

Stepwise and best subset variable selection

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```
library(MASS)
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
## v dplyr      1.1.4      v readr      2.1.5
## v forcats    1.0.0      v stringr   1.5.1
## v ggplot2    3.5.1      v tibble    3.2.1
## v lubridate  1.9.3      v tidyr     1.3.1
## v purrr      1.0.2
## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()     masks stats::lag()
## x dplyr::select() masks MASS::select()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(caret)
```

```
## Loading required package: lattice
##
## Attaching package: 'caret'
##
## The following object is masked from 'package:purrr':
##
##     lift
```

```
library(glmnet)
```

```
## Loading required package: Matrix
##
## Attaching package: 'Matrix'
##
## The following objects are masked from 'package:tidyr':
##
##     expand, pack, unpack
##
## Loaded glmnet 4.1-8
```

```
library(leaps)
```

```

# Use variables except G1 and G2 to predict G3
# y: G3
# x: all the other variables
data <- read.csv('math.csv', sep = ';')
data <- subset(data, select = - c(G1,G2))
str(data)

```

```

## 'data.frame': 395 obs. of 31 variables:
## $ school : chr "GP" "GP" "GP" "GP" ...
## $ sex : chr "F" "F" "F" "F" ...
## $ age : int 18 17 15 15 16 16 16 17 15 15 ...
## $ address : chr "U" "U" "U" "U" ...
## $ famsize : chr "GT3" "GT3" "LE3" "GT3" ...
## $ Pstatus : chr "A" "T" "T" "T" ...
## $ Medu : int 4 1 1 4 3 4 2 4 3 3 ...
## $ Fedu : int 4 1 1 2 3 3 2 4 2 4 ...
## $ Mjob : chr "at_home" "at_home" "at_home" "health" ...
## $ Fjob : chr "teacher" "other" "other" "services" ...
## $ reason : chr "course" "course" "other" "home" ...
## $ guardian : chr "mother" "father" "mother" "mother" ...
## $ traveltime: int 2 1 1 1 1 1 1 2 1 1 ...
## $ studytime : int 2 2 2 3 2 2 2 2 2 2 ...
## $ failures : int 0 0 3 0 0 0 0 0 0 0 ...
## $ schoolsup : chr "yes" "no" "yes" "no" ...
## $ famsup : chr "no" "yes" "no" "yes" ...
## $ paid : chr "no" "no" "yes" "yes" ...
## $ activities: chr "no" "no" "no" "yes" ...
## $ nursery : chr "yes" "no" "yes" "yes" ...
## $ higher : chr "yes" "yes" "yes" "yes" ...
## $ internet : chr "no" "yes" "yes" "yes" ...
## $ romantic : chr "no" "no" "no" "yes" ...
## $ famrel : int 4 5 4 3 4 5 4 4 4 5 ...
## $ freetime : int 3 3 3 2 3 4 4 1 2 5 ...
## $ goout : int 4 3 2 2 2 2 4 4 2 1 ...
## $ Dalc : int 1 1 2 1 1 1 1 1 1 1 ...
## $ Walc : int 1 1 3 1 2 2 1 1 1 1 ...
## $ health : int 3 3 3 5 5 5 3 1 1 5 ...
## $ absences : int 6 4 10 2 4 10 0 6 0 0 ...
## $ G3 : int 6 6 10 15 10 15 11 6 19 15 ...

```

```

set.seed(123)
training.samples <- data$G3 %>% createDataPartition(p = 0.75, list = FALSE) # caret pkg
# Uses createDataPartition() from the caret package to split the data into training (75%) and test (25%)
# The %>% operator (pipe) is from the tidyverse pkg
train.data <- data[training.samples, ]
test.data <- data[-training.samples, ]

```

stepwise model selection using the Bayesian Information Criterion (BIC)

Backward: 1. Starts with the full model (all predictors). 2. Removes the least significant predictor (one with the highest p-value) iteratively. 3. Stops when removing predictors no longer improves BIC. 4. Returns the final optimal model (fit_step).

```
fit <- lm(G3~., data = train.data) # a full model
fit_step <- stepAIC(fit, k = log(nrow(train.data)), trace = 1) # MASS pkg
```

```
## Start: AIC=1034.07
## G3 ~ school + sex + age + address + famsize + Pstatus + Medu +
##      Fedu + Mjob + Fjob + reason + guardian + traveltime + studytime +
##      failures + schoolsup + famsup + paid + activities + nursery +
##      higher + internet + romantic + famrel + freetime + goout +
##      Dalc + Walc + health + absences
##
##              Df Sum of Sq    RSS    AIC
## - Fjob         4    41.387 4499.2 1014.0
## - Mjob         4    97.705 4555.6 1017.7
## - reason       3    27.452 4485.3 1018.8
## - guardian     2     4.311 4462.2 1023.0
## - traveltime   1     0.045 4457.9 1028.4
## - Pstatus      1     0.082 4457.9 1028.4
## - activities   1     5.276 4463.1 1028.7
## - school       1     7.353 4465.2 1028.9
## - Fedu         1     8.054 4465.9 1028.9
## - address      1     9.158 4467.0 1029.0
## - paid         1    11.233 4469.1 1029.1
## - freetime     1    15.892 4473.7 1029.4
## - internet     1    15.967 4473.8 1029.4
## - Dalc         1    19.126 4477.0 1029.7
## - higher       1    19.372 4477.2 1029.7
## - nursery      1    20.591 4478.4 1029.8
## - Walc         1    21.954 4479.8 1029.8
## - goout        1    26.597 4484.4 1030.2
## - Medu         1    32.380 4490.2 1030.5
## - health       1    34.929 4492.8 1030.7
## - famrel       1    34.994 4492.8 1030.7
## - studytime    1    37.209 4495.1 1030.8
## - famsup       1    41.951 4499.8 1031.2
## - sex          1    43.454 4501.3 1031.3
## - age          1    51.687 4509.5 1031.8
## - schoolsup     1    53.305 4511.2 1031.9
## - famsize      1    66.075 4523.9 1032.8
## - absences     1    74.544 4532.4 1033.3
## <none>                     4457.8 1034.1
## - romantic     1   109.816 4567.7 1035.6
## - failures     1   254.495 4712.3 1044.9
##
## Step: AIC=1014.04
## G3 ~ school + sex + age + address + famsize + Pstatus + Medu +
##      Fedu + Mjob + reason + guardian + traveltime + studytime +
##      failures + schoolsup + famsup + paid + activities + nursery +
##      higher + internet + romantic + famrel + freetime + goout +
##      Dalc + Walc + health + absences
##
##              Df Sum of Sq    RSS    AIC
## - Mjob         4    90.713 4589.9  997.20
## - reason       3    37.315 4536.5  999.41
```

```

## - guardian      2      6.620 4505.9 1003.08
## - traveltime    1      0.004 4499.2 1008.34
## - Pstatus       1      0.014 4499.2 1008.34
## - Fedu          1      1.541 4500.8 1008.44
## - activities    1      8.044 4507.3 1008.87
## - school        1      8.559 4507.8 1008.91
## - paid          1      8.889 4508.1 1008.93
## - address       1     11.199 4510.4 1009.08
## - internet      1     11.208 4510.4 1009.08
## - Dalc          1     12.244 4511.5 1009.15
## - Walc          1     12.458 4511.7 1009.16
## - freetime      1     13.421 4512.7 1009.23
## - higher        1     17.188 4516.4 1009.48
## - nursery       1     19.765 4519.0 1009.65
## - goout         1     22.359 4521.6 1009.82
## - Medu          1     29.220 4528.5 1010.27
## - famrel        1     32.787 4532.0 1010.50
## - health        1     32.908 4532.1 1010.51
## - sex           1     39.533 4538.8 1010.95
## - studytime     1     40.155 4539.4 1010.99
## - famsup        1     40.789 4540.0 1011.03
## - schoolsup     1     44.419 4543.7 1011.27
## - age           1     50.844 4550.1 1011.69
## - famsize       1     58.276 4557.5 1012.17
## - absences      1     70.580 4569.8 1012.98
## <none>          1     4499.2 1014.04
## - romantic      1    101.250 4600.5 1014.97
## - failures      1    247.187 4746.4 1024.28
##
## Step:  AIC=997.2
## G3 ~ school + sex + age + address + famsize + Pstatus + Medu +
##      Fedu + reason + guardian + traveltime + studytime + failures +
##      schoolsup + famsup + paid + activities + nursery + higher +
##      internet + romantic + famrel + freetime + goout + Dalc +
##      Walc + health + absences
##
##              Df Sum of Sq   RSS   AIC
## - reason      3    67.742 4657.7  984.47
## - guardian    2     9.081 4599.0  986.39
## - traveltime  1     0.127 4590.1  991.51
## - Pstatus     1     0.705 4590.7  991.55
## - Fedu        1     2.664 4592.6  991.67
## - school      1     7.446 4597.4  991.98
## - activities  1     8.718 4598.7  992.07
## - paid        1     9.325 4599.3  992.10
## - freetime    1    11.680 4601.6  992.26
## - goout       1    13.269 4603.2  992.36
## - address     1    13.898 4603.8  992.40
## - Walc        1    16.346 4606.3  992.56
## - internet    1    16.583 4606.5  992.57
## - nursery     1    19.575 4609.5  992.77
## - Dalc        1    22.564 4612.5  992.96
## - higher      1    23.022 4613.0  992.99
## - health      1    27.256 4617.2  993.26

```

```

## - famrel      1      32.023 4622.0 993.57
## - sex         1      32.258 4622.2 993.59
## - famsup      1      35.123 4625.1 993.77
## - studytime   1      36.898 4626.8 993.89
## - age         1      47.197 4637.1 994.55
## - schoolsup    1      50.192 4640.1 994.74
## - Medu        1      51.533 4641.5 994.83
## - absences    1      62.739 4652.7 995.55
## - famsize     1      65.170 4655.1 995.70
## <none>                4589.9 997.20
## - romantic    1     106.790 4696.7 998.35
## - failures    1     230.587 4820.5 1006.11
##
## Step: AIC=984.47
## G3 ~ school + sex + age + address + famsize + Pstatus + Medu +
##      Fedu + guardian + traveltime + studytime + failures + schoolsup +
##      famsup + paid + activities + nursery + higher + internet +
##      romantic + famrel + freetime + goout + Dalc + Walc + health +
##      absences
##
##           Df Sum of Sq   RSS   AIC
## - guardian    2      9.785 4667.5 973.70
## - Pstatus      1      0.146 4657.8 978.78
## - traveltime   1      0.368 4658.1 978.80
## - Fedu         1      3.678 4661.4 979.01
## - school       1      6.475 4664.2 979.19
## - activities   1      6.959 4664.6 979.22
## - address      1     10.186 4667.9 979.43
## - paid         1     12.148 4669.8 979.55
## - higher       1     13.122 4670.8 979.61
## - internet     1     14.679 4672.4 979.71
## - freetime     1     15.091 4672.8 979.74
## - goout        1     18.097 4675.8 979.93
## - Dalc         1     18.681 4676.4 979.97
## - Walc         1     19.883 4677.6 980.04
## - nursery      1     21.917 4679.6 980.17
## - sex          1     22.507 4680.2 980.21
## - famrel       1     31.653 4689.3 980.79
## - studytime    1     35.681 4693.4 981.05
## - health       1     36.394 4694.1 981.09
## - famsup       1     40.659 4698.3 981.36
## - schoolsup    1     45.873 4703.6 981.70
## - age          1     47.479 4705.2 981.80
## - famsize      1     62.488 4720.2 982.75
## - absences     1     64.142 4721.8 982.85
## - Medu         1     66.728 4724.4 983.01
## <none>                4657.7 984.47
## - romantic    1     103.281 4761.0 985.31
## - failures    1     239.621 4897.3 993.72
##
## Step: AIC=973.7
## G3 ~ school + sex + age + address + famsize + Pstatus + Medu +
##      Fedu + traveltime + studytime + failures + schoolsup + famsup +
##      paid + activities + nursery + higher + internet + romantic +

```

