Modelling the Effect of Business Cycles on Ocean Acidification

A Non-Constant Coefficient Lotka Volterra System Approach

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1 Abstract

The aim of this project is to model the effect of business cycles on ocean acidification as a Lotka Volterra competition system with a non-constant coefficient.

The Lotka Volterra system proposed by Mendoza-Mendoza et al. [1] uses constant coefficients to describe competition in their paper on Modeling the Link between Carbon Emissions and Ocean Acidification using a Lotka-Volterra Dynamical System. However, due to the complex dynamic nature of competition, this project aims to replace the constant coefficient that characterizes the effect of carbon emissions on ocean acidity with a non-constant function, modeled after the fluctuation of GDP due to business cycles.

Potential GDP increases linearly due to economic growth, and real GDP oscillates about potential GDP due to business cycles [2]. To capture these cyclical fluctuations, a sinusoidal family of functions was chosen to characterize parameter f, in the form:

$$f(t) = A_f \sin(\omega_f t + \phi_f)$$

Upon data acquisition and normalization in accordance with scenario 10 from Mendoza-Mendoza, our method to determine the non-constant $f(t)$ function was to use a Fast Fourier Transform to denoise data, a fourth-order Runge-Kutta solver to generate a numerical solution, and a Least Squares algorithm to determine the function with the least error compared to the reference paper's dataset. These steps were executed for both the set of parameters (GA and MCMC). With MCMC parameters, the resulting function was

$$f(t) = -1.27 \times 10^{-4} \sin((8.16 \times 10^{-3})t + 9.95 \times 10^{-3})$$

with a 22.3% decrease in percentage error compared to the reference paper, and a reduced χ^2 value of 3.125 compared to reference model's 1.375. With GA parameters, the resulting function was

$$f(t) = -4.08 \times 10^{-4} \sin((9.04 \times 10^{-3})t + 1.01 \times 10^{-2})$$

with a 27.4% decrease in percentage error compared to the reference paper, and a reduced χ^2 value of 6.250 compared to reference model's 2.875.

These results indicate that aragonite concentration exhibits a strong correlation with business cycles, and a non-constant function was effective at describing competition in this system, validating our aim. Due to the pertinence of ocean acidity as a planetary boundary threatened by fossil-fuel driven climate change [3], we believe that improved modeling of emissions and ocean systems can inform better climate policy and action, encouraging the safegaurding of planet Earth.

Contents

2 Introduction

The aim of this project is to formulate a Lotka-Volterra (LV) system with a non-constant coefficient as a more accurate way to describe the interdependence between CO_2 levels and ocean acidification, by incorporating the impact of business cycles on carbon emissions and consequently on ocean acidification.

The Lotka-Volterra model assumes that the coefficients that describe inter-specific competition are constant. However, competition is a complex process affected by several dynamic factors; it cannot be reduced to a static constant in the real world [4]. By characterizing the reciprocal relationality and interdependence of carbon emissions and ocean acidification as a *competition* between the two, a Lotka-Volterra model has been adapted by Mendoza-Mendoza et al. to describe their relationship [1]. We propose that the parameter used to relate carbon emissions with ocean acidification, namely parameter f, can be modified to be a time-dependent function by considering the effect that economic changes, specifically business cycles, have on carbon emissions.

Through a least squares optimization process, the $f(t)$ that had the least percentage error was the sinusoidal function, $-1.27\times10^{-4}\sin((8.16\times10^{-3})t+9.95\times10^{-3})$ along with the MCMC parameters from the reference paper, and $-4.08\times10^{-4}\sin((9.04\times10^{-3})t+1.01\times10^{-2})$ along with the GA parameters from the reference paper. The percentage error of this fitting compared to the constant Mendoza-Mendoza system is 22.3% and 27.4% respectively.

3 Motivation

"Ocean acidification refers to a reduction in the pH of the ocean over an extended period, typically decades or longer, caused primarily by the uptake of CO_2 from the atmosphere"[5]. CO_2 reacts with water to form carbonic acid (H_2CO_3) to release hydrogen atoms, which increases acidity of the ocean, thereby decreasing pH. Ocean pH is currently around 8.1 [6], and has decreased by 0.1 pH units over the 200 years since the industrial revolution [6]. Marine life exists within a narrow range of optimal pH conditions, and disruptions to these conditions can significantly impact the health and survival of aquatic ecosystems.

