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Tutorial 8 - CS303

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① Show that $L = (a^n b^j : n \le j^2)$ is not context-free language using pumping lemma.

⇒ Assume for contradiction that L is context-free-language. Given L is infinite.
We apply the pumping lemma. Let m be the parameter of the pumping lemma.

we take string $a^{m^2} b^m \in L$

$$a^{m^2} b^m = uvxyz \; ; \quad |vxy| \le m, \quad |vy| \ge 1.$$

we have to examine all possible positions of vxy.
First, we can see that the string v cannot span simultaneously both a^{m^2} and b^m, since if we pump up v (i.e. repeat v), the resulting string is not in the language (as a's will be mixed with b's). Therefore, v must be totally within a^{m^2} or b^m. Same holds for string y.

Case 1: v is within a^{m^2} and y is in b^m

$$v = a^{k_1} \text{ and } y = b^{k_2} \; ; \quad 1 \le k_1 + k_2 \le m$$
$$(\because |vxy| \le m \text{ and } |vy| \ge 1)$$

Subcase 1: Consider case when $k_2 \ge 1$;

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From PL; $uv^0xy^0z \in L$

$\therefore \; a^{m^2-k_1} b^{m-k_2} \in L$

$\therefore \; m^2-k_1 \leq (m-k_2)^2$ (from $n<=j^2$)

However this is incorrect, since

$$(m-k_2)^2 <= (m-1)^2 \quad (\because k_2 \geq 1)$$
$$= m^2-2m+1$$
$$< m^2-k_1 \quad (\because k_1 \leq m)$$

$\therefore \; a^{m^2-k_1} b^{m-k_2} \notin L$.

Subcase 2: $k_2 = 0$.

$\Rightarrow \; k_1 \geq 1 \quad (\because k_1+k_2 \geq 1)$.

from PL; $uv^2xy^2z \in L$

$\therefore \; a^{m^2+k_1} b^m \in L \quad (\because k_2 = 0)$

$\Rightarrow \; m^2+k_1 \leq m$.

This is impossible $\because \; m^2+k_1 > m$.

Case 2: v is in a^{m^2} and y is also in a^{m^2}.

if we repeat v & y; we obtain a string of form

$v = a^{k_1}$ & $y = a^{k_2}$, $1 \leq k_1+k_2 \leq m$.

$a^{m^2+k} b^m$; $k \geq 1$. ($k = k_1+k_2$)

But $a^{m^2+k} b^m \notin L$. as $m^2+k > m$.

Case 3: v and y both are in b^m

Let $v = b^{k_1}$ & $y = b^{k_2}$

From PL; $uv^0xy^0z \in L$

$\therefore \; a^{m^2} b^{m-(k_1+k_2)} \in L$; $k_1+k_2 \geq 1$

$\Rightarrow \; m^2 <= (m-k_1+k_2)$ (from $n<=j^2$)

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which is incorrect, since $m^2 > m-(k_1+k_2)$
; $k_1+k_2 \geq 1$

$\therefore a^{m^2} b^{m-(k_1+k_2)} \notin L$.

As we obtain contradiction in all valid cases, given language $L = (a^n b^j : n <= j^2)$ is not a context-free language

(2) Show that $L = (w : na(w) < nb(w) < nc(w))$ is not context-free language using pumping lemma.

$\Rightarrow$ Given, L is infinite.
Assume that L is a context-free language.
We will apply the pumping lemma. Let m be the parameter of the pumping lemma

we take string $a^m b^{m+1} c^{m+2} \in L$

we can write $a^m b^{m+1} c^{m+2} = uvxyz$;
$|vxy| \leq m$ & $|vy| \geq 1$.

we examine all the possible cases for the string vxy.

Case 1: v is within a^m & y is in b^{m+1}.

Let $v = a^{k_1}$ & $y = b^{k_2}$; $1 \leq k_1+k_2 \leq m$
as $|vxy| \leq m$ & $|vy| \geq 1$

From PL ; $uv^3xy^3z \in L$

$\therefore \quad a^{m+2k_1} b^{m+1+2k_2} c^{m+2} \in L$

since $\quad k_1 + k_2 \geq 1$

Subcase 1 : $\quad k_1 \geq 1$

$\Rightarrow \quad m + 2k_1 \geq m + 2$

$\therefore \quad a^{m+2k_1} b^{m+1+2k_2} c^{m+2} \notin L$

contradiction

Subcase 2 : $\quad k_2 \geq 1$

$\Rightarrow \quad m + 2k_2 + 1 > m + 2$

$\therefore \quad a^{m+2k_1} b^{m+1+2k_2} c^{m+2} \notin L$

contradiction

Case 2 : $\quad v$ is in a^m & y is in a^m and b^{m+1}

let $\quad v = a^{k_1}$ & $y = a^{k_2k_3}$

$k_1 + k_2k_3 \geq 1$.

From PL ; $\quad uv^3xy^3z \in L$

$a^{m+2(k_1+k_2)} b^{m+1+2(k_3)} c^{m+2} \in L$

Subcase1 : $k_1 \geq 1$

$m + 2(k_1 + k_2) \geq m + 2$

Subcase2 : $k_2k_3 \geq 1 \Rightarrow k_2 \geq 1$ & $k_3 \geq 1$

$m + 1 + 2(k_3) > m + 2$

$\Rightarrow \quad a^{m+2(k_1+k_2)} b^{m+1+2(k_3)} c^{m+2} \notin L$

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Case 3 : v spans a^m and b^{m+1}, and y is within b^{m+1}

Similar to previous case.

Case.4 : v and y are within a^m

If we pump up v and y we obtain a string of the form $a^{m+k}b^{m+1}c^{m+2}$, with $k \geq 1$, which obviously is not in the language.

Case 5 : v and y are within b^{m+1}

If we pump up v and y we obtain a string of the form $a^m b^{m+k+1} c^{m+2}$, with $k \geq 1$, which obvi-ously is not in the language.

Case 6 : v and y are somewhere with $b^{m+1}c^{m+2}$

Similar to previous cases.

In all cases, we obtain a contradiction ; therefo-re language L is not context-free

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(3) Let L be the language $\{a^i b^j c^k \mid 0 \le i \le j \le k\}$
Show that language is not CFL

$\Rightarrow$

L is infinite.
Assume the L is a context-free language.
We will apply the pumping lemma. we take String

$$a^m b^m c^m \in L$$
$$= uvxyz \; ; \; |vxy| \le m \; \& \; |vy| \ge 1$$

vxy cannot contain a's, b's and c's. we then pump the string depending on string vxy as follows:

Case 1: There are no a's in vxy.
From PL; $uv^0xy^0z \in L$
this contains same number of a's, but fewer b's or c's. Therefore, it is not in L.

Case 2: There are no b's but there are a's.
From PL; $uv^2xy^2z \in L$
this contains more a's than b's. Therefore, it is not is L.

Case 3: There are no b's but there are c's.
From PL; $uv^0xy^0z \in L$
this string contains same number of b's but fewer c's. Therefore, this is not in L

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Case 4 : There are no c's.

From PL ; $uv^2xy^2z \in L$

this string contains more b's or more a's than there are c's. Therefore, it is not in L.

As we have contradiction for every case, Language L is not a CFL.