Fourier Series

Harmonic Analysis :

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{p} a_n \cos nx + \sum_{n=1}^{p} b_n \sin nx$$

D.C component

$a_1 \cos x$ | $b_1 \sin x$
$a_2 \cos 2x$ | $b_2 \sin 2x$

a_1, b_1 ← fundamental frequencies

a_2, b_2 ← Second harmonics

Example 24. *Find the Fourier series as far as the second harmonic to represent the function given by table below :*

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
$f(x)$	2.34	3.01	3.69	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

Solution

$x°$	$\sin x$	$\sin 2x$	$\cos x$	$\cos 2x$	$f(x)$	$f(x).\sin x$	$f(x).\sin 2x$	$f(x).\cos x$	$f(x).\cos 2x$
0°	0	0	1	1	2.34	0	0	2.340	2.340
30°	0.50	0.87	0.87	0.50	3.01	1.505	2.619	2.619	1.505
60°	0.87	0.87	0.50	−0.50	3.69	3.210	3.210	1.845	−1.845
90°	1.00	0	0	−1.00	4.15	4.150	0	0	−4.150
120°	0.87	−0.87	−0.50	−0.50	3.69	3.210	−3.210	−1.845	−1.845
150°	0.50	−0.87	−0.87	0.50	2.20	1.100	−1.914	−1.914	1.100
180°	0	0	−1	1.00	0.83	0	0	−0.830	0.830
210°	−0.50	0.87	−0.87	0.50	0.51	−0.255	0.444	−0.444	0.255
240°	−0.87	0.87	−0.50	−0.50	0.88	−0.766	0.766	−0.440	−0.440
270°	−1.00	0	0	−1.00	1.09	−1.090	0	0	−1.090
300°	−0.87	−0.87	0.50	−0.50	1.19	−1.035	−1.035	0.595	−0.595
330°	−0.50	−0.87	0.87	0.50	1.64	−0.820	−1.427	1.427	0.820
					25.22	9.209	−0.547	3.353	−3.115

$$a_0 = 2 \times \text{Mean of } f(x) = 2 \times \frac{25.22}{12} = 4.203 \quad a_0$$

$$a_1 = 2 \times \text{Mean of } f(x)\cos x = 2 \times \frac{3.353}{12} = 0.559 \quad a_1$$

$$a_2 = 2 \times \text{Mean of } f(x)\cos 2x = 2 \times \frac{-3.115}{12} = -0.519 \quad a_2$$

$$b_1 = 2 \times \text{Mean of } f(x)\sin x = 2 \times \frac{9.209}{12} = 1.535 \quad b_1$$

$$b_2 = 2 \times \text{Mean of } f(x)\sin 2x = 2 \times \frac{-0.547}{12} = -0.091 \quad b_2$$

Fourier series is

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \ldots + b_1 \sin x + b_2 \sin 2x + \ldots$$

$$= 2.1015 + 0.559 \cos x - 0.519 \cos 2x + \ldots + 1.535 \sin x - 0.091 \sin 2x + \ldots$$ **Ans.**