```

##      famrel + freetime + goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - Pstatus      1      0.064 4667.5 968.01
## - traveltime    1      0.741 4668.2 968.05
## - Fedu          1      3.881 4671.4 968.25
## - activities    1      6.633 4674.1 968.43
## - school        1      8.291 4675.8 968.53
## - address       1      9.709 4677.2 968.63
## - higher        1     10.318 4677.8 968.66
## - paid          1     10.472 4677.9 968.67
## - freetime      1     13.671 4681.1 968.88
## - internet      1     16.231 4683.7 969.04
## - nursery       1     17.863 4685.3 969.14
## - goout         1     17.994 4685.5 969.15
## - Dalc          1     18.655 4686.1 969.19
## - Walc          1     22.208 4689.7 969.42
## - sex           1     23.557 4691.0 969.51
## - famrel        1     29.950 4697.4 969.91
## - health        1     32.610 4700.1 970.08
## - studytime     1     36.001 4703.5 970.30
## - famsup        1     41.609 4709.1 970.65
## - schoolsup      1     45.532 4713.0 970.90
## - absences      1     57.759 4725.2 971.67
## - famsize       1     61.606 4729.1 971.91
## - Medu          1     66.154 4733.6 972.20
## - age           1     66.396 4733.9 972.22
## <none>                                4667.5 973.70
## - romantic      1    106.326 4773.8 974.72
## - failures      1    288.196 4955.7 985.86
##
## Step:  AIC=968.01
## G3 ~ school + sex + age + address + famsize + Medu + Fedu + traveltime +
##      studytime + failures + schoolsup + famsup + paid + activities +
##      nursery + higher + internet + romantic + famrel + freetime +
##      goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - traveltime    1      0.777 4668.3 962.36
## - Fedu          1      3.827 4671.4 962.56
## - activities     1      6.886 4674.4 962.75
## - school         1      8.280 4675.8 962.84
## - address        1      9.697 4677.2 962.93
## - higher         1     10.333 4677.9 962.97
## - paid           1     10.409 4677.9 962.98
## - freetime       1     13.673 4681.2 963.18
## - internet       1     16.192 4683.7 963.34
## - nursery        1     17.855 4685.4 963.45
## - goout          1     18.085 4685.6 963.47
## - Dalc           1     18.595 4686.1 963.50
## - Walc           1     22.208 4689.7 963.73
## - sex            1     23.513 4691.1 963.81
## - famrel         1     30.221 4697.8 964.24
## - health         1     32.822 4700.4 964.40

```

```

## - studytime 1 36.093 4703.6 964.61
## - famsup 1 41.546 4709.1 964.95
## - schoolsup 1 45.496 4713.0 965.20
## - absences 1 59.337 4726.9 966.08
## - famsize 1 63.213 4730.8 966.32
## - Medu 1 66.566 4734.1 966.53
## - age 1 66.584 4734.1 966.53
## <none> 4667.5 968.01
## - romantic 1 106.274 4773.8 969.02
## - failures 1 288.151 4955.7 980.16
##
## Step: AIC=962.36
## G3 ~ school + sex + age + address + famsize + Medu + Fedu + studytime +
## failures + schoolsup + famsup + paid + activities + nursery +
## higher + internet + romantic + famrel + freetime + goout +
## Dalc + Walc + health + absences
##
## Df Sum of Sq RSS AIC
## - Fedu 1 3.505 4671.8 956.89
## - activities 1 7.111 4675.4 957.12
## - school 1 7.648 4676.0 957.15
## - higher 1 10.593 4678.9 957.34
## - paid 1 10.837 4679.2 957.36
## - address 1 12.423 4680.7 957.46
## - freetime 1 14.508 4682.8 957.59
## - internet 1 16.559 4684.9 957.72
## - nursery 1 18.223 4686.5 957.83
## - goout 1 18.374 4686.7 957.84
## - Dalc 1 19.867 4688.2 957.93
## - Walc 1 22.344 4690.7 958.09
## - sex 1 23.459 4691.8 958.16
## - famrel 1 29.829 4698.1 958.56
## - health 1 32.534 4700.8 958.73
## - studytime 1 37.255 4705.6 959.03
## - famsup 1 43.715 4712.0 959.44
## - schoolsup 1 46.150 4714.5 959.60
## - absences 1 59.413 4727.7 960.43
## - famsize 1 62.691 4731.0 960.64
## - age 1 65.898 4734.2 960.84
## - Medu 1 67.769 4736.1 960.96
## <none> 4668.3 962.36
## - romantic 1 108.614 4776.9 963.52
## - failures 1 287.427 4955.7 974.47
##
## Step: AIC=956.89
## G3 ~ school + sex + age + address + famsize + Medu + studytime +
## failures + schoolsup + famsup + paid + activities + nursery +
## higher + internet + romantic + famrel + freetime + goout +
## Dalc + Walc + health + absences
##
## Df Sum of Sq RSS AIC
## - school 1 7.228 4679.0 951.65
## - activities 1 7.979 4679.8 951.70
## - higher 1 10.431 4682.3 951.86

```

```

## - paid      1      12.085 4683.9 951.96
## - address   1      13.386 4685.2 952.04
## - freetime  1      15.227 4687.0 952.16
## - internet  1      16.041 4687.9 952.21
## - goout     1      19.358 4691.2 952.42
## - Dalc      1      19.553 4691.4 952.44
## - nursery   1      19.828 4691.6 952.45
## - Walc      1      22.149 4694.0 952.60
## - sex       1      24.876 4696.7 952.77
## - famrel    1      29.988 4701.8 953.10
## - health    1      34.198 4706.0 953.37
## - studytime 1      39.144 4711.0 953.68
## - schoolsup  1      46.635 4718.5 954.15
## - famsup    1      47.391 4719.2 954.20
## - absences  1      61.972 4733.8 955.12
## - famsize   1      64.452 4736.3 955.27
## - age       1      64.557 4736.4 955.28
## - Medu      1      77.401 4749.2 956.09
## <none>              4671.8 956.89
## - romantic  1     109.334 4781.2 958.09
## - failures  1     284.237 4956.1 968.79
##
## Step: AIC=951.65
## G3 ~ sex + age + address + famsize + Medu + studytime + failures +
##      schoolsup + famsup + paid + activities + nursery + higher +
##      internet + romantic + famrel + freetime + goout + Dalc +
##      Walc + health + absences
##
##              Df Sum of Sq    RSS    AIC
## - activities  1       9.353 4688.4 946.55
## - address     1       9.530 4688.6 946.56
## - higher      1      10.887 4689.9 946.65
## - paid        1      12.918 4692.0 946.78
## - internet    1      16.202 4695.2 946.99
## - freetime    1      16.364 4695.4 947.00
## - Dalc        1      17.739 4696.8 947.08
## - goout       1      19.874 4698.9 947.22
## - nursery     1      20.514 4699.6 947.26
## - Walc        1      20.767 4699.8 947.28
## - sex         1      23.336 4702.4 947.44
## - famrel      1      28.011 4707.1 947.73
## - studytime   1      35.392 4714.4 948.20
## - health      1      35.418 4714.5 948.20
## - schoolsup    1      47.592 4726.6 948.97
## - famsup      1      53.342 4732.4 949.33
## - absences    1      56.334 4735.4 949.52
## - age         1      57.329 4736.4 949.58
## - famsize     1      67.750 4746.8 950.24
## - Medu        1      77.169 4756.2 950.83
## <none>              4679.0 951.65
## - romantic    1     108.686 4787.7 952.80
## - failures    1     287.868 4966.9 963.75
##
## Step: AIC=946.55

```



```

## G3 ~ sex + age + address + famsize + Medu + studytime + failures +
##      schoolsup + famsup + paid + nursery + higher + internet +
##      romantic + famrel + freetime + goout + Dalc + Walc + health +
##      absences
##
##      Df Sum of Sq    RSS    AIC
## - higher      1      9.674 4698.1 941.47
## - address      1     10.791 4699.2 941.54
## - paid         1     13.422 4701.8 941.71
## - freetime     1     14.524 4702.9 941.78
## - internet     1     15.410 4703.8 941.83
## - Dalc         1     16.596 4705.0 941.91
## - nursery      1     19.731 4708.1 942.10
## - Walc         1     20.576 4709.0 942.16
## - sex          1     21.070 4709.5 942.19
## - goout        1     21.959 4710.4 942.25
## - famrel       1     29.880 4718.3 942.75
## - studytime    1     30.073 4718.5 942.76
## - health       1     37.868 4726.3 943.25
## - famsup       1     50.546 4738.9 944.05
## - schoolsup    1     50.669 4739.1 944.06
## - age          1     54.697 4743.1 944.31
## - absences     1     58.566 4747.0 944.55
## - famsize      1     64.537 4752.9 944.93
## - Medu         1     71.932 4760.3 945.39
## <none>                4688.4 946.55
## - romantic     1    108.989 4797.4 947.70
## - failures     1    292.019 4980.4 958.86
##
## Step:  AIC=941.47
## G3 ~ sex + age + address + famsize + Medu + studytime + failures +
##      schoolsup + famsup + paid + nursery + internet + romantic +
##      famrel + freetime + goout + Dalc + Walc + health + absences
##
##      Df Sum of Sq    RSS    AIC
## - address      1     10.177 4708.3 936.42
## - internet     1     14.050 4712.1 936.66
## - freetime     1     15.496 4713.6 936.75
## - paid         1     16.179 4714.3 936.80
## - Dalc         1     16.243 4714.3 936.80
## - sex          1     17.353 4715.4 936.87
## - Walc         1     19.344 4717.4 937.00
## - goout        1     20.839 4718.9 937.09
## - nursery      1     21.074 4719.1 937.10
## - famrel       1     28.753 4726.8 937.59
## - studytime    1     32.684 4730.8 937.84
## - health       1     35.971 4734.0 938.04
## - famsup       1     49.822 4747.9 938.91
## - schoolsup    1     50.848 4748.9 938.98
## - absences     1     55.275 4753.3 939.26
## - age          1     61.064 4759.1 939.62
## - famsize      1     67.757 4765.8 940.04
## - Medu         1     78.972 4777.0 940.74
## <none>                4698.1 941.47

```

```

## - romantic    1    116.967 4815.0 943.10
## - failures    1    313.684 5011.8 955.03
##
## Step:  AIC=936.42
## G3 ~ sex + age + famsize + Medu + studytime + failures + schoolsup +
##      famsup + paid + nursery + internet + romantic + famrel +
##      freetime + goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - paid      1      15.44 4723.7 931.69
## - freetime   1      15.75 4724.0 931.71
## - sex        1      15.76 4724.0 931.71
## - Dalc       1      17.11 4725.4 931.80
## - Walc       1      17.31 4725.6 931.81
## - goout      1      18.05 4726.3 931.86
## - nursery    1      20.25 4728.5 932.00
## - internet   1      21.33 4729.6 932.07
## - famrel     1      28.37 4736.6 932.51
## - studytime  1      29.42 4737.7 932.57
## - health     1      35.00 4743.3 932.93
## - famsup     1      48.53 4756.8 933.77
## - schoolsup  1      51.08 4759.3 933.93
## - absences   1      53.39 4761.6 934.08
## - age        1      67.83 4776.1 934.98
## - famsize    1      74.73 4783.0 935.41
## - Medu       1      81.13 4789.4 935.81
## <none>                4708.3 936.42
## - romantic   1     117.73 4826.0 938.08
## - failures   1     319.20 5027.5 950.27
##
## Step:  AIC=931.69
## G3 ~ sex + age + famsize + Medu + studytime + failures + schoolsup +
##      famsup + nursery + internet + romantic + famrel + freetime +
##      goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - sex        1      13.50 4737.2 926.85
## - freetime    1      14.23 4737.9 926.89
## - Dalc        1      14.61 4738.3 926.92
## - goout       1      18.04 4741.7 927.13
## - nursery     1      18.17 4741.9 927.14
## - Walc        1      19.51 4743.2 927.23
## - internet    1      25.80 4749.5 927.62
## - famrel      1      30.07 4753.8 927.89
## - studytime   1      34.14 4757.8 928.14
## - famsup      1      36.61 4760.3 928.30
## - health      1      37.50 4761.2 928.35
## - absences    1      53.67 4777.4 929.36
## - schoolsup   1      57.98 4781.7 929.63
## - age         1      68.81 4792.5 930.31
## - famsize     1      75.07 4798.8 930.70
## - Medu        1      82.27 4806.0 931.14
## <none>                4723.7 931.69
## - romantic    1     122.05 4845.7 933.60

```

