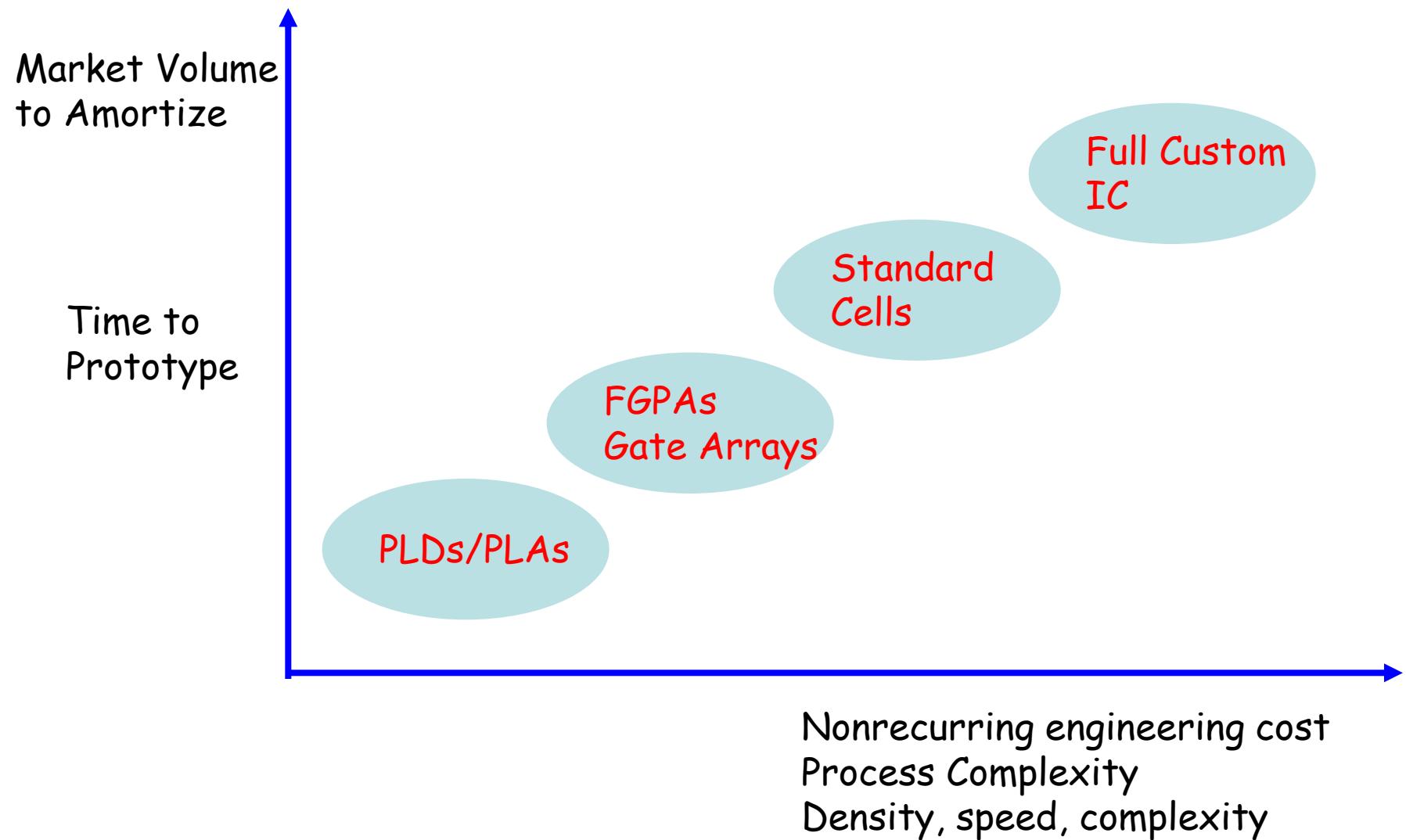
