PH 201

OPTICS & LASERS

Lecture_Coherence

Coherence

Dictionary meaning:

- Quality or state of cohering or sticking together, especially a logical, orderly, & aesthetically consistent relationship of parts.
- Property of moving in unison.
- Logical or natural connection or consistency.
- Property of being coherent, as of waves. Constant phase difference in two or more waves over time.
- Existence of correlation between phases of two or more waves.

Coherence

Coherence is a property of waves that enables stationary (temporally & spatially) interference. Two sources which vibrate with a fixed phase difference between them are said to be **coherent**.

If phase difference changes with such great rapidity that a stationary interference cannot be observed then sources are said to be **incoherent**.

Consider displacements produced by two sources S_1 & S_2

$$y_1 = a\cos\omega t$$

$$y_2 = a\cos(\omega t + \phi)$$

Resultant displacement:

$$y = y_1 + y_2 = 2a\cos(\phi/2)\cos(\omega t + \phi/2)$$

Intensity:

$$I = 4I_0\cos^2(\phi/2)$$

I$_0$ is intensity produced by each one of sources individually.

$$If \quad \phi = \pm\pi, \pm 3\pi, ...$$

$$\Rightarrow I = 0 \qquad Minima$$

$$If \quad \phi = \pm 2\pi, \pm 4\pi, ...$$

$$\Rightarrow I = 4I_0 \qquad Maxima$$

If phase difference between sources S_1 & S_2 is changing with time, then

$$I = 4I_0 < \cos^2 \frac{\phi}{2} >$$

$$= 4I_0 \times \frac{1}{2}$$

$$= 2I_0$$

Time average

$$< f(t) > = \frac{1}{\tau} \int_{-\tau/2}^{+\tau/2} f(t)\,dt$$

If sources are incoherent then resultant intensity is sum of two intensities & there is no variation of intensity.

If there is a constant phase relation between two or more disturbances they are said to be coherent & if there is no fixed phase relation they are incoherent.

To add coherent disturbances: **Add complex amplitudes**

To add incoherent disturbances: **Add intensities**

For any source average length of a wave train is called **coherence length**, & time taken by light to travel this distance (i.e. interval of time during which mean wave train is emitted) is called **coherence time**.

Degree of coherence (1^{st} degree, 2^{nd} degree, …) is measured by interference visibility, a measure of how perfectly waves can cancel due to destructive interference.

$$Visibility = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

For interference, it is assumed that displacement associated with a wave remained sinusoidal for all values of time.

$$E = A\cos(kx - \omega t + \phi)$$

This Eq. predicts that at any value of *x*, displacement is sinusoidal for $-\infty < t < \infty$.

At a given point, electric field at times t & $t + \Delta t$ will, in general, have a definite phase relationship if $\Delta t \ll \tau_c$ & will never have any phase relationship if $\Delta t \gg \tau_c$.

Time duration τ_c is known as **coherence time** of source & field is said to remain coherent for times $\sim \tau_c$.

Length of wave train, $L = c\,\tau_c$, **Coherence length**

Reasons of finite value of coherence time:

- If a radiating atom undergoes collision with another atom, then wave train undergoes an abrupt phase shift.

- Random motion of atoms.

- An atom has a finite life time in energy level from which it drops to lower energy level while radiating.

Commercially available He-Ne lasers have

Coherence time τ_c ~ 50 nsec

Coherence length L ~ 15 m

Decrease in contrast of fringes: due to fact that source is not emitting at a single frequency but over a narrow band of frequencies.

- ❖ When path difference between two interfering beams is zero or very small, different wavelength components produce fringes superimposed on one another & fringe contrast is good.
- ❖ When path difference is increased, different wavelength components produce fringe patterns which are slightly displaced with respect to one another & fringe contrast becomes poorer.

