Assignment 4

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May 30, 2022

Question

Papoulli Chapter 6(6.36) :
The moment generating function of x and y is given by

$$\Phi(s_1, s_2) = E\{e^{s_1 x + s_2 y}\}$$

Show that if the random variables x and y are jointly normal and have zero mean,

$$E\{x^2 y^2\} = E\{x^2\}E\{y^2\} + 2E^2\{xy\}$$

Solution Page 1

The joint characteristic function of two jointly normal random variables is given by

$$\Phi(s_1, s_2) = e^{-A} \tag{1}$$

where,

$$A = \frac{1}{2}(\sigma_1^2 s_1^2 + 2Cs_1 s_2 + \sigma_2^2 s_2^2) \tag{2}$$

where $C = E\{xy\} = r\sigma_1\sigma_2$
Expanding the exponential and using the linearity of expected values, we get

Solution Page 2

$$\Phi(s_1, s_2) = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{n=0}^{n} \binom{n}{k} E\{x^k y^{n-k}\} s_1^k s_2^{n-k} \tag{3}$$

$$= 1 + m_{10}s_1 + m_{01}s_2 + \frac{1}{2}(m_{20}s_1^2 + 2m_{11}s_2 + m_{02}s_2^2) + \ldots \tag{4}$$

By equating of coefficients of $s_1^2 s_2^2$ in (4) with the corresponding coefficient of the expansion of e^{-A}, we get

Solution Page 3

$$\frac{A^2}{2} = \frac{1}{8}(\sigma_1^2 s_1^2 + 2Cs_1 s_2 + \sigma_2^2 s_2^2)^2 \tag{5}$$

Hence

$$\frac{1}{4!}\binom{4}{2} E\{x^2y^2\} = \frac{1}{8}(2\sigma_1^2\sigma_2^2 + 4C^2) \tag{6}$$

$$E\{x^2y^2\} = \sigma_1^2\sigma_2^2 + 2C^2 = E\{x^2\}E\{y^2\} + 2E^2\{xy\} \tag{7}$$

The equation (7) is the equation that is asked to prove.