$$\vec{\nabla}\phi\Big|_{(1,-2,1)} = \hat{j} + 6\hat{k}$$

$$\vec{d} = 2\hat{i} - \hat{j} - 2\hat{k} \Rightarrow \hat{d} = \frac{2\hat{i} - \hat{j} - 2\hat{k}}{\sqrt{2^2 + 1^2 + 2^2}}$$

$$= \frac{1}{3}(2\hat{i} - \hat{j} - 2\hat{k})$$

D.D of ϕ along $\hat{d}$

$$\vec{\nabla}\phi \cdot \hat{d} = (0\hat{i} + \hat{j} + 6\hat{k}) \cdot \frac{(2\hat{i} - \hat{j} - 2\hat{k})}{3}$$

$$= \frac{1}{3} \cdot (-1 - 12)$$

$$D.D = \frac{-13}{3}$$

$$\vec{\nabla}\phi = \hat{j} + 6\hat{k}$$

$$|\vec{\nabla}\phi| = \sqrt{1^2 + 6}$$

$$|\vec{\nabla}\phi| = \sqrt{37}$$

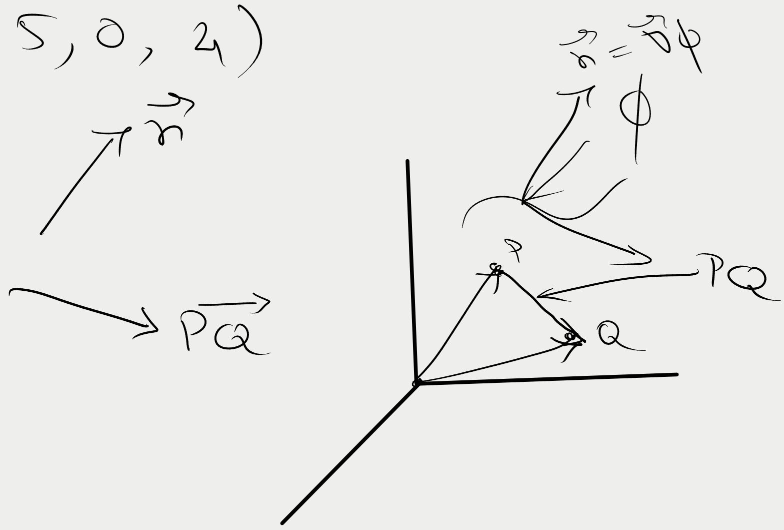
Normal $\rightarrow \vec{\nabla}\phi$

DD along $\vec{d}$ $\rightarrow \vec{\nabla}\phi \cdot \hat{d}$

Greatest rate of change of ϕ $\rightarrow |\vec{\nabla}\phi|$

Example 27: Find the DD of the fn: $\phi = x^2 - y^2 + 2z^2$ @ the pt: P (1, 2, 3) in the direction of the line PQ where Q (5, 0, 4)

$\vec{\nabla}\phi$



$\vec{PQ} =$

$\vec{P} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{Q} = 5\hat{i} + 0\hat{j} + 4\hat{k}$

$$\underline{\underline{\overline{PQ}}} = \vec{Q} - \vec{P} = 4\hat{i} - 2\hat{j} + \hat{k}$$

D.D of ϕ along $\vec{PQ}$

$\vec{\nabla}\phi$; $\hat{PQ}$

$$\vec{\nabla}\phi \cdot \hat{PQ}$$

H.W → <u>Example 29</u>

Example 21. *Find the constants m and n such that the surface $mx^2 - 2nyz = (m+4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, –1, 2).*

$$\phi_1 = mx^2 - 2nyz - (m+4)x$$

$$\phi_2 = 4x^2y + z^3 - 4$$

$P = (1, -1, 2)$

$$\phi_1\big|_P \perp \phi_2\big|_P$$

$$\vec{\nabla}\phi_1\Big|_{P} \quad \vec{\nabla}\phi_1 = \left[2mx-(m+4)\right]\hat{i} - 2nz\hat{j} - 2ny\hat{k}$$

