CS 303 - Tut 8

Name: P.V.SRIRAM
Roll No.: 1801CS37

1) RTP: $L = (a^n b^j : n <= j^2)$ is not context free language.

We can prove this with pumping lemma and contradiction,

(i) Assume that L is a context free language.

This means that for a pumping length 'p', A string from L, say S, can be written in form of uvwxy where $|vx| \geq 1$, $|vwx| \leq p$. and $\forall i \geq 0$ $uv^i w x^i y \in L$ where $|S| \geq P$.

Consider $S = a^q b^p$ where $q = p^2$

Two cases,

Case 1: v, x contain only one type of symbol, say 'a' here.

$vx = a^K$, Then $S = uvwxy = \underbrace{a}_{u}\underbrace{a}_{v}\underbrace{a}_{w}\underbrace{a}_{x}\underbrace{bb}_{y}$

$uv^2wx^2y = a^{q+K} b^p$

we cant say that $q <= p^2 \Rightarrow q+k <= p^2$

$\therefore$ Doesn't hold in case 1

Case (ii) $vx = a^j b^k$ (contain more than one kind of character a, b).

$S = uvwxy$, $s' = uv^0wx^0y$

$\Rightarrow s' = a^{q-j} b^{p-k}$

now, we know $q \leq p^2$

now $q - j$ | $(p-k)^2$

$= q - j$ | $= p^2 + k^2 - 2pk$

$\leq p^2 - j$

Consider $pk - j$, we know $1 \leq j \leq p$ (as $k > 1$ from case)

$pk - j > pk - p$

$= p(k-1) \geq 0$

consider $k^2 - kp = k(k-p)$, we know $1 \leq k < p$

$= k(k-p) \leq 0$

$\Rightarrow pk - j > k^2 - kp$

$\Rightarrow -j > k^2 - 2kp$

$\Rightarrow p^2 - j > k^2 + p^2 - 2kp \Rightarrow p^2 - j > (k-p)^2$

for the case where $q = p^2$ we have

a contradiction

$\therefore$ contradiction for both cases, Not CFL.

2) RTP: $L = (w : n_a(w) < n_b(w) < n_c(w))$ is not CFL

Proof through pumping lemma.

Assume that given language is CFL.

i.e) For a pumping length p, A string from L, say s can be written as $uvwxy$ where $|vx| \geq 1$, $|vwx| \leq p$ and $\forall i \geq 0$, $uv^iwx^iy \in L$ where $|s| \geq p$.

Consider $S = a^p \, b^{p+k} \, c^{p+q}$

case (i) $vx = b^{k+1}$ (within same character b.) (~~same~~ ~~character~~ ~~in~~ ~~b~~

i.e) $v = b^k$ $w = b$ $x = b$

$s' = uv^0wx^0y = a^p \, b^{p-1} \, c^{p+q}$

∴ There is a contradiction in first case

case (ii) $vx = b^k c^q$ (spans accross more than one kind)

$s' = uv^0wx^0y = a^p b^p c^p$

∴ There is a contradiction in second case

∴ Given language is not context free.

3) RTP: $L = \{ a^i b^j c^k \mid 0 \le i \le j \le k \}$ is not CFL

Assume that L is indeed a CFL.

i.e) For a pumping length 'p'. A string from L, say s, can be written in form $s = uvwxy$ where

$|vx| \geq 0$, $|vwx| \leq p$ and $uv^iwx^iy \in L$

$\forall i \geq 0$ where $|s| \geq p$.

Let $s = a^p b^p c^p$. Since $|vwx| \leq p$, it cant contain both a, c at same time

Case (i) $\quad vx = b^k c^j \quad$ (no a's) $\left(0 \le k,\ 0 \le j \text{ and } k+j \le p \right)$

$$s' = uv^0wx^0y = a^p b^{p-k} c^{p-j} \notin L$$

$\therefore$ Contradiction

Case ii $\quad vx = a^k b^j \quad$ (no c's) $\left(0 \le k, 0 \le j, k+j \le p \right)$

$$s' = uv^0wx^0y = a^{p-k} b^{p-j} c^p$$

In the case where $k < j$, there is a contradiction

$\therefore$ L is not a context free grammar.