Cold Fermions in Artificial Gauge Fields

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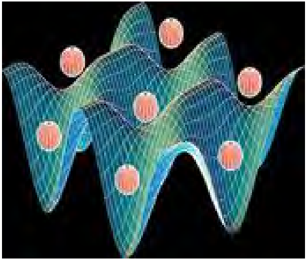
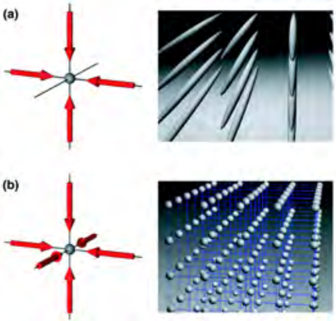
Plan of the talk

- Introduction
  - Cold atoms
  - Roadblocks in cold atoms
  - Necessity of synthetic gauge fields
  - Background material
  - Interacting fermions in artificial gauge fields
- Main outcomes of our research – Outlined in detail later
  - New kinds of excitations
  - Implications to high temperature superfluidity

Cold atoms

- Paradigm of condensed matter physics
- Progress hampered by technical difficulties
- Cold Atoms can tailor Hamiltonians ! – Quantum emulation
- Eg: Optical lattice:



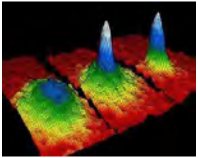


(Bloch, 2008)

- Cold is Gold: Trap + Quantum

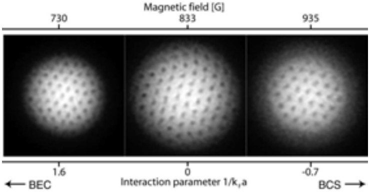
Cold atoms help CMP

- In bosonic systems :



(Colorado group (1995))

- In fermionic systems :



(Ketterle et. al. (2008))

- Then are all problems solved?

Problems in cold atoms

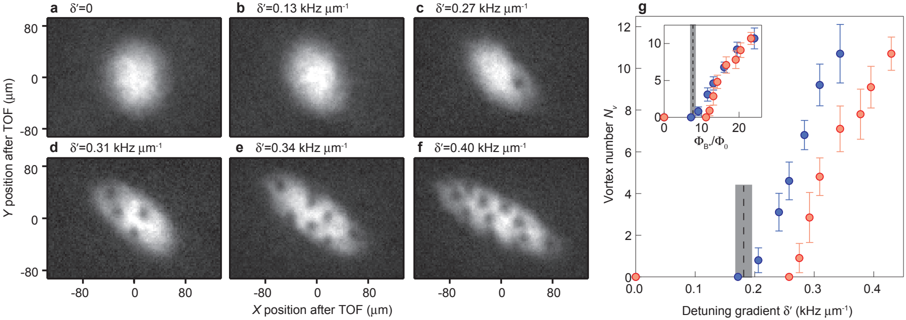
- Problem : How to cool cold atoms?
- A separate industry . . . (McKay et. al. 2011)
- Problem : Neutrality of particles – Emulating electromagnetic response?
- Artificial Fields: Coriolis – a possibility

$$\begin{aligned} \boldsymbol{F}_{Magnetic} &= q\,(\boldsymbol{v} \times \boldsymbol{B}) \\ \boldsymbol{F}_{Coriolis} &= 2m\,(\boldsymbol{v} \times \boldsymbol{\omega}) \end{aligned}$$

- But,
    - Stability of the rotational set up
    - Nonhomogeneous magnetic fields
    - Achieving high strengths

- Enter: Artificial Gauge Fields.

Artificial gauge fields



(Spielman's group at NIST)

- Internal Degrees of freedom
- What happens if the ground state is degenerate?