Thus, ocean acidification increases the risk of biodiversity loss in marine ecosystems and causes the degradation of marine habitats [5][7]. In 2025, the global mean aragonite saturation state was 2.86, exceeding the 'Safe Operating Space' and crossing the Planetary Boundary set at $\Omega_{ar} = 2.85$ [8][3]. Minimizing ocean acidification is a primary objective in the UN Sustainable Development Goal of protecting marine ecosystems (Goal 14.3) [9], providing the motivation for our analysis.

Research on ocean acidification and its effects is an area of great interest in modern science, and increasing our understanding of human impacts on ocean acidification can inform climate action and encourage sustainable development [8].

4 Mathematical Model and Theoretical Foundation

4.1 Reference System of Equations

Business cycles are the periodic intervals of expansion and recession in an economy. Because of the fossil-fuel dependence of modern economies, the fossil-fuel based CO_2 emissions that cause ocean acidification "move together over the business cycle" [10]. Hence, business cycles, CO_2 emissions and ocean acidification are deeply interlinked.

The Lotka-Volterra model describes the relationship between predator and prey - here, these actors are substituted with CO_2 and ocean acidity levels, and their interdependency is analyzed. The reference Lotka Volterra equation is:

$$\frac{dx}{dt} = r_1 x - \alpha x^2 - \beta xy \qquad (1)$$

$$\frac{dy}{dt} = r_2 y - \gamma xy - \delta y^2 \qquad (2)$$

These equations were adapted for carbon emissions and ocean acidification by Mendoza-Mendoza et al.[1] as:

$$\frac{dCO_2}{dt} = aCO_2 + b(CO_2)^2 \quad (3)$$

$$\frac{d\Omega_{ar}}{dt} = -d\Omega_{ar} + e(\Omega_{ar})^2 - f\Omega_{ar}CO_2 + g\sin(\omega_A t + \Phi) \quad (4)$$

where ocean acidity is characterized using the aragonite saturation state (Ω_{ar}), and the $g\sin(\omega_A t + \Phi)$ term is an oscillatory component in the aragonite equation, to account for oscillations in ocean temperature [3].

Symbol	reference LV	Symbol	Modified LV
x	Prey population density	CO_2	Total CO_2 emissions
y	Predator population density	Ω_{ar}	Aragonite saturation in the ocean
$\frac{dx}{dt}$	Instantaneous prey growth rate	$\frac{dCO_2}{dt}$	Rate of carbon emission
$\frac{dy}{dt}$	Instantaneous predator growth rate	$\frac{d\Omega_{ar}}{dt}$	Rate of change of aragonite concentration
r_1	Maximum prey growth rate per capita	a	CO_2 injection rate
r_2	Predator death rate per capita	d	Rate of aragonite saturation
t	Time	t	Time
α	Individual carrying capacity	b	Stabilizing non-linear terms
β	Effect of predator presence on prey	c	Aragonite effect on CO_2 emissions (omitted)
γ	Effect of prey presence on predator	f	Effect of CO_2 on Ω_{ar}
δ	Individual carrying capacity	e	Stabilizing non-linear terms

Table 1: Explanations of LV variables and coefficients. All parameters are positive and real.

4.2 Modified System of Equations

To incorporate the effect of business cycles on CO_2 emissions, the function $f(t)$ must be a non-constant function.

The carbon emissions in this system are dependent on GDP. Potential GDP (maximum output considering a nation's resources at full employment) displays linearly increasing behavior due to economic growth. Real GDP (a nation's actual output, the total market value of goods and services) oscillates about potential GDP in a sinusoidal manner due to business cycles [2]. As a result, the function $f(t)$ must exhibit oscillatory behavior, so the chosen family of functions was $f(t) = A_f \sin(\omega_f t + \phi_f)$. Here, ω represents the angular frequency of business cycles [yr^{-1}], and ϕ represents the phase shift of the sine function [rad]. All other parameters are kept identical to those used by Mendoza-Mendoza.

5 Methodology

This project is divided into three phases: data acquisition and processing, model simulation, and data visualization.

5.1 Data Acquisition and Processing

Mendoza-Mendoza propose eighteen scenarios of CO_2 emission and aragonite saturation, determined with the Markov Chain Monte Carlo (MCMC) method, and the Genetic Algorithm (GA) method. Each scenario featured CO_2 emissions from two different data sources for fossil-fuel and land-use emissions, as well as a different source of aragonite saturation. In accordance with data availability, the scenario chosen in this project is scenario 10 - EDGAR-OSCAR-ALOHA.

a$\times 10^{-2}$	b$\times 10^{-5}$	d$\times 10^{-4}$	e$\times 10^{-4}$	f$\times 10^{-11}$	g	ϕ	$CO_2(0)$	$\Omega_{ar}(0)$
14.28837	4.89799	2.36865	6.41611	5.78787	0.48753	-1986.9587	27.07203	3.77466

Table 2: Parameters for scenario 10 (EDGAR-OSCAR-ALOHA, using MCMC) from Mendoza-Mendoza. In this project, the constant f is replaced with a non-constant function.

