

Assignment 6

Jupally Sriram, CS21BTECH11025

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Question

Papoulli Chapter 6(Ex 6.67) : Show that, if the random variable x is of discrete type taking the values x_n with $P\{x = x_n\} = p_n$ and $z = g(x, y)$, then

$$E\{z\} = \sum_n E(g(x_n, y))p_n$$

$$f_z(z) = \sum_n f_z(z|x_n)p_n$$

Theory

$$E\{g(X, Y)|x\} = \int_{-\infty}^{\infty} g(x, y)f(y|x)dy \quad (1)$$

The conditional mean of $g(y)$

$$E\{g(y)|M\} = \int_{-\infty}^{\infty} g(y)f(y|M)dy \quad (2)$$

From (2), we can say that

$$E\{g(x, y)|x\} = \int_{-\infty}^{\infty} g(x, y)f(y|x)dy \quad (3)$$

Theory

From (1) and (3) we can say that

$$E\{g(X, Y)|x\} = E\{g(x, y)|x\} \quad (4)$$

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From (4), we can say that

$$E\{g(x, y)\} = E\{E\{g(x, y)|y\}\} = E\{g(x_n, y)P\{x = x_n\}\} \quad (5)$$

$$E\{z\} = \sum_n E(g(x_n, y))p_n \quad (6)$$

we know from Bayes theorem that,

$$f(x) = f(x|A_1)P(A_1) + f(x|A_2)P(A_2) + + f(x|A_n)P(A_n) \quad (7)$$

putting $A_n = \{x = x_n\}$

$$f_z(z) = \sum_n f_z(z|x = x_n)P\{x = x_n\}$$