Department of Medical Physics
Bharathidasan University

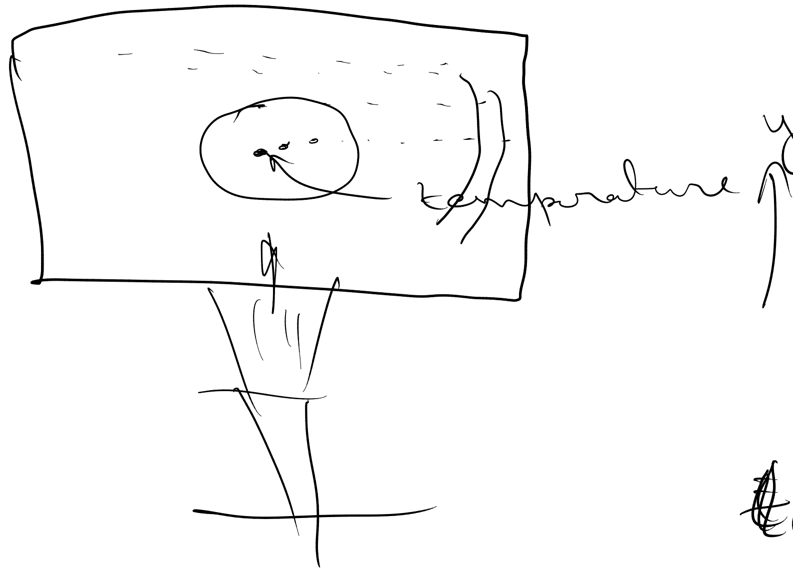
Mathematical Physics (MP101)

Date : **08-10-2020**

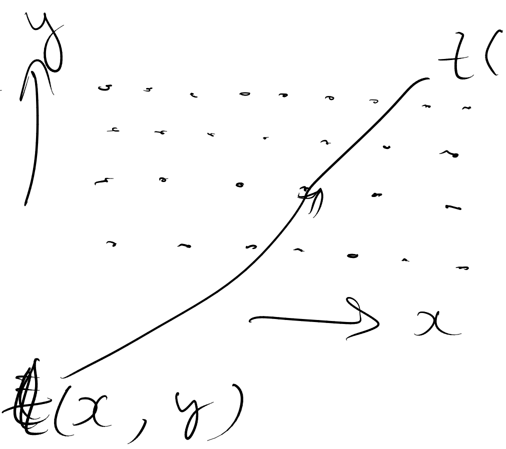
Attendance : **8**

Vector Calculus :

- → Coordinate System
- → Scalar and Vector fn.
- → Scalar and Vector product
- → Gradient, Divergence, Curl and Laplacian.



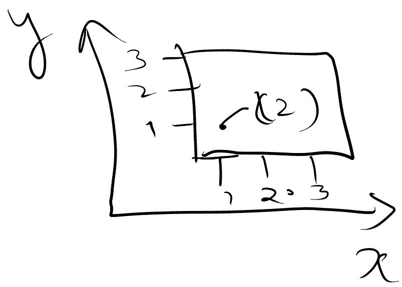
Value:



"t" temperature

"p" pressure

"D" density   "m" mass



$$\boxed{t(x, y) = x^2 + y^2}$$

$x = 1$

$y = 1$

$\boxed{t = 2}$ $t(1, 1) = 2$

Magnitude & direction (vector quantities)

Eg: 1) Velocity

2) momentum

3) Force

4) Magnetization etc...

Ex for vector quan.