EDGAR (the Emissions Database for Global Atmospheric Research) was the source for fossil-fuel emissions (Gt CO_2) [11], and OSCAR (the Online Solution for Carbon Analysis and Reporting) was the source for land-use emissions (Gt CO_2) [12]. Aragonite concentration was determined from ocean data from Station ALOHA in the North Pacific Subtropical Gyre (NPSG) [13].

Carbon emissions data from both sources ranged from 1990 to 2022, identical to the range used by Mendoza-Mendoza. The annual emission value is the normalized summation of that of each country included in the data set. To calculate aragonite saturation, we and Mendoza applied CO2SYS to the ALOHA dataset. CO2SYS is a group of software programs that calculate chemical equilibria for aquatic inorganic carbon species and parameters. We used PyCO2SYS [14], a python toolbox to accomplish this, as explained in appendix section 10.1.

5.2 Model Simulation and Data Visualization

To generate both a replica of Mendoza-Mendoza's solution, as well as our final solution, a fourth-order Runge-Kutta (RK-4) solver was created in Python. The parameters from Mendoza-Mendoza (Table 2) and the data we acquired were used to generate the plots discussed in this project. The global truncation error of RK-4 is $\mathcal{O}(h^4)$. The RK-4 solver was implemented with a step size of 6.250 years, which gives a global error on the order of 4×10^{-3}, which is appropriate for the resolution of our project. To obtain a smooth output similar to the reference paper, the acquired data was denoised using a Fast Fourier Transform (FFT) using Python, shown in appendix Figure 6.

The Least Squares Optimization algorithm was utilized to find the coefficients of the non-constant $f(t)$ function that produce a model with the least error compared to the reference paper's dataset. Convergence for the parameters was achieved within a tolerance of 1×10^{-8}, verifying that the parameter vector had stabilized. This resulted in a root mean squared error of 14% and 11% for MCMC and GA parameters, demonstrating a statistically good fit when compared with the scale of the data.

6 Results

In this section, we present the plots that compare the Mendoza-Mendoza LV system with constant coefficients, as well as our system with a non-constant f function. The graph shows Ω_{ar} for every year between 1990 and 2022, along with their respective percentage errors.