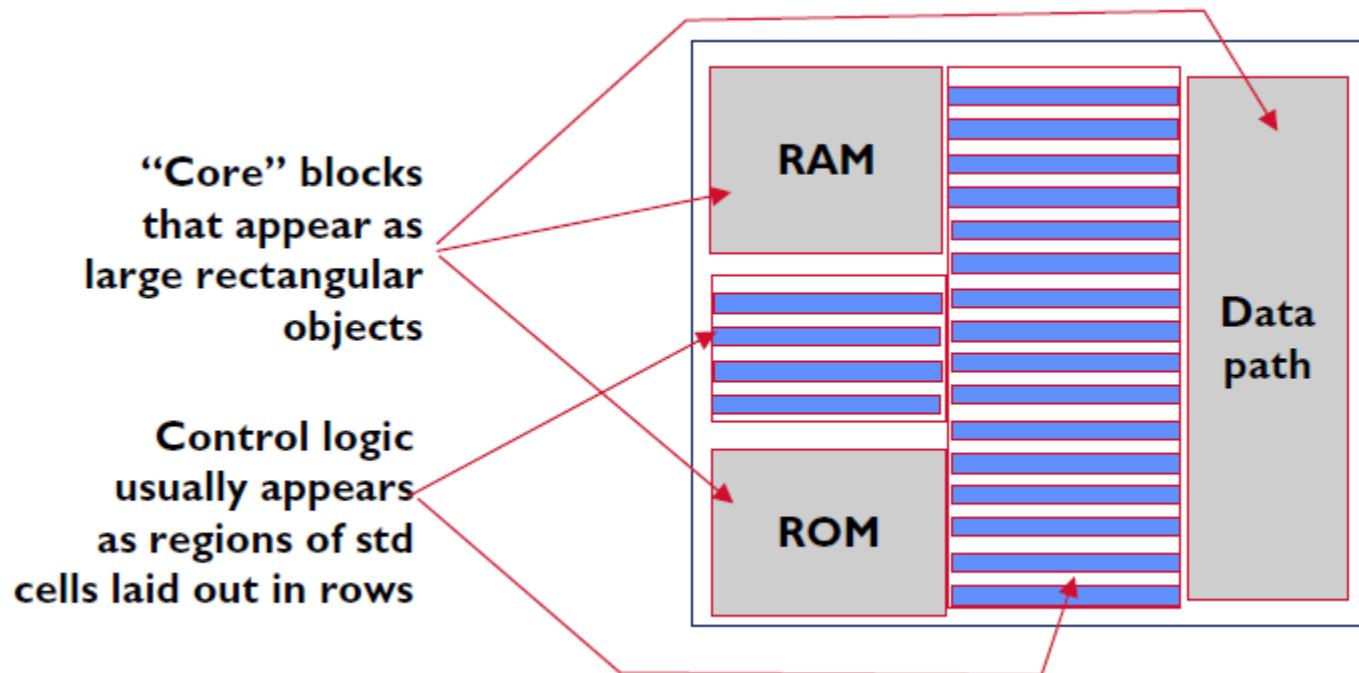