Spectral width of a source,

Temporal coherence τ_c of beam is directly related to spectral width $\Delta\lambda$

$$\Delta\lambda = \frac{\lambda^2}{L} = \frac{\lambda^2}{c\tau_c}$$

Frequency spread of a spectral line, $\Delta\nu = \frac{1}{\tau_c}$

Monochromaticity or spectral purity is defined as $\frac{\Delta\nu}{\nu}$

Coherence Time & Length

Examples:

In case of an incandescent lamp/bulb: Life time (Δt) of electron in excited state ~ 10^{-8} s

So, Coherence time (τ_c) of light pulse from incandescent bulb : ~ 10^{-8} s

Coherence length ($L \sim c\tau_c$) = $\Delta x = c\Delta t = 3 \times 10^{8} \times 10^{-8} = 3$ m

incandescent bulb

In case of a Neon line ($\lambda = 6328 \times 10^{-10}$ m):

Coherence time (τ_c) : ~ 10^{-10} s

Coherence length ($L \sim c\tau_c$): ~ 3 cm

In case of a red Cd line ($\lambda = 6438 \times 10^{-10}$ m):

Coherence time (τ_c) : ~ 10^{-9} s

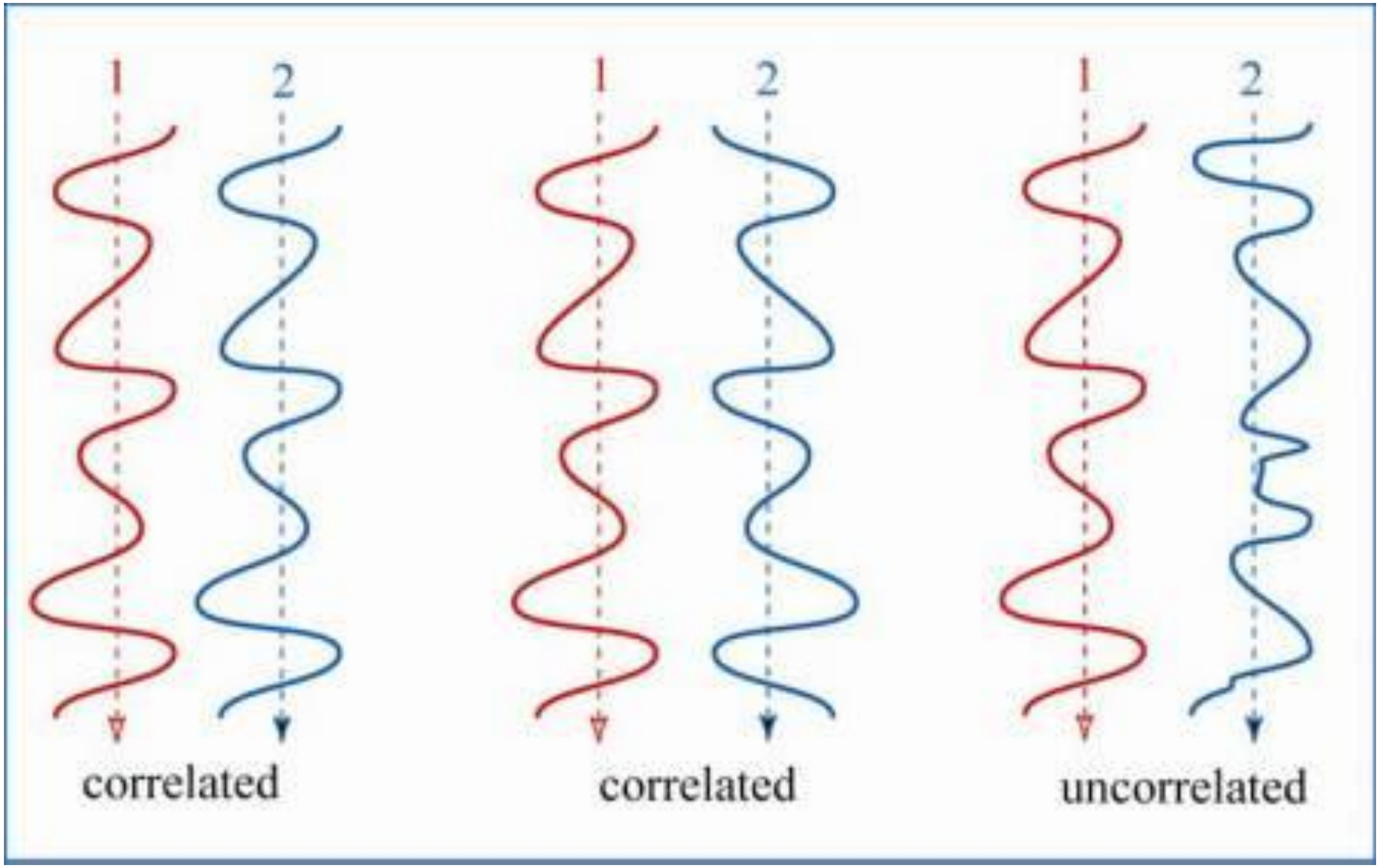
Coherence length ($L \sim c\tau_c$): ~ 30 cm

