Charge to Mass Ratio for the Electron

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Abstract

This experiment was conducted to empirically determine the specific charge of an electron, by setting up a magnetic field parallel to a beam of electrons, forming a circular electron charge orbit. Current and voltage are varied, and the radius of the resulting charge orbit is measured using an illuminated scale. The data of current, voltage and radius are applied to a linear fit to determine the charge per unit mass of the electron.

The value of specific charge determined by our calculations for the first dataset is

$$e/m = 2.21 \times 10^{11} \pm 0.01 \times 10^{11} C/kg$$

And for the second dataset is

$$e/m = 2.3 \times 10^{11} \pm 0.2 \times 10^{11} C/kg$$

The percentage difference between our value and the published value of specific charge is 25.6% for the first dataset, and 27.7% for the second data set.

The average is $e/m = 2.3 \times 10^{11} \pm 0.2 \times 10^{11} C/kg$ (27.7% error). The additional parameter of magnetic field due to the earth, buildings and other laboratory equipment is calculated to be $B_e = 0.00004 \pm 0.00001\, T$.

1 Introduction

The aim of this experiment is to determine the *specific charge* (charge per mass ratio) of an electron. First measured in a similar experiment by JJ Thomspon, this experiment is conducted using a Helmholtz coil set-up. The effects of varying current and voltage on the radius of a charge orbit are measured, and the resulting data sets are applied to a linear fit model to determine the charge per unit mass of the electron.

The governing equation of this experiment is:

$$\frac{1}{r} = \sqrt{\frac{e}{m}} k \left(\frac{I + \frac{1}{\sqrt{2}} I_0}{\sqrt{\Delta V}}\right) \quad (1)$$

where r is the radius of the charge orbit, I is the measured current, ΔV is the measured voltage, $\sqrt{e/m}$ is the charge per mass ratio, k is the characteristic of the coil dimensions, $I_0 = B_e/k$, the constant proportional to the external magnetic field B_e where $\vec{B}_e = \vec{B} - \vec{B}_c$. This governing equation is derived from the following two equations.

$$evB = m\frac{v^2}{r} \text{ and } e\Delta V = \frac{1}{2}mv^2 \quad (2),(3)$$

The characteristic of the coil dimensions, k is found through:

$$k = \frac{1}{\sqrt{2}} \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n}{R} \quad (4)$$

where n is the number of turns in each coil and R is the radius of the Helmholtz Coils.

The constant proportional to the external magnetic field, I_o is found through: $I_o = B_e/k$ (5)

where the external magnetic field, $\vec{B}_e = \vec{B} - \vec{B}_c$ with $\vec{B}$ as the total axial magnetic field in electron beam region and $\vec{B}_c = (\frac{4}{5})^{3/2} \frac{\mu_0 n I}{R}$ as the magnetic field from the coils.

The sources of error in our experiment, as well as the measures taken to reduce their effects are discussed, in addition with the effects of external factors.