FPGA - Field Programmable Gate Array

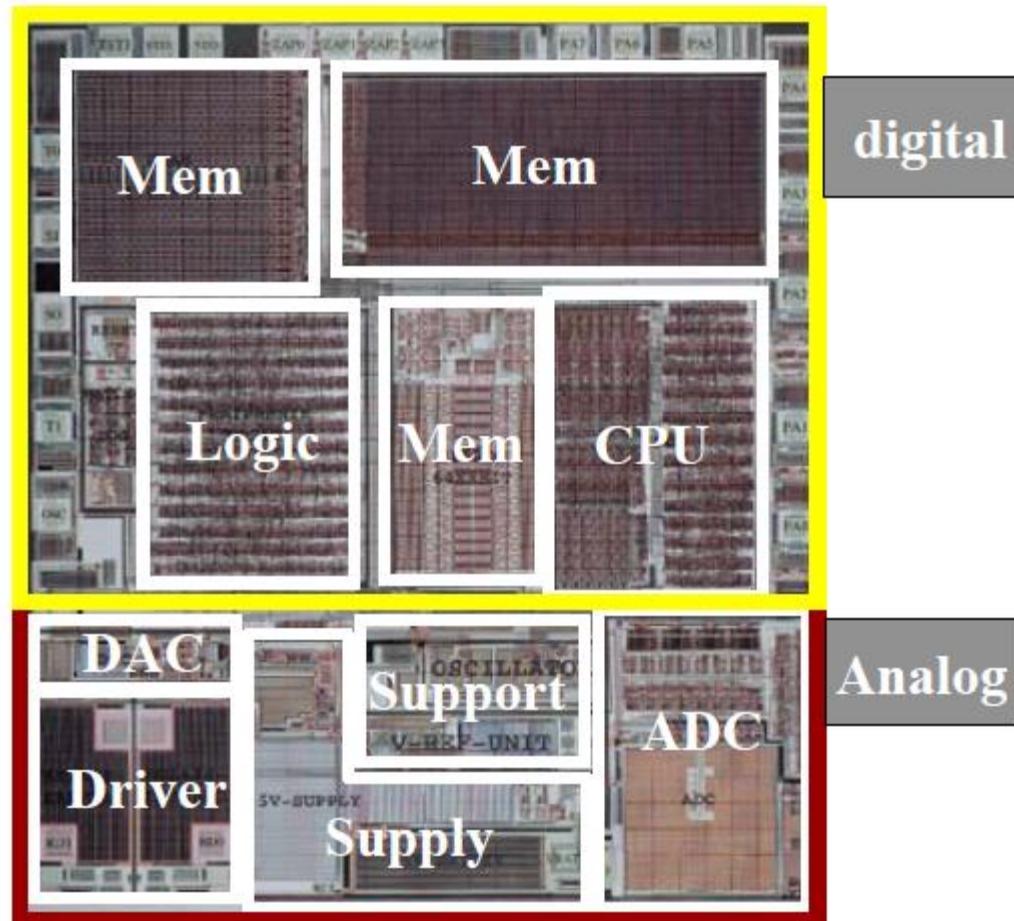
Alternative Technologies for IC Implementation