Non-Abelian gauge field

- Non-Abelian gauge fields possible (Jaksch/Zoller, (2003),Osterloh et al., (2005), Ruseckas et al. (2005), Gerbier/Dalibard (2010)...)
- The general structure of a gauge-coupled Hamiltonian in 3D:

$$\mathcal{H} = \int \mathrm{d}\boldsymbol{r}\, \psi_{\sigma}^{\dagger}(\boldsymbol{r}) \left[\frac{(\boldsymbol{p}-\boldsymbol{A})^2}{2} \right]_{\sigma\,\sigma'} \psi_{\sigma'}(\boldsymbol{r}) + \mathcal{H}_{int}$$

- Abelian gauge field :
  - If components of A commute
  - Uniform Abelian gauge field $\equiv$ Galilean transformation
- non-Abelian gauge field :
  - If components of A do not commute
  - In case of bosons, even a uniform non-Abelian gauge field nurtures interesting physics.

(Ho and Zhang, (2011), Wang et al., (2010))

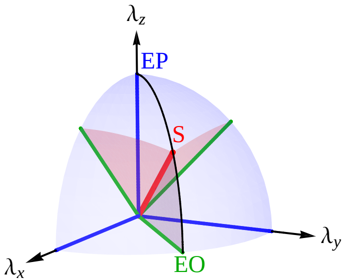
Gauge field configuration

- $\boldsymbol{A} = (\lambda_x\tau_x, \lambda_y\tau_y, \lambda_z\tau_z)$ is a spatially uniform non-Abelian gauge field. τs $\rightarrow$ Pauli matrices.

$$\mathcal{H} = \int \mathrm{d}\boldsymbol{r}\, \psi^{\dagger}_{\sigma}(\boldsymbol{r}) \left(\underbrace{\frac{\boldsymbol{p}^2}{2}}_{\text{Usual Piece}} - \overbrace{\boldsymbol{p}_{\lambda} \cdot \boldsymbol{\tau}}^{\text{RSOC}} \right) \psi_{\sigma'}(\boldsymbol{r}) + \mathcal{H}_{int}\,,$$

$$\boldsymbol{p}_{\lambda} = (\lambda_x p_x, \lambda_y p_y, \lambda_z p_z)$$

- $\boldsymbol{\lambda} = (\lambda_x, \lambda_y, \lambda_z) = \lambda\hat{\boldsymbol{\lambda}}$ specifies a gauge field configuration (GFC).



Special GFCs :

- Extreme Prolate
  EP, $\boldsymbol{\lambda} = \lambda(0,0,1)$
  (Expt: Shanxi,MIT,NIST)
- Spherical
  S, $\boldsymbol{\lambda} = \frac{\lambda}{\sqrt{3}}(1,1,1)$
  (Proposal: Anderson et al.)
- Extreme Oblate
  EO, $\boldsymbol{\lambda} = \frac{\lambda}{\sqrt{2}}(1,1,0)$
  (Proposal: Campbell et al.)

Interaction between fermions

- Contact interaction in singlet channel

$$\begin{aligned}\mathcal{H}_{int} &= \frac{v}{2}\int d\boldsymbol{r}\, S^\dagger(\boldsymbol{r})S(\boldsymbol{r}) = v\int d\boldsymbol{r}\,\psi^\dagger_\uparrow(\boldsymbol{r})\psi^\dagger_\downarrow(\boldsymbol{r})\psi_\downarrow(\boldsymbol{r})\psi_\uparrow(\boldsymbol{r}) \\ &= \frac{v}{2V}\sum_{\boldsymbol{q}} S^\dagger(\boldsymbol{q})S(\boldsymbol{q}) = \frac{v}{V}\sum_{\boldsymbol{q}\,\boldsymbol{k}\,\boldsymbol{k}'} C^\dagger_{\frac{\boldsymbol{q}}{2}+\boldsymbol{k}\uparrow}\, C^\dagger_{\frac{\boldsymbol{q}}{2}-\boldsymbol{k}\downarrow}\, C_{\frac{\boldsymbol{q}}{2}-\boldsymbol{k}'\downarrow}\, C_{\frac{\boldsymbol{q}}{2}+\boldsymbol{k}'\uparrow}\end{aligned}$$

$S^\dagger(\boldsymbol{r}) = \frac{1}{\sqrt{2}}\left(\psi^\dagger_\uparrow(\boldsymbol{r})\psi^\dagger_\downarrow(\boldsymbol{r}) - \psi^\dagger_\downarrow(\boldsymbol{r})\psi^\dagger_\uparrow(\boldsymbol{r})\right) \to$ singlet creation operator,
$v \to$ bare interaction parameter and $V \to$ volume.

