The language is:

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\} \qquad n \geq 0$$

We will show:

- L is context-free
- L is **not** deterministic context-free

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

Language L is context-free

Context-free grammar for L :

$$S \to S_1 \mid S_2 \qquad \{a^n b^n\} \cup \{a^n b^{2n}\}$$

$$S_1 \to a S_1 b \mid \lambda \qquad \{a^n b^n\}$$

$$S_2 \to a S_2 bb \mid \lambda \qquad \{a^n b^{2n}\}$$

Theorem:

The language $L = \{a^n b^n\} \cup \{a^n b^{2n}\}$

is **not** deterministic context-free

(there is **no** DPDA that accepts L)

Proof: Assume for contradiction that

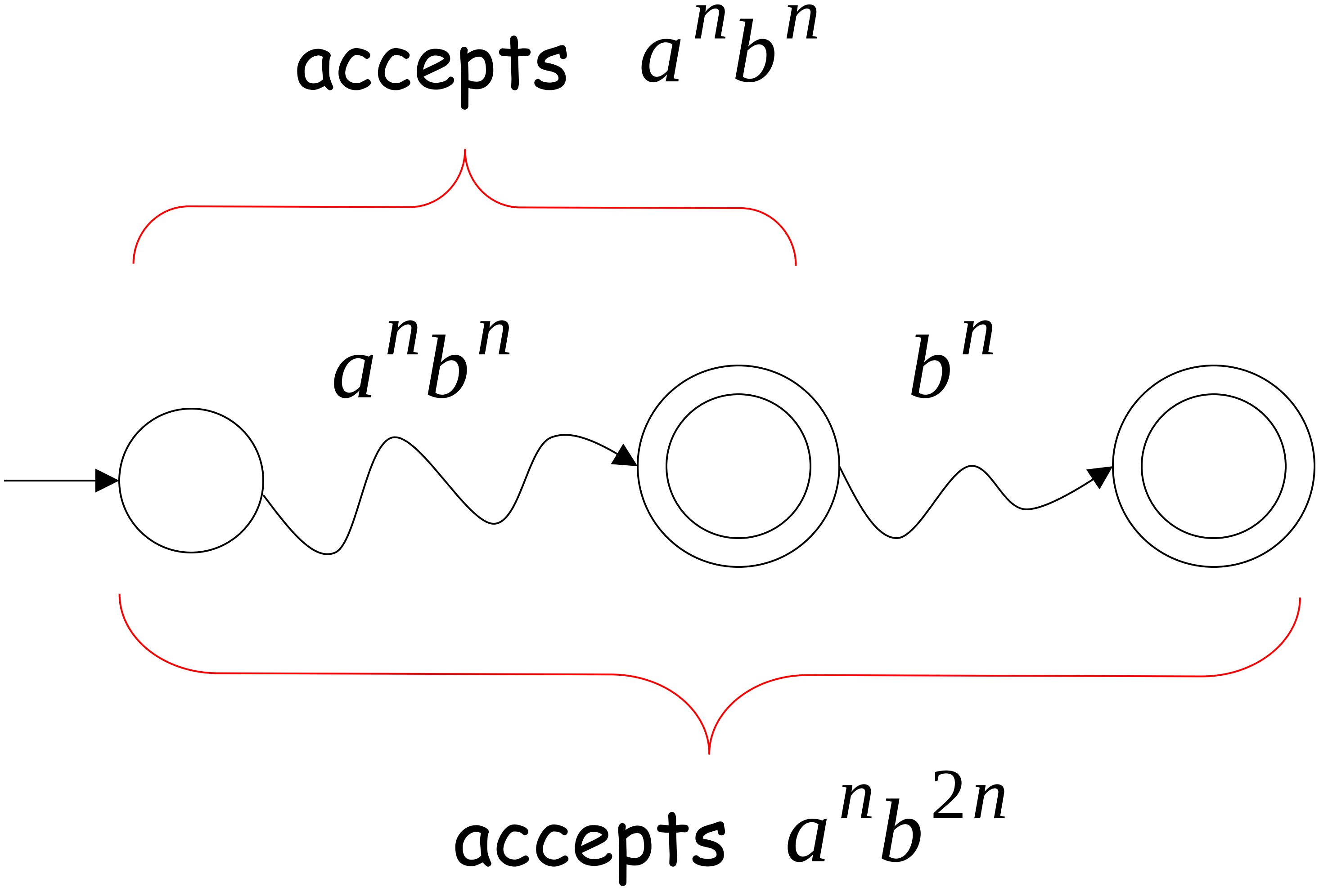
$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

is deterministic context free

Therefore:

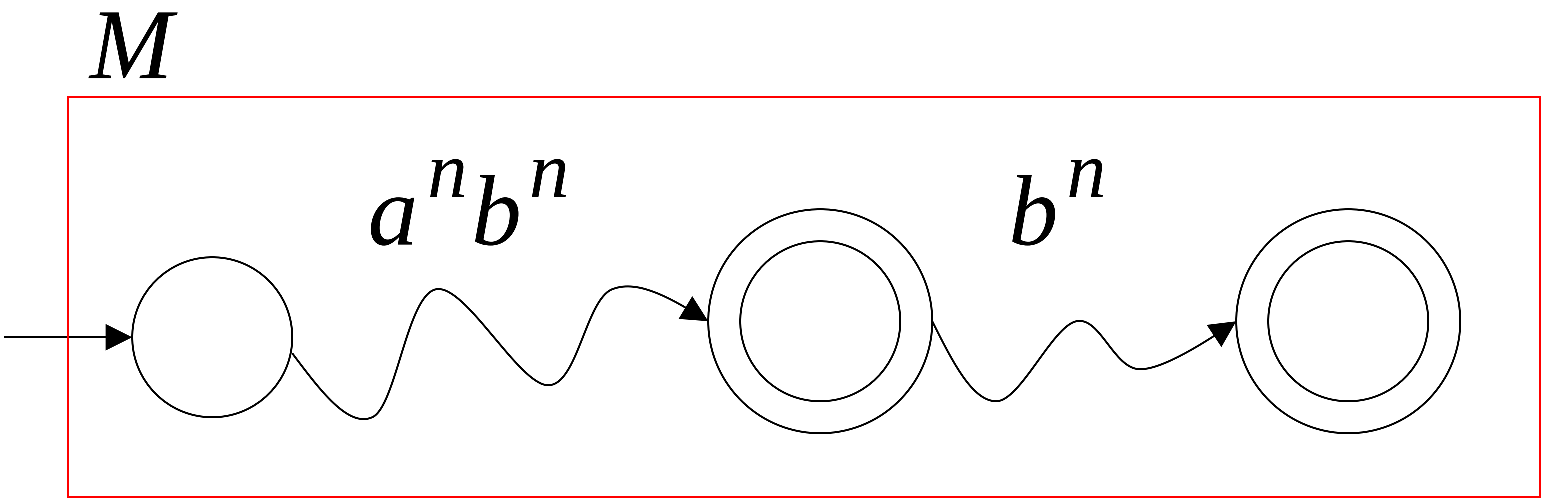
there is a DPDA M that accepts L

DPDA M with $L(M) = \{a^n b^n\} \cup \{a^n b^{2n}\}$



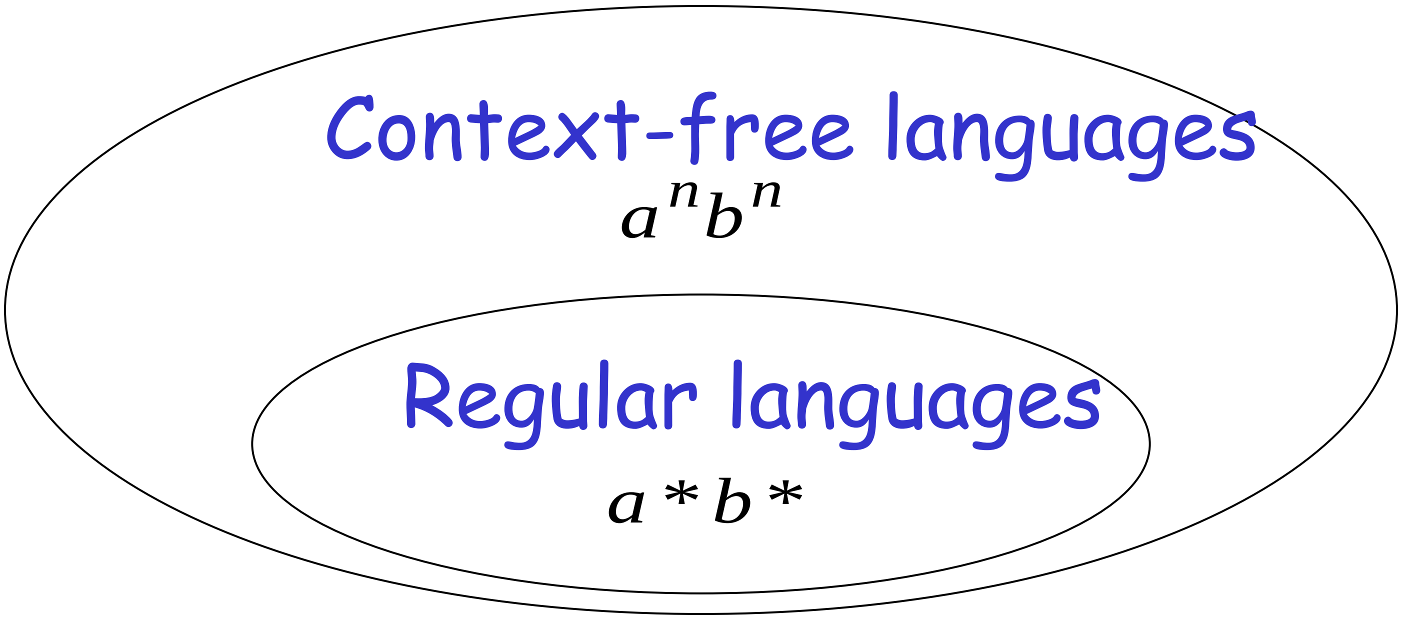
DPDA M with $L(M) = \{a^n b^n\} \cup \{a^n b^{2n}\}$