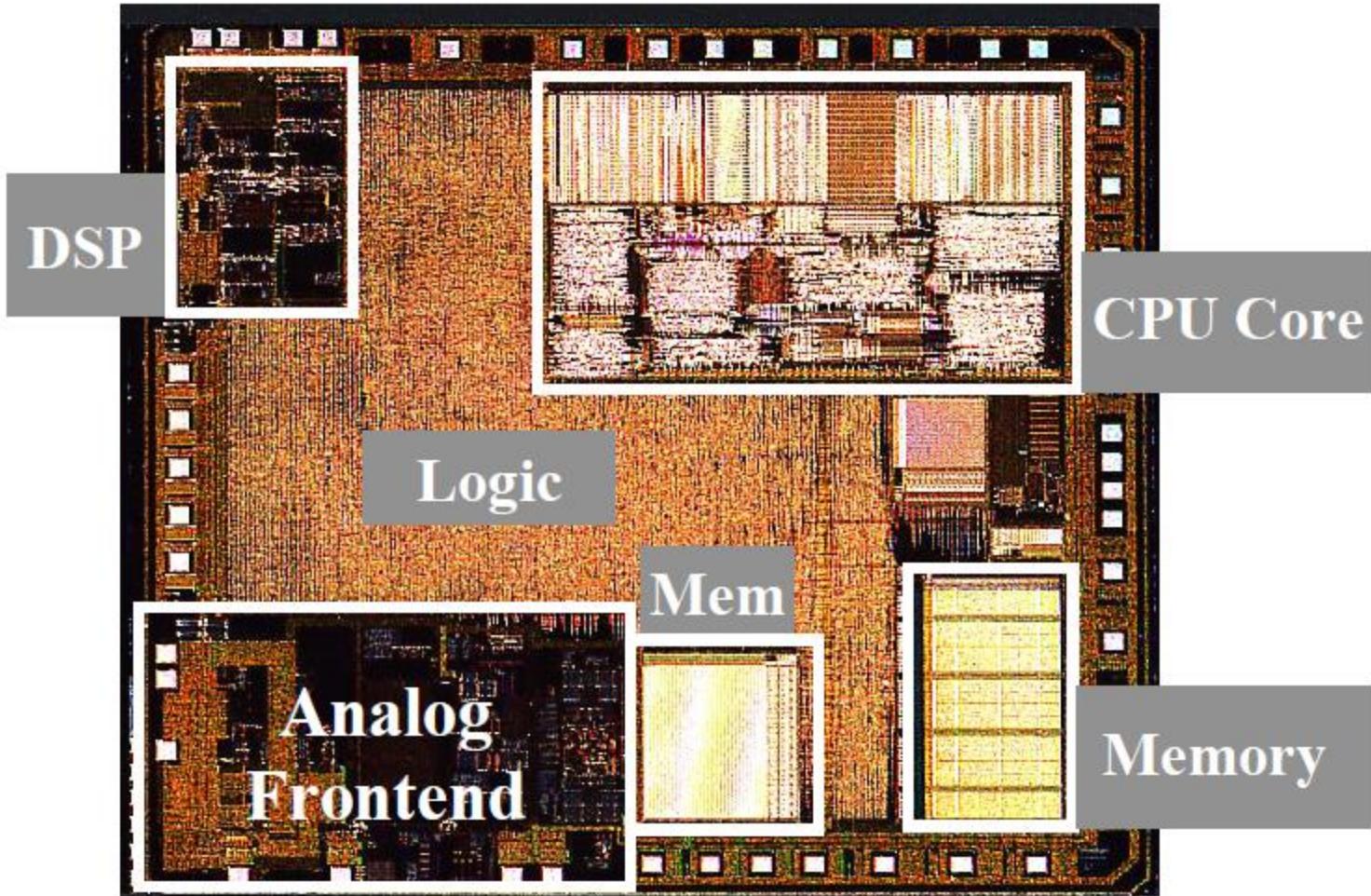
System on Chip



System on chip – example(automotive)