- Field theory in terms of v has ultraviolet divergence – artificial
- Regularization : $\frac{1}{v} + \frac{1}{V}\sum_{\boldsymbol{k}} \frac{1}{k^2} = \frac{1}{4\pi a_s}$ where a_s is the s-wave scattering length tunable by the Feshbach resonance.

Our Study - An outline

- What is the fate of interacting fermions in a non-Abelian gauge field ($\boldsymbol{\lambda}$)?
  1. One body problem
  2. Two body problem
  3. Noninteracting fermions
  4. Ground state of interacting fermions
  5. Collective excitations of the superfluid
  6. The phase diagram
- Scales

$$T_F = E_F = \frac{{k_F}^2}{2} = \frac{1}{2}(3\pi^2\rho)^{2/3}$$

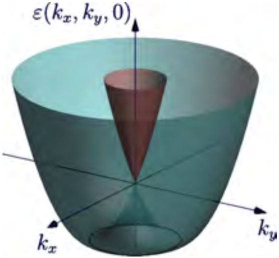
- Dimensionless parameters : $k_F a_s$, λa_s, λ / k_F, $\hat{\boldsymbol{\lambda}}$.
- Spherical GFC, $\boldsymbol{\lambda} = \frac{\lambda}{\sqrt{3}}(1,1,1)$, is chosen here as a default example to present some of the results.

One body problem

- Momentum continues to be a good quantum number ...
- ...but spin does not.
- Its role is taken up by generalized helicity α $(=\pm)$

$$
\begin{aligned}
|\boldsymbol{k}\alpha\rangle &= |\boldsymbol{k}\rangle \otimes |\alpha\hat{\boldsymbol{k}}_\lambda\rangle \equiv C^\dagger_{\boldsymbol{k}\alpha}|0\rangle \\
\boldsymbol{k}_\lambda &= (\lambda_x k_x, \lambda_y k_y, \lambda_z k_z)
\end{aligned}
$$

$$\varepsilon_\alpha(\boldsymbol{k}) = \frac{k^2}{2} - \alpha|\boldsymbol{k}_\lambda|$$