In case of LASERS :

Coherence time (τ_c) : ~ 10^{-3} s

Coherence length ($L \sim c\tau_c$): ~ $3 \times 10^{8} \times 10^{-3} = 300$ km

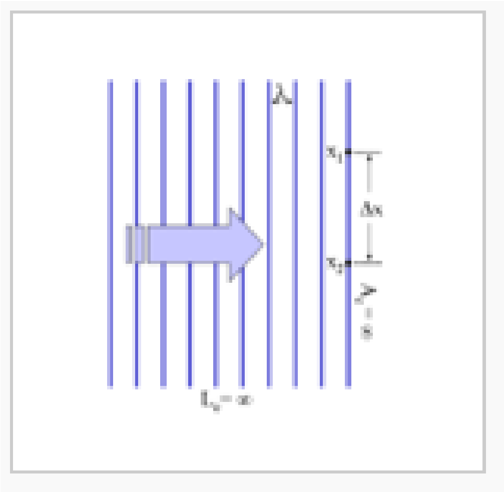
Existence of correlation between phases of two or more waves.



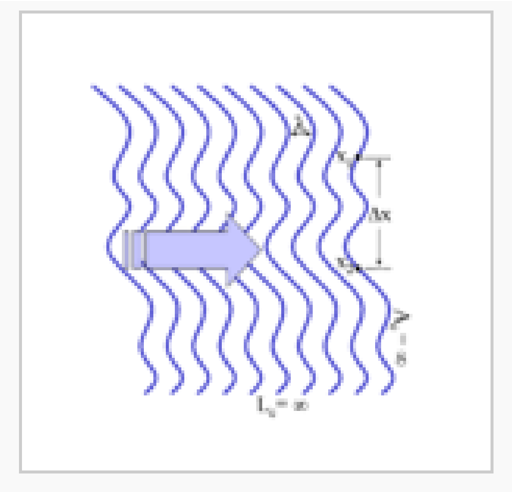
Spatial coherence

Spatial (lateral or transverse) coherence describes ability for two points in space, in the extent of a wave to interfere, when averaged over time. If a wave has only ONE value of amplitude over an infinite length, it is perfectly spatially coherent.

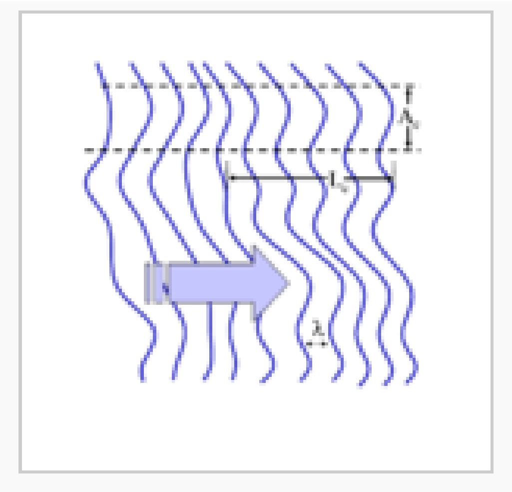
It is coherence property of field associated with **finite dimension of source**.



A plane wave with an infinite coherence length.



A wave with a varying profile (wavefront) & infinite coherence length.



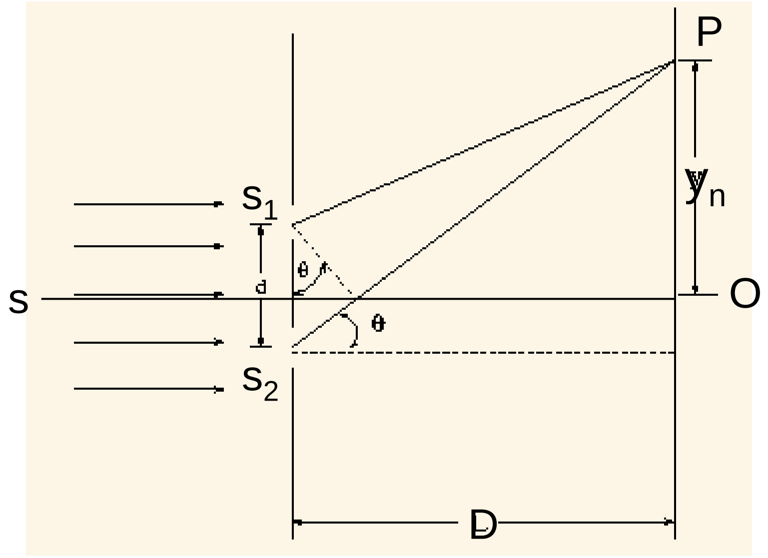
A wave with a varying profile (wavefront) & finite coherence length.

