Eigenvalues and Eigenvector

$$A = \begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$$

This is a $\underline{\underline{3 \times 3}}$ matrix

Characteristic Equation / polynomial : $|A - \lambda I| = 0$

Eigen value

$$\underbrace{\begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}}_{A} - \underbrace{\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}}_{\lambda I}$$

$$\begin{vmatrix} -2-\lambda & -4 & 2 \\ -2 & 1-\lambda & 2 \\ 4 & 2 & 5-\lambda \end{vmatrix} = 0$$

$$(-2-\lambda)\left[(1-\lambda)(5-\lambda) - 4\right] + 4\left[-2(5-\lambda) - 8\right] + 2\left[-4 - 4(1-\lambda)\right] = 0$$

$$-\lambda^3 + 4\lambda^2 + 27\lambda - 90 = 0$$

$$\lambda^3 - 4\lambda^2 - 27\lambda + 90 = 0$$

$$\begin{array}{r|rrrr} 3 & 1 & -4 & -27 & 90 \\ & \downarrow & 3 & -3 & -90 \\ \hline & 1 & -1 & -30 & \underline{0} \end{array}$$

$1, 3, 5, 7, 11, \dots$

$\lambda = 1$

$1^3 - 4 - 27 + 90 \neq 0$

$\lambda = 3$

$27 - 36 - 81 + 90$

$117 - 117 = 0$ ✓

$$(\lambda - 3)\underline{(\lambda^2 - \lambda - 30)} = 0$$

$$(\lambda - 3)(\lambda + 5)(\lambda - 6) = 0$$

1	30 ×
2	15 ×
3	10 ×
5	6

30

5 -6

$$\left.\begin{array}{l} \lambda_1 = 3 \\ \lambda_2 = -5 \\ \lambda_3 = 6 \end{array}\right\} \leftarrow \text{Eigen Values}$$

Eigen Vectors:

$$(A - \lambda I)\left(\underset{\text{Eigen vector}}{X}\right) = (0)$$

$\lambda = 3$

$$\begin{pmatrix} -2-3 & -4 & 2 \\ -2 & 1-3 & 2 \\ 4 & 2 & 5-3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -5 & -4 & 2 \\ -2 & -2 & 2 \\ 4 & 2 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-5x_1 - 4x_2 + 2x_3 = 0 \quad \text{—— (1a)}$$

$$-2x_1 - 2x_2 + 2x_3 = 0 \quad \text{—— (1b)}$$

$$4x_1 + 2x_2 + 2x_3 = 0 \quad \text{—— (1c)}$$

$\underline{x_1 = 1}$ in (1a) − (1b)

$$\begin{array}{r} -5 - 4x_2 + \cancel{2x_3} = 0 \\ \overset{(+)}{-2} \overset{(+)}{-2x_2} \overset{(-)}{+} \cancel{2x_3} = 0 \\ \hline -3 - 2x_2 = 0 \end{array}$$

$$-2x_2 = 3$$

$$\underline{x_2 = -\tfrac{3}{2}}$$

Sub. value of x_1 & x_2 in (1c) we have

$$4 - 3 + 2x_3 = 0$$

$$1 + 2x_3 = 0$$

$$x_3 = -\tfrac{1}{2}$$

Eigenvector for the Eigenvalue $\lambda = 3$ $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\begin{pmatrix} 1 \\ -3/2 \\ -1/2 \end{pmatrix} \leftarrow \text{Eigenvector for Eigenvalue } \lambda = 3$$

$$\text{Norm} = \sqrt{1^2 + (-3/2)^2 + (-1/2)^2}$$

$$= \sqrt{1 + \tfrac{10}{4}} = \sqrt{\tfrac{14}{4}} = \tfrac{\sqrt{14}}{2} \leftarrow \text{norm}$$

Normalized Eigen vector for $\lambda = 3$ is

$$\begin{pmatrix} \frac{2}{\sqrt{14}} \\ \frac{-3}{\sqrt{14}} \\ -\frac{1}{\sqrt{14}} \end{pmatrix}$$

normalized Eigen vector.

$\lambda = -5$; $\lambda = 6$

$x_1 = 1$; $x_2 = \frac{1}{2}$; $x_3 = -\frac{1}{2}$

$x_1 = 1$; $x_2 = 6$; $x_3 = 16$

$n = \sqrt{\frac{3}{2}}$

$n = \sqrt{293}$

$x_1 = \frac{\sqrt{2}}{3}$; $x_2 = \frac{1}{\sqrt{6}}$; $x_3 = -\frac{1}{\sqrt{6}}$

$x_1 = \frac{1}{\sqrt{293}}$; $x_2 = \frac{6}{\sqrt{293}}$

$x_3 = \frac{6}{\sqrt{293}}$

Diagonalized matrix D :

$$D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 6 \end{pmatrix}.$$

$$P^{-1}AP = D$$

Similarity Transformation

$$P = \begin{pmatrix} \frac{1}{\sqrt{14}} & \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{293}} \\ \frac{-3}{\sqrt{14}} & \frac{1}{\sqrt{6}} & \frac{6}{\sqrt{293}} \\ \frac{-1}{\sqrt{14}} & \frac{-1}{\sqrt{6}} & \frac{16}{\sqrt{293}} \end{pmatrix}$$

In the video I made a mistake

Instead of $\sqrt{3}$ I put 3 there.

It should be $\sqrt{3}$ //.