- Observe : ground state degeneracy

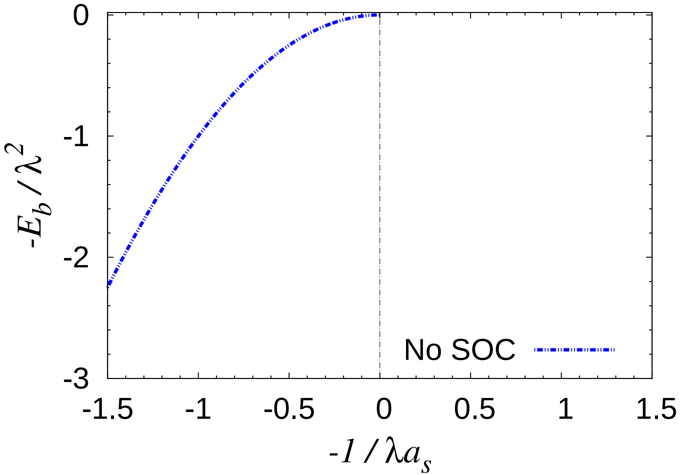
Two body problem in gauge fields

- Non-interacting states : $|\boldsymbol{q}\,\boldsymbol{k}\,\alpha\,\beta\rangle \equiv C^{\dagger}_{\left(\frac{\boldsymbol{q}}{2}+\boldsymbol{k}\right)\alpha} C^{\dagger}_{\left(\frac{\boldsymbol{q}}{2}-\boldsymbol{k}\right)\beta}|0\rangle$
- Non-interacting energies : $\varepsilon_{\alpha\beta}(\boldsymbol{q},\boldsymbol{k}) = \varepsilon_{\alpha}\left(\frac{\boldsymbol{q}}{2}+\boldsymbol{k}\right) + \varepsilon_{\beta}\left(\frac{\boldsymbol{q}}{2}-\boldsymbol{k}\right)$
- Singlet amplitude : $A_{\alpha\beta}(\boldsymbol{q},\boldsymbol{k}) = \frac{1}{\sqrt{2}}\langle 0|\left(C_{\frac{\boldsymbol{q}}{2}+\boldsymbol{k}\uparrow}C_{\frac{\boldsymbol{q}}{2}-\boldsymbol{k}\downarrow} - C_{\frac{\boldsymbol{q}}{2}+\boldsymbol{k}\downarrow}C_{\frac{\boldsymbol{q}}{2}-\boldsymbol{k}\uparrow}\right)|\boldsymbol{q}\,\boldsymbol{k}\,\alpha\,\beta\rangle$
- Secular equation :

$$\frac{1}{4\pi a_s} = \frac{1}{V}\sum_{\boldsymbol{k}}\left[\left(\sum_{\alpha\beta}\frac{|A_{\alpha\beta}(\boldsymbol{q},\boldsymbol{k})|^2}{E(\boldsymbol{q}) - \varepsilon_{\alpha\beta}(\boldsymbol{q},\boldsymbol{k})}\right) + \frac{1}{k^2}\right]$$

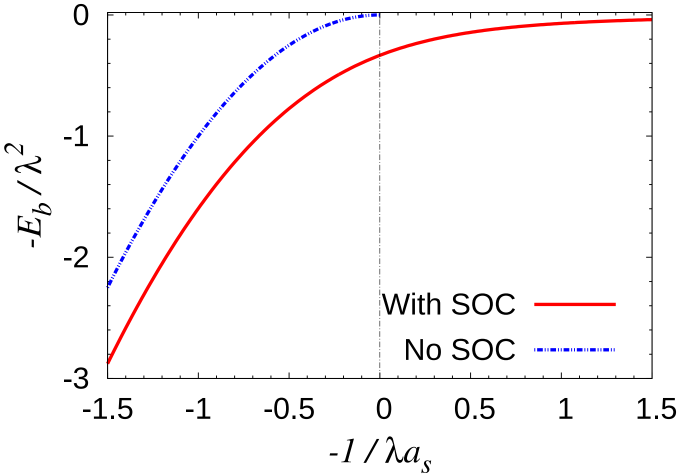
- Bound state wave function : $|\psi_b\rangle = |\psi_s\rangle + |\psi_t\rangle$
- Triplet content : $\eta_t = \langle\psi_t|\psi_t\rangle$

Two body problem: $q = 0$



- Critical attraction for a two-body bound state in 3D

Two body problem: $q = 0$

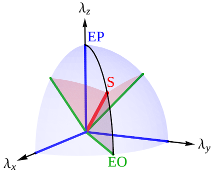


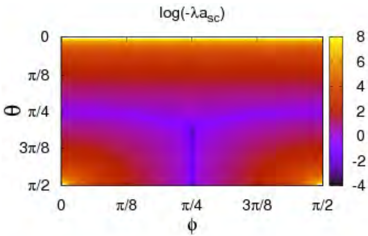
• The two body problem has a bound state for arbitrarily small attractions in presence of λ, however small even in 3 spatial dimensions !

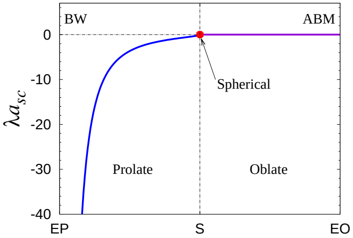
(Spherical GFC)

Generic GFC

- $\lambda a_{sc} = \mathcal{F}(\hat{\boldsymbol{\lambda}})$
- $\mathcal{F}(\hat{\boldsymbol{\lambda}}) \leq 0$
- Gauge fields are therefore, **attraction amplifiers**

