

Assignment 3

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Question

Papoulli Chapter 2(Ex.2.17) :

If we toss a coin twice we generate the four outcomes hh, ht, th, and tt.

$$H_1 = \{\text{heads at first toss}\} = \{hh, ht\}$$

$$H_2 = \{\text{heads at second toss}\} = \{hh, th\}$$

show that H_1 and H_2 are independent.

Solution Page 1

If we assign the probability to elementary events, then we can construct an experiment with these outcomes.

Let a and b be two positive numbers such that $a + b = 1$.
we assume that,

$$P(hh) = a^2 \quad (1)$$

$$P(ht) = P(th) = ab \quad (2)$$

$$P(tt) = b^2 \quad (3)$$

These are consistent with the axioms because

$$a^2 + ab + ab + b^2 = (a + b)^2 = 1 \quad (4)$$

Solution Page 2

The events

$$H_1 = \{\text{heads at first toss}\} = \{hh, ht\}$$

$$H_2 = \{\text{heads at second toss}\} = \{hh, th\}$$

consists of two elements each and their respective probabilities are

$$P(H_1) = P\{hh\} + P\{ht\} = a^2 + ab = a \quad (5)$$

$$P(H_2) = P\{hh\} + P\{th\} = a^2 + ab = a \quad (6)$$

The intersection of events H_1 and H_2 consists of only single element $\{hh\}$.

$$P(H_1 H_2) = P[hh] = a^2 = P(H_1)P(H_2) \quad (7)$$

The above equation proves that H_1 and H_2 are independent