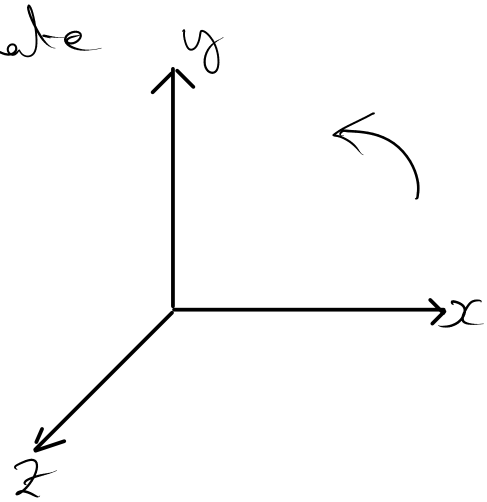
$$M(x, y) = x^2 + y^2$$

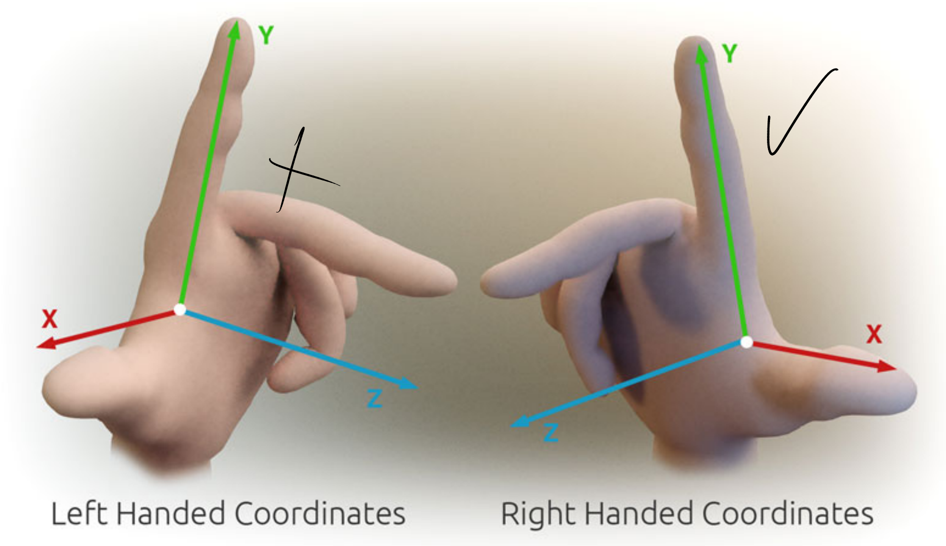
1) Scalar & Vector fn:

Coordinate System:

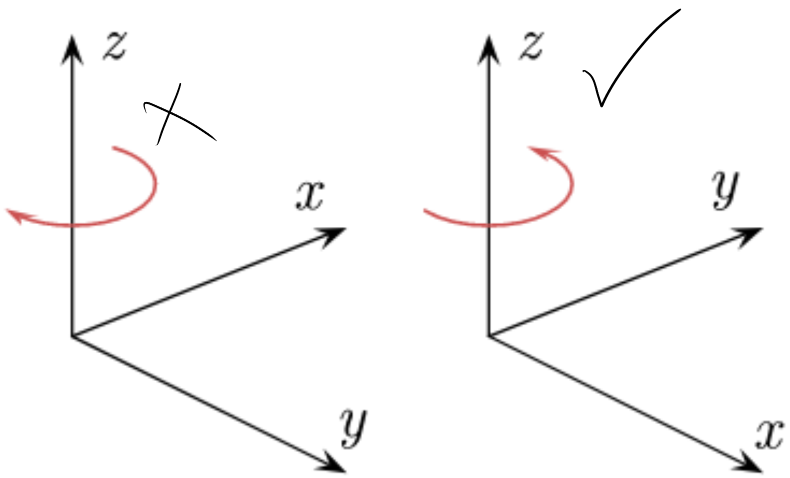
Cartesian Coordinate

Right handed Rule





for our convention



for our convention

Unit vector along

$x \rightarrow \hat{i}$

$y \rightarrow \hat{j}$

$z \rightarrow \hat{k}$

→ Scalar & Vector Intro

→ Coordinate System (Cartesian System)

→ Axis labelling convection (Right handed ~~labelling~~ Rule)

—x—

→ Scalar & Vector production:

$\vec{a}$ & $\vec{b}$

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\hat{i}\cdot\hat{i} = 1$	$\hat{i}\cdot\hat{j} = 0$
$\hat{j}\cdot\hat{j} = 1$	$\hat{j}\cdot\hat{k} = 0$
$\hat{k}\cdot\hat{k} = 1$	$\hat{k}\cdot\hat{i} = 0$

$\vec{a} \cdot \vec{b}$ = Scalar

$\vec{a} \cdot \vec{b} \Rightarrow$ Scalar quantity

dot product

$\vec{a}\cdot\vec{b} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \cdot (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

$= a_1\hat{i}\cdot b_1\hat{i} + 0 + 0 + 0 + a_2\hat{j}\cdot b_2\hat{j} + 0 + 0$

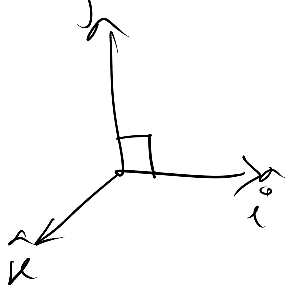
$+ 0 + a_3\hat{k}\cdot b_3\hat{k}$

$\vec{a}\cdot\vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\hat{i}\cdot\hat{j}$ or $\hat{j}\cdot\hat{k}$ or $\hat{k}\cdot\hat{i} = 0$?