```

## - failures    1    342.67 5066.4 946.87
##
## Step: AIC=926.85
## G3 ~ age + famsize + Medu + studytime + failures + schoolsup +
##       famsup + nursery + internet + romantic + famrel + freetime +
##       goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - Dalc      1      11.94 4749.1 921.90
## - nursery   1      18.16 4755.4 922.29
## - freetime  1      18.75 4755.9 922.33
## - goout     1      19.16 4756.4 922.35
## - Walc      1      24.59 4761.8 922.69
## - studytime 1      26.22 4763.4 922.80
## - internet  1      27.28 4764.5 922.86
## - famrel    1      34.17 4771.4 923.29
## - health    1      35.80 4773.0 923.39
## - famsup    1      41.93 4779.1 923.78
## - absences  1      51.31 4788.5 924.36
## - schoolsup 1      67.16 4804.4 925.35
## - age       1      72.51 4809.7 925.68
## - famsize   1      80.03 4817.2 926.14
## <none>              4737.2 926.85
## - Medu      1      95.46 4832.7 927.10
## - romantic  1     139.96 4877.2 929.83
## - failures  1     336.01 5073.2 941.57
##
## Step: AIC=921.9
## G3 ~ age + famsize + Medu + studytime + failures + schoolsup +
##       famsup + nursery + internet + romantic + famrel + freetime +
##       goout + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - Walc      1      13.09 4762.2 917.02
## - freetime  1      14.80 4763.9 917.13
## - nursery   1      16.53 4765.7 917.24
## - goout     1      17.72 4766.8 917.31
## - studytime 1      26.15 4775.3 917.84
## - internet  1      26.36 4775.5 917.85
## - health    1      36.47 4785.6 918.48
## - famrel    1      37.07 4786.2 918.52
## - famsup    1      43.91 4793.0 918.95
## - absences  1      50.61 4799.7 919.36
## - schoolsup 1      73.30 4822.4 920.77
## - famsize   1      77.91 4827.0 921.05
## - age       1      79.69 4828.8 921.16
## - Medu      1      91.26 4840.4 921.88
## <none>              4749.1 921.90
## - romantic  1     140.73 4889.9 924.91
## - failures  1     343.43 5092.6 937.01
##
## Step: AIC=917.02
## G3 ~ age + famsize + Medu + studytime + failures + schoolsup +
##       famsup + nursery + internet + romantic + famrel + freetime +

```

```

##      goout + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - goout      1      8.82 4771.0 911.88
## - freetime    1     15.16 4777.4 912.27
## - studytime   1     20.34 4782.6 912.60
## - nursery     1     20.45 4782.7 912.60
## - internet    1     26.73 4788.9 912.99
## - famrel      1     31.73 4793.9 913.31
## - health      1     31.82 4794.0 913.31
## - famsup      1     45.51 4807.7 914.16
## - absences    1     53.73 4815.9 914.67
## - age         1     77.91 4840.1 916.16
## - schoolsup    1     78.78 4841.0 916.22
## - famsize     1     82.65 4844.9 916.45
## - Medu        1     89.31 4851.5 916.86
## <none>                4762.2 917.02
## - romantic    1    145.52 4907.7 920.30
## - failures    1    343.02 5105.2 932.05
##
## Step: AIC=911.88
## G3 ~ age + famsize + Medu + studytime + failures + schoolsup +
##      famsup + nursery + internet + romantic + famrel + freetime +
##      health + absences
##
##           Df Sum of Sq    RSS    AIC
## - freetime    1     10.08 4781.1 906.81
## - studytime    1     19.93 4791.0 907.42
## - nursery      1     20.88 4791.9 907.48
## - internet     1     26.67 4797.7 907.84
## - health       1     31.51 4802.5 908.14
## - famrel       1     32.62 4803.7 908.21
## - famsup       1     44.83 4815.9 908.97
## - absences     1     52.96 4824.0 909.47
## - schoolsup    1     80.25 4851.3 911.15
## - famsize      1     82.23 4853.3 911.27
## - age          1     83.01 4854.0 911.32
## - Medu         1     84.76 4855.8 911.43
## <none>                4771.0 911.88
## - romantic     1    143.67 4914.7 915.02
## - failures     1    359.32 5130.4 927.82
##
## Step: AIC=906.81
## G3 ~ age + famsize + Medu + studytime + failures + schoolsup +
##      famsup + nursery + internet + romantic + famrel + health +
##      absences
##
##           Df Sum of Sq    RSS    AIC
## - studytime    1     15.69 4796.8 902.09
## - nursery      1     20.01 4801.1 902.36
## - health       1     29.23 4810.3 902.93
## - internet     1     31.42 4812.5 903.06
## - famrel       1     39.56 4820.7 903.57
## - famsup       1     44.66 4825.8 903.88

```

```

## - absences      1      47.82 4828.9 904.08
## - schoolsup     1      79.28 4860.4 906.01
## - age           1      79.50 4860.6 906.03
## - famsize       1      80.89 4862.0 906.11
## - Medu          1      86.17 4867.3 906.43
## <none>          4781.1 906.81
## - romantic      1     143.13 4924.2 909.90
## - failures      1     353.33 5134.4 922.36
##
## Step: AIC=902.09
## G3 ~ age + famsize + Medu + failures + schoolsup + famsup + nursery +
##       internet + romantic + famrel + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - nursery   1      17.69 4814.5 897.49
## - internet   1      31.09 4827.9 898.32
## - health     1      33.77 4830.6 898.48
## - famsup     1      39.78 4836.6 898.85
## - famrel     1      43.59 4840.4 899.09
## - absences   1      43.82 4840.6 899.10
## - age        1      74.67 4871.5 900.99
## - schoolsup   1      75.89 4872.7 901.07
## - famsize    1      76.83 4873.6 901.13
## - Medu       1      83.89 4880.7 901.56
## <none>       4796.8 902.09
## - romantic   1     136.24 4933.0 904.74
## - failures   1     396.48 5193.3 920.06
##
## Step: AIC=897.49
## G3 ~ age + famsize + Medu + failures + schoolsup + famsup + internet +
##       romantic + famrel + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - health     1      30.41 4844.9 893.67
## - internet    1      34.19 4848.7 893.90
## - absences    1      40.95 4855.4 894.32
## - famrel      1      41.21 4855.7 894.33
## - famsup      1      44.03 4858.5 894.50
## - age         1      67.36 4881.9 895.93
## - famsize     1      68.21 4882.7 895.98
## - Medu        1      76.03 4890.5 896.46
## - schoolsup    1      77.64 4892.1 896.56
## <none>       4814.5 897.49
## - romantic    1     139.45 4953.9 900.30
## - failures    1     390.81 5205.3 915.05
##
## Step: AIC=893.67
## G3 ~ age + famsize + Medu + failures + schoolsup + famsup + internet +
##       romantic + famrel + absences
##
##           Df Sum of Sq    RSS    AIC
## - famrel      1      32.40 4877.3 889.96
## - absences     1      34.46 4879.4 890.08
## - internet     1      40.03 4884.9 890.42

```

```

## - famsup      1      47.43 4892.3 890.87
## - age         1      54.51 4899.4 891.30
## - schoolsup   1      67.63 4912.5 892.10
## - famsize     1      70.45 4915.3 892.27
## - Medu        1      80.10 4925.0 892.86
## <none>                4844.9 893.67
## - romantic    1     147.34 4992.2 896.90
## - failures    1     412.26 5257.2 912.31
##
## Step:  AIC=889.96
## G3 ~ age + famsize + Medu + failures + schoolsup + famsup + internet +
##       romantic + absences
##
##           Df Sum of Sq    RSS    AIC
## - absences  1      29.39 4906.7 886.05
## - internet  1      42.10 4919.4 886.82
## - famsup    1      47.65 4925.0 887.16
## - age       1      49.65 4926.9 887.28
## - schoolsup  1      65.82 4943.1 888.25
## - famsize   1      73.12 4950.4 888.69
## - Medu      1      81.49 4958.8 889.20
## <none>                4877.3 889.96
## - romantic  1     151.57 5028.9 893.38
## - failures  1     402.76 5280.1 907.90
##
## Step:  AIC=886.05
## G3 ~ age + famsize + Medu + failures + schoolsup + famsup + internet +
##       romantic
##
##           Df Sum of Sq    RSS    AIC
## - age       1      36.07 4942.8 882.54
## - famsup    1      48.23 4954.9 883.27
## - internet  1      49.39 4956.1 883.34
## - schoolsup  1      56.14 4962.8 883.74
## - famsize   1      71.01 4977.7 884.63
## <none>                4906.7 886.05
## - Medu      1      96.15 5002.8 886.14
## - romantic  1     141.09 5047.8 888.80
## - failures  1     397.95 5304.6 903.59
##
## Step:  AIC=882.54
## G3 ~ famsize + Medu + failures + schoolsup + famsup + internet +
##       romantic
##
##           Df Sum of Sq    RSS    AIC
## - schoolsup  1      38.61 4981.4 879.16
## - famsup    1      43.39 4986.2 879.44
## - internet  1      53.16 4995.9 880.03
## - famsize   1      63.68 5006.4 880.65
## <none>                4942.8 882.54
## - Medu      1     112.56 5055.3 883.55
## - romantic  1     166.38 5109.1 886.70
## - failures  1     493.04 5435.8 905.17
##

```

```
## Step: AIC=879.16
## G3 ~ famsize + Medu + failures + famsup + internet + romantic
##
##           Df Sum of Sq   RSS   AIC
## - internet  1    48.76 5030.1 876.36
## - famsup    1    54.85 5036.2 876.72
## - famsize   1    65.56 5046.9 877.36
## <none>                4981.4 879.16
## - Medu      1   115.88 5097.3 880.31
## - romantic  1   159.04 5140.4 882.83
## - failures  1   505.19 5486.6 902.25
##
## Step: AIC=876.36
## G3 ~ famsize + Medu + failures + famsup + romantic
##
##           Df Sum of Sq   RSS   AIC
## - famsup    1    48.61 5078.8 873.53
## - famsize   1    66.91 5097.1 874.60
## <none>                5030.1 876.36
## - Medu      1   145.22 5175.4 879.15
## - romantic  1   147.76 5177.9 879.29
## - failures  1   521.78 5551.9 900.08
##
## Step: AIC=873.53
## G3 ~ famsize + Medu + failures + romantic
##
##           Df Sum of Sq   RSS   AIC
## - famsize   1    78.09 5156.8 872.38
## <none>                5078.8 873.53
## - Medu      1   114.79 5193.5 874.50
## - romantic  1   147.25 5226.0 876.35
## - failures  1   526.64 5605.4 897.24
##
## Step: AIC=872.38
## G3 ~ Medu + failures + romantic
##
##           Df Sum of Sq   RSS   AIC
## <none>                5156.8 872.38
## - Medu      1   102.28 5259.1 872.54
## - romantic  1   136.67 5293.5 874.48
## - failures  1   545.76 5702.6 896.66
```

```
# stepwise selection to find the best subset of predictors step by step.
# backward selection starts with all predictors and iteratively removes the least important one.
# k = 2: AIC
# k = log(n) BIC
# trace = 1: print the selection process. Larger values may give more information on the fitting process.
# direction = 'both'/'backward'/'forward'
pred <- fit_step %>% predict(test.data)
data.frame(
  RMSE = RMSE(pred, test.data$G3),
  Rsquare = R2(pred, test.data$G3)
)
```

```
##      RMSE  Rsquare
## 1 4.32272 0.147046
```

forward: 1. Starts with the null model (intercept only). 2. Adds the most significant predictor iteratively. 3. Stops when removing predictors no longer improves BIC. 4. Returns the final optimal model (fit_step).