2 Materials and Methods

2.1 Materials

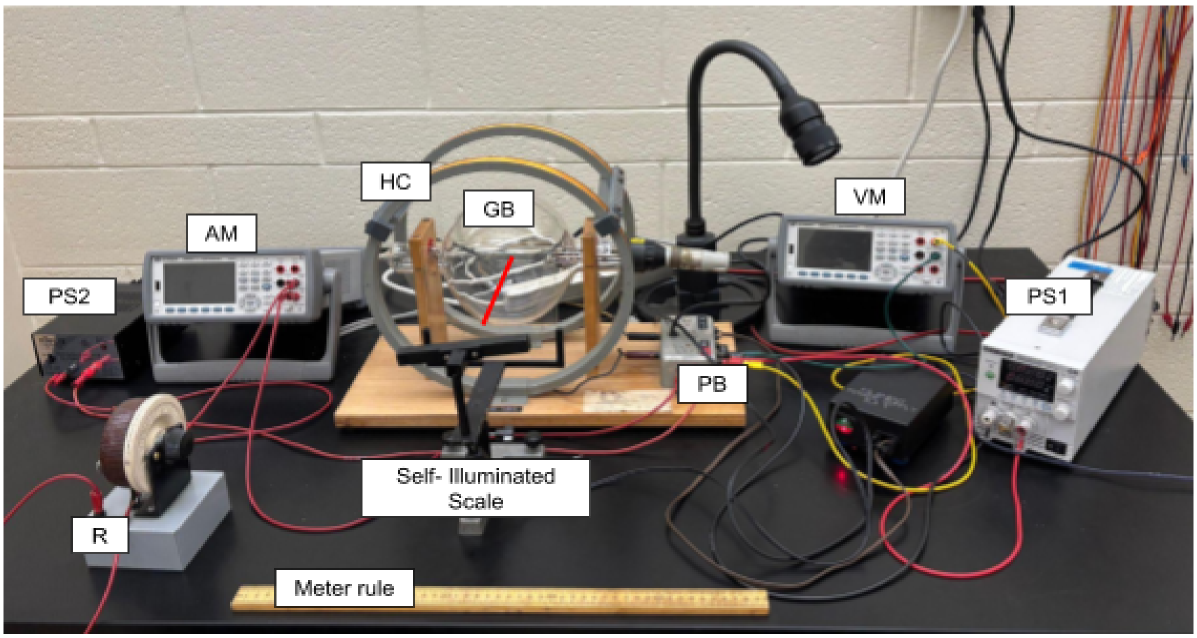

Figure 1: Experimental apparatus to measure specific charge of an electron. The coil axis is indicated with a red line in GB.

The apparatus to conduct this experiment is shown in figure 1.

The components of this setup are:

1. Helmholtz coils (HC, with a radius, $R = 15.1 \pm 0.1\ cm$)
2. Glass bulb (GB)
3. Electron gun (EG)
4. Rheostat (R)
5. Power box (PB, set at an anode voltage of 6.3V DC.)
6. Power sources (PS1 varying between 0 and 300V, PS2 set at 13.8V DC as described on the box)
7. Multimeters (VM - used as a voltmeter, AM - used as an ammeter)
   The multimeters had a least count of 0.00001 V or A. However, due to continuous fluctuations in the last two digits, only measurements up to 0.001 V or A were recorded. Hence, the uncertainty of the multimeters is taken to be 0.0005 V or A.
8. Meter rule
9. Self-illuminating scale with a plastic reflector

2.2 Methods

The two Helmholtz coils (HC), powered by PS2, were used to create a magnetic field along the coil axis of GB. The electron gun, powered by PS1, generated an electron beam directed into the glass bulb. The magnetic field distorted the beam, making it a helical shape; this shape was manipulated into a circle by rotating the electron gun. VM, set as a voltmeter, was connected in parallel with PS1, and AM, set as an ammeter, was connected in series with PS2. The rheostat (R) was connected in series with PS2. The order of powering-on the circuit is (1) turn on PB (2) after 30 seconds, turn on PS1 (3) turn on PS2. This order ensured the filament in EG had time to heat up and the tube was not damaged. The reverse order was followed when powering-off the circuit.

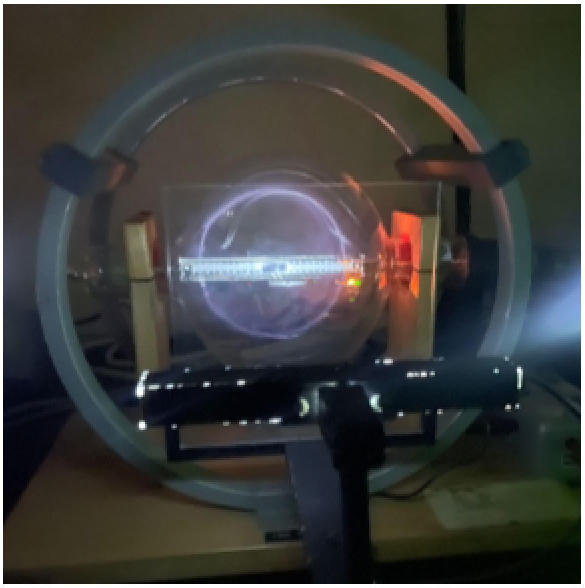

Figure 2: The charge orbit produced by following the procedure above. The illuminated scale reflects off the plastic reflector and is used to measure radius.