Young's Experiment

Interference pattern observed around P at time t is due to superposition of waves emanating from S_1 & S_2 at times $t - r_1/c$ and $t - r_2/c$ respectively, where $r_1 = S_1P$ & $r_2 = S_2P$

$$If \quad \frac{r_2 - r_1}{c} << \tau_c$$

$\Rightarrow$ then waves arriving at P from S_1 & S_2 will have a definite phase relationship & an interference pattern of good contrast will be observed.



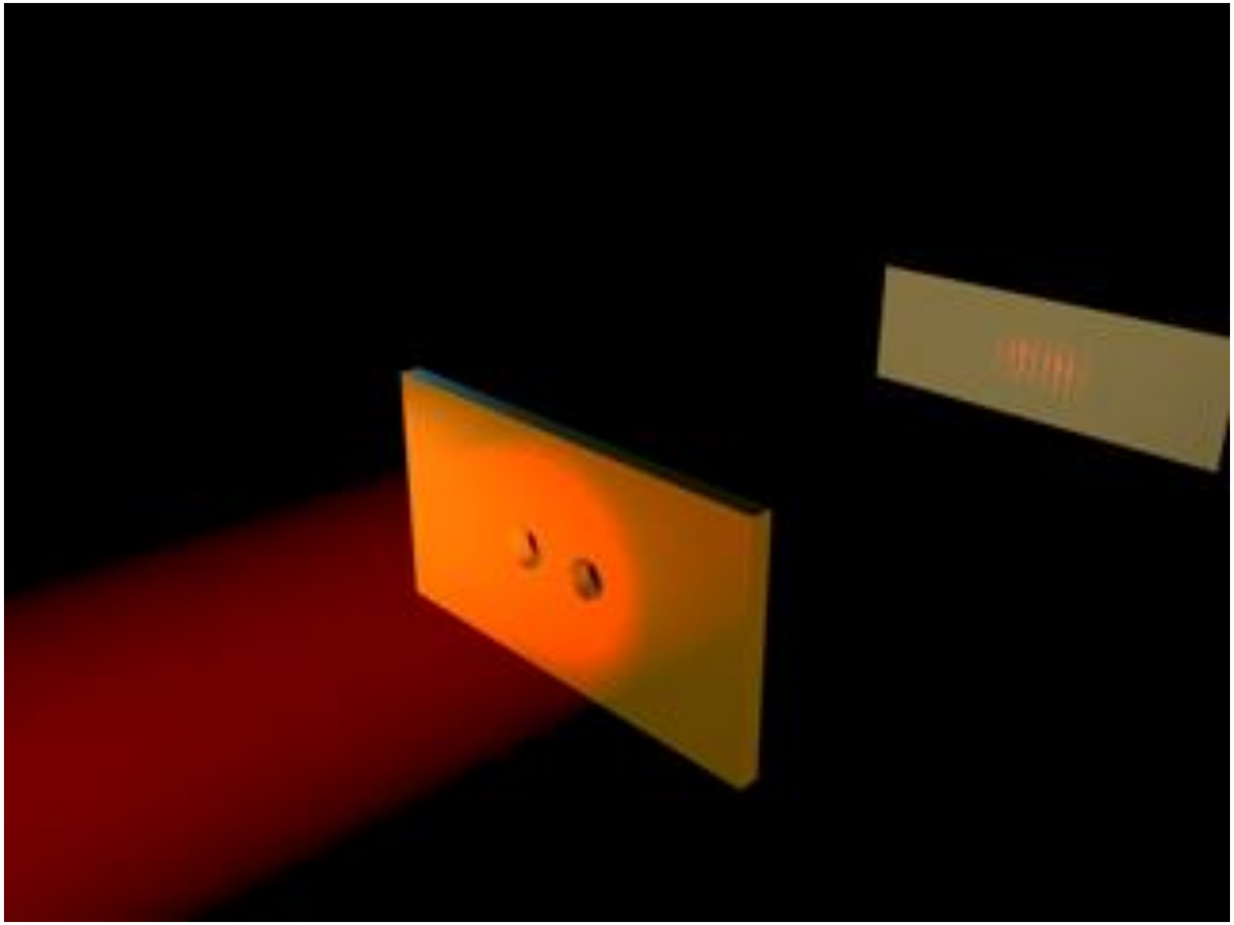
$$If \quad \frac{r_2 - r_1}{c} >> \tau_c$$