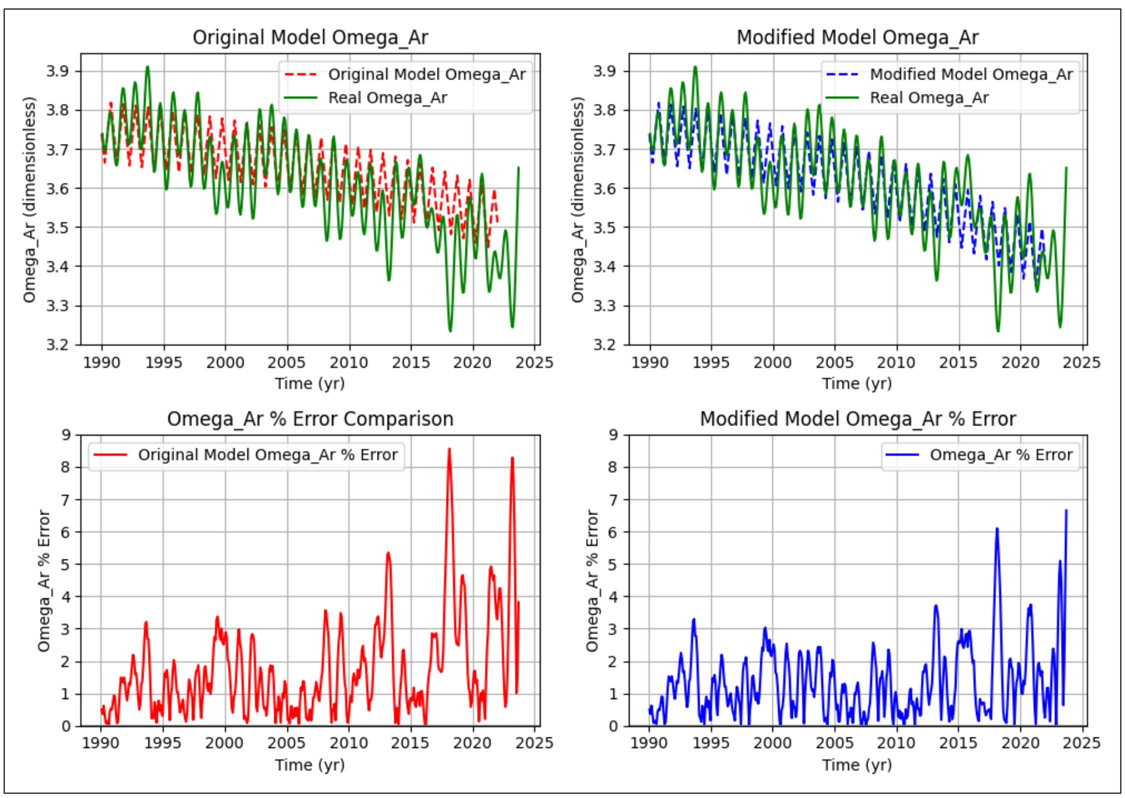

Figure 1: A comparison plot of the aragonite saturation fitting of the reference model (left, red) with the MCMC parameters and the modified non-constant coefficient model (right, blue). The maximum percentage error decreases from 8.5% to 7.6%.

The plots for the original system with MCMC 3 and GA parameters 4 and the comparison plot between original and modified for the GA parameters 5 are listed in the appendix (10.3).

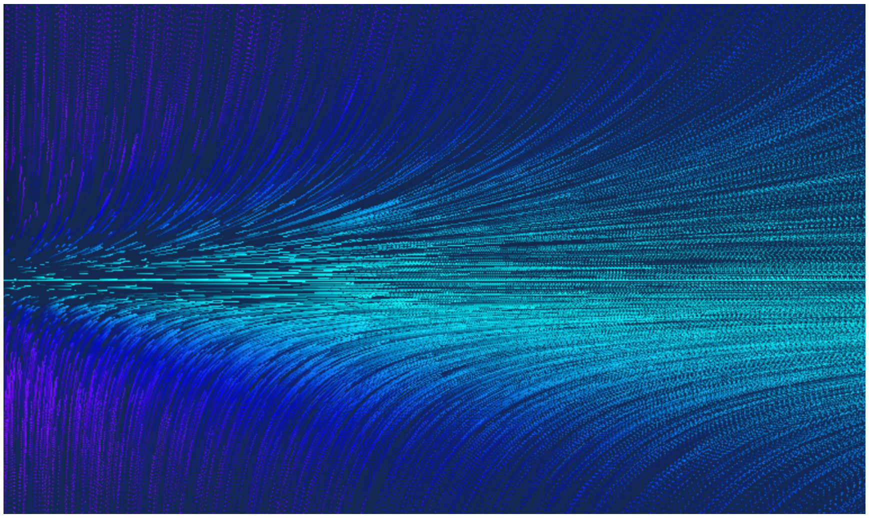

Figure 2: A snapshot of the dynamic vector field of our modified Lotka Volterra system for MCMC (simulation in section 10.4). The x-axis is CO_2 emissions, and the y-axis is Ω_{ar}. The vector arrows represent $(\dot{x}, \dot{y}) = (\dot{CO_2}, \dot{\Omega_{ar}})$.

7 Discussion

7.1 Findings

The reduced χ^2 Goodness-of-fit values increased from 1.375 to 3.125 for MCMC, and from 2.875 to 6.250 for AG. The methodology of reduced χ^2 calculation and critical values, are explained in Appendix section 10.5. The decrease in percentage error between the reference system and our modified system is 22.3% for MCMC and 27.4% for GA. A time-varying phase plot of our modified system is plotted on Field Play, a link to which is listed in the appendix.

In the analysis of the dynamic vector phase plot, two semi-stable equilibria are seen at $\Omega_{ar} = 0$, and $CO_2 = 0$, demonstrating that the coupled rate of change falls to zero when either species is zero. At small CO_2 and Ω_{ar} values (i.e., close to the origin), trajectories move rapidly in favor of $(\dot{\Omega_{ar}})$, but as both variables increase, trajectories stabilize, and vector field becomes smoothly aligned, with flow following consistent curved paths. This validates the model's characterization of the sensitivity of aragonite saturation to changes in carbon emissions, especially at low aragonite concentration values.

The above-critical reduced χ^2 value confirms that business cycles have a strong influence on aragonite saturation, and when coupled with the ecological interpretation of the phase plot, our results strongly suggest that having a non-constant function improves this LV system.

