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1)

a) $S \to BSB \mid B \mid \varepsilon$
   $B \to 00 \mid \varepsilon$

Step-1 : Remove Null Production

$S' \to S$
$S \to BSB \mid B \mid \varepsilon$
$B \to 00 \mid \varepsilon$

$\xrightarrow[\quad S \to \varepsilon \quad]{\text{Remove}}$

$S' \to S \mid \varepsilon$
$S \to BSB \mid B \mid BB$
$B \to 00 \mid \varepsilon$

$\downarrow \begin{array}{l}\text{Remove}\\ B \to \varepsilon\end{array}$

$S' \to S$
$S \to BSB \mid B \mid SB \mid BS \mid BB$
$B \to 00$

$\xleftarrow[\quad S' \to \varepsilon \quad]{\text{Remove}}$

$S' \to S \mid \varepsilon$
$S \to BSB \mid B \mid SB \mid BS \mid BB$
$B \to 00$

Step : Remove unit Production

$S' \to S$
$S \to BSB \mid B \mid SB \mid BS \mid BB$
$B \to 00$

$\xrightarrow[\quad S \to B \quad]{\text{Remove}}$

$S' \to S$
$S \to BSB \mid SB \mid BS \mid BB \mid 00$
$B \to 00$

$$\underrightarrow{\text{Remove}}$$
$$S' \to S$$

$S' \to BSB \mid SB \mid BS \mid BB \mid 00$

$S \to BSB \mid SB \mid BS \mid BB \mid 00$

$B \to 00$

Step 3 : Add Variables

$S' \to V_1 B \mid SB \mid BS \mid BB \mid 00$

$V_1 \to BS$

$B \to 00$

using Terminal variables,

$S' \to V_1 B \mid SB \mid BS \mid BB \mid T_0 T_0$

$V_1 \to BS$

$T_0 \to 0$

$B \to T_0 T_0$

(b)

$S \to S1 | S2$
$S1 \to S1b | Ab | \lambda$
$A \to aAb | ab$
$S2 \to S2a | Ba | \lambda$
$B \to bBa | ba$

Step 1 : Remove Null production

$S \to S1 | S2$
$S1 \to S1b | Ab | \lambda$
$A \to aAb | ab$
$S2 \to S2a | Ba | \lambda$
$B \to bBa | ba$

$\xrightarrow[\substack{S1 \to \lambda \\ \text{and} \\ S2 \to \lambda}]{\text{Remove}}$

$S \to S1 | S2 | \lambda$
$S1 \to S1b | Ab | b$
$A \to aAb | ab$
$S2 \to S2a | Ba | a$
$B \to bBa | ba$

$\Big\downarrow \substack{\text{Remove} \\ S \to \lambda}$

$S \to S1 | S2$
$S1 \to S1b | Ab | b$
$A \to aAb | ab$
$S2 \to S2a | Ba | a$
$B \to bBa | ba$

Step 2: Remove unit production

$\xrightarrow[\substack{S \to S1 \\ \text{and} \\ S \to S2}]{\text{Remove}}$

$S \to S1b | Ab | b | S2a | Ba | a$
$S1 \to S1b | Ab | b$
$A \to aAb | ab$
$S2 \to S2a | Ba | a$
$B \to bBa | ba$

Step 3 : Add variables

$S\$ \rightarrow S1\, T_b \mid A\, T_b \mid b \mid S2\, T_a \mid B\, T_a \mid a$

$S1 \rightarrow S1\, T_b \mid A\, T_b \mid b$

$A \rightarrow V_1\, T_b \mid T_a\, T_b$

$S2 \rightarrow S2\, T_a \mid B\, T_a \mid a$

$B \rightarrow V_2\, T_a \mid T_b\, T_a$

$V_1 \rightarrow T_a\, A$

$V_2 \rightarrow T_b\, B$

$T_a \rightarrow a$

$T_b \rightarrow b$

2)

