CS-303

Tutorial - 8

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1 Ans:-

let us assume

$L = \{a^n b^j \mid n \le j^2\}$ is context free, then pumping lemma hold.

Let p be the number in pumping lemma.

Let $z = a^q b^p$ and $q = p^2$. clearly, $|z| \ge p$ and $z \in L$.

Therefore, $z = uvwxy$ and $|vx| \ge 1$, $|vwx| \le p$, and

$(\forall i \ge 0)$ $uv^iwx^iy \in L$. Let us consider

2 cases,

case 1) $vx = a^k$, i.e, containing no b's. Then, $z' = uv^2wx^2y$

$= a^{q+k}b^p$, and $k \ge 1$.

Therefore, $z' \notin L$, contradiction.

case 2) $vx = a^j b^k$, and $k \ge 1$, i.e, containing at least one b. Then, consider $z' = uv^0wx^0y$.

$n_a(z') = q - j = p^2 - j$, and $1 \le j \le p-1$

$n_b(z') = p - k$, and $1 \le k \le p$

so, $(n_b(z'))^2 = (p-k)^2 = p^2 - 2kp + k^2$

clearly $kp - j > k^2 - kp$, since $kp - j \ge p - (p-1) = 1$,

but $k^2 - kp = k(k-p) \le 0$. Now, it is easy to verify that, $p^2 - j > p^2 - 2kp + k^2$.

$\therefore z' \notin L$, contradiction.

$\therefore L$ is not a context free language.

Ans:-

We have to show

$L = (W : n_a(W) < n_b(W) < n_c(W))$ is not a

Assume the contradiction that the given language L is content free. We apply the pumping lemma we take a string of language as

$W = a^m b^{2m} c^{3m} = uv^i x y^i z$

where u, v, x, y and z are substrings and $i = 1, 2, 3, 4, 5, 6, \ldots$

$|vy| >= 1$ where n is an arbitary integer $<= |W|$

$|vxy| <= n$

Taking m=2, we get the string

$W = aa\ bbbb\ cc\ cccc\ (uv^i x y^i z)$

Hence, decomposing the string we get

$u \to a$

$v \to a$

$x \to bbbbc$

$y \to cccc$

$z \to c$

Here taking i = 3, the string obtained is,

$W = a\ aaa\ bbbbc\ cccccccc$

We can see the string has $n_a(W) = n_b(W)$, which doesn't belong to L.

Thus, the given language L is not context free.

3 Ans:-

We will use the pumping for context-free languages to show that the language $L = \{a^i b^j c^k \mid 0 \le i \le j \le k\}$ is not context-free.

Suppose for a contradiction that L is context-free. Let p be the pumping length given by the pumping lemma for context-free languages.

Let $s = a^p b^p c^p$. Then $s \in L$ and $|s| \geq p$. By the pumping lemma, s can be split into 5 pieces, $s = uvxyz$, such that $|vy| > 0$, $|vxy| \le p$, and $uv^i xy^i z \in L$ for all $i \geq 0$.

Since $|vxy| \le p$, vxy does not contain both a's and c's.

First suppose that vxy does not contain a's. Since $|vy| > 0$, vy contains at least one b or c. It follows that $uv^0 xy^0 z = uxz$ contains p a's and [less than p b's or less than p c's]. But then $uxz \notin L$. This is contradiction.

Finally suppose that vxy does not contain c's. Since $|vy| > 0$, vy contains at least one a or b. It follows that $uv^2 xy^2 z$ contains p c's and [more than p a's or more than p b's].

But then $uv^2xy^2z \notin L$. This is contradiction.

It follows that the assumption that L is context-free is wrong.

$\therefore$ L is not a context free language.