```
null_model <- lm(G3 ~ 1, data = train.data)
full_model <- lm(G3 ~ ., data = train.data)
fit_step_fwd <- stepAIC(null_model, scope = list(lower = null_model, upper = full_model),
                      direction = "forward", k = log(nrow(train.data)), trace = 1)
```

```
## Start:  AIC=907.64
## G3 ~ 1
##
##           Df Sum of Sq   RSS   AIC
## + failures    1    769.04 5378.1 873.51
## + Medu        1    221.38 5925.8 902.41
## + romantic    1    199.54 5947.6 903.50
## + age         1    189.81 5957.3 903.99
## <none>                6147.2 907.64
## + higher      1    114.35 6032.8 907.74
## + internet    1     90.84 6056.3 908.90
## + paid        1     73.49 6073.7 909.75
## + Fedu        1     68.04 6079.1 910.02
## + famsize     1     67.93 6079.2 910.02
## + address     1     59.52 6087.6 910.44
## + schoolsup   1     51.75 6095.4 910.82
## + sex         1     51.68 6095.5 910.82
## + traveltime  1     51.10 6096.1 910.85
## + health      1     37.55 6109.6 911.51
## + studytime   1     34.85 6112.3 911.64
## + goout       1     26.94 6120.2 912.03
## + Dalc        1     22.55 6124.6 912.24
## + famsup      1     18.08 6129.1 912.46
## + famrel      1     17.78 6129.4 912.47
## + school      1     15.20 6132.0 912.60
## + guardian    2    127.89 6019.3 912.77
## + absences    1      3.34 6143.8 913.17
## + Pstatus     1      2.81 6144.3 913.20
## + nursery     1      1.12 6146.0 913.28
## + activities  1      0.36 6146.8 913.32
## + Walc        1      0.14 6147.0 913.33
## + freetime    1      0.07 6147.1 913.33
## + Mjob        4    250.92 5896.2 918.01
## + reason      3     75.09 6072.1 921.07
## + Fjob        4     31.69 6115.5 928.89
##
## Step:  AIC=873.51
## G3 ~ failures
##
##           Df Sum of Sq   RSS   AIC
## + romantic    1    118.992 5259.1 872.54
## <none>                5378.1 873.51
```



```

## + Medu      1      84.608 5293.5 874.48
## + sex       1      83.851 5294.3 874.52
## + famsize   1      57.522 5320.6 876.00
## + internet  1      54.119 5324.0 876.19
## + schoolsup  1      39.330 5338.8 877.02
## + age       1      38.444 5339.7 877.07
## + famrel    1      32.297 5345.8 877.41
## + traveltime 1      29.786 5348.3 877.55
## + famsup    1      27.135 5351.0 877.70
## + address   1      24.929 5353.2 877.82
## + health    1      18.493 5359.6 878.18
## + higher    1      14.875 5363.2 878.38
## + paid      1      14.486 5363.6 878.40
## + absences  1      14.312 5363.8 878.41
## + freetime  1       9.915 5368.2 878.66
## + Walc      1       7.637 5370.5 878.78
## + nursery   1       2.719 5375.4 879.05
## + Fedu      1       2.351 5375.8 879.08
## + school    1       1.710 5376.4 879.11
## + goout     1       1.646 5376.5 879.11
## + studytime 1       1.231 5376.9 879.14
## + Dalc      1       0.956 5377.2 879.15
## + Pstatus   1       0.846 5377.3 879.16
## + activities 1       0.025 5378.1 879.20
## + Mjob      4     213.619 5164.5 884.22
## + guardian  2       8.141 5370.0 884.45
## + reason    3      68.703 5309.4 886.77
## + Fjob      4      22.093 5356.0 895.07
##
## Step:  AIC=872.54
## G3 ~ failures + romantic
##
##           Df Sum of Sq  RSS    AIC
## + Medu      1    102.284 5156.8 872.38
## <none>                5259.1 872.54
## + internet  1     68.759 5190.4 874.31
## + famsize   1     65.588 5193.5 874.50
## + sex       1     55.699 5203.4 875.06
## + schoolsup  1     45.919 5213.2 875.62
## + famrel    1     28.950 5230.2 876.59
## + absences  1     27.519 5231.6 876.67
## + famsup    1     25.789 5233.3 876.77
## + address   1     25.469 5233.7 876.79
## + traveltime 1     23.755 5235.4 876.89
## + age       1     22.174 5237.0 876.98
## + health    1     15.415 5243.7 877.36
## + paid      1     13.616 5245.5 877.46
## + freetime  1       9.617 5249.5 877.69
## + higher    1       8.563 5250.6 877.75
## + Walc      1       5.416 5253.7 877.93
## + Fedu      1       4.763 5254.4 877.97
## + studytime 1       3.709 5255.4 878.02
## + Pstatus   1       2.284 5256.8 878.11
## + nursery   1       1.813 5257.3 878.13

```

```

## + goout      1      1.759 5257.4 878.14
## + Dalc       1      0.950 5258.2 878.18
## + school     1      0.759 5258.4 878.19
## + activities 1      0.040 5259.1 878.23
## + Mjob       4     208.046 5051.1 883.30
## + guardian   2      3.406 5255.7 883.74
## + reason     3     76.939 5182.2 885.24
## + Fjob       4     25.502 5233.6 893.88
##
## Step: AIC=872.38
## G3 ~ failures + romantic + Medu
##
##           Df Sum of Sq  RSS   AIC
## <none>                5156.8 872.38
## + famsize      1      78.092 5078.8 873.53
## + famsup       1     59.791 5097.1 874.60
## + schoolsup    1     48.122 5108.7 875.29
## + internet     1     43.225 5113.6 875.57
## + sex          1     37.879 5119.0 875.88
## + famrel       1     29.259 5127.6 876.38
## + Fedu         1     28.700 5128.1 876.42
## + absences     1     16.867 5140.0 877.10
## + address      1     16.756 5140.1 877.11
## + health       1     14.844 5142.0 877.22
## + age          1     10.287 5146.6 877.48
## + traveltime   1     10.192 5146.7 877.49
## + nursery      1      8.190 5148.7 877.61
## + freetime     1      7.906 5148.9 877.62
## + paid         1      6.776 5150.1 877.69
## + Walc         1      5.903 5150.9 877.74
## + goout        1      5.217 5151.6 877.78
## + studytime    1      4.333 5152.5 877.83
## + higher       1      3.651 5153.2 877.87
## + activities   1      2.177 5154.7 877.95
## + Dalc         1      1.935 5154.9 877.97
## + school       1      0.312 5156.5 878.06
## + Pstatus      1      0.088 5156.8 878.07
## + guardian     2      5.273 5151.6 883.47
## + reason       3     59.250 5097.6 886.03
## + Mjob         4    129.070 5027.8 887.62
## + Fjob         4     15.463 5141.4 894.28

```

```

pred_fwd <- fit_step_fwd %>% predict(test.data)
data.frame(
  RMSE = RMSE(pred_fwd, test.data$G3),
  Rsquare = R2(pred_fwd, test.data$G3)
)

```

```

##           RMSE  Rsquare
## 1 4.32272 0.147046

```

Bidirectional (Both)

```
fit_step_both <- stepAIC(fit, k = log(nrow(train.data)), direction = "both", trace = 1)
```

```
## Start:  AIC=1034.07
## G3 ~ school + sex + age + address + famsize + Pstatus + Medu +
##      Fedu + Mjob + Fjob + reason + guardian + traveltime + studytime +
##      failures + schoolsup + famsup + paid + activities + nursery +
##      higher + internet + romantic + famrel + freetime + goout +
##      Dalc + Walc + health + absences
##
##           Df Sum of Sq   RSS   AIC
## - Fjob      4    41.387 4499.2 1014.0
## - Mjob      4    97.705 4555.6 1017.7
## - reason    3    27.452 4485.3 1018.8
## - guardian  2     4.311 4462.2 1023.0
## - traveltime 1     0.045 4457.9 1028.4
## - Pstatus   1     0.082 4457.9 1028.4
## - activities 1     5.276 4463.1 1028.7
## - school    1     7.353 4465.2 1028.9
## - Fedu      1     8.054 4465.9 1028.9
## - address   1     9.158 4467.0 1029.0
## - paid      1    11.233 4469.1 1029.1
## - freetime  1    15.892 4473.7 1029.4
## - internet  1    15.967 4473.8 1029.4
## - Dalc      1    19.126 4477.0 1029.7
## - higher    1    19.372 4477.2 1029.7
## - nursery   1    20.591 4478.4 1029.8
## - Walc      1    21.954 4479.8 1029.8
## - goout     1    26.597 4484.4 1030.2
## - Medu      1    32.380 4490.2 1030.5
## - health    1    34.929 4492.8 1030.7
## - famrel    1    34.994 4492.8 1030.7
## - studytime 1    37.209 4495.1 1030.8
## - famsup    1    41.951 4499.8 1031.2
## - sex       1    43.454 4501.3 1031.3
## - age       1    51.687 4509.5 1031.8
## - schoolsup  1    53.305 4511.2 1031.9
## - famsize   1    66.075 4523.9 1032.8
## - absences  1    74.544 4532.4 1033.3
## <none>                      4457.8 1034.1
## - romantic  1   109.816 4567.7 1035.6
## - failures  1   254.495 4712.3 1044.9
##
## Step:  AIC=1014.04
## G3 ~ school + sex + age + address + famsize + Pstatus + Medu +
##      Fedu + Mjob + reason + guardian + traveltime + studytime +
##      failures + schoolsup + famsup + paid + activities + nursery +
##      higher + internet + romantic + famrel + freetime + goout +
##      Dalc + Walc + health + absences
##
##           Df Sum of Sq   RSS   AIC
## - Mjob      4    90.713 4589.9  997.20
## - reason    3    37.315 4536.5  999.41
## - guardian  2     6.620 4505.9 1003.08
```

```

## - traveltime 1      0.004 4499.2 1008.34
## - Pstatus    1      0.014 4499.2 1008.34
## - Fedu       1      1.541 4500.8 1008.44
## - activities 1      8.044 4507.3 1008.87
## - school     1      8.559 4507.8 1008.91
## - paid       1      8.889 4508.1 1008.93
## - address    1     11.199 4510.4 1009.08
## - internet   1     11.208 4510.4 1009.08
## - Dalc       1     12.244 4511.5 1009.15
## - Walc       1     12.458 4511.7 1009.16
## - freetime   1     13.421 4512.7 1009.23
## - higher     1     17.188 4516.4 1009.48
## - nursery    1     19.765 4519.0 1009.65
## - goout      1     22.359 4521.6 1009.82
## - Medu       1     29.220 4528.5 1010.27
## - famrel     1     32.787 4532.0 1010.50
## - health     1     32.908 4532.1 1010.51
## - sex        1     39.533 4538.8 1010.95
## - studytime  1     40.155 4539.4 1010.99
## - famsup     1     40.789 4540.0 1011.03
## - schoolsup   1     44.419 4543.7 1011.27
## - age        1     50.844 4550.1 1011.69
## - famsize    1     58.276 4557.5 1012.17
## - absences   1     70.580 4569.8 1012.98
## <none>              4499.2 1014.04
## - romantic   1    101.250 4600.5 1014.97
## - failures   1    247.187 4746.4 1024.28
## + Fjob       4     41.387 4457.8 1034.07
##
## Step: AIC=997.2
## G3 ~ school + sex + age + address + famsize + Pstatus + Medu +
##      Fedu + reason + guardian + traveltime + studytime + failures +
##      schoolsup + famsup + paid + activities + nursery + higher +
##      internet + romantic + famrel + freetime + goout + Dalc +
##      Walc + health + absences
##
##              Df Sum of Sq  RSS    AIC
## - reason      3    67.742 4657.7  984.47
## - guardian    2     9.081 4599.0  986.39
## - traveltime  1     0.127 4590.1  991.51
## - Pstatus     1     0.705 4590.7  991.55
## - Fedu        1     2.664 4592.6  991.67
## - school      1     7.446 4597.4  991.98
## - activities  1     8.718 4598.7  992.07
## - paid        1     9.325 4599.3  992.10
## - freetime    1    11.680 4601.6  992.26
## - goout       1    13.269 4603.2  992.36
## - address     1    13.898 4603.8  992.40
## - Walc        1    16.346 4606.3  992.56
## - internet    1    16.583 4606.5  992.57
## - nursery     1    19.575 4609.5  992.77
## - Dalc        1    22.564 4612.5  992.96
## - higher     1    23.022 4613.0  992.99
## - health      1    27.256 4617.2  993.26

```