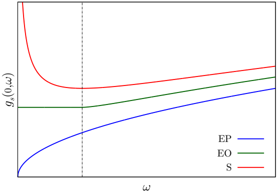






Qualitative understanding – a simple model

- Singlet density of states : $g_s(\boldsymbol{q},\omega) = \frac{1}{V}\sum_{\boldsymbol{k}\,\alpha\,\beta} |A_{\alpha\beta}(\boldsymbol{q},\boldsymbol{k})|^2\,\delta(\omega - \varepsilon_{\alpha\beta}(\boldsymbol{q},\boldsymbol{k}))$
- Actual SDOS



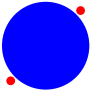
$$g_s(\mathbf{0},\omega) \sim \begin{cases} \sqrt{\omega} & \text{for EP} \\ \lambda \text{ (constant)} & \text{for EO} \\ \frac{1}{\sqrt{\omega}} & \text{for S} \end{cases}$$

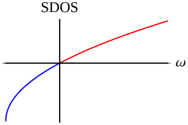
$$g_s(\mathbf{0},\omega) \to \sqrt{\omega} \quad \text{as} \quad \omega \to \infty$$

- Model SDOS (called $\varepsilon_0 - \gamma$ model) with $\varepsilon_0 \sim \lambda^2$

$$g_s(\mathbf{0},\omega) = \begin{cases} \frac{\sqrt{\varepsilon_0}}{4\pi^2}\left(\frac{\omega}{\varepsilon_0}\right)^{\gamma}\Theta(\omega) & \text{if } \omega < \varepsilon_0, \\ \frac{\sqrt{\omega}}{4\pi^2} & \text{if } \omega \geq \varepsilon_0. \end{cases} \Bigg| \implies \sqrt{\varepsilon_0}a_{sc} = \frac{\pi\gamma}{2\gamma - 1}\Theta(\gamma)$$

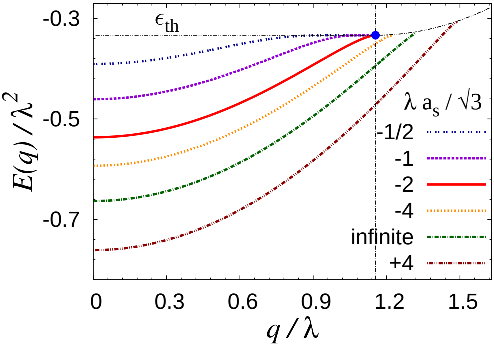
- Gauge fields $\Rightarrow$ Enhanced infrared SDOS $\Rightarrow$ Bound state formation
- Connection: Cooper problem

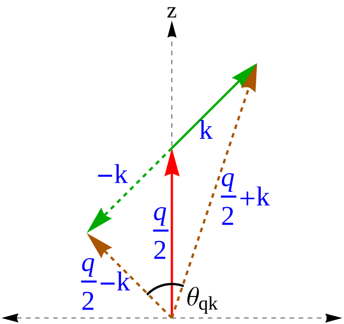




Two body problem at a finite q

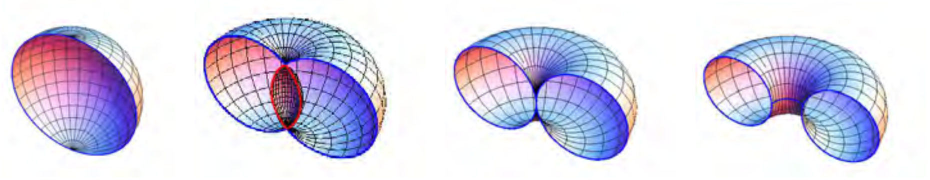
- For any attraction, however large, there exists a critical center of mass momentum, beyond which the bound state dies.





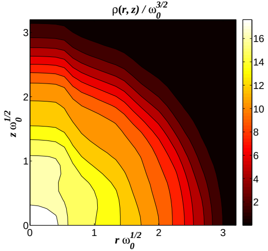
Many body problem: Noninteracting fermions

- The Fermi surface undergoes a change in topology.



