### Assignment 7

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### Question

**Papoulli Chapter 8(Ex 8.28)**: Brand A batteries cost more than brand B batteries. Their life lenghts are two normal and independent random variables x and y. we test 16 batteries of brand A and 26 batteries of brand B and find these values.(in hours)

$$\bar{x} = 4.6$$

$$s_{x} = 1.1$$

$$\bar{y} = 4.2$$

$$s_v = 0.9$$

Test the hypothesis  $\eta_x = \eta_y$  against  $\eta_x > \eta_y$  with  $\alpha = 0.05$ 

### Theory

Under hypothesis  $H_0$ , q is N(0,1). Replacing the  $q_u$  percentile by the standard normal percentile  $z_u$ , we obtain the following test  $H_1: \eta \neq \eta_0$ . Accept  $H_0$  iff  $z_{\alpha/2} < q < z_{1-\alpha/2}$ 

$$\beta\{\eta\} = P\{|q| < z_{1-\alpha/2}|H_1\} = G(z_{1-\alpha/2} - \eta_q) - G(z_{\alpha/2} - \eta_q)$$
 (1)

 $H_1: \eta > \eta_0$  Accept  $H_0$  iff  $q < z_{1-\alpha}$ 

$$\beta\{\eta\} = P\{q < z_{1-\alpha}|H_1\} = G(z_{1-\alpha} - \eta_q)$$
 (2)

 $H_1: \eta < \eta_0$  Accept  $H_0$  iff  $q > z_\alpha$ 

$$\beta\{\eta\} = P\{q > z_{\alpha} | H_1\} = 1 - G(z_{\alpha} - \eta_q) \tag{3}$$

## Solution Page 1

Let w be the difference of their sample means

$$w = \bar{x} - \bar{y} \tag{4}$$

$$\bar{x} = \frac{1}{16} \sum_{i=1}^{16} x_i \tag{5}$$

$$\bar{y} = \frac{1}{26} \sum_{i=1}^{26} y_i \tag{6}$$

Let q be an another R.V such that

$$q = \frac{w}{\sigma_w} \qquad \sigma_w^2 = \frac{\sigma_x^2}{16} + \frac{\sigma_y^2}{26} \tag{7}$$

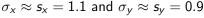


# Solution Page 2

The R.V q is normal with  $\sigma_q = 1$  and under hypothesis  $H_0, E\{q\} = 0$ . we can therefore use (2) because  $q_u = z_u$ .

To find q, we must determine  $\sigma_w$ .

Since  $\sigma_x$  and  $\sigma_y$  are not specified, we shall use the approximations



# Solution Page 3

$$\sigma_w^2 \approx \frac{1.1^2}{16} + \frac{0.9^2}{26} = 0.107 \tag{8}$$

$$\sigma_w^2 \approx \frac{1.1^2}{16} + \frac{0.9^2}{26} = 0.107$$

$$q = \frac{\bar{x} - \bar{y}}{\sigma_w} = \frac{0.4}{0.327} = 1.223$$
(9)

since  $z_{0.95} = 1.645 > 1.223$ , we accept  $H_0$ 

