Capital Asset Price Model:

If the market portfolio M is efficient, the expected return $\bar{r}_i$ of any asset 'i' satisfies

$$\bar{r}_i - r_f = \beta_i (\bar{r}_m - r_f)$$

where $\beta_i = \dfrac{\sigma_{iM}}{\sigma_M^2}$

Proof: Assume 'α' is invested in asset 'i'
$(1-\alpha)$ is " " market 'M'

$$0 \leq \alpha \leq 1$$

Portfolio's return $= \alpha \bar{r}_i + (1-\alpha)\bar{r}_M = \bar{r}_\alpha$

" risk (SD) $= \left[\alpha^2 \sigma_i^2 + 2\alpha(1-\alpha)\sigma_{iM} + (1-\alpha)^2 \sigma_M^2\right]^{1/2}$

$$\frac{d r_\alpha}{d\alpha} = \bar{r}_i - \bar{r}_M$$

$$\frac{d\sigma_\alpha}{d\alpha} = \frac{\alpha\sigma_i^2 + (1-2\alpha)\sigma_{iM} + (\alpha-1)\sigma_M^2}{\sigma_\alpha}$$

Thus,

$$\left.\frac{d\sigma_\alpha}{d\alpha}\right|_{\alpha=0} = \frac{\sigma_{iM} - \sigma_M^2}{\sigma_M}$$