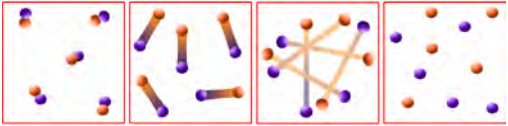
(a) $\lambda = 0$ (b) $0 < \lambda < \lambda_T$ (c) $\lambda = \lambda_T$ (d) $\lambda > \lambda_T$

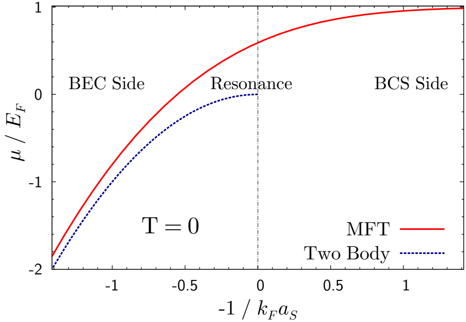
- $+$ a harmonic trap: Characteristic anisotropies



(Extreme Oblate GFC, $\boldsymbol{\lambda} = \frac{\lambda}{\sqrt{2}}(1, 1, 0)$)

BCS-BEC crossover



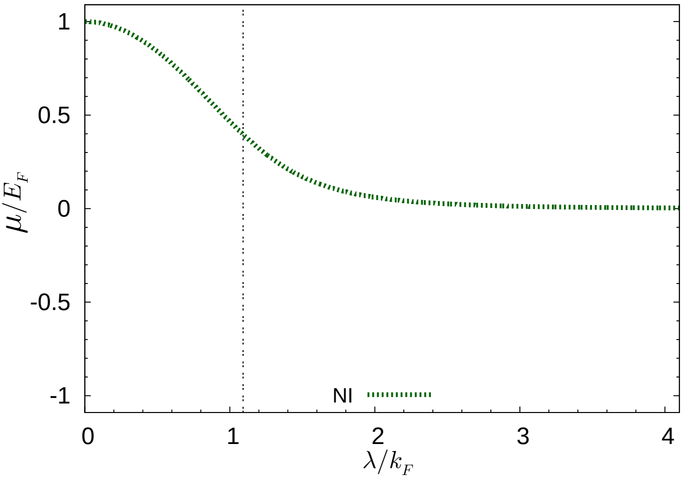


$$\mu \approx E_F \qquad \text{(BCS limit)}$$

$$\mu \approx -\frac{1}{2}\left(\frac{1}{{a_s}^2}\right) + \pi \rho a_s \qquad \text{(BEC limit)}$$

BCS-BEC crossover induced by the gauge field

Spherical GFC, $\boldsymbol{\lambda} = \frac{\lambda}{\sqrt{3}}(1,1,1)$; $\quad k_F a_s = -1/4$



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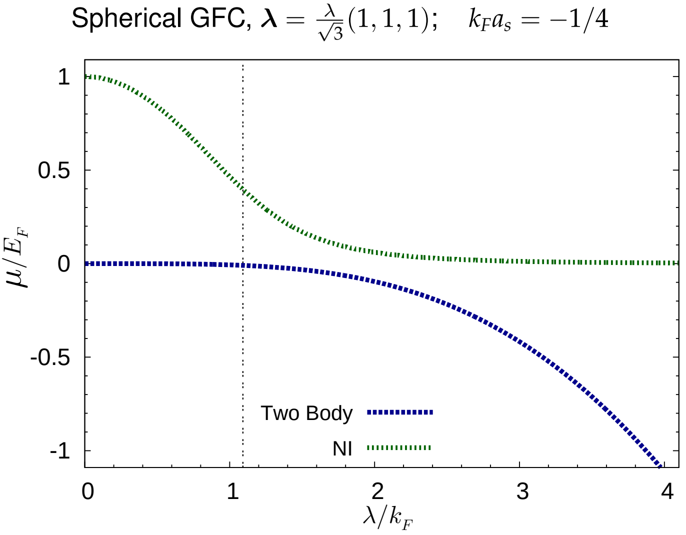