```

## - famrel      1      32.023 4622.0 993.57
## - sex         1      32.258 4622.2 993.59
## - famsup      1      35.123 4625.1 993.77
## - studytime   1      36.898 4626.8 993.89
## - age         1      47.197 4637.1 994.55
## - schoolsup   1      50.192 4640.1 994.74
## - Medu        1      51.533 4641.5 994.83
## - absences    1      62.739 4652.7 995.55
## - famsize     1      65.170 4655.1 995.70
## <none>                4589.9 997.20
## - romantic    1     106.790 4696.7 998.35
## - failures    1     230.587 4820.5 1006.11
## + Mjob        4      90.713 4499.2 1014.04
## + Fjob        4      34.396 4555.6 1017.74
##
## Step:  AIC=984.47
## G3 ~ school + sex + age + address + famsize + Pstatus + Medu +
##      Fedu + guardian + traveltime + studytime + failures + schoolsup +
##      famsup + paid + activities + nursery + higher + internet +
##      romantic + famrel + freetime + goout + Dalc + Walc + health +
##      absences
##
##           Df Sum of Sq   RSS   AIC
## - guardian    2      9.785 4667.5 973.70
## - Pstatus      1      0.146 4657.8 978.78
## - traveltime   1      0.368 4658.1 978.80
## - Fedu         1      3.678 4661.4 979.01
## - school       1      6.475 4664.2 979.19
## - activities   1      6.959 4664.6 979.22
## - address      1     10.186 4667.9 979.43
## - paid         1     12.148 4669.8 979.55
## - higher       1     13.122 4670.8 979.61
## - internet     1     14.679 4672.4 979.71
## - freetime     1     15.091 4672.8 979.74
## - goout        1     18.097 4675.8 979.93
## - Dalc         1     18.681 4676.4 979.97
## - Walc         1     19.883 4677.6 980.04
## - nursery      1     21.917 4679.6 980.17
## - sex          1     22.507 4680.2 980.21
## - famrel       1     31.653 4689.3 980.79
## - studytime    1     35.681 4693.4 981.05
## - health       1     36.394 4694.1 981.09
## - famsup       1     40.659 4698.3 981.36
## - schoolsup    1     45.873 4703.6 981.70
## - age          1     47.479 4705.2 981.80
## - famsize      1     62.488 4720.2 982.75
## - absences     1     64.142 4721.8 982.85
## - Medu         1     66.728 4724.4 983.01
## <none>                4657.7 984.47
## - romantic    1    103.281 4761.0 985.31
## - failures     1    239.621 4897.3 993.72
## + reason       3      67.742 4589.9 997.20
## + Mjob         4     121.140 4536.5 999.41
## + Fjob         4      49.738 4608.0 1004.06

```

```

##
## Step: AIC=973.7
## G3 ~ school + sex + age + address + famsize + Pstatus + Medu +
##      Fedu + traveltime + studytime + failures + schoolsup + famsup +
##      paid + activities + nursery + higher + internet + romantic +
##      famrel + freetime + goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - Pstatus    1      0.064 4667.5 968.01
## - traveltime  1      0.741 4668.2 968.05
## - Fedu        1      3.881 4671.4 968.25
## - activities  1      6.633 4674.1 968.43
## - school      1      8.291 4675.8 968.53
## - address     1      9.709 4677.2 968.63
## - higher      1     10.318 4677.8 968.66
## - paid        1     10.472 4677.9 968.67
## - freetime    1     13.671 4681.1 968.88
## - internet    1     16.231 4683.7 969.04
## - nursery     1     17.863 4685.3 969.14
## - goout       1     17.994 4685.5 969.15
## - Dalc        1     18.655 4686.1 969.19
## - Walc        1     22.208 4689.7 969.42
## - sex         1     23.557 4691.0 969.51
## - famrel      1     29.950 4697.4 969.91
## - health      1     32.610 4700.1 970.08
## - studytime   1     36.001 4703.5 970.30
## - famsup      1     41.609 4709.1 970.65
## - schoolsup   1     45.532 4713.0 970.90
## - absences    1     57.759 4725.2 971.67
## - famsize     1     61.606 4729.1 971.91
## - Medu        1     66.154 4733.6 972.20
## - age         1     66.396 4733.9 972.22
## <none>                4667.5 973.70
## - romantic    1    106.326 4773.8 974.72
## + guardian    2       9.785 4657.7 984.47
## - failures    1    288.196 4955.7 985.86
## + reason      3     68.446 4599.0 986.39
## + Mjob        4    124.275 4543.2 988.45
## + Fjob        4     55.095 4612.4 992.95
##
## Step: AIC=968.01
## G3 ~ school + sex + age + address + famsize + Medu + Fedu + traveltime +
##      studytime + failures + schoolsup + famsup + paid + activities +
##      nursery + higher + internet + romantic + famrel + freetime +
##      goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - traveltime  1      0.777 4668.3 962.36
## - Fedu        1      3.827 4671.4 962.56
## - activities  1      6.886 4674.4 962.75
## - school      1      8.280 4675.8 962.84
## - address     1      9.697 4677.2 962.93
## - higher      1     10.333 4677.9 962.97
## - paid        1     10.409 4677.9 962.98

```

```

## - freetime      1      13.673 4681.2 963.18
## - internet      1      16.192 4683.7 963.34
## - nursery       1      17.855 4685.4 963.45
## - goout         1      18.085 4685.6 963.47
## - Dalc          1      18.595 4686.1 963.50
## - Walc          1      22.208 4689.7 963.73
## - sex           1      23.513 4691.1 963.81
## - famrel        1      30.221 4697.8 964.24
## - health        1      32.822 4700.4 964.40
## - studytime     1      36.093 4703.6 964.61
## - famsup        1      41.546 4709.1 964.95
## - schoolsup     1      45.496 4713.0 965.20
## - absences      1      59.337 4726.9 966.08
## - famsize       1      63.213 4730.8 966.32
## - Medu          1      66.566 4734.1 966.53
## - age           1      66.584 4734.1 966.53
## <none>          1      4667.5 968.01
## - romantic      1      106.274 4773.8 969.02
## + Pstatus       1         0.064 4667.5 973.70
## + guardian      2         9.702 4657.8 978.78
## - failures      1      288.151 4955.7 980.16
## + reason        3       67.994 4599.5 980.73
## + Mjob          4      124.226 4543.3 982.76
## + Fjob          4       54.756 4612.8 987.28
##
## Step:  AIC=962.36
## G3 ~ school + sex + age + address + famsize + Medu + Fedu + studytime +
##      failures + schoolsup + famsup + paid + activities + nursery +
##      higher + internet + romantic + famrel + freetime + goout +
##      Dalc + Walc + health + absences
##
##           Df Sum of Sq   RSS   AIC
## - Fedu      1      3.505 4671.8 956.89
## - activities 1      7.111 4675.4 957.12
## - school     1      7.648 4676.0 957.15
## - higher     1     10.593 4678.9 957.34
## - paid       1     10.837 4679.2 957.36
## - address    1     12.423 4680.7 957.46
## - freetime   1     14.508 4682.8 957.59
## - internet   1     16.559 4684.9 957.72
## - nursery    1     18.223 4686.5 957.83
## - goout      1     18.374 4686.7 957.84
## - Dalc       1     19.867 4688.2 957.93
## - Walc       1     22.344 4690.7 958.09
## - sex        1     23.459 4691.8 958.16
## - famrel     1     29.829 4698.1 958.56
## - health     1     32.534 4700.8 958.73
## - studytime  1     37.255 4705.6 959.03
## - famsup     1     43.715 4712.0 959.44
## - schoolsup   1     46.150 4714.5 959.60
## - absences   1     59.413 4727.7 960.43
## - famsize    1     62.691 4731.0 960.64
## - age        1     65.898 4734.2 960.84
## - Medu       1     67.769 4736.1 960.96

```

```

## <none> 4668.3 962.36
## - romantic 1 108.614 4776.9 963.52
## + traveltime 1 0.777 4667.5 968.01
## + Pstatus 1 0.099 4668.2 968.05
## + guardian 2 10.075 4658.2 973.11
## - failures 1 287.427 4955.7 974.47
## + reason 3 68.309 4600.0 975.06
## + Mjob 4 124.742 4543.6 977.08
## + Fjob 4 55.342 4613.0 981.60
##
## Step: AIC=956.89
## G3 ~ school + sex + age + address + famsize + Medu + studytime +
## failures + schoolsup + famsup + paid + activities + nursery +
## higher + internet + romantic + famrel + freetime + goout +
## Dalc + Walc + health + absences
##
## Df Sum of Sq RSS AIC
## - school 1 7.228 4679.0 951.65
## - activities 1 7.979 4679.8 951.70
## - higher 1 10.431 4682.3 951.86
## - paid 1 12.085 4683.9 951.96
## - address 1 13.386 4685.2 952.04
## - freetime 1 15.227 4687.0 952.16
## - internet 1 16.041 4687.9 952.21
## - goout 1 19.358 4691.2 952.42
## - Dalc 1 19.553 4691.4 952.44
## - nursery 1 19.828 4691.6 952.45
## - Walc 1 22.149 4694.0 952.60
## - sex 1 24.876 4696.7 952.77
## - famrel 1 29.988 4701.8 953.10
## - health 1 34.198 4706.0 953.37
## - studytime 1 39.144 4711.0 953.68
## - schoolsup 1 46.635 4718.5 954.15
## - famsup 1 47.391 4719.2 954.20
## - absences 1 61.972 4733.8 955.12
## - famsize 1 64.452 4736.3 955.27
## - age 1 64.557 4736.4 955.28
## - Medu 1 77.401 4749.2 956.09
## <none> 4671.8 956.89
## - romantic 1 109.334 4781.2 958.09
## + Fedu 1 3.505 4668.3 962.36
## + traveltime 1 0.454 4671.4 962.56
## + Pstatus 1 0.023 4671.8 962.58
## + guardian 2 10.256 4661.6 967.63
## - failures 1 284.237 4956.1 968.79
## + reason 3 69.240 4602.6 969.53
## + Mjob 4 125.294 4546.5 971.58
## + Fjob 4 43.430 4628.4 976.89
##
## Step: AIC=951.65
## G3 ~ sex + age + address + famsize + Medu + studytime + failures +
## schoolsup + famsup + paid + activities + nursery + higher +
## internet + romantic + famrel + freetime + goout + Dalc +
## Walc + health + absences

```



```

##
##           Df Sum of Sq    RSS    AIC
## - activities  1      9.353 4688.4 946.55
## - address     1      9.530 4688.6 946.56
## - higher      1     10.887 4689.9 946.65
## - paid        1     12.918 4692.0 946.78
## - internet    1     16.202 4695.2 946.99
## - freetime    1     16.364 4695.4 947.00
## - Dalc        1     17.739 4696.8 947.08
## - goout       1     19.874 4698.9 947.22
## - nursery     1     20.514 4699.6 947.26
## - Walc        1     20.767 4699.8 947.28
## - sex         1     23.336 4702.4 947.44
## - famrel      1     28.011 4707.1 947.73
## - studytime   1     35.392 4714.4 948.20
## - health      1     35.418 4714.5 948.20
## - schoolsup    1     47.592 4726.6 948.97
## - famsup      1     53.342 4732.4 949.33
## - absences    1     56.334 4735.4 949.52
## - age         1     57.329 4736.4 949.58
## - famsize     1     67.750 4746.8 950.24
## - Medu        1     77.169 4756.2 950.83
## <none>                4679.0 951.65
## - romantic    1    108.686 4787.7 952.80
## + school      1      7.228 4671.8 956.89
## + Fedu        1      3.085 4676.0 957.15
## + traveltime  1      0.042 4679.0 957.35
## + Pstatus     1      0.011 4679.0 957.35
## + guardian    2     11.715 4667.3 962.30
## - failures    1    287.868 4966.9 963.75
## + reason      3     67.909 4611.1 964.39
## + Mjob        4    123.421 4555.6 966.47
## + Fjob        4     45.043 4634.0 971.56
##
## Step:  AIC=946.55
## G3 ~ sex + age + address + famsize + Medu + studytime + failures +
##      schoolsup + famsup + paid + nursery + higher + internet +
##      romantic + famrel + freetime + goout + Dalc + Walc + health +
##      absences
##
##           Df Sum of Sq    RSS    AIC
## - higher      1      9.674 4698.1 941.47
## - address      1     10.791 4699.2 941.54
## - paid         1     13.422 4701.8 941.71
## - freetime     1     14.524 4702.9 941.78
## - internet     1     15.410 4703.8 941.83
## - Dalc         1     16.596 4705.0 941.91
## - nursery      1     19.731 4708.1 942.10
## - Walc         1     20.576 4709.0 942.16
## - sex          1     21.070 4709.5 942.19
## - goout        1     21.959 4710.4 942.25
## - famrel       1     29.880 4718.3 942.75
## - studytime    1     30.073 4718.5 942.76
## - health       1     37.868 4726.3 943.25

```