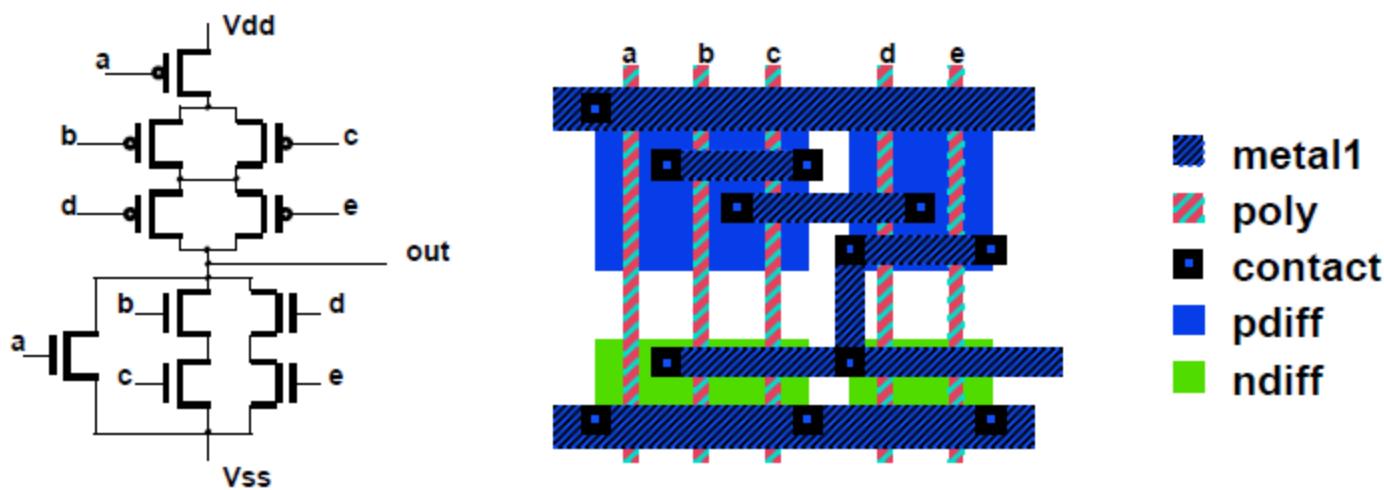


System on chip –example(Telecom)

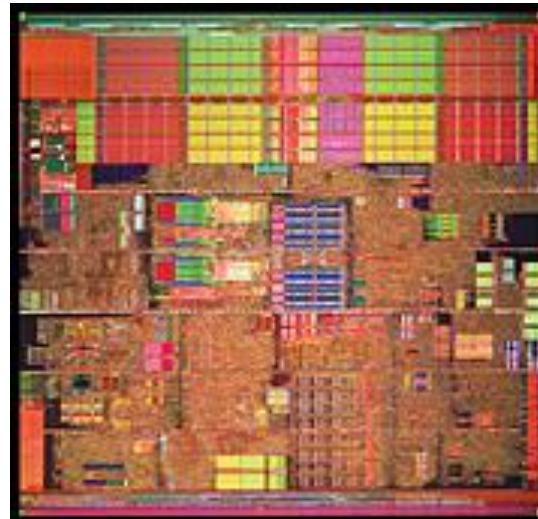
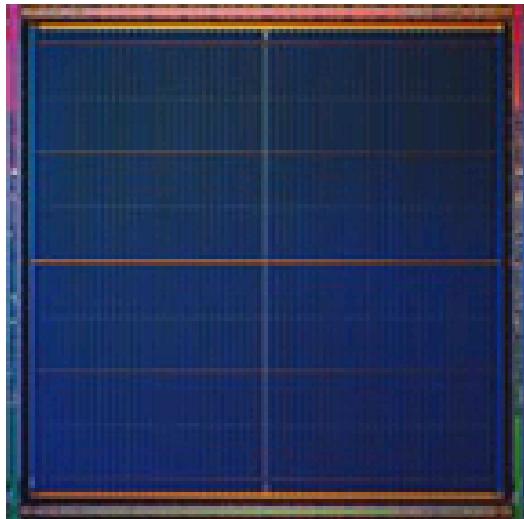


Courtesy Frank Op't Eynde, Alcatel

Full custom- for Implementation



Die Photos: Vertex vs. Pentium IV



- FPGA Vertex chip looks remarkably well structured
 - Very dense, very regular structure
- Full Custom Pentium chip somewhat more random in structure
 - Large on-chip memories (caches) are visible

Shannon's Expansion Theorem

Review

x_2	x_1	x_0	$F(x_2, x_1, x_0)$
0	0	0	0 = $F(0, 0, 0)$
0	0	1	1 = $F(0, 0, 1)$
0	1	0	0 = $F(0, 1, 0)$
0	1	1	0 = $F(0, 1, 1)$
1	0	0	1 = $F(1, 0, 0)$
1	0	1	1 = $F(1, 0, 1)$
1	1	0	1 = $F(1, 1, 0)$
1	1	1	0 = $F(1, 1, 1)$