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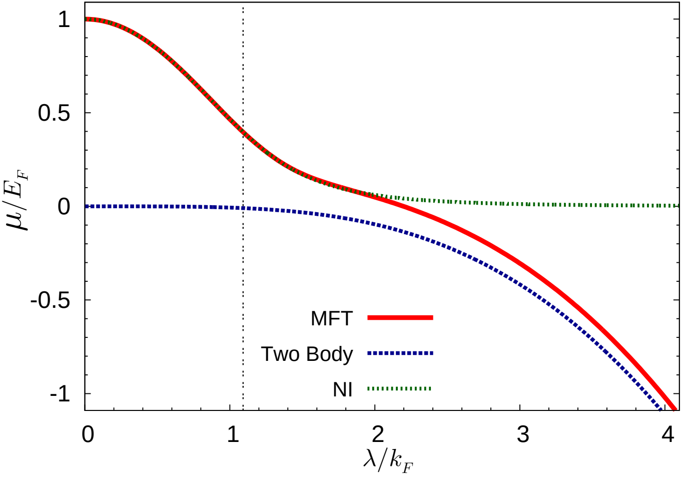

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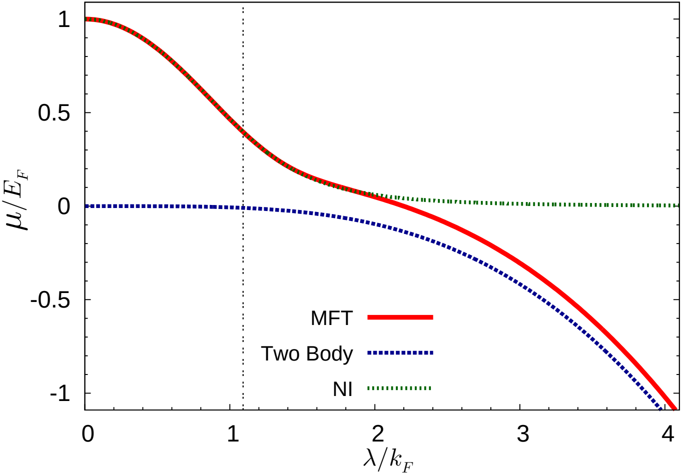
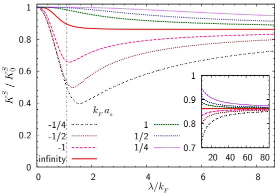


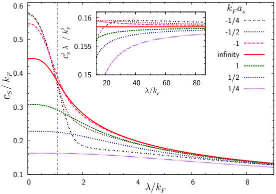
- Gauge field induces BCS-BEC crossover for a fixed (weak) attraction (a_s)!

Gaussian fluctuation theory: Collective excitations

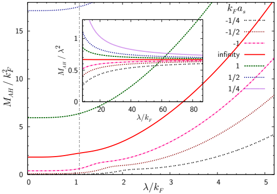
Phase stiffness ($K_s^0 = \frac{\rho}{4m}$)

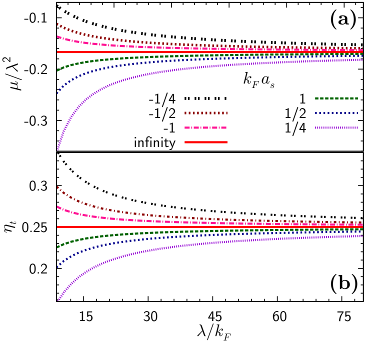


Sound speed



Anderson-Higgs Mass





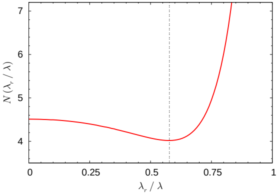
- The BEC obtained at large λ is a condensate of a novel kind of bosons whose properties are determined solely by $\boldsymbol{\lambda}$.
- We call these **Rashbons**.
- Emergent Galilean invariance: $K^s(\lambda \to \infty) = \frac{\rho^R}{m^R}$