$$\frac{\partial\phi_1}{\partial x} = 2mx-(m+4)$$

$$\frac{\partial\phi_1}{\partial y} = -2nz$$

$$\frac{\partial\phi_1}{\partial z} = -2ny$$

$$\vec{\nabla}\phi_1\Big|_{(1,-1,2)} = (2m-m-4)\hat{i} - 4n\hat{j} + 2n\hat{k}$$

$$2m-m$$

$$\vec{\nabla}\phi_1\Big|_{P} = (m-4)\hat{i} - 4n\hat{j} + 2n\hat{k}$$

$$\vec{\nabla}\phi_2\Big|_{P} = -8\hat{i} + 4\hat{j} + 12\hat{k}$$

$$\vec{n}_1 \cdot \vec{n}_2 = 0$$

$$\vec{\nabla}\phi_1 \cdot \vec{\nabla}\phi_2 = 0$$

$$\vec{n}_1 = \vec{\nabla}\phi_1$$

$$\vec{n}_2 = \vec{\nabla}\phi_2$$

$$\underbrace{\left((m-4)\hat{i} - 4n\hat{j} + 2n\hat{k}\right) \cdot \left(-8\hat{i} + 4\hat{j} + 12\hat{k}\right) = 0}_{\text{because } \vec{\nabla}\phi_1 \text{ \& } \vec{\nabla}\phi \text{ are orthogonal}}$$

$$-8(m-4) - 16n + 24n = 0$$

$$\boxed{-8m + 32 + 8n = 0} \quad \text{--- (1)}$$

2 unknowns but 1 Equation

$\phi_1|_p \Rightarrow mx^2 - 2nyz - (m+4)x = 0$

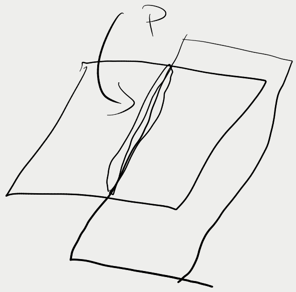
$\phi_1|_p \Rightarrow m(1)^2 - 2n(-1)(2) - (m+4)(1)$

$\Rightarrow \cancel{m} + 4n - \cancel{m} - 4$

$4n - 4 = 0$

$4n = 4$

$\boxed{n = 1}$



$P(1, -1, 2)$
$\quad\; x \quad y \quad z$

Subs. Value of n in Eqn. (1) we have

$-8m + 32 + 8(1) = 0$

$-8m + 40 = 0$

$\cancel{-}8m = \cancel{-}40$

$m = \frac{40}{8} = 5$

$\boxed{m = 5;\ n = 1}$

EXERCISE 5.7

1. Evaluate grad ϕ if $\phi = \log(x^2 + y^2 + z^2)$ **Ans.** $\frac{2(x\hat{i} + y\hat{j} + z\hat{k})}{x^2 + y^2 + z^2}$

2. Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 5$ at the point (0, 1, 2). **Ans.** $\frac{1}{\sqrt{5}}(\hat{j} + 2\hat{k})$

(AMIETE, June 2010)

3. Calculate the directional derivative of the function $\phi(x, y, z) = xy^2 + yz^3$ at the point (1, –1, 1) in the direction of (3, 1, –1) *(A.M.I.E.T.E. Winter 2009, 2000)* **Ans.** $\frac{5}{\sqrt{11}}$

4. Find the direction in which the directional derivative of $f(x, y) = (x^2 - y^2)/xy$ at (1, 1) is zero.

(Nagpur Winter 2000) **Ans.** $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

7. If $\phi(x, y, z) = 3xz^2y - y^3z^2$, find grad ϕ at the point (1, –2, –1) **Ans.** $-(16\hat{i} + 9\hat{j} + 4\hat{k})$

8. Find a unit vector normal to the surface $x^2y + 2xz = 4$ at the point (2, –2, 3).

Ans. $\frac{1}{3}(-\hat{i} + 2\hat{j} + 2\hat{k})$

9. What is the greatest rate of increase of the function $u = xyz^2$ at the point (1, 0, 3)? **Ans.** 9

11. Find the values of constants a, b, c so that the maximum value of the directional directive of $\phi = axy^2 + byz + cz^2x^3$ at (1, 2, –1) has a maximum magnitude 64 in the direction parallel to the axis of z. **Ans.** $a = b$, $b = 24$, $c = -8$

12. Find the values of λ and μ so that surfaces $\lambda x^2 - \mu y z = (\lambda + 2)x$ and $4x^2y + z^3 = 4$ intersect orthogonally at the point (1, –1, 2). **Ans.** $\lambda = \frac{9}{2}, \mu = 1$