Shannon showed that the logic expression for the above F can be obtained by the following expansion

$$\begin{aligned}
 F(x_2, x_1, x_0) &= \overline{x_2} \cdot \overline{x_1} \cdot \overline{x_0} \cdot F(0, 0, 0) + \overline{x_2} \cdot \overline{x_1} \cdot x_0 \cdot F(0, 0, 1) + \overline{x_2} \cdot x_1 \cdot \overline{x_0} \cdot F(0, 1, 0) \\
 &\quad + \overline{x_2} \cdot x_1 \cdot x_0 \cdot F(0, 1, 1) + x_2 \cdot \overline{x_1} \cdot \overline{x_0} \cdot F(1, 0, 0) + x_2 \cdot \overline{x_1} \cdot x_0 \cdot F(1, 0, 1) \\
 &\quad + x_2 \cdot x_1 \cdot \overline{x_0} \cdot F(1, 1, 0) + x_2 \cdot x_1 \cdot x_0 \cdot F(1, 1, 1)
 \end{aligned}$$

$$F(x_2, x_1, x_0) = \overline{x_2} \cdot \overline{x_1} \cdot x_0 + x_2 \cdot \overline{x_1} \cdot \overline{x_0} + x_2 \cdot \overline{x_1} \cdot x_0 + x_2 \cdot x_1 \cdot \overline{x_0} = \sum m(1, 4, 5, 6)$$

POS form

$$\begin{aligned} F(x_2, x_1, x_0) = & (x_2 + x_1 + x_0 + F(0, 0, 0)) \cdot (x_2 + x_1 + \overline{x_0} + F(0, 0, 1)) \cdot (x_2 + \overline{x_1} + x_0 + F(0, 1, 0)) \\ & \cdot (x_2 + \overline{x_1} + \overline{x_0} + F(0, 1, 1)) \cdot (\overline{x_2} + x_1 + x_0 + F(1, 0, 0)) \cdot (\overline{x_2} + x_1 + \overline{x_0} + F(1, 0, 1)) \\ & \cdot (\overline{x_2} + \overline{x_1} + x_0 + F(1, 1, 0)) \cdot (\overline{x_2} + \overline{x_1} + \overline{x_0} + F(1, 1, 1)) \end{aligned}$$

$$F(x_2, x_1, x_0) = (x_2 + x_1 + x_0) \cdot (x_2 + \overline{x_1} + x_0) \cdot (x_2 + \overline{x_1} + \overline{x_0}) \cdot (\overline{x_2} + \overline{x_1} + \overline{x_0}) = \prod M(0, 2, 3, 7)$$

Shannon's Expansion Theorem

- C. E. Shannon, “A Symbolic Analysis of Relay and Switching Circuits,” *Trans. AIEE*, vol. 57, pp. 713-723, 1938.
- Consider:
 - Boolean variables, X_1, X_2, \dots, X_n
 - Boolean function, $F(X_1, X_2, \dots, X_n)$
- Then $F = X_i F(X_i=1) + X'_i F(X_i=0)$
- Where
 - X'_i is complement of X_i
 - Cofactors, $F(X_i=j) = F(X_1, X_2, \dots, X_i=j, \dots, X_n)$, $j = 0$ or 1

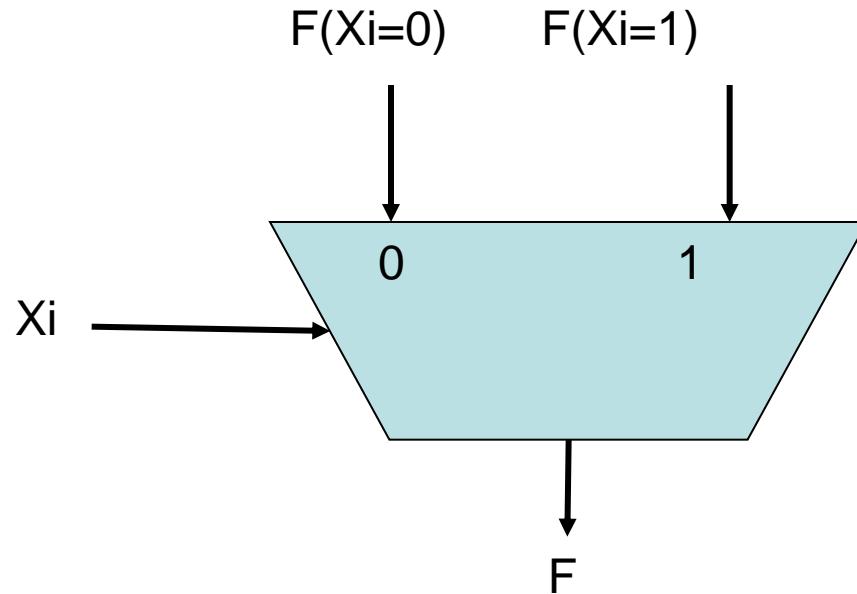
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Theorem

