Assignment 9

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LaTeX

Question

Papoulli Chapter 9(Ex 9.43) : The process x(t) is WSS and $R_{xx}(\tau) = 5\delta(\tau)$
(a) Find $E\{y^2(t)\}$ and $S_{yy}(\omega)$ if y'(t) +3y(t) = x(t)
(b) Find $E\{y^2(t)\}$ and $R_{xy}(t_1, t_2)$ if y'(t) + 3y(t) =x(t)U(t).

Theory Page 1

If h(t) is of short duration relative to the variations of q(t) ,then

$$E\{y^2(t)\} \approx q(t) \int_{-\infty}^{\infty} |h(\alpha)|^2 d\alpha = Eq(t) \tag{1}$$

This relationship justifies the term average intensity used to describe the function q(t)

Theory Page 2

If a stationary process v(t) with autocorrelation $R_{vv}(\tau) = q\delta(t)$ is applied at t=0 to a linear system with

$$h(t) = e^{-ct}U(t)$$

The autocorrelation of the resulting output y(t) equals

$$E\{y^2(t)\} = R_{yy}(t,t) = \frac{q}{2c}(1 - e^{-2ct}) = q\int_0^t h^2(\alpha)d\alpha \tag{2}$$

Solution Page 1

(a) y'(t) + 3y(t) = x(t), $R_{XX}(\tau) = 5\delta(\tau)$. The process y(t) is the output of the system

$$H(s) = \frac{1}{s+3} \tag{3}$$
$$h(t) = e^{-3t}U(t) \tag{4}$$

Hence from (1)

$$E\{y^2(t)\} = 5\int_0^\infty e^{-6t}dt = \frac{5}{6} \tag{5}$$
$$S_{yy}(\omega) = \frac{5}{\omega^2+9} \qquad R_{yy}(\tau) = \frac{5}{6}e^{-3|\tau|} \tag{6}$$

Solution Page 2

(b) From (2)

$$E\{y^2(t)\} = 5\int_0^t e^{-6\alpha} d\alpha = \frac{5}{6}(1 - e^{-6t}) \qquad t > 0 \tag{7}$$

$$R_{xy}(t_1, t_2) = 5e^{-2|t_2 - t_1|} U(t_1)U(t_2)U(t_2 - t_1) \tag{8}$$