7.2 Limitations

Mendoza-Mendoza chose to compare the aragonite saturation calculated at the coast of Hawaii with carbon emissions from all the countries in the world. While the North Pacific subtropical Gyre is the largest ocean ecosystem in the world [15], it covers only the Pacific ocean, which may not be representative of the true effect of carbon emissions on the acidity of all the world's water bodies.

Furthermore, only one of eighteen combinations of emissions-aragonite saturation data sets was tested with our modified system. The same non-constant function may not be the function with least error for all combinations, since they have been reported to have different constant coefficients.

7.3 Next Steps

To take this project further, the modified LV model can be tested against the other datasets used by Mendoza-Mendoza (CEDS, GCP and H&C) in the other seventeen combinations.

Currently, only f was modified to be a non-constant coefficient. To improve the accuracy of LV fitting, the other competition coefficients can also be made into time-varying functions depending on the ecological factors that shape them. Furthermore, only the parameters in $f(t)$ were fit using Least Squares; to increase fit accuracy, all the coefficients can be fit instead of being lifted from the reference system. Finally, the effect of world-wide carbon emissions on aragonite saturation from the other four major oceanic gyres can be analyzed to get an impression of the holistic effect of business cycles on ocean acidification.

8 Conclusion

The aim of this project was to formulate a non-constant function to replace the constant that characterizes the effect of business cycles on ocean acidity through a Lotka Volterra system.

Through a process of data acquisition and processing, model simulation and data visualization, both a replica of original LV model proposed by Mendoza-Mendoza et al., and a modified model were created. A Fast Fourier Transform was used to denoise data, a fourth-order Runge-Kutta solver was created to generate a numerical solution.

Due to the cyclic fluctations of real GDP about potential GDP due to business cycles, a base function in the form $f(t) = A_f \sin(\omega_f t + \phi_f)$ was chosen. The fit parameters for this new function were calculated using Least Squares Optimization algorithm, such that the percentage error between the fitting and data set was the smallest.

This process was repeated for both the MCMC and the GA coefficients, resulting in the functions respectively: $f(t) = -1.27 \times 10^{-4} \sin((8.16 \times 10^{-3})t + 9.95 \times 10^{-3})$ and $f(t) = -4.08 \times 10^{-4} \sin((9.04 \times 10^{-3})t + 1.01 \times 10^{-2})$. The modified models had 22.3% and 27.4% decrease in percentage error, compared to the reference paper, and increase in reduced χ^2 values of 3.125 and 1.375 respectively. The above-critical reduced χ^2 value, along with the physical interpretations of the resulting phase plots indicate a strong correlation between business cycles and ocean acidity, confirming our proposal.

The next steps of this project are to test the modified model with the other data sets used in Mendoza-Mendoza, and fit all the parameters, not just the coefficients of the added $f(t)$ function. To further improve the model, the other competition coefficients can also be replaced with time-dependent functions to model the impact of other ecological factors influencing competition. Finally, to improve the scope of the analysis, several ocean acidity datasets can be incorporated to get a holistic view of the effect of business cycles on ocean acidity.

9 References

[1] L. F. Mendoza-Mendoza, I. Álvarez Ríos, F. Z. León-Altamirano, and F. S. Guzmán-Murillo, "Modeling the link between carbon emissions and ocean acidification using a lotka-volterra dynamical system," *Revista Mexicana de Física*, vol. 71, p. 051401 1–, Sep. 2025.

[2] L. R. Weinstock, "Introduction to u.s. economy: The business cycle and growth."

[3] H. S. Findlay, R. A. Feely, L.-Q. Jiang, G. Pelletier, and N. Bednaršek, "Ocean acidification: Another planetary boundary crossed," *Global Change Biology*, vol. 31, no. 6, p. e70238, 2025. e70238 GCB-24-3918.R1.

[4] A. Marasco, A. Picucci, and A. Romano, "Market share dynamics using lotka–volterra models," *Technological Forecasting and Social Change*, vol. 105, pp. 49–62, 2016.

[5] E. by Jean-Pierre Gattuso and L. Hansson, *Ocean Acidification*. Oxford University Press, 2011.

[6] N. Oceanic and A. Administration, "Ocean acidification," 2025.

[7] C. E. Cornwall, S. Comeau, N. A. Kornder, C. T. Perry, R. van Hooidonk, T. M. DeCarlo, M. S. Pratchett, K. D. Anderson, N. Browne, R. Carpenter, G. Diaz-Pulido, J. P. D'Olivo, S. S. Doo, J. Figueiredo, S. A. V. Fortunato, E. Kennedy, C. A. Lantz, M. T. McCulloch, M. González-Rivero, V. Schoepf, S. G. Smithers, and R. J. Lowe, "Global declines in coral reef calcium carbonate production under ocean acidification and warming," *Proceedings of the National Academy of Sciences*, vol. 118, no. 21, p. e2015265118, 2021.