```

## - famsup      1      50.546 4738.9 944.05
## - schoolsup   1      50.669 4739.1 944.06
## - age         1      54.697 4743.1 944.31
## - absences    1      58.566 4747.0 944.55
## - famsize     1      64.537 4752.9 944.93
## - Medu        1      71.932 4760.3 945.39
## <none>                4688.4 946.55
## - romantic    1     108.989 4797.4 947.70
## + activities  1       9.353 4679.0 951.65
## + school      1       8.602 4679.8 951.70
## + Fedu        1       3.939 4684.5 952.00
## + Pstatus     1       0.205 4688.2 952.23
## + traveltime  1       0.074 4688.3 952.24
## + guardian    2      11.499 4676.9 957.21
## - failures    1     292.019 4980.4 958.86
## + reason      3      65.755 4622.6 959.43
## + Mjob        4     123.627 4564.8 961.38
## + Fjob        4      46.180 4642.2 966.39
##
## Step:  AIC=941.47
## G3 ~ sex + age + address + famsize + Medu + studytime + failures +
##      schoolsup + famsup + paid + nursery + internet + romantic +
##      famrel + freetime + goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - address      1      10.177 4708.3 936.42
## - internet      1      14.050 4712.1 936.66
## - freetime      1      15.496 4713.6 936.75
## - paid          1      16.179 4714.3 936.80
## - Dalc          1      16.243 4714.3 936.80
## - sex           1      17.353 4715.4 936.87
## - Walc          1      19.344 4717.4 937.00
## - goout         1      20.839 4718.9 937.09
## - nursery       1      21.074 4719.1 937.10
## - famrel        1      28.753 4726.8 937.59
## - studytime     1      32.684 4730.8 937.84
## - health        1      35.971 4734.0 938.04
## - famsup        1      49.822 4747.9 938.91
## - schoolsup     1      50.848 4748.9 938.98
## - absences      1      55.275 4753.3 939.26
## - age           1      61.064 4759.1 939.62
## - famsize       1      67.757 4765.8 940.04
## - Medu          1      78.972 4777.0 940.74
## <none>                4698.1 941.47
## - romantic      1     116.967 4815.0 943.10
## + higher        1       9.674 4688.4 946.55
## + school        1       8.972 4689.1 946.60
## + activities    1       8.140 4689.9 946.65
## + Fedu          1       3.701 4694.4 946.93
## + Pstatus       1       0.221 4697.9 947.15
## + traveltime    1       0.150 4697.9 947.16
## + guardian      2       8.384 4689.7 952.33
## + reason        3      56.345 4641.7 954.96
## - failures      1     313.684 5011.8 955.03

```

```

## + Mjob          4    125.835 4572.2 956.17
## + Fjob          4     42.359 4655.7 961.56
##
## Step:  AIC=936.42
## G3 ~ sex + age + famsize + Medu + studytime + failures + schoolsup +
##      famsup + paid + nursery + internet + romantic + famrel +
##      freetime + goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - paid      1     15.44 4723.7 931.69
## - freetime   1     15.75 4724.0 931.71
## - sex        1     15.76 4724.0 931.71
## - Dalc       1     17.11 4725.4 931.80
## - Walc       1     17.31 4725.6 931.81
## - goout      1     18.05 4726.3 931.86
## - nursery    1     20.25 4728.5 932.00
## - internet   1     21.33 4729.6 932.07
## - famrel     1     28.37 4736.6 932.51
## - studytime  1     29.42 4737.7 932.57
## - health     1     35.00 4743.3 932.93
## - famsup     1     48.53 4756.8 933.77
## - schoolsup  1     51.08 4759.3 933.93
## - absences   1     53.39 4761.6 934.08
## - age        1     67.83 4776.1 934.98
## - famsize    1     74.73 4783.0 935.41
## - Medu       1     81.13 4789.4 935.81
## <none>                4708.3 936.42
## - romantic   1    117.73 4826.0 938.08
## + address    1     10.18 4698.1 941.47
## + activities  1      9.32 4698.9 941.52
## + higher     1      9.06 4699.2 941.54
## + Fedu       1      4.79 4703.5 941.81
## + school     1      4.52 4703.7 941.83
## + traveltime  1      1.99 4706.3 941.99
## + Pstatus    1      0.28 4708.0 942.09
## + guardian   2      7.47 4700.8 947.34
## + reason     3     53.78 4654.5 950.08
## - failures   1    319.20 5027.5 950.27
## + Mjob       4    130.49 4577.8 950.83
## + Fjob       4     44.39 4663.9 956.38
##
## Step:  AIC=931.69
## G3 ~ sex + age + famsize + Medu + studytime + failures + schoolsup +
##      famsup + nursery + internet + romantic + famrel + freetime +
##      goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - sex        1     13.50 4737.2 926.85
## - freetime    1     14.23 4737.9 926.89
## - Dalc        1     14.61 4738.3 926.92
## - goout       1     18.04 4741.7 927.13
## - nursery     1     18.17 4741.9 927.14
## - Walc        1     19.51 4743.2 927.23
## - internet    1     25.80 4749.5 927.62

```

```

## - famrel      1      30.07 4753.8 927.89
## - studytime   1      34.14 4757.8 928.14
## - famsup      1      36.61 4760.3 928.30
## - health      1      37.50 4761.2 928.35
## - absences    1      53.67 4777.4 929.36
## - schoolsup   1      57.98 4781.7 929.63
## - age         1      68.81 4792.5 930.31
## - famsize     1      75.07 4798.8 930.70
## - Medu        1      82.27 4806.0 931.14
## <none>                4723.7 931.69
## - romantic    1     122.05 4845.7 933.60
## + paid        1      15.44 4708.3 936.42
## + higher      1      11.69 4712.0 936.65
## + activities  1       9.65 4714.0 936.78
## + address     1       9.44 4714.3 936.80
## + Fedu        1       6.35 4717.3 936.99
## + school      1       5.43 4718.3 937.05
## + traveltime  1       2.46 4721.2 937.24
## + Pstatus     1       0.05 4723.6 937.39
## + guardian    2       5.69 4718.0 942.73
## + reason       3      56.45 4667.2 945.20
## + Mjob         4     132.27 4591.4 946.02
## - failures    1     342.67 5066.4 946.87
## + Fjob        4      43.09 4680.6 951.75
##
## Step:  AIC=926.85
## G3 ~ age + famsize + Medu + studytime + failures + schoolsup +
##       famsup + nursery + internet + romantic + famrel + freetime +
##       goout + Dalc + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - Dalc      1      11.94 4749.1 921.90
## - nursery   1      18.16 4755.4 922.29
## - freetime  1      18.75 4755.9 922.33
## - goout     1      19.16 4756.4 922.35
## - Walc      1      24.59 4761.8 922.69
## - studytime 1      26.22 4763.4 922.80
## - internet  1      27.28 4764.5 922.86
## - famrel    1      34.17 4771.4 923.29
## - health    1      35.80 4773.0 923.39
## - famsup    1      41.93 4779.1 923.78
## - absences  1      51.31 4788.5 924.36
## - schoolsup 1      67.16 4804.4 925.35
## - age       1      72.51 4809.7 925.68
## - famsize   1      80.03 4817.2 926.14
## <none>                4737.2 926.85
## - Medu      1      95.46 4832.7 927.10
## - romantic  1     139.96 4877.2 929.83
## + sex       1      13.50 4723.7 931.69
## + paid      1      13.19 4724.0 931.71
## + address   1       8.06 4729.1 932.04
## + activities 1       7.84 4729.4 932.05
## + higher    1       7.81 4729.4 932.05
## + Fedu      1       7.45 4729.7 932.08

```

```

## + school      1      4.44 4732.8 932.26
## + traveltime  1      2.08 4735.1 932.41
## + Pstatus     1      0.01 4737.2 932.54
## + guardian    2      6.77 4730.4 937.82
## + reason      3     51.77 4685.4 940.66
## - failures    1    336.01 5073.2 941.57
## + Mjob        4    123.96 4613.2 941.73
## + Fjob        4     39.67 4697.5 947.13
##
## Step:  AIC=921.9
## G3 ~ age + famsize + Medu + studytime + failures + schoolsup +
##       famsup + nursery + internet + romantic + famrel + freetime +
##       goout + Walc + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - Walc      1     13.09 4762.2 917.02
## - freetime   1     14.80 4763.9 917.13
## - nursery    1     16.53 4765.7 917.24
## - goout      1     17.72 4766.8 917.31
## - studytime  1     26.15 4775.3 917.84
## - internet   1     26.36 4775.5 917.85
## - health     1     36.47 4785.6 918.48
## - famrel     1     37.07 4786.2 918.52
## - famsup     1     43.91 4793.0 918.95
## - absences   1     50.61 4799.7 919.36
## - schoolsup   1     73.30 4822.4 920.77
## - famsize    1     77.91 4827.0 921.05
## - age        1     79.69 4828.8 921.16
## - Medu       1     91.26 4840.4 921.88
## <none>                        4749.1 921.90
## - romantic   1    140.73 4889.9 924.91
## + Dalc       1     11.94 4737.2 926.85
## + paid       1     11.29 4737.8 926.89
## + sex        1     10.83 4738.3 926.92
## + address    1      8.89 4740.2 927.04
## + higher     1      7.67 4741.5 927.12
## + activities  1      7.15 4742.0 927.15
## + Fedu       1      6.89 4742.2 927.16
## + traveltime  1      3.67 4745.5 927.37
## + school     1      3.18 4746.0 927.40
## + Pstatus    1      0.01 4749.1 927.60
## + guardian   2      6.77 4742.4 932.87
## + reason     3     49.82 4699.3 935.85
## + Mjob       4    131.51 4617.6 936.32
## - failures    1    343.43 5092.6 937.01
## + Fjob       4     34.35 4714.8 942.53
##
## Step:  AIC=917.02
## G3 ~ age + famsize + Medu + studytime + failures + schoolsup +
##       famsup + nursery + internet + romantic + famrel + freetime +
##       goout + health + absences
##
##           Df Sum of Sq    RSS    AIC
## - goout      1      8.82 4771.0 911.88

```

```

## - freetime      1      15.16 4777.4 912.27
## - studytime     1      20.34 4782.6 912.60
## - nursery       1      20.45 4782.7 912.60
## - internet      1      26.73 4788.9 912.99
## - famrel        1      31.73 4793.9 913.31
## - health        1      31.82 4794.0 913.31
## - famsup        1      45.51 4807.7 914.16
## - absences      1      53.73 4815.9 914.67
## - age           1      77.91 4840.1 916.16
## - schoolsup     1      78.78 4841.0 916.22
## - famsize       1      82.65 4844.9 916.45
## - Medu          1      89.31 4851.5 916.86
## <none>          1      4762.2 917.02
## - romantic      1     145.52 4907.7 920.30
## + sex           1      16.45 4745.8 921.69
## + paid          1      14.20 4748.0 921.83
## + Walc          1      13.09 4749.1 921.90
## + Fedu          1       7.26 4755.0 922.27
## + activities    1       7.20 4755.0 922.27
## + higher        1       6.37 4755.8 922.32
## + address       1       5.97 4756.2 922.35
## + school        1       3.34 4758.9 922.51
## + traveltime    1       2.29 4759.9 922.58
## + Dalc          1       0.43 4761.8 922.69
## + Pstatus       1       0.00 4762.2 922.72
## + guardian      2       8.75 4753.5 927.87
## + reason        3      54.19 4708.0 930.70
## + Mjob          4     128.91 4633.3 931.63
## - failures      1     343.02 5105.2 932.05
## + Fjob          4      30.11 4732.1 937.92
##
## Step:  AIC=911.88
## G3 ~ age + famsize + Medu + studytime + failures + schoolsup +
##       famsup + nursery + internet + romantic + famrel + freetime +
##       health + absences
##
##           Df Sum of Sq    RSS    AIC
## - freetime      1      10.08 4781.1 906.81
## - studytime     1      19.93 4791.0 907.42
## - nursery       1      20.88 4791.9 907.48
## - internet      1      26.67 4797.7 907.84
## - health        1      31.51 4802.5 908.14
## - famrel        1      32.62 4803.7 908.21
## - famsup        1      44.83 4815.9 908.97
## - absences      1      52.96 4824.0 909.47
## - schoolsup     1      80.25 4851.3 911.15
## - famsize       1      82.23 4853.3 911.27
## - age           1      83.01 4854.0 911.32
## - Medu          1      84.76 4855.8 911.43
## <none>          1      4771.0 911.88
## - romantic      1     143.67 4914.7 915.02
## + sex           1      14.82 4756.2 916.65
## + paid          1      12.98 4758.1 916.76
## + goout         1       8.82 4762.2 917.02

```