Such a path exists due to determinism



Fact 1: The language $\{a^n b^n c^n\}$

is not context-free



(we will prove this at a later class using pumping lemma for context-free languages)

Fact 2: The language $L \cup \{a^n b^n c^n\}$ is **not** context-free

$$(L = \{a^n b^n\} \cup \{a^n b^{2n}\})$$

(we can prove this using pumping lemma for context-free languages)

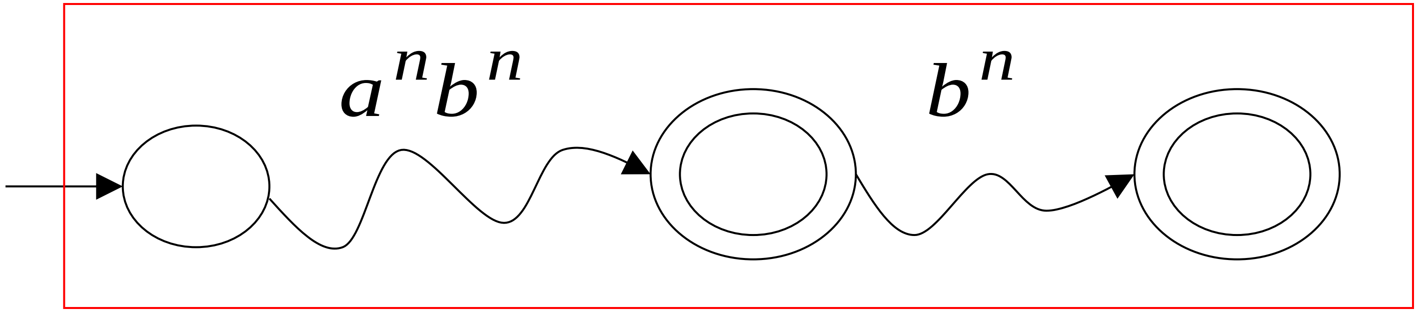
We will construct a PDA that accepts:

$$L \cup \{a^n b^n c^n\}$$

$$(L = \{a^n b^n\} \cup \{a^n b^{2n}\})$$

which is a contradiction!

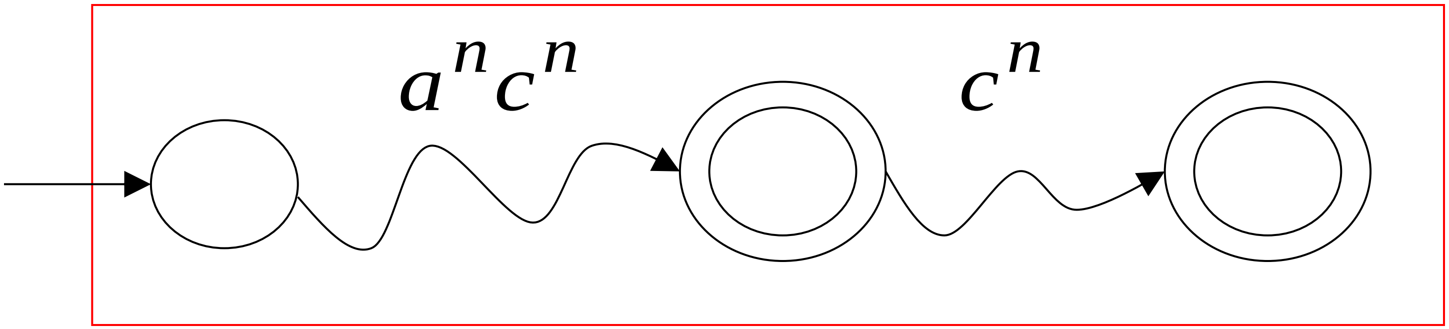
DPDA M $\quad L(M) = \{a^n b^n\} \cup \{a^n b^{2n}\}$



