



$$\hat{y} = \hat{\alpha} + \hat{\beta}x \quad (y \text{ is dependent} \\ x \text{ is independent})$$

\hat{y} represents values on the line

$$\text{Error} = y - \hat{y} \quad (\text{Actual} - \text{fitted})$$

$$\sum (\text{Error})^2 = \sum (y - \hat{y})^2 = \sum (y - \hat{\alpha} - \hat{\beta}x)^2$$

Minimize $(\text{error})^2$ w.r.t. parameters
(α, β)

$$\frac{\partial (\text{Error})^2}{\partial \alpha} = 0 = -2 \sum (y_i - \hat{\alpha} - \hat{\beta}x_i) \rightarrow \textcircled{1}$$

$$\frac{\partial (\text{Error})^2}{\partial \beta} = 0 = -2 \sum x_i (y_i - \hat{\alpha} - \hat{\beta}x_i) \rightarrow \textcircled{2}$$

Solving $\textcircled{1}$ and $\textcircled{2} \Rightarrow$

$$\hat{\beta} = \frac{\sum x_i' y_i'}{\sum x_i'^2}$$

$$x_i' = x_i - \bar{x} \\ y_i' = y_i - \bar{y}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x}$$

\bar{x} = Mean of x
 \bar{y} = Mean of y .

$$\left\{ \hat{\beta} = \frac{\text{Cov}(x, y)}{\text{Var}(x)} \right\} \rightarrow \text{risk factor in CAPM}$$