$\Rightarrow$ then waves arriving at P from S_1 & S_2 will have no fixed phase relationship & no interference pattern will be observed.

Central fringe (for which $r_1 = r_2$) will in general, have a good contrast & as we move towards higher order fringes contrast of fringes will gradually become poorer.

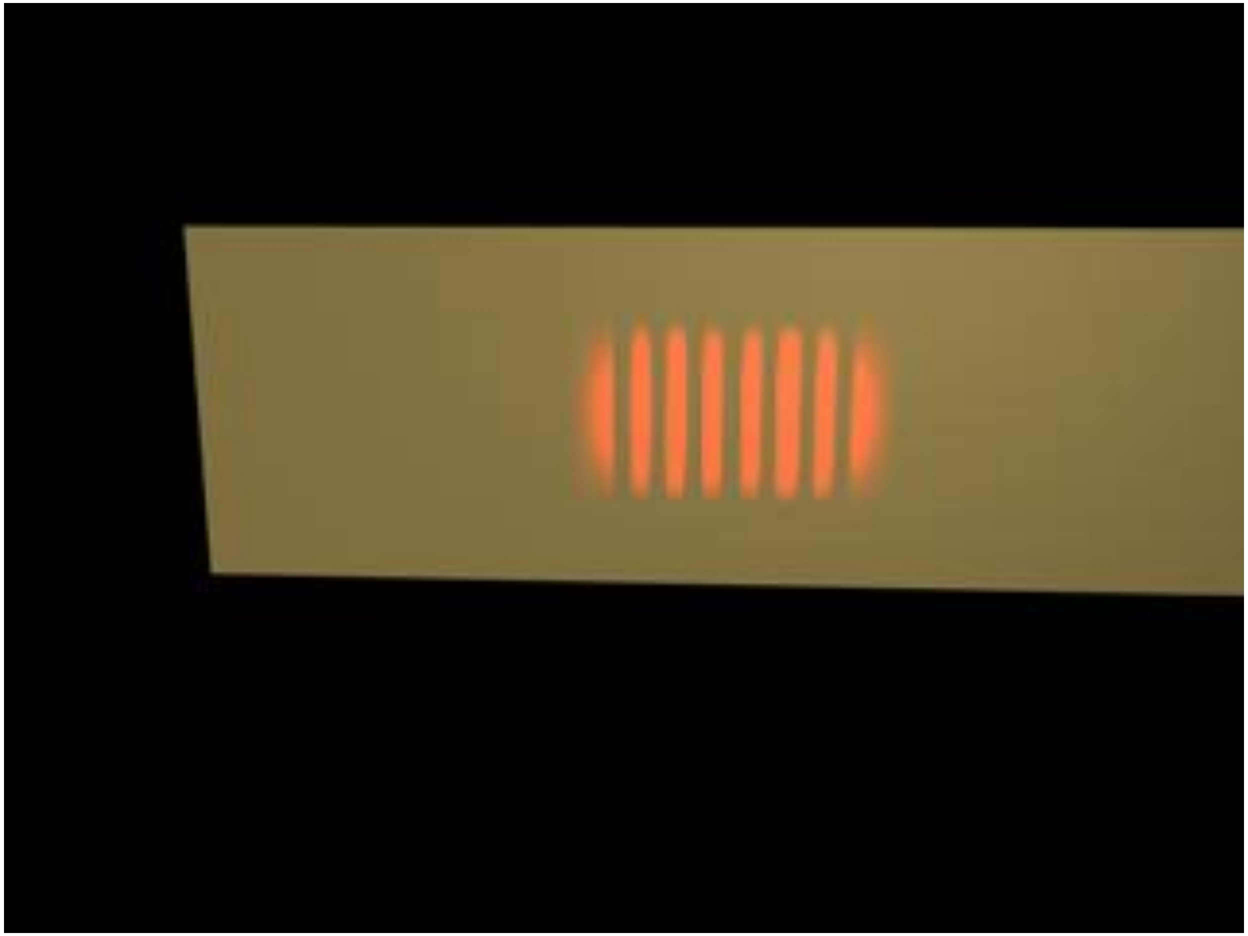
Young's Double-Slit Experiment

`The most beautiful experiment ever…' [1]



[1] New York Times (Sept. 24, 2002).