[8] B. Sakschewski, L. Caesar, L. Andersen, M. Bechthold, L. Bergfeld, A. Beusen, M. Billing, B. Bodirsky, S. Botsyun, D. Dennis, J. Donges, X. Dou, A. Eriksson, I. Fetzer, D. Gerten, T. Häyhä, S. Hebden, T. Heckmann, A. Heilemann, and J. Rockström, "Planetary health check 2025 a scientific assessment of the state of the planet," Sep. 2025.

[9] United Nations, "Transforming our world: The 2030 agenda for sustainable development," 2015.

[10] B. Doda, "Evidence on business cycles and co2 emissions," *Journal of Macroeconomics*, vol. 40, pp. 214–227, 2014.

[11] E. Comission, "Ghg emissions of all world countries," 2023.

[12] "Historical co2 emissions from land use and land cover change and their uncertainty," 2020.

[13] U. S. D. Scripps Institution of Oceanography, "Scripps co2 program seawater carbon data."

[14] S. A. J. S. D. E. G. L. P. D. v. H. S. M. A. C. L. E. R. Humphreys, M. P. and D. W. R. Wallace, "Pyco2sys: marine carbonate system calculations in python."

[15] M. J. C. David M. Karl, "Ecosystem structure and dynamics in the north pacific subtropical gyre: New views of an old ocean."

10 Appendix

10.1 Information about PyCO2SYS

PyCO2SYS is the python toolbox that performs calculations in the marine carbonate system using seawater properties to determine aragonite saturation. [14]. The input is a csv file with depth, salinity, temperature, average dissolved inorganic carbon (AVGDIC), and average alkalinity (AVGALK). This data is collected by the Hawaii Ocean Time-Series Station ALOHA (HOT). The database contains time-series data for inorganic carbon chemistry from surface seawater samples. The resulting data set is normalized and averaged in accordance with Mendoza-Mendoza, and matched to the associated year and carbon emission.

10.2 GA Reference Coefficients

a$\times 10^{-2}$	b$\times 10^{-5}$	d$\times 10^{-4}$	e$\times 10^{-4}$	f$\times 10^{-11}$	g	ϕ	$CO_2(0)$	$\Omega_{ar}(0)$
1.3652	1.4411	3.49188	2.46133	2.3697	-0.1535	-1988.33277	26.56956	3.79583

Table 3: Parameters for scenario 10 (EDGAR-OSCAR-ALOHA, using GA) from Mendoza-Mendoza, applied to the RK-4 solver. Function f is replaced with a non-constant function.