```

## + activities 1      8.33 4762.7 917.05
## + Fedu      1      8.11 4762.9 917.07
## + higher    1      6.14 4764.9 917.19
## + address   1      5.15 4765.9 917.25
## + Walc      1      4.18 4766.8 917.31
## + school    1      3.94 4767.1 917.33
## + traveltime 1      2.55 4768.5 917.42
## + Dalc      1      1.69 4769.3 917.47
## + Pstatus   1      0.01 4771.0 917.57
## + guardian  2      8.04 4763.0 922.77
## + reason    3     56.15 4714.9 925.44
## + Mjob      4    121.39 4649.6 926.99
## - failures  1    359.32 5130.4 927.82
## + Fjob      4     30.44 4740.6 932.76
##
## Step:  AIC=906.81
## G3 ~ age + famsize + Medu + studytime + failures + schoolsup +
##      famsup + nursery + internet + romantic + famrel + health +
##      absences
##
##           Df Sum of Sq    RSS    AIC
## - studytime 1     15.69 4796.8 902.09
## - nursery   1     20.01 4801.1 902.36
## - health    1     29.23 4810.3 902.93
## - internet  1     31.42 4812.5 903.06
## - famrel    1     39.56 4820.7 903.57
## - famsup    1     44.66 4825.8 903.88
## - absences  1     47.82 4828.9 904.08
## - schoolsup 1     79.28 4860.4 906.01
## - age       1     79.50 4860.6 906.03
## - famsize   1     80.89 4862.0 906.11
## - Medu      1     86.17 4867.3 906.43
## <none>                      4781.1 906.81
## - romantic  1    143.13 4924.2 909.90
## + sex       1     19.01 4762.1 911.32
## + paid      1     12.16 4768.9 911.75
## + freetime  1     10.08 4771.0 911.88
## + Fedu      1      8.74 4772.4 911.96
## + higher    1      6.40 4774.7 912.11
## + activities 1      6.26 4774.8 912.12
## + Walc      1      6.04 4775.1 912.13
## + address   1      5.32 4775.8 912.17
## + school    1      4.34 4776.8 912.24
## + goout     1      3.74 4777.4 912.27
## + traveltime 1      3.08 4778.0 912.31
## + Dalc      1      0.42 4780.7 912.48
## + Pstatus   1      0.15 4781.0 912.50
## + guardian  2      7.63 4773.5 917.73
## + reason    3     57.98 4723.1 920.26
## + Mjob      4    120.27 4660.8 922.01
## - failures  1    353.33 5134.4 922.36
## + Fjob      4     28.90 4752.2 927.79
##
## Step:  AIC=902.09

```

```

## G3 ~ age + famsize + Medu + failures + schoolsup + famsup + nursery +
##      internet + romantic + famrel + health + absences
##
##      Df Sum of Sq    RSS    AIC
## - nursery      1      17.69 4814.5 897.49
## - internet     1      31.09 4827.9 898.32
## - health       1      33.77 4830.6 898.48
## - famsup       1      39.78 4836.6 898.85
## - famrel       1      43.59 4840.4 899.09
## - absences     1      43.82 4840.6 899.10
## - age         1      74.67 4871.5 900.99
## - schoolsup    1      75.89 4872.7 901.07
## - famsize     1      76.83 4873.6 901.13
## - Medu        1      83.89 4880.7 901.56
## <none>                4796.8 902.09
## - romantic     1     136.24 4933.0 904.74
## + studytime    1      15.69 4781.1 906.81
## + paid         1      15.15 4781.7 906.84
## + Fedu         1       9.78 4787.0 907.18
## + higher       1       9.31 4787.5 907.21
## + sex          1       8.80 4788.0 907.24
## + freetime     1       5.84 4791.0 907.42
## + traveltime   1       4.40 4792.4 907.51
## + goout        1       4.32 4792.5 907.52
## + address      1       4.13 4792.7 907.53
## + activities   1       3.30 4793.5 907.58
## + Walc         1       2.74 4794.1 907.61
## + school       1       2.56 4794.2 907.63
## + Dalc         1       1.43 4795.4 907.70
## + Pstatus      1       0.08 4796.7 907.78
## + guardian     2       6.77 4790.0 913.06
## + reason       3      58.60 4738.2 915.52
## + Mjob         4     120.71 4676.1 917.28
## - failures     1     396.48 5193.3 920.06
## + Fjob         4      31.87 4764.9 922.89
##
## Step:  AIC=897.49
## G3 ~ age + famsize + Medu + failures + schoolsup + famsup + internet +
##      romantic + famrel + health + absences
##
##      Df Sum of Sq    RSS    AIC
## - health       1      30.41 4844.9 893.67
## - internet     1      34.19 4848.7 893.90
## - absences     1      40.95 4855.4 894.32
## - famrel       1      41.21 4855.7 894.33
## - famsup       1      44.03 4858.5 894.50
## - age         1      67.36 4881.9 895.93
## - famsize     1      68.21 4882.7 895.98
## - Medu        1      76.03 4890.5 896.46
## - schoolsup    1      77.64 4892.1 896.56
## <none>                4814.5 897.49
## - romantic     1     139.45 4953.9 900.30
## + nursery      1      17.69 4796.8 902.09
## + paid         1      13.50 4801.0 902.35

```



```

## + studytime 1 13.37 4801.1 902.36
## + Fedu 1 11.86 4802.6 902.45
## + sex 1 9.97 4804.5 902.57
## + higher 1 9.87 4804.6 902.57
## + freetime 1 5.48 4809.0 902.85
## + goout 1 4.70 4809.8 902.89
## + Walc 1 4.40 4810.1 902.91
## + traveltime 1 4.20 4810.3 902.93
## + address 1 3.43 4811.1 902.97
## + school 1 3.18 4811.3 902.99
## + activities 1 3.06 4811.4 903.00
## + Dalc 1 0.52 4814.0 903.15
## + Pstatus 1 0.08 4814.4 903.18
## + guardian 2 4.20 4810.3 908.62
## + reason 3 60.92 4753.6 910.78
## + Mjob 4 119.48 4695.0 912.79
## - failures 1 390.81 5205.3 915.05
## + Fjob 4 28.39 4786.1 918.51
##
## Step: AIC=893.67
## G3 ~ age + famsize + Medu + failures + schoolsup + famsup + internet +
## romantic + famrel + absences
##
## Df Sum of Sq RSS AIC
## - famrel 1 32.40 4877.3 889.96
## - absences 1 34.46 4879.4 890.08
## - internet 1 40.03 4884.9 890.42
## - famsup 1 47.43 4892.3 890.87
## - age 1 54.51 4899.4 891.30
## - schoolsup 1 67.63 4912.5 892.10
## - famsize 1 70.45 4915.3 892.27
## - Medu 1 80.10 4925.0 892.86
## <none> 4844.9 893.67
## - romantic 1 147.34 4992.2 896.90
## + health 1 30.41 4814.5 897.49
## + studytime 1 17.58 4827.3 898.28
## + paid 1 15.76 4829.1 898.39
## + Fedu 1 14.46 4830.4 898.47
## + nursery 1 14.32 4830.6 898.48
## + higher 1 9.15 4835.8 898.80
## + sex 1 6.46 4838.4 898.97
## + goout 1 5.02 4839.9 899.06
## + activities 1 4.29 4840.6 899.10
## + school 1 3.82 4841.1 899.13
## + traveltime 1 3.61 4841.3 899.14
## + freetime 1 3.48 4841.4 899.15
## + address 1 3.21 4841.7 899.17
## + Dalc 1 1.80 4843.1 899.25
## + Walc 1 1.72 4843.2 899.26
## + Pstatus 1 0.00 4844.9 899.36
## + guardian 2 2.06 4842.8 904.93
## + reason 3 69.94 4775.0 906.43
## + Mjob 4 112.92 4732.0 909.43
## - failures 1 412.26 5257.2 912.31

```

```

## + Fjob      4      22.35 4822.6 915.08
##
## Step: AIC=889.96
## G3 ~ age + famsize + Medu + failures + schoolsup + famsup + internet +
##      romantic + absences
##
##           Df Sum of Sq    RSS    AIC
## - absences  1      29.39 4906.7 886.05
## - internet  1      42.10 4919.4 886.82
## - famsup    1      47.65 4925.0 887.16
## - age       1      49.65 4926.9 887.28
## - schoolsup  1      65.82 4943.1 888.25
## - famsize   1      73.12 4950.4 888.69
## - Medu      1      81.49 4958.8 889.20
## <none>                      4877.3 889.96
## - romantic  1     151.57 5028.9 893.38
## + famrel    1      32.40 4844.9 893.67
## + health    1      21.60 4855.7 894.33
## + studytime  1      20.66 4856.6 894.39
## + paid      1      16.01 4861.3 894.67
## + Fedu      1      15.17 4862.1 894.73
## + nursery   1      12.85 4864.5 894.87
## + higher    1       8.42 4868.9 895.14
## + sex       1       8.41 4868.9 895.14
## + freetime  1       7.47 4869.8 895.20
## + activities 1       4.53 4872.8 895.38
## + goout     1       4.49 4872.8 895.38
## + Dalc      1       3.75 4873.5 895.42
## + traveltime 1       3.74 4873.6 895.43
## + address   1       3.28 4874.0 895.45
## + school    1       2.11 4875.2 895.52
## + Walc      1       0.41 4876.9 895.63
## + Pstatus   1       0.01 4877.3 895.65
## + guardian  2       1.34 4876.0 901.27
## + reason    3      68.01 4809.3 902.86
## + Mjob      4     112.20 4765.1 905.81
## - failures  1     402.76 5280.1 907.90
## + Fjob      4      21.00 4856.3 911.46
##
## Step: AIC=886.05
## G3 ~ age + famsize + Medu + failures + schoolsup + famsup + internet +
##      romantic
##
##           Df Sum of Sq    RSS    AIC
## - age       1      36.07 4942.8 882.54
## - famsup    1      48.23 4954.9 883.27
## - internet  1      49.39 4956.1 883.34
## - schoolsup  1      56.14 4962.8 883.74
## - famsize   1      71.01 4977.7 884.63
## <none>                      4906.7 886.05
## - Medu      1      96.15 5002.8 886.14
## - romantic  1     141.09 5047.8 888.80
## + absences  1      29.39 4877.3 889.96
## + famrel    1      27.33 4879.4 890.08

```

```

## + Fedu      1      17.96 4888.7 890.65
## + health    1      17.19 4889.5 890.70
## + paid      1      16.45 4890.2 890.75
## + studytime 1      16.25 4890.4 890.76
## + nursery   1      11.15 4895.5 891.07
## + sex       1       7.69 4899.0 891.28
## + activities 1      6.35 4900.3 891.36
## + higher    1      6.00 4900.7 891.38
## + goout     1      4.90 4901.8 891.45
## + freetime  1      4.24 4902.5 891.49
## + traveltime 1      3.62 4903.1 891.53
## + Dalc      1      2.85 4903.8 891.57
## + address   1      2.54 4904.2 891.59
## + Walc      1      1.07 4905.6 891.68
## + Pstatus   1      0.87 4905.8 891.69
## + school    1      0.34 4906.4 891.73
## + guardian  2      0.56 4906.1 897.41
## + reason    3     71.65 4835.0 898.76
## + Mjob      4     108.33 4798.4 902.19
## - failures  1     397.95 5304.6 903.59
## + Fjob      4      18.50 4888.2 907.71
##
## Step: AIC=882.54
## G3 ~ famsize + Medu + failures + schoolsup + famsup + internet +
##      romantic
##
##           Df Sum of Sq    RSS    AIC
## - schoolsup  1      38.61 4981.4 879.16
## - famsup     1      43.39 4986.2 879.44
## - internet   1      53.16 4995.9 880.03
## - famsize    1      63.68 5006.4 880.65
## <none>                4942.8 882.54
## - Medu       1     112.56 5055.3 883.55
## + age        1      36.07 4906.7 886.05
## - romantic   1     166.38 5109.1 886.70
## + famrel     1      24.55 4918.2 886.75
## + absences   1      15.81 4926.9 887.28
## + paid       1      15.59 4927.2 887.29
## + Fedu       1      14.32 4928.4 887.37
## + studytime  1      12.91 4929.8 887.45
## + health     1      10.54 4932.2 887.60
## + higher     1       9.89 4932.9 887.64
## + sex        1       9.67 4933.1 887.65
## + goout      1       7.71 4935.0 887.77
## + nursery    1       7.55 4935.2 887.78
## + address    1       5.84 4936.9 887.88
## + Dalc       1       5.70 4937.1 887.89
## + traveltime 1       4.86 4937.9 887.94
## + activities 1       4.34 4938.4 887.97
## + freetime   1       3.74 4939.0 888.01
## + school     1       2.26 4940.5 888.10
## + Pstatus    1       0.78 4942.0 888.19
## + Walc       1       0.50 4942.3 888.20
## + guardian   2       3.88 4938.9 893.70

```