In this experiment, two sets of data were recorded to determine the charge per unit mass ratio of an electron. The data collected was for the diameter of the charge orbit. This was measured using the self-illuminated scale with the plastic reflector. The distance from HC to the scale was kept at a constant distance using the meter rule.

For the first set, current was varied while keeping voltage constant. The variation of current is achieved by changing the resistance using the rheostat (R), reflected in AM. The effect of varying current was to vary the strength of the magnetic fields generated by the Helmholtz coils.

For the second set, voltage was varied while keeping current constant. The variation of voltage is achieved by changing the input voltage on PS1, reflected in VM. The effect of varying voltage was to vary the amount of kinetic energy in the electrons.

During each set, quick data collection was done to avoid charge accumulation in VM. Between the two data collection rounds, power sources were turned off, to allow any charge accumulation in the VM to discharge.

3 Data and Analysis

The effect of axial field was examined for off-axis distances ρ, lying in the range $0.2R \leq \rho \leq 0.5R$, using the equation (Calculation in Appendix 2)

$$\frac{B(\rho)}{B(0)} = 1 - \frac{\rho^4}{R^4 (0.6583+0.29 \frac{\rho^2}{R^2})^2} \tag{6}$$

From this, the range of correctional factors, dependent on I, is derived. The factors range from 0.97 to 0.99, as seen in Table 3 (Appendix 1). The corresponding factor is applied to B_c and I for the collected data.

To determine the value of specific charge, the following process is followed. The formula below is applied.

$$B = B_c + B_e = \sqrt{\frac{2m}{e}\Delta V}\frac{1}{r}, \text{ where}$$

$$B_c = \alpha\frac{1}{r} - B_e \text{ and } \alpha = \sqrt{\frac{2m}{e}\Delta V} \tag{7}$$

B_c was calculated using the formula $B_c = (\frac{4}{5})^{\frac{3}{2}} \frac{\mu_0 n I}{r}$ for every known current (I) value (Appendix 2). B_c versus $1/r$ is plotted (Figure 3), where B_e is the y-intercept. B_e was found to be $0.00004 \pm 0.00001\,T$.

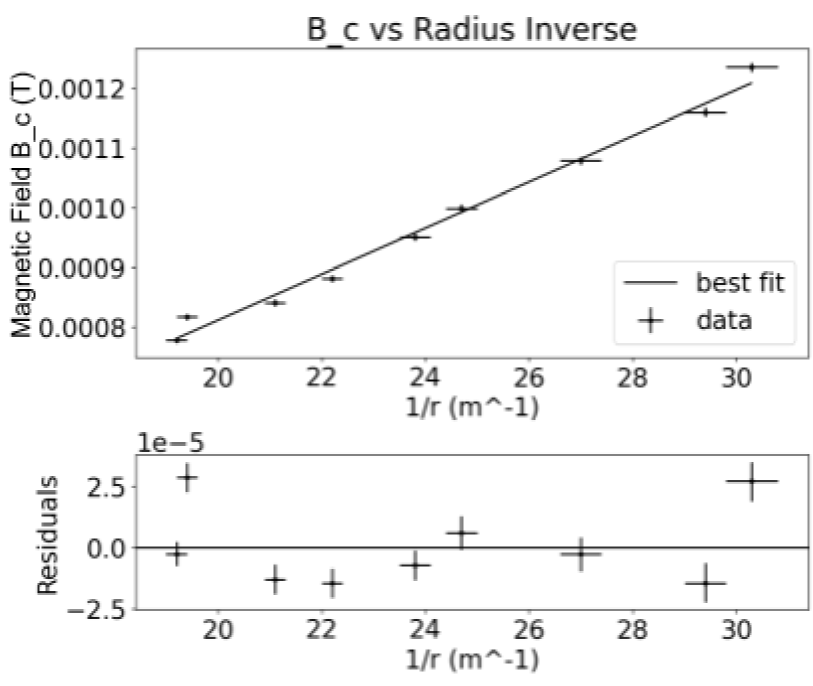

Figure 3: Variation of B_c as a function of $1/r$. Vertical uncertainties are too small to be visible. Fitting the data to equation 7 gives $\alpha = 0.0000386 \pm 0.0000006\,Tm$ and $B_e = 0.00004 \pm 0.00001\,T$. From the Residual plot, most data points are within 2 error bars. Hence, this is a good fit.