Increasing Spatial Coherence



Increasing the spatial coherence of incident field increases the ‘sharpness’ or ‘visibility’ of the interference fringes.

Temporal coherence

Temporal (longitudinal) coherence tells us how monochromatic a source is. It characterizes how well a **wave can interfere with itself at a different time.**

It is measured in an interferometer such as Michelson Interferometer or Mach-Zehnder Interferometer.

A wave is combined with a copy of itself that is delayed by time *τ*.

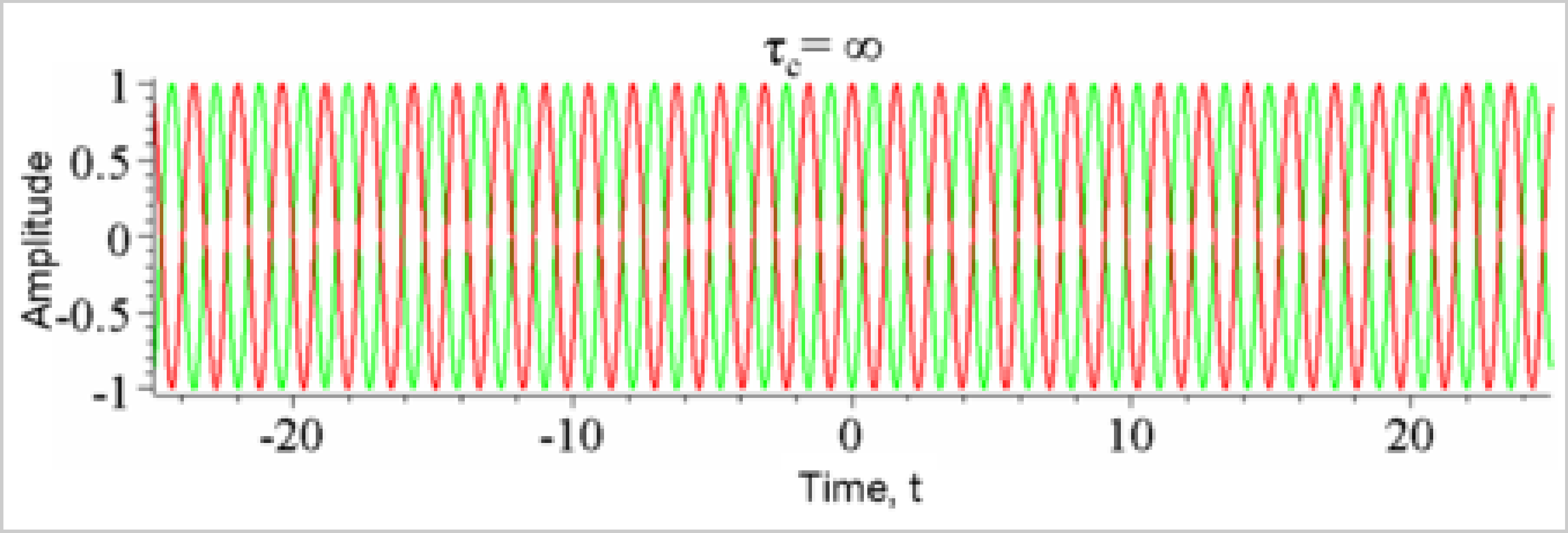


Figure 1: The amplitude of a single frequency wave as a function of time *t* (red) and a copy of the same wave delayed by τ(green). The coherence time of the wave is infinite since it is perfectly correlated with itself for all delays τ.