Angle btn $\hat{i}$ & $\hat{j}$ = 90°

$\angle \hat{i}$ & $\hat{j} = 90^\circ$

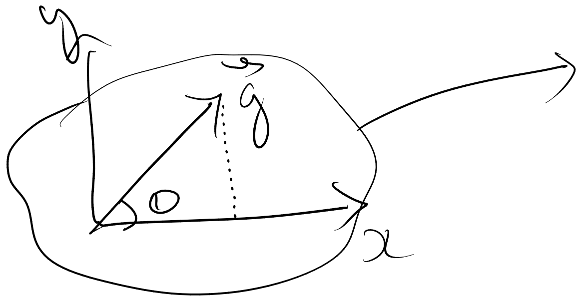


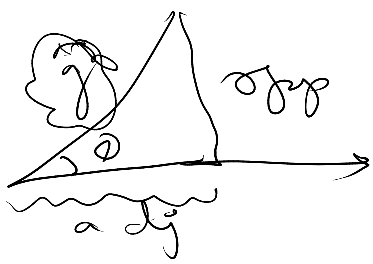
$\hat{i}\cdot\hat{j} = 0$

$\hat{i}\cdot\hat{j} = \hat{i}\hat{j}\cos\theta$

$= \hat{i}\hat{j}\cos 90^\circ$

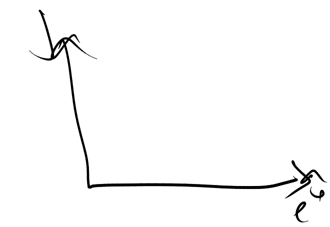
Dot product $\Rightarrow$ projection of one vector on the other.



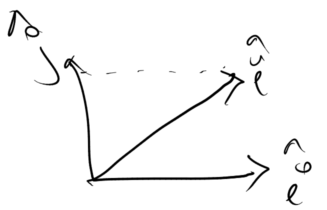


adj = hy. cos θ

$= g\cos\theta$



$$\boxed{\hat{i}\cdot\hat{j} = 0}$$



$\hat{k}\cdot\hat{j} = 0$

$\hat{k}\cdot\hat{i} = 0$

$$\boxed{\begin{array}{lll} \hat{i}\times\hat{j}=\hat{k} & \text{and} & \hat{j}\times\hat{i}=-\hat{k} \\ \hat{j}\times\hat{k}=\hat{i} & \text{and} & \hat{k}\times\hat{j}=-\hat{i} \\ \hat{k}\times\hat{i}=\hat{j} & \text{and} & \hat{i}\times\hat{k}=-\hat{j} \end{array}}$$

Next we can discuss about Vector product

$$\vec{a} \,\&\, \vec{b} \Rightarrow \vec{a}\times\vec{b} \cancel{= |\vec{a}\vec{b}|}$$

Cross product

$|\vec{a}|$ modulus $\not\equiv$ Determinant

$$\overbrace{|\vec{a}|}^{\text{modulus}} = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Determinant:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\vec{a}\times\vec{b} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$

$$= a_1\hat{i}\times b_1\hat{i} + a_1\hat{i}\times b_2\hat{j} + a_1\hat{i}\times b_3\hat{k} +$$
$$a_2\hat{j}\times b_1\hat{i} + a_2\hat{j}\times b_2\hat{j} + a_2\hat{j}\times b_3\hat{k} +$$
$$a_3\hat{k}\times b_1\hat{i} + a_3\hat{k}\times b_2\hat{j} + a_3\hat{k}\times b_3\hat{k}$$

$$\boxed{\begin{array}{l} \hat{i}\times\hat{i}=0 \\ \hat{j}\times\hat{j}=0 \\ \hat{k}\times\hat{k}=0 \end{array}}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \hat{i}(a_2b_3 - b_2a_3) - \hat{j}(a_1b_3 - b_1a_3) + \hat{k}(a_1b_2 - b_2a_2)$$

→ Scalar & Vector product.

Vector Calculus:

Vector differential operator:

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Gradient → Vector ~~product~~ operation
Divergent → Scalar ~~product~~ operation
Curl → Vector ~~product~~ operation
Laplacian → Scalar operation

Result quantities

——— × ———

Gradient: Vector operation on a scalar fn. function

$$\vec{\nabla} f = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) f$$

$f(x, y, z)$

⇓

$x^3 + 2xy + 3z$

scalar fn.

$$\boxed{\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}}$$

Divergence : Its a scalar operation on a Vector fn.

We cannot perform Divergence on a scalar fn.

We need a vector fn. to find

Divergence : scalar value

$$\vec{F} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$$

$$\vec{\nabla}\cdot\vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(f_1\hat{i} + f_2\hat{j} + f_3\hat{k}\right)$$

$$\boxed{\underbrace{\vec{\nabla}\cdot\vec{F}}_{\text{vector operation}} = \underbrace{\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}}_{\text{scalar fn.}}}$$

Curl : Its a vector operation on a vector fn.

$$\vec{F} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$\underbrace{\vec{\nabla} \times \vec{F}}_{\text{Vector operation}} = \hat{i}\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right) - \underbrace{\hat{j}\left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z}\right)}_{\text{Vector fn}} + \hat{k}\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)$$

Vector operation

Vector fn.

