Tutorial - 7

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1) Proof by contradiction :

We initially consider two context free languages and prove their intersection is not context free.

Let $L1 = \{a^m b^n c^n \mid n \geq 0 \text{ and } m \geq 0\}$

$L2 = \{a^n b^n c^m \mid n \geq 0 \text{ and } m \geq 0\}$

L1:

$S \rightarrow aS$

$S \rightarrow bAC$

$A \rightarrow bAc \mid \lambda$

L2:

$S \rightarrow Sc$

$S \rightarrow aAb$

$A \rightarrow aAb \mid \lambda$

Here both are context free grammars.

although $L1 \cap L2 = \{a^n b^n c^n \mid n \geq 0\}$

In PDA, we can compare only two characters at a time using the stack. But as we have to ensure equal number of 'a', 'b', 'c', we will not be able to do so using context free grammar

$\therefore L_1 \cap L_2$ is not context free

2) Given,

$L1 = \{a^n b^n c^m\}$ and $L2 = \{a^m b^n c^n\}$, two

context free grammar.

<u>$L_1 \cap L_2$</u>

$L1$ requires equal number of 'a', 'b'. And $L2$ requires equal number of 'b', 'c'. Now, considering these conditions together, we need the intersection language to have both equal no. of 'a', 'b' and 'b', 'c'.

i.e) $n(a) = n(b) = K_1$

$n(b) = n(c) = K_2$

$K_1 = K_2 = n(b) \Rightarrow n(a) = n(b) = n(c)$

$\therefore L_1 \cap L_2$ should have equal number of 'a', 'b', 'c'.

$\Rightarrow \quad L_1 \cap L_2 = \{a^n b^n c^n\}$

Since in a PDA we cannot ensure that more than 2 characters to have same frequency this is a non context free grammar.

3) Proof by contradiction

RTP: Context free languages are not closed under complement.

i.e) If $L1$ is a CFL then $\bar{L}_1$ may not be a CFL.

Let us consider that statement is true.

and L_1, L_2 are CFL

we know that $L_1 \cup L_2$ is CFL.

Now $\overline{L_1 \cup L_2} = \bar{L}_1 \cap \bar{L}_2$.

Now from our initial assumption, we get that $\bar{L}_1, \bar{L}_2$ are also CFL.

But as intersection of two CFL may not be CFL, there is no guarantee that $\bar{L}_1 \cap \bar{L}_2$ is CFL

$\Rightarrow$ No guarantee that $\overline{L_1 \cup L_2}$ is CFL

$\therefore \underset{CFL}{L_1 \cup L_2} \not\Rightarrow \overline{L_1 \cup L_2}$ CFL

Hence $L_1 (CFL) \not\Rightarrow \bar{L}_1 (CFL)$

Hence proved.