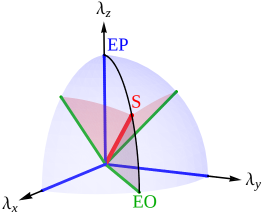
Interaction among rashbons

- From the Bogoliubov theory of bosons (AGD, 1965)

$$\mu_B = \frac{4\pi a_B}{m_B}\rho_B \; ; \qquad c_s^B = \sqrt{\frac{\mu_B}{m_B}} = \sqrt{\frac{4\pi a_B \rho_B}{m_B^2}}$$

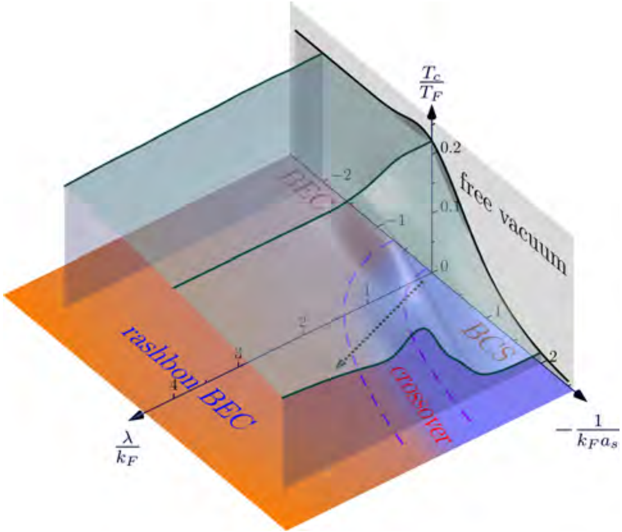
- Can we describe rashbons by a Bogoliubov theory of bosons?
- A positive yes: $c_s{}^2 = \mu^R / m_i{}^R$! ...but with the anisotropic theory
- Importantly, this result allows to extract the rashbon-rashbon scattering length $a^R = \frac{N(\hat{\boldsymbol{\lambda}})}{\lambda}$ – a competitor for Feshbach resonance(?)





- Remarkable state...the interaction between emergent bosons is determined by the parameter λ that enters the kinetic energy of the constituent fermions and independent of the constituent fermion-fermion interaction !

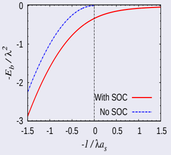
A beyond Gaussian theory: Transition temperature



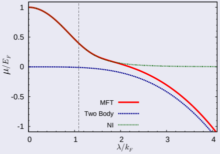
- The transition temperature of a weak superfluid can be enhanced upto order of Fermi temperature by tuning λ even for fixed weak attraction !

Conclusions

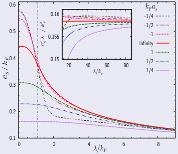
Two-body bound state for any attraction



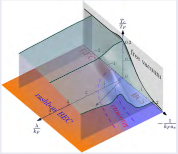
BCS - Rashbon BEC crossover



Novel excitations



A clue to realize high T_c superfluids



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Publications based on this work

1. J. P. Vyasanakere, V. B. Shenoy, **Phys. Rev. B**, 83, 094515.
2. J. P. Vyasanakere, S. Zhang, V. B. Shenoy, **Phys. Rev. B**, 84, 014512.
3. S. K. Ghosh, J. P. Vyasanakere, V. B. Shenoy, **Phys. Rev. A**, 84, 053629.
4. J. P. Vyasanakere, V. B. Shenoy, **New J. Phys.**, 14, 043041.
5. V. B. Shenoy, J. P. Vyasanakere, and S. K. Ghosh, **Current Science**, 103, 525.
6. J. P. Vyasanakere, V. B. Shenoy, **Phys. Rev. A**, 86, 053617.
7. V. B. Shenoy and J. P. Vyasanakere, **J. Phys. B: AMO Physics**, 46, 134009.
8. S. Sanyal, S. K. Ghosh, J. P. Vyasanakere, **J. IISc.**, 94:2,199.
9. J. P. Vyasanakere, V. B. Shenoy, **Phys. Rev. B (Rapid Com.)**, 92, 121111(R).
10. J. P. Vyasanakere, a book chapter in "Synthetic Spin-Orbit Coupling in Cold Atoms" published by **World Scientific**.

THANK YOU