$S \to XA \mid BB$
$B \to b \mid SB$
$X \to b$
$A \to a$

Step 1 : Use A_i variables for Existent variables

i.e) $A_1 = S, A_2 = X, A_3 = A, A_4 = B$

$A_1 \to A_2 A_3 \mid A_4 A_4$
$A_4 \to b \mid A_1 A_4$
$A_2 \to b$
$A_3 \to a$

Step 2 : For all $A_i \to A_j X$; make sure $i < j$

$A_1 \to A_2 A_3 \mid A_4 A_4$
$A_4 \to b \mid A_1 A_4$
$A_2 \to b$
$A_3 \to a$

$\xrightarrow[A_4 \to A_1 A_4]{\text{Replace}}$

$A_1 \to A_2 A_3 \mid A_4 A_4$
$A_4 \to b \mid A_2 A_3 A_4 \mid A_4 A_4 A_4$
$A_2 \to b$
$A_3 \to a$

Replace
$\downarrow$ ~~$A_4 \to A_4 A_4$, $A_4 \to A_4 A_4 Z$~~
$A_4 \to A_2 A_3 A_4$

$A_1 \to A_2 A_3 \mid A_4 A_4$
$A_4 \to b \mid b A_3 A_4 \mid A_2 A_4 A_4$
$A_2 \to b$
$A_3 \to a$

$(A_4 \to A_4 A_4 A_4$ is a left recursion$)$

To remove a left recursion, we need to add a new variable

$$Z' \to A_4 A_4 \mid A_4 A_4 Z'$$

$A_1 \to A_2 A_3 \mid A_4 A_4$

$A_4 \to b \mid bA_3 A_4 \mid bZ' \mid bA_3 A_4 Z'$

$Z \to A_4 A_4 \mid A_4 A_4 Z$

$A_2 \to b$

$A_3 \to a$

$\underrightarrow{\text{Replace}}$

$A_1 \to A_2 A_3$

and

$A_1 \to A_4 A_4$

$A_1 \to bA_3 \mid bA_4 \mid$

$\qquad bA_3 A_4 A_4 \mid$

$\qquad bZ' A_4 \mid$

$\qquad b A_3 A_4 Z' A_4$

$A_4 \to b \mid bA_3 A_4 \mid bZ' \mid$

$\qquad bA_3 A_4 Z'$

$Z' \to A_4 A_4 \mid A_4 A_4 Z'$

$\underrightarrow{\text{Replace}}$

$Z \to A_4 A_4 , \quad Z \to A_4 A_4 Z$

$A_2 \to b$

$A_3 \to a$

$A_1 \to bA_3 \mid bA_4 \mid bA_3 A_4 A_4 \mid bZ' A_4 \mid bA_3 A_4 Z' A_4$

$A_4 \to b \mid bA_3 A_4 \mid bZ' \mid bA_3 A_4 Z'$

$Z' \to bA_4 \mid bA_3 A_4 A_4 \mid bZ' A_4 \mid bA_3 A_4 Z' A_4$

$Z' \to bA_4 Z' \mid bA_3 A_4 A_4 Z' \mid bZ' A_4 Z' \mid bA_3 A_4 Z' A_4 A_4$

$A_2 \to b$

$A_3 \to a$

(e) $S \to bA \mid bB \mid bABD \mid bZ'B \mid bABZ'B$

$B \to b \mid bAB \mid bZ' \mid bABZ'$

$Z' \to bB \mid bABB \mid bZ'B \mid bABZ'B$

$Z' \to bBZ' \mid bABBZ' \mid bZ'BZ' \mid bABZ'AB$

$A_2 \to b$

$A_3 \to a$

3) C is a nullable because $C \to \varepsilon$ is a given production

And since $B \to C$ too, B can be considered as a nullable.

$S \to BC$, so S is also a nullable

There is no path leading A to ε so its non-nullable.

$\therefore$ for $S \to ABC | BC$
$A \to aA | a$
$B \to b | C$
$C \to cc | dd | \varepsilon$

$N_\varepsilon = \{S, B, C\}$