Modify M

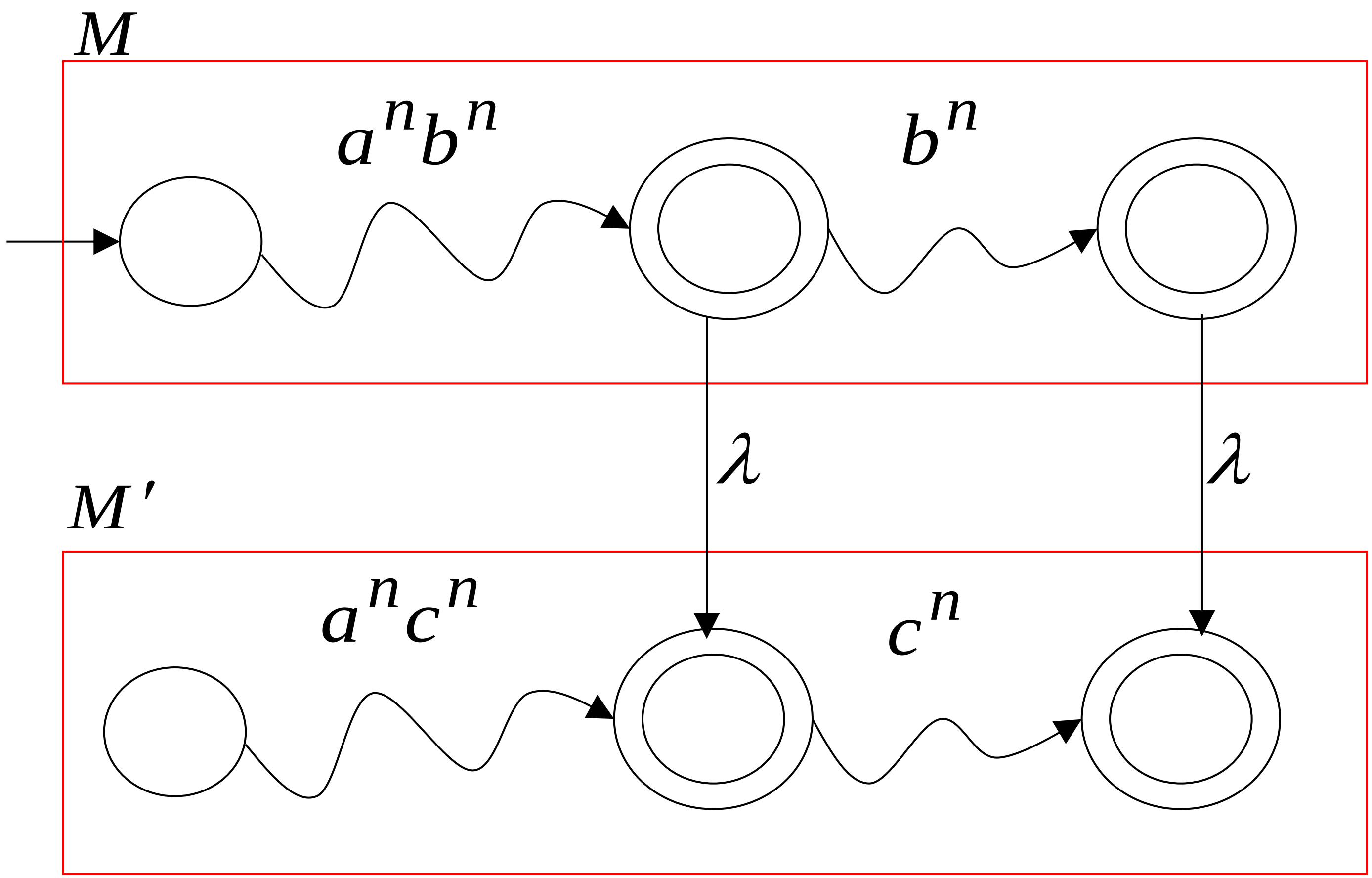
Replace b with c

DPDA M' $\quad L(M') = \{a^n c^n\} \cup \{a^n c^{2n}\}$



A PDA that accepts $L \cup \{a^n b^n c^n\}$

Connect the final states of M
with the final states of M'



Since $L \cup \{a^n b^n c^n\}$ is accepted by a PDA

it is context-free

Contradiction!

(since $L \cup \{a^n b^n c^n\}$ is not context-free)

Therefore:

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

Is not deterministic context free

There is **no** DPDA that accepts it

End of Proof