$$(1) \quad F = X_i \cdot F(X_i=1) + X_i' \cdot F(X_i=0) \quad \forall i=1,2,3, \dots n$$

$$(2) \quad F = (X_i + F(X_i=0)) \cdot (X_i' + F(X_i=1)) \quad \forall i=1,2,3, \dots n$$



Example

$$f(a, b, c, d) = ab + b'cd + acd$$

$$f(a, 1, c, d) \stackrel{b=1}{=} a + acd = \underbrace{a + acd}_{\text{covering}} = a$$

$$f(a, 0, c, d) \stackrel{b=0}{=} cd + acd = \underbrace{cd + acd}_{\text{covering}} = cd$$

$$\begin{aligned} f(a, b, c, d) &= b \cdot f(a, 1, c, d) + b' \cdot f(a, 0, c, d) \quad \text{Shannon's expansion} \\ &= b \cdot a + b' \cdot cd \\ &= ab + b'cd \end{aligned}$$

Example-POS

$$f(a, b, c, d) = (a + b + c)(a' + d)(b + c + d)$$

$$f(1, b, c, d) \stackrel{a=1}{=} (1 + b + c)(0 + d)(b + c + d) = \underbrace{(d)(a + c + d)}_{covering} = d$$

$$f(0, b, c, d) \stackrel{a=0}{=} (0 + b + c)(1 + d)(a + c + d) = \underbrace{(b + c)(b + c + d)}_{covering} = (b + c)$$

$$\begin{aligned} f(a, b, c, d) &= (a + f(\bar{0}, \bar{b}, c, d))(a' + f(\bar{1}, \bar{b}, c, d)) \text{ Shannon's expansion} \\ &= (a + b + c)(a' + d) \end{aligned}$$

Shannon's Expansion Theorem

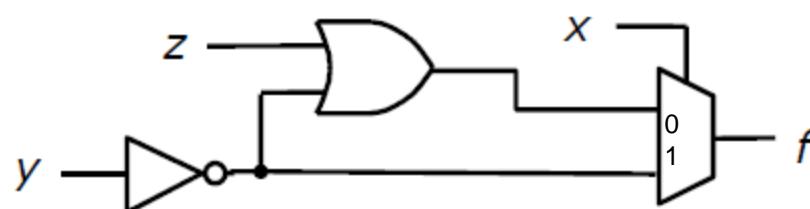
More examples

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$$f = x'y'z' + x'y'z + x'yz + xy'z' + xy'z$$

choose x as the expansion variable

$$\begin{aligned} f &= x'(y'z' + y'z + yz) + x(y'z' + y'z) \\ f &= x'(y' + z) + x(y') \end{aligned}$$

