Assignment 7

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Question

Papoulli Chapter 8(Ex 8.28) : Brand A batteries cost more than brand B batteries. Their life lenghts are two normal and independent random variables x and y. we test 16 batteries of brand A and 26 batteries of brand B and find these values.(in hours)

$$\bar{x} = 4.6$$

$$s_x = 1.1$$

$$\bar{y} = 4.2$$

$$s_y = 0.9$$

Test the hypothesis $\eta_x = \eta_y$ against $\eta_x > \eta_y$ with $\alpha = 0.05$

Theory

Under hypothesis H_0, q is N(0,1). Replacing the q_u percentile by the standard normal percentile z_u, we obtain the following test
$H_1 : \eta \neq \eta_0$. Accept H_0 iff $z_{\alpha/2} < q < z_{1-\alpha/2}$

$$\beta\{\eta\} = P\{|q| < z_{1-\alpha/2}|H_1\} = G(z_{1-\alpha/2} - \eta_q) - G(z_{\alpha/2} - \eta_q) \tag{1}$$

$H_1 : \eta > \eta_0$ Accept H_0 iff $q < z_{1-\alpha}$

$$\beta\{\eta\} = P\{q < z_{1-\alpha}|H_1\} = G(z_{1-\alpha} - \eta_q) \tag{2}$$

$H_1 : \eta < \eta_0$ Accept H_0 iff $q > z_\alpha$

$$\beta\{\eta\} = P\{q > z_\alpha|H_1\} = 1 - G(z_\alpha - \eta_q) \tag{3}$$

Solution Page 1

Let w be the difference of their sample means

$$w = \bar{x} - \bar{y} \tag{4}$$

$$\bar{x} = \frac{1}{16}\sum_{i=1}^{16} x_i \tag{5}$$

$$\bar{y} = \frac{1}{26}\sum_{i=1}^{26} y_i \tag{6}$$

Let q be an another R.V such that

$$q = \frac{w}{\sigma_w} \qquad \sigma_w^2 = \frac{\sigma_x^2}{16} + \frac{\sigma_y^2}{26} \tag{7}$$

Solution Page 2

The R.V q is normal with $\sigma_q = 1$ and under hypothesis $H_0, E\{q\} = 0.$ we can therefore use (2) because $q_u = z_u$.
To find q, we must determine σ_w.
Since σ_x and σ_y are not specified, we shall use the approximations $\sigma_x \approx s_x = 1.1$ and $\sigma_y \approx s_y = 0.9$

Solution Page 3

$$\sigma_w^2 \approx \frac{1.1^2}{16} + \frac{0.9^2}{26} = 0.107 \tag{8}$$

$$q = \frac{\bar{x} - \bar{y}}{\sigma_w} = \frac{0.4}{0.327} = 1.223 \tag{9}$$

since $z_{0.95} = 1.645 > 1.223$, we accept H_0