```

## + reason      3      66.35 4876.4 895.60
## + Mjob        4     109.95 4832.8 898.62
## + Fjob        4      22.35 4920.4 903.97
## - failures    1     493.04 5435.8 905.17
##
## Step: AIC=879.16
## G3 ~ famsize + Medu + failures + famsup + internet + romantic
##
##           Df Sum of Sq  RSS   AIC
## - internet  1      48.76 5030.1 876.36
## - famsup    1      54.85 5036.2 876.72
## - famsize   1      65.56 5046.9 877.36
## <none>                      4981.4 879.16
## - Medu      1     115.88 5097.3 880.31
## + schoolsup  1      38.61 4942.8 882.54
## - romantic  1     159.04 5140.4 882.83
## + famrel    1      24.64 4956.7 883.38
## + paid      1      20.29 4961.1 883.64
## + age       1      18.54 4962.8 883.74
## + Fedu      1      16.38 4965.0 883.87
## + sex       1      16.09 4965.3 883.89
## + absences  1      12.39 4969.0 884.11
## + studytime 1      11.23 4970.1 884.18
## + nursery   1       9.84 4971.5 884.27
## + higher    1       8.51 4972.9 884.35
## + health    1       7.90 4973.5 884.38
## + goout     1       7.81 4973.6 884.39
## + activities 1       6.29 4975.1 884.48
## + traveltime 1       5.80 4975.6 884.51
## + Dalc      1       5.37 4976.0 884.53
## + address   1       4.63 4976.7 884.58
## + freetime  1       4.21 4977.2 884.60
## + Walc      1       1.66 4979.7 884.76
## + school    1       0.48 4980.9 884.83
## + Pstatus   1       0.09 4981.3 884.85
## + guardian  2       3.04 4978.3 890.37
## + reason    3      63.26 4918.1 892.44
## + Mjob      4     114.31 4867.1 895.03
## + Fjob      4      14.07 4967.3 901.10
## - failures  1     505.19 5486.6 902.25
##
## Step: AIC=876.36
## G3 ~ famsize + Medu + failures + famsup + romantic
##
##           Df Sum of Sq  RSS   AIC
## - famsup     1      48.61 5078.8 873.53
## - famsize    1      66.91 5097.1 874.60
## <none>                      5030.1 876.36
## - Medu       1     145.22 5175.4 879.15
## + internet   1      48.76 4981.4 879.16
## - romantic   1     147.76 5177.9 879.29
## + schoolsup   1      34.22 4995.9 880.03
## + paid       1      26.90 5003.2 880.46
## + famrel     1      25.92 5004.2 880.52

```

```

## + age      1      21.87 5008.3 880.76
## + sex      1      19.03 5011.1 880.93
## + Fedu     1      16.97 5013.2 881.05
## + absences 1      16.87 5013.3 881.06
## + address  1      13.59 5016.5 881.25
## + nursery  1      11.81 5018.3 881.36
## + studytime 1      10.71 5019.4 881.43
## + health   1      10.70 5019.4 881.43
## + traveltime 1      10.55 5019.6 881.43
## + freetime 1       7.75 5022.4 881.60
## + goout    1       6.64 5023.5 881.67
## + higher   1       6.12 5024.0 881.70
## + activities 1      5.48 5024.7 881.74
## + Dalc     1       4.03 5026.1 881.82
## + Walc     1       2.02 5028.1 881.94
## + school   1       1.45 5028.7 881.97
## + Pstatus  1       0.15 5030.0 882.05
## + guardian 2       4.54 5025.6 887.49
## + reason   3      65.24 4964.9 889.56
## + Mjob     4     126.88 4903.3 891.54
## + Fjob     4      13.17 5017.0 898.37
## - failures 1     521.78 5551.9 900.08
##
## Step: AIC=873.53
## G3 ~ famsize + Medu + failures + romantic
##
##           Df Sum of Sq  RSS   AIC
## - famsize    1      78.09 5156.8 872.38
## <none>                5078.8 873.53
## - Medu       1     114.79 5193.5 874.50
## - romantic   1     147.25 5226.0 876.35
## + famsup     1      48.61 5030.1 876.36
## + schoolsup   1      44.64 5034.1 876.60
## + internet   1      42.52 5036.2 876.72
## + sex        1      28.01 5050.7 877.58
## + famrel     1      26.20 5052.6 877.69
## + Fedu       1      23.22 5055.5 877.86
## + absences   1      17.25 5061.5 878.22
## + age        1      16.02 5062.7 878.29
## + nursery    1      15.92 5062.8 878.29
## + traveltime 1      12.69 5066.1 878.48
## + health     1      12.52 5066.2 878.49
## + address    1      10.90 5067.8 878.59
## + freetime   1       7.97 5070.8 878.76
## + paid       1       7.33 5071.4 878.80
## + studytime  1       6.40 5072.4 878.85
## + goout      1       5.90 5072.8 878.88
## + activities 1       3.85 5074.9 879.00
## + Dalc       1       3.69 5075.1 879.01
## + Walc       1       3.62 5075.1 879.02
## + higher     1       3.32 5075.4 879.03
## + Pstatus    1       0.46 5078.3 879.20
## + school     1       0.01 5078.7 879.23
## + guardian   2       6.00 5072.8 884.57

```

```

## + reason      3      67.56 5011.2 886.63
## + Mjob        4     119.60 4959.1 889.22
## + Fjob        4      13.30 5065.4 895.54
## - failures    1     526.64 5605.4 897.24
##
## Step:  AIC=872.38
## G3 ~ Medu + failures + romantic
##
##           Df Sum of Sq   RSS   AIC
## <none>                5156.8 872.38
## - Medu          1     102.28 5259.1 872.54
## + famsize       1      78.09 5078.8 873.53
## - romantic      1     136.67 5293.5 874.48
## + famsup        1      59.79 5097.1 874.60
## + schoolsup     1      48.12 5108.7 875.29
## + internet      1      43.22 5113.6 875.57
## + sex           1      37.88 5119.0 875.88
## + famrel        1      29.26 5127.6 876.38
## + Fedu          1      28.70 5128.1 876.42
## + absences      1      16.87 5140.0 877.10
## + address       1      16.76 5140.1 877.11
## + health        1      14.84 5142.0 877.22
## + age           1      10.29 5146.6 877.48
## + traveltime    1      10.19 5146.7 877.49
## + nursery       1       8.19 5148.7 877.61
## + freetime      1       7.91 5148.9 877.62
## + paid          1       6.78 5150.1 877.69
## + Walc          1       5.90 5150.9 877.74
## + goout         1       5.22 5151.6 877.78
## + studytime     1       4.33 5152.5 877.83
## + higher        1       3.65 5153.2 877.87
## + activities    1       2.18 5154.7 877.95
## + Dalc          1       1.94 5154.9 877.97
## + school        1       0.31 5156.5 878.06
## + Pstatus       1       0.09 5156.8 878.07
## + guardian      2       5.27 5151.6 883.47
## + reason        3      59.25 5097.6 886.03
## + Mjob          4     129.07 5027.8 887.62
## + Fjob          4      15.46 5141.4 894.28
## - failures      1     545.76 5702.6 896.66

```

```

pred_both <- fit_step_both %>% predict(test.data)
data.frame(
  RMSE = RMSE(pred_both, test.data$G3),
  Rsquare = R2(pred_both, test.data$G3)
)

```

```

##      RMSE  Rsquare
## 1 4.32272 0.147046

```

Backward

Full model (G3 ~ .) Removes least significant predictors iteratively
When you have many predictors and want to simplify the model

Forward Intercept-only ($G3 \sim 1$) Adds most significant predictors iteratively
When you have few predictors and want to build a model from scratch

Both Directions Either ($G3 \sim .$ or $G3 \sim 1$) Adds and removes predictors iteratively When unsure whether to start with a full or empty model

These three stepwise regression methods (forward, backward, and bidirectional) may not always give the same final model.

1. Stepwise methods are greedy algorithms, meaning they make the best choice at each step but do not guarantee the best overall model.
2. The sequence of adding/removing predictors can affect which predictors remain in the final model.
3. Backward selection removes predictors based on p-values, but if two variables are correlated, removing one may make the other significant.

Bestsubset Variable Selection

It selects the best set of predictors based on BIC (or AIC or R square) Tries all possible combinations of predictors to find the best model # Global Optima! Unlike stepwise selection, which follows a specific path (adding/removing one variable at a time), best subset selection evaluates all possible models.

```
fit_bs <- regsubsets(G3 ~ ., data = train.data, nvmax = 30)
# nvmax = 30: sets the maximum number of predictors to consider.
result <- summary(fit_bs)
which.min(result$bic)
```

```
## [1] 3
```

```
result$which[3,]
```

```
##      (Intercept)      schoolMS      sexM      age
##      TRUE      FALSE      FALSE      FALSE
##      addressU      famsizeLE3      PstatusT      Medu
##      FALSE      FALSE      FALSE      FALSE
##      Fedu      Mjobhealth      Mjobother      Mjobservices
##      FALSE      TRUE      FALSE      FALSE
##      Mjobteacher      Fjobhealth      Fjobother      Fjobservices
##      FALSE      FALSE      FALSE      FALSE
##      Fjobteacher      reasonhome      reasonother      reasonreputation
##      FALSE      FALSE      FALSE      FALSE
##      guardianmother      guardianother      traveltime      studytime
##      FALSE      FALSE      FALSE      FALSE
##      failures      schoolsupyes      famsupyes      paidyes
##      TRUE      FALSE      FALSE      FALSE
##      activitiesyes      nurseryyes      higheryes      internetyes
##      FALSE      FALSE      FALSE      FALSE
##      romanticsyes      famrel      freetime      goout
##      TRUE      FALSE      FALSE      FALSE
##      Dalc      Walc      health      absences
##      FALSE      FALSE      FALSE      FALSE
```

```
n <- nrow(train.data) # Sample size
p <- 1:(length(result$rss)) # Model sizes (number of predictors + intercept)
aic_values <- n * log(result$rss / n) + 2 * p # Compute AIC for each model
which.min(aic_values) # Get the model size with the lowest AIC
```

```
## [1] 12
```

```
result$which[12,]
```

```
##      (Intercept)      schoolMS      sexM      age
##      TRUE      FALSE      FALSE      TRUE
##      addressU      famsizeLE3      PstatusT      Medu
##      FALSE      TRUE      FALSE      TRUE
##      Fedu      Mjobhealth      Mjobother      Mjobservices
##      FALSE      TRUE      FALSE      TRUE
##      Mjobteacher      Fjobhealth      Fjobother      Fjobservices
##      FALSE      FALSE      FALSE      FALSE
##      Fjobteacher      reasonhome      reasonother      reasonreputation
##      FALSE      FALSE      FALSE      FALSE
##      guardianmother      guardianother      traveltime      studytime
##      FALSE      FALSE      FALSE      FALSE
##      failures      schoolsupyes      famsupyes      paidyes
##      TRUE      TRUE      TRUE      FALSE
##      activitiesyes      nurseryyes      higheryes      internetyes
##      FALSE      FALSE      FALSE      FALSE
##      romanticsyes      famrel      freetime      goout
##      TRUE      TRUE      FALSE      FALSE
##      Dalc      Walc      health      absences
##      FALSE      FALSE      TRUE      TRUE
```

```
which.min(result$adjr2)
```

```
## [1] 1
```

```
result$which[1,]
```

```
##      (Intercept)      schoolMS      sexM      age
##      TRUE      FALSE      FALSE      FALSE
##      addressU      famsizeLE3      PstatusT      Medu
##      FALSE      FALSE      FALSE      FALSE
##      Fedu      Mjobhealth      Mjobother      Mjobservices
##      FALSE      FALSE      FALSE      FALSE
##      Mjobteacher      Fjobhealth      Fjobother      Fjobservices
##      FALSE      FALSE      FALSE      FALSE
##      Fjobteacher      reasonhome      reasonother      reasonreputation
##      FALSE      FALSE      FALSE      FALSE
##      guardianmother      guardianother      traveltime      studytime
##      FALSE      FALSE      FALSE      FALSE
##      failures      schoolsupyes      famsupyes      paidyes
##      TRUE      FALSE      FALSE      FALSE
##      activitiesyes      nurseryyes      higheryes      internetyes
```


##	FALSE	FALSE	FALSE	FALSE
##	romanticyes	famrel	freetime	goout
##	FALSE	FALSE	FALSE	FALSE
##	Dalc	Walc	health	absences
##	FALSE	FALSE	FALSE	FALSE

Best Subset Selection (regsubsets())

Tries all possible combinations of predictors
 Finds the truly best model
 Computationally expensive for many predictors

Stepwise Selection (stepAIC())

Adds/removes one predictor at a time
 Faster, works well for moderate predictor sets
 Can miss the best model due to greedy approach