Laplacian: Scalar operation on a scalar fn.

$$\nabla^2 f \equiv \underbrace{\vec{\nabla} \cdot}_{\text{divergence}} \underbrace{\vec{\nabla} f}_{\text{gradient}}$$

Vector fn.

gradient

divergence

$$= \underbrace{\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)}_{\vec{\nabla}} \cdot \underbrace{\left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}\right)}_{\vec{\nabla} f}$$

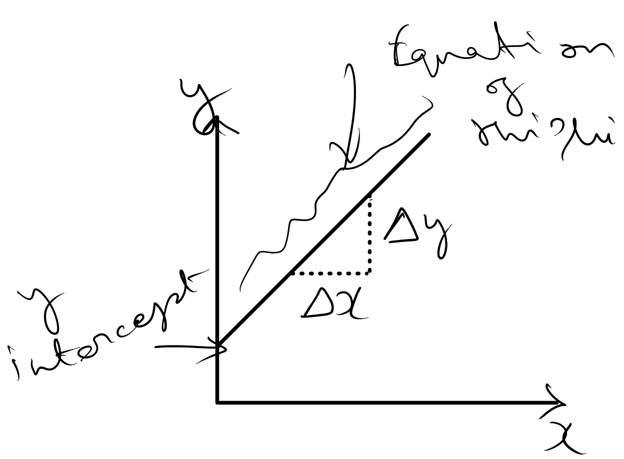
$$\boxed{\underbrace{\nabla^2 f}_{\text{Scalar operation}} = \underbrace{\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}}_{\text{Scalar fn}}}$$

Scalar operation

Scalar fn.

- → Gradient
- → Divergence
- → Curl
- → Laplacian.

Basics



$$\text{Slope } (m) = \frac{\Delta y}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} \; \{ \text{ slope}$$

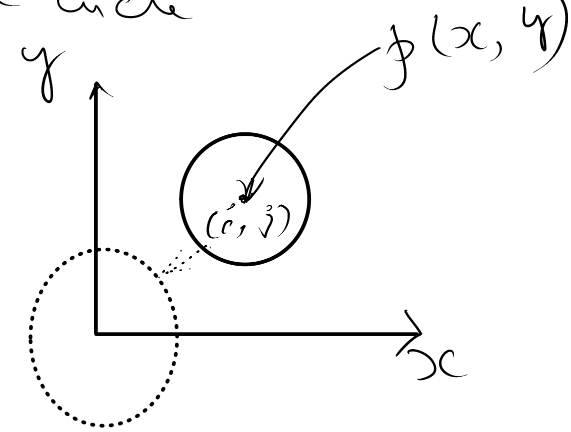
Equation of a straight:

$$y = \underbrace{m}_{\text{slope}} x + \underbrace{c}_{\text{intercept}}$$

Equation for a Circle: 2D case

$$x^2 + y^2 = r^2 \quad \text{radius of the circle}$$

$$(x - i)^2 + (y - j)^2 = r^2$$



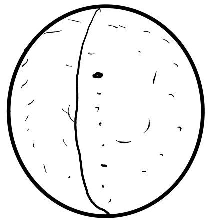
i & $j = 0$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

3D case: In 3D we have sphere

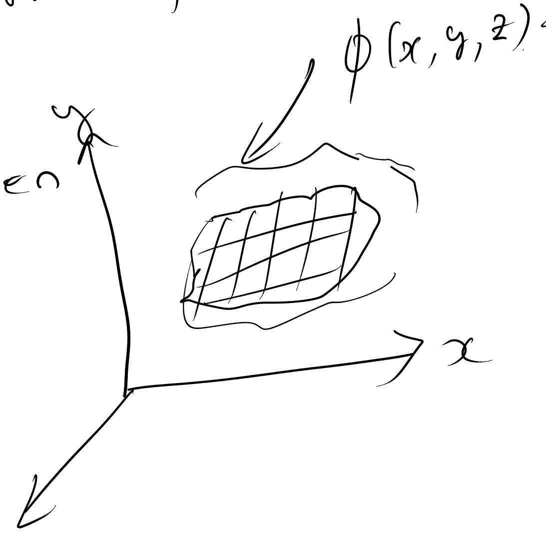
$$x^2+y^2+z^2 = r^2$$



$$f(x,y,z) = c$$

$$\boxed{\phi(x,y,z) = k}$$

Represent surface by this form



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