Michelson Interferometer

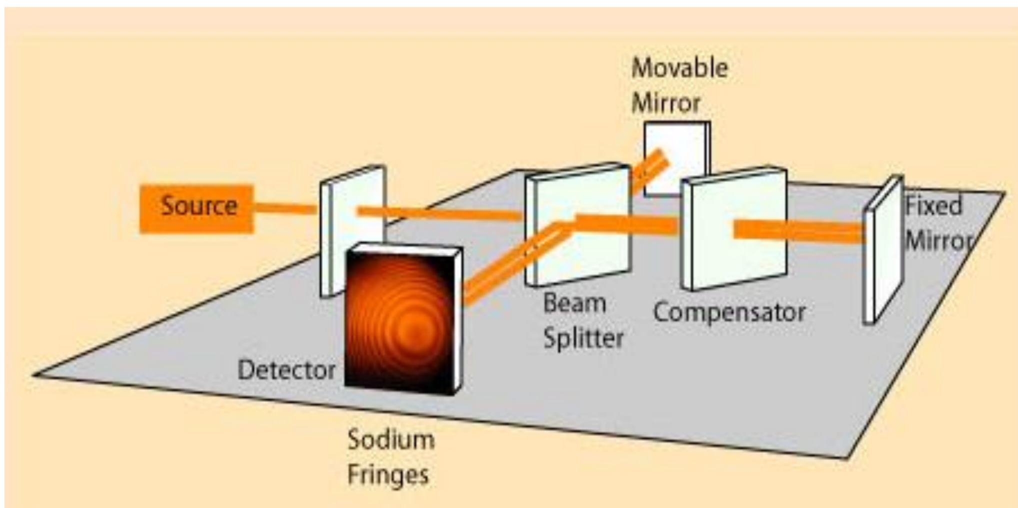
Michelson interferometer produces interference fringes by splitting a beam of monochromatic light so that one beam strikes a fixed mirror & other a movable mirror. When reflected beams are brought back together, interference pattern results.

If distance *d* is such that

$$\frac{2d}{c} << \tau_c$$

$\Rightarrow$

then a definite phase relationship exists between two beams & well-defined interference fringes are observed.



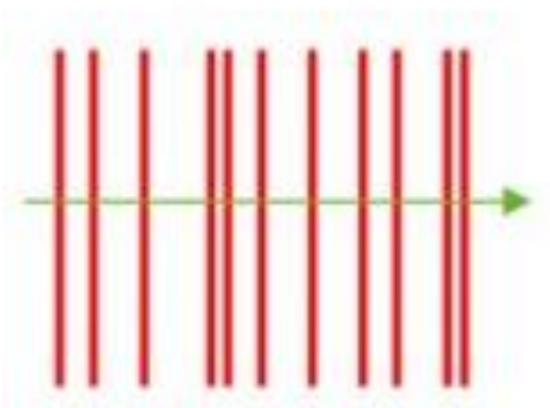
If distance *d* is such that

$$\frac{2d}{c} >> \tau_c$$

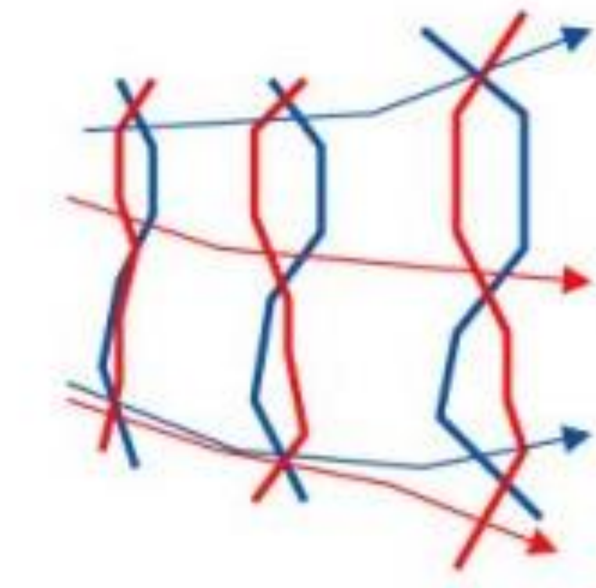
$\Rightarrow$

then there is no definite phase relationship between two beams & no interference pattern is observed.

There is no definite distance at which interference pattern disappears; as distance increases, contrast of fringes becomes gradually poorer & eventually fringe system disappears.



Temporal coherence: random fluctuations in spacing of wavefronts



Spatial coherence: random fluctuations in shape of wavefronts