

Assignment 5

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Question

Papoulli Chapter 6(Ex 6.66) : Let x and y be independent random variables with variances σ_1^2 and σ_2^2 , respectively. consider the sum

$$z = aX + (1 - a)Y \quad 0 \leq a \leq 1$$

Find a that minimizes the variance of z .

Solution Page 1

Given that,

$$z = aX + (1 - a)Y \quad 0 \leq a \leq 1$$

$$\text{Var}(Z) = \sigma_z^2 = a^2\sigma_1^2 + (1 - a)^2\sigma_2^2 \quad (1)$$

By differentiating on both sides with respect to a , we get

$$\frac{d\sigma_z^2}{da} = 2a\sigma_1^2 - 2(1 - a)\sigma_2^2 \quad (2)$$

For σ_z^2 to be minimum, $\frac{d\sigma_z^2}{da}$ must be zero
equating (2) to zero, we get

$$2a\sigma_1^2 - 2(1 - a)\sigma_2^2 = 0 \quad (3)$$

Solution Page 2

$$a(\sigma_1^2 + \sigma_2^2) = \sigma_2^2 \quad (4)$$

$$a = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \quad 0 \leq a \leq 1 \quad (5)$$