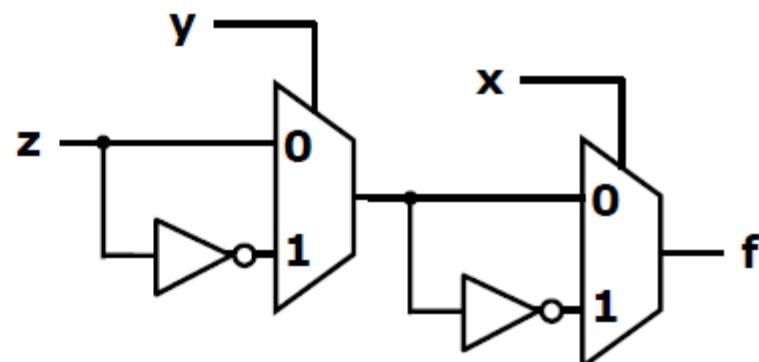


More examples

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

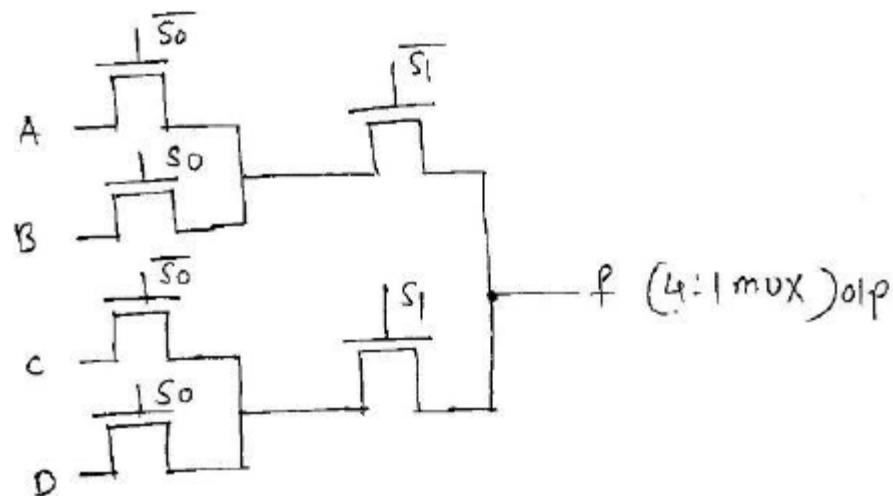
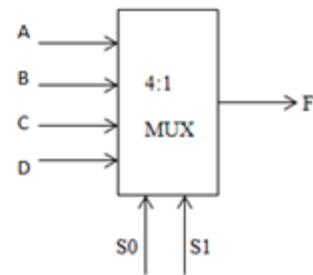
$y \oplus z$

$(y \oplus z)'$

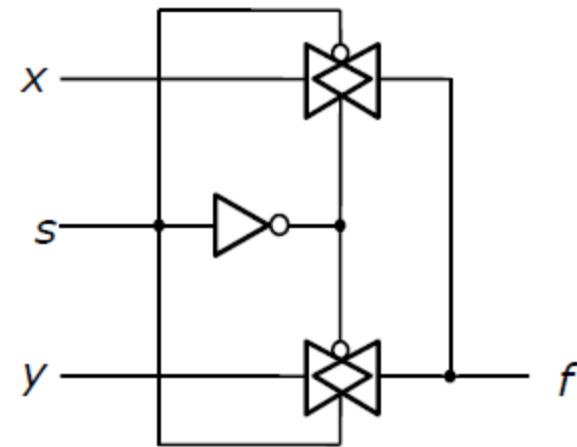
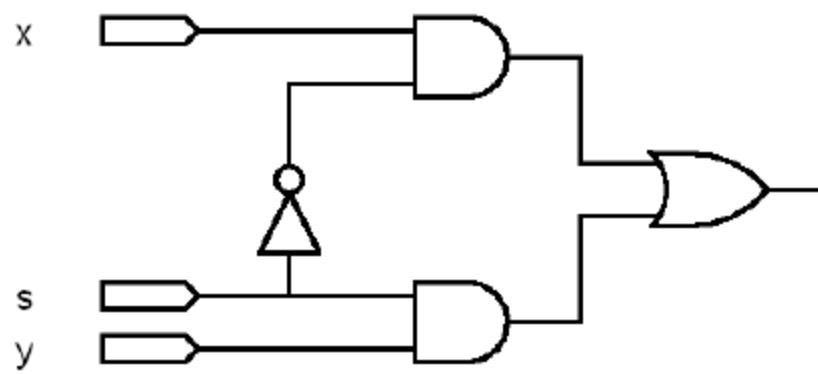


MUX implementation

S1	S0	F
0	0	A
0	1	B
1	0	C
1	1	D



MUX implementation



The preferred
implementation

