MID-SEMESTER

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Q1) Given,

$$Y_t = 2 + \underbrace{0.5}_{a_1} \times Y_{t-1} + \underbrace{0.75}_{\beta_1} \times \varepsilon_{t-1} + \varepsilon_t$$

ACF for a lag 1 ARMA model is

$$\rho_1 = \frac{(1 + a_1\beta_1)(a_1+\beta_1)}{1 + \beta_1^2 + 2a_1\beta_1}$$

Substituting values,

$$\rho_1 = \frac{(1 + 0.5\times 0.75)(1.25)}{1 + (0.75)^2 + 2(0.5)(0.75)} = \frac{1.71875}{2.3125}$$

$$= \boxed{0.74324}$$

2) Given,

$$Y_t = 2 - 0.5 * Y_{t-1}$$

i.e) $Y_t = CF + PI$

<u>For PI</u>

consider $\overline{Y_t} = \overline{Y_{t-1}} = \mu$

$$\overline{Y_t} = 2 - 0.5\,\overline{Y} \Rightarrow 1.5\,\overline{y} = 2$$

$$\overline{y} = \frac{2}{3/2} = \frac{4}{3}$$

<u>For CF</u>

$$Y_t = Ab^t$$

$$Ab^t = 2 - 0.5 \times Ab^{t-1}$$

$$\Rightarrow Ab^{t-1}(b + 0.5) = 2$$

only possible when $b = -0.5$

which means, $A(-0.5)^2 = y_t$

Clearly this is a <u>convergent</u> form

as $t \to \infty$, $CF \to 0$

3) $Corr(X, Y) = 0.5$

RTF: $Corr(3X, (Y-5)) = \dfrac{Cov(3X, (Y-5))}{SD_{(3X)} \cdot SD_{(Y-5)}}$

$$= \frac{E((3X)(Y-5)) - E(3X)\,E(Y-5)}{SD_{(3X)} \cdot SD_{(Y-5)}}$$

$$\Rightarrow \frac{3E(XY-5X) - 3E(X)(E(Y)-5)}{SD_{(3X)} \cdot SD_{(Y-5)}}$$

$$\Rightarrow \frac{3\left[E(XY) - 5\cancel{E(X)} - E(X)\cdot E(Y) + 5\cancel{(E(X))}\right]}{9\, SD_{(X)} \cdot SD_{(Y)}}$$

$$\Rightarrow \frac{1}{3} \times Corr(X, Y) = \frac{0.5}{3} = \boxed{0.1666}$$

$$\Rightarrow \boxed{\frac{1}{6}}$$

4) Dicky Fuller test is a practical way to Test stationarity

For Eg: $y_t = \alpha + \beta y_{t-1} + \varepsilon_t$

$$\Rightarrow y_t - y_{t-1} = \alpha + (\beta - 1) y_{t-1} + \varepsilon_t$$

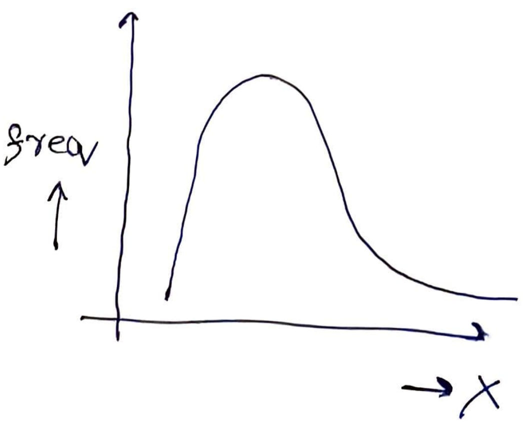
$$\Delta y_t = \alpha + \overset{\gamma = (\beta-1)}{\gamma} y_{t-1} + \varepsilon_t$$

$\Rightarrow$ If $\beta \to 1$, $\gamma \to 0$

$\therefore$ In essence we are doing the same test of coefficient 1 but doing a transformation to perform regression.

5) Yes, since skewness increases the tail size on one side.

So even though the tail is extended in the positive side, there is higher volatility in the market, meaning higher risk.



Here we can see that mean, median > mode