10.3 Plots

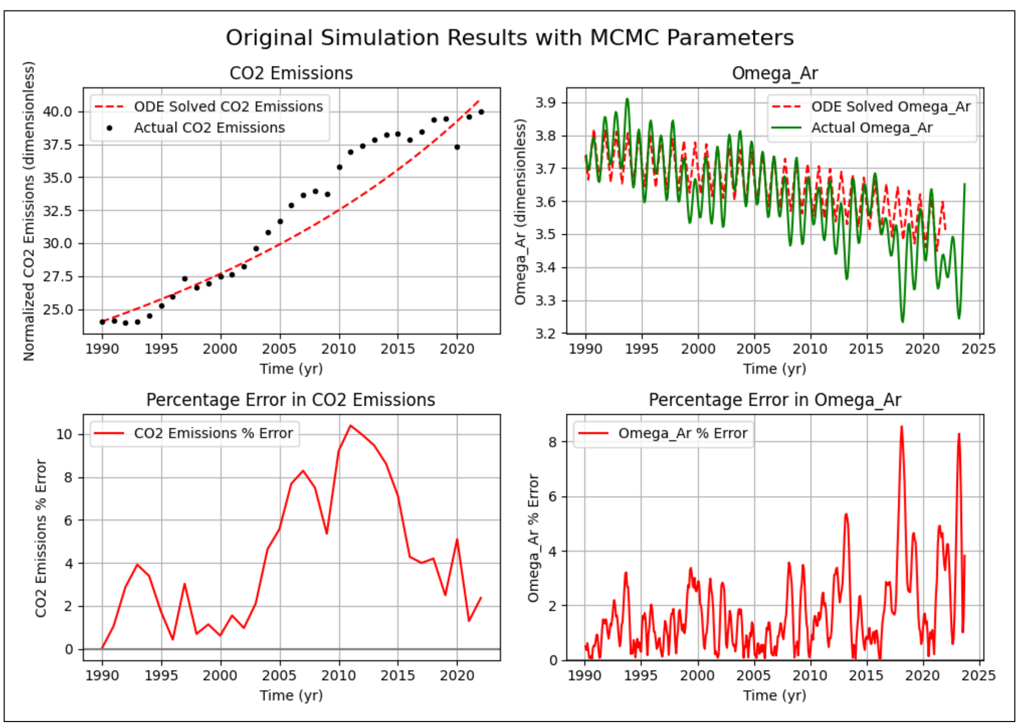

Figure 3: Original Model Numerical Solution with the MCMC parameters found by the reference paper. The error in the CO_2 emissions and Ω_{ar} fitting is 10.4% and 8.6%.

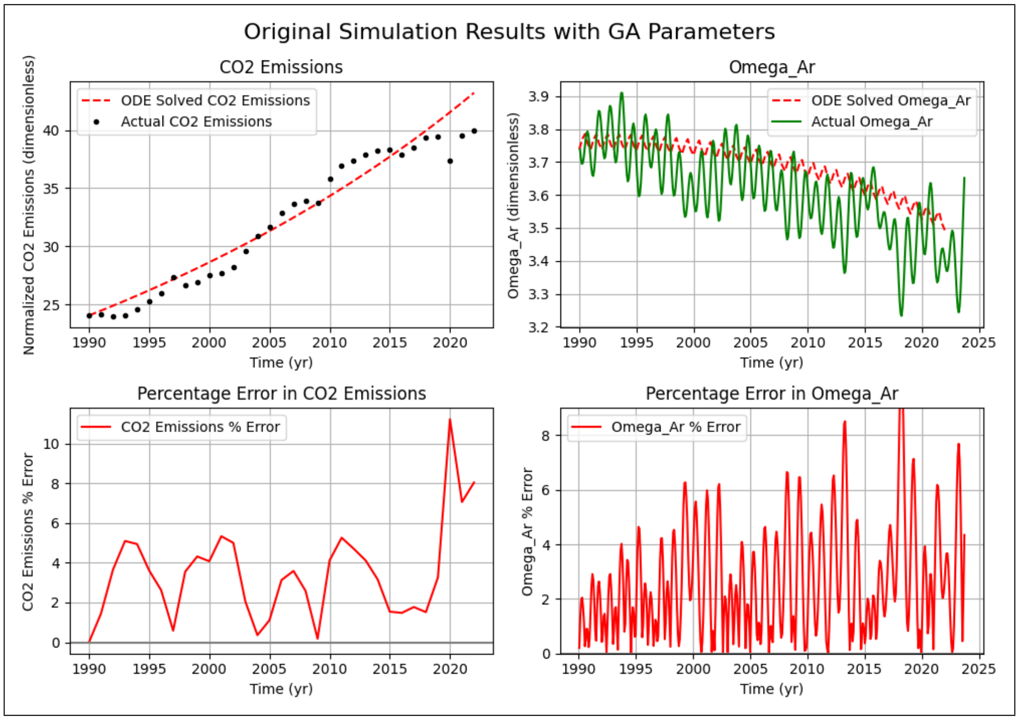

Figure 4: Original Model Numerical Solution with the GA parameters found by the reference paper. The error in the CO_2 emissions and Ω_{ar} fitting is 11.2% and 10.8%.

