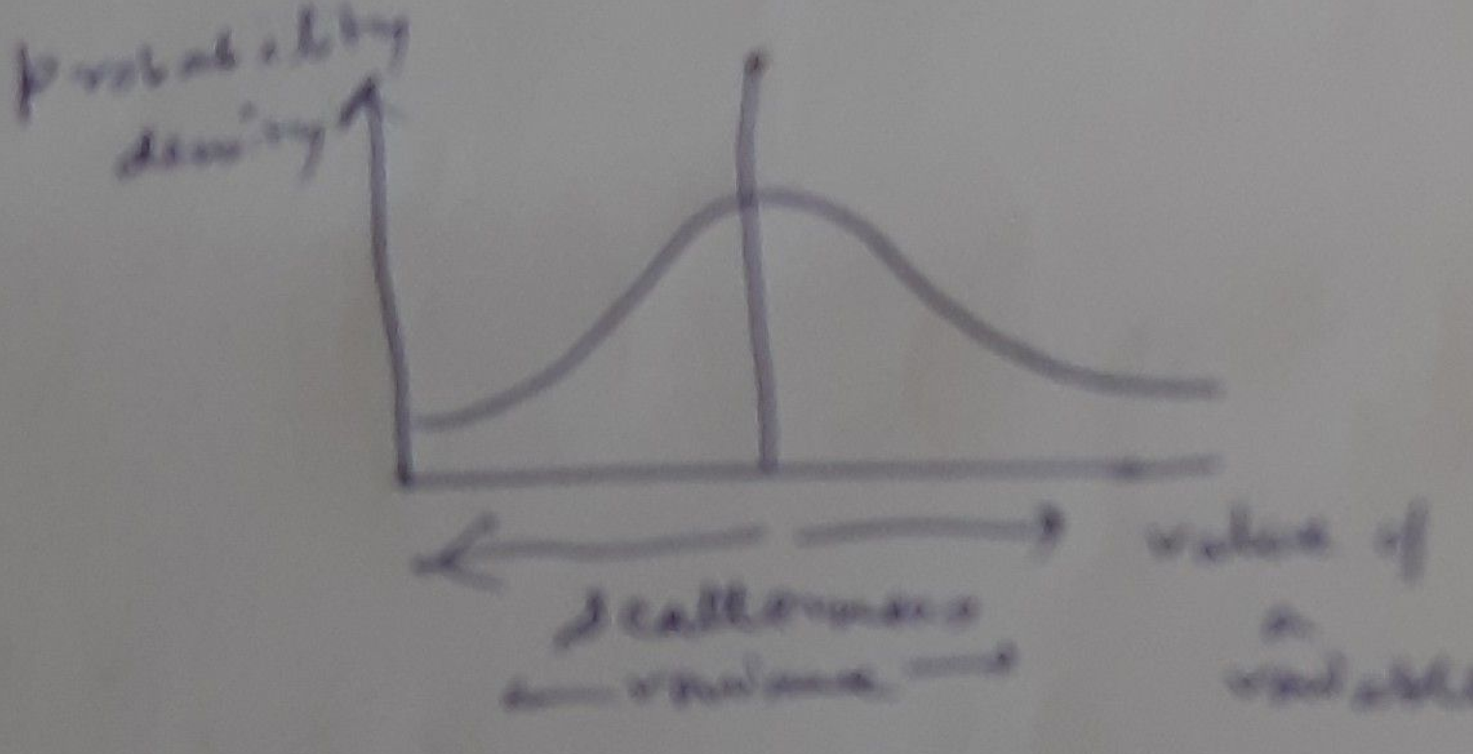
If we assume stock price = 100 INR (₹)
the simulation of the stock price will be

Stock price	'ε' (random sample)	ΔS (change in stock price)
100.00	0.52	2.45
102.45	0.44	6.43
108.88	−0.86	−3.58
105.30	1.46	6.70
...	...	...



$$p.d. = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$-\infty < x < +\infty$$

$$\int_{x=-\infty}^{+\infty} f(x)\,dx = 1$$ (Total area under curve)

When we are generating 'ε' we have
to draw random numbers b/w 0 and 1
first and then inverse cumulative normal
distb. afterward.

ε follows a p.d. of a standard normal distb^n.
∴ $\varepsilon \sim N(0,1)$
Mean 0 and variance 1

Application of Itô's Lemma:-

we saw $S_T = S_0 e^{rT}$ (under constant drift assumption)

$\therefore$ $F_0 = S_0 e^{rT}$ (r is risk free interest rate)

F is forward ~~rate~~ price at $t=0$)

S_0 = spot price.

T = time to maturity.

If we take gap/interval of time b/w t and T then

$$F = S e^{r(T-t)}$$

$$\frac{\partial F}{\partial S} = e^{r(T-t)} \qquad \frac{\partial^2 F}{\partial S^2} = 0$$

$$\frac{\partial F}{\partial t} = -rSe^{r(T-t)}$$

$\therefore$ Using Itô's Lemma,

$$dF = \left[e^{r(T-t)} \mu S - rSe^{-r(T-t)}\right] dt + e^{r(T-t)} \sigma S \, dz$$

If we use $F = Se^{r(T-t)}$,

$$dF = (\mu - r) F \, dt + \sigma F \, dz$$

F (forward price) follows a Geometric Brownian motion with $(\mu - r)$, σ.

Process of a stock price:-

A stock price follows a generalized Wiener process.
i.e. it has a constant drift rate and a constant variance rate.
However, this type of model fails to capture a key aspect of stock price movement.
People look at return rather than absolute value. (think of GDP growth rate!)

If we assume the drift rate is constant then

$\Delta S = \mu S \Delta t$ (where μ is the constant drift)

here we believe there is no volatility term.

As $\Delta t \to 0$ then

$$\frac{dS}{S} = \mu dt$$

or % change in $S = \mu dt$ (if multiplied by 100)

$$\therefore \int \frac{dS}{S} = \mu \int dt$$

$\ln S = \mu t +$ constant of integration

or $S_T = S_0 e^{\mu T}$ (between $t=0$ and

But in reality stock price exhibits volati

$$\therefore dS = \mu S dt + \sigma S dz$$

where σ is volatility (SD) fact

$$\therefore \frac{dS}{S} = \mu dt + \sigma dz$$

(z follows Wiener process
$\therefore \Delta z = \varepsilon \sqrt{\Delta t}$

Monte Carlo (named after city of casino)

$$\frac{dS}{S} = \mu\, dt + \sigma\, dz$$

For a ~~small~~ time interval Δt the change in stock price ΔS will be

$$\frac{\Delta S}{S} = \mu\, \Delta t + \sigma\, \Delta z$$

Let's assume $\left\{ \begin{array}{l} \mu = 0.15 \\ \sigma = 0.3 \end{array} \right.$ (From past data of stock we calculate historical figures)

Expressed in annual % form.

$$\therefore \quad \frac{\Delta S}{S} = 0.15\, \Delta t + 0.30\, \varepsilon \sqrt{\Delta t}$$

$\varepsilon \sim N(0,1)$, Let's assume a time interval of one week

one week = 0.0192 year.

$\therefore \Delta t = 0.0192.$

$$\therefore \quad \Delta S = (S \times 0.15 \times 0.0192) + (S \times 0.30 \times \sqrt{0.01}$$

$$= (0.002888)S + (0.0416)S\varepsilon$$

Now, ε's value can be attained/simulated from repeated sampling.

In 'excel' we can artificially generate 'ε' values -

=NORMSINV(RAND()) → 0.52, 0.44, −0.88, 1.46