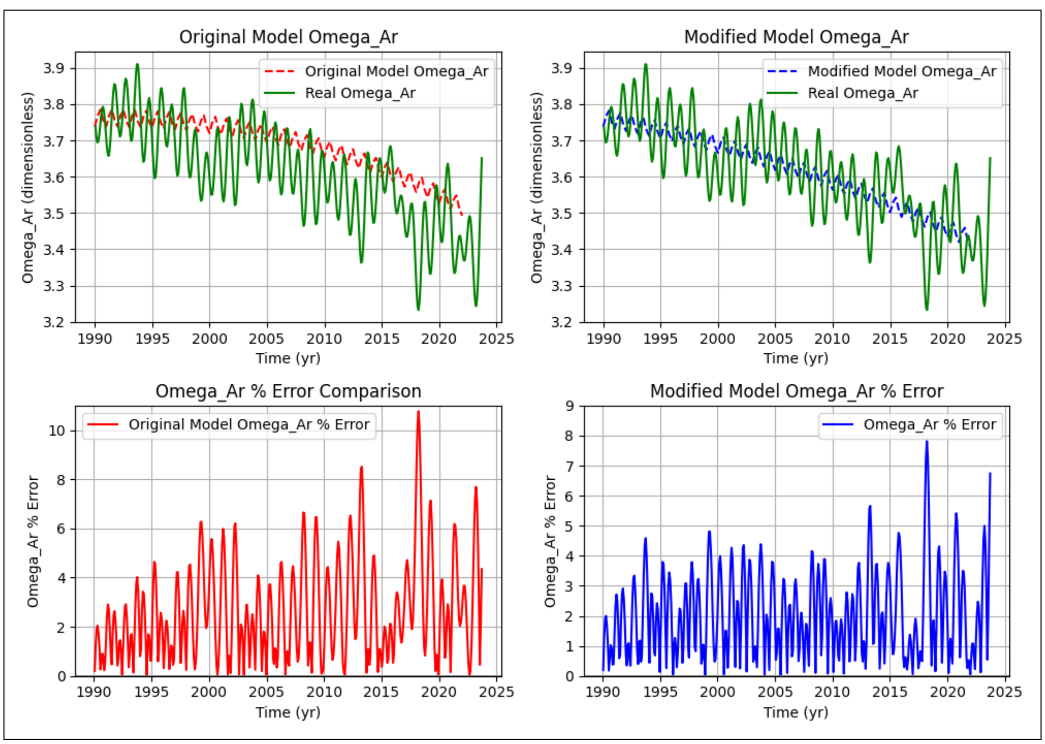

Figure 5: A comparison plot of Ω_{ar} with coefficients computed by the GA method for the reference model (right, red) and modified model (left, blue). The maximum percentage error decreases from 10.8% to 7.8%.

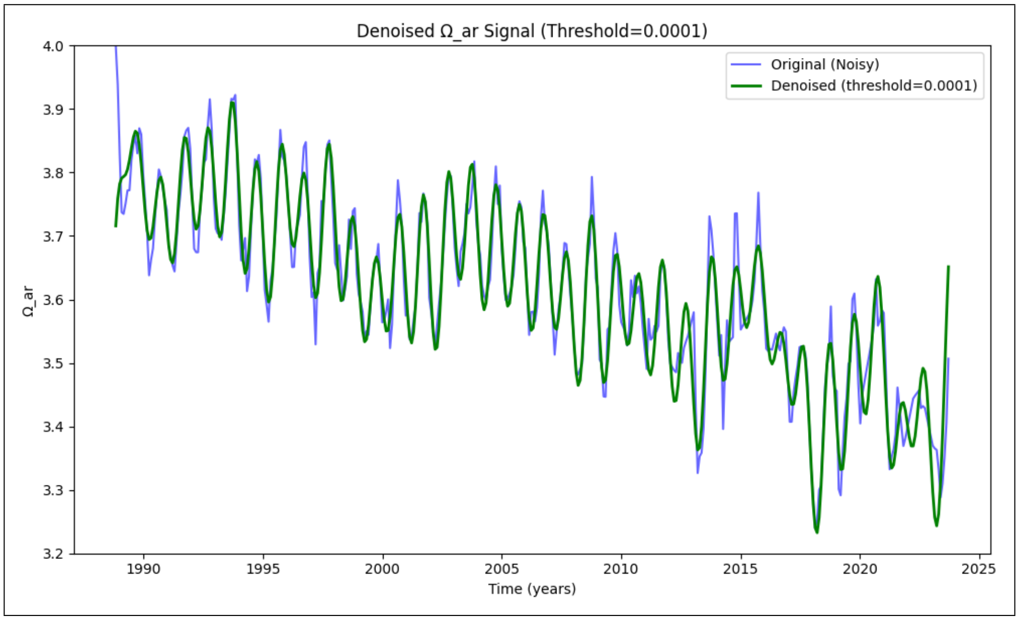

Figure 6: The aragonite concentration after Fast Fourier Transform denoising has been applied.

10.4 Field Play

The links to view the dynamic phase plots of the modified LV systems for MCMC and GA are listed here.

10.5 χ^2 Squared Analysis

This project uses reduced χ^2 squared values to determine goodness-of-fit. Reduced χ^2 squared values are determined by dividing the original χ^2 squared values by the relevant degrees of freedom.

With a P-value of 0.95 (95% confidence) and 8 degrees of freedom for the reference model and 10 for the modified model (due to the 8 variable parameters in both system), the critical reduced χ^2 squared value is 0.342.

Hence, values above the critical value show a correlation between the dataset and the fitting, and the larger the reduced χ^2 squared value, the stronger the correlation.