The Chi-squared goodness of fit value for Figure 3 is 2.359. With 8 degrees of freedom, and a P value of 0.99, the critical value is 1.647. Since our Chi-squared value is larger than this value, there is no significant difference between our expected and observed values of B_c for this calculation.

This is a significant B_e value as it is 4 times its uncertainty, and hence, needs to be accounted for in equation (1) through I_0 like below:

$$I_0 = \frac{B_e}{k} = 0.04 \pm 0.02\,A,$$

$$\text{where } k = \frac{1}{\sqrt{2}}\left(\frac{4}{5}\right)^{\frac{3}{2}}\frac{\mu_0 n}{R} = 0.0005474 \pm 0.0000036\,T/A\,.$$

Now, calculating the specific charge. In the first half, the value of ΔV is kept constant while I is varied (Appendix 1, Table 1), and the graph for $\frac{1}{r}$ by $(I + I_0/\sqrt{2})$ is plotted (Figure 4). From this, the charge per mass ratio value determined is $e/m = 2.21 \times 10^{11} \pm 0.01 \times 10^{11}\,C/kg$ (Appendix 2).

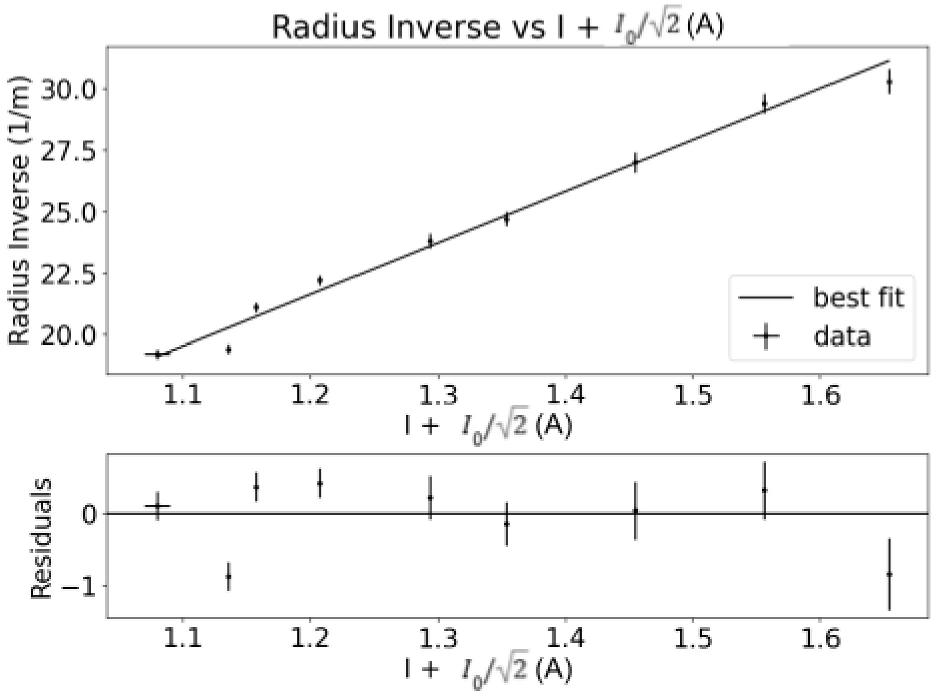

Figure 4: The graph for for holding Variation of $1/r$ *as a function of* $I + I_0/\sqrt{2}$. *Vertical uncertainties are too small to be visible. The fitting gives a gradient value of* $21.0 \pm 0.6\,(mA)^{-1}$ *and a y-intercept of* $-3.6 \pm 0.7\,m^{-1}$. *The specific charge determined from this is* $e/m = 2.21 \times 10^{11} \pm 0.01 \times 10^{11}\,C/kg$ *(Appendix 2). From the Residual plot, all data points are within 2 error bars. Hence, this is a good fit.*

The Chi-squared goodness of fit value for Figure 4 is 7.202. With 8 degrees of freedom, and a P value of 0.99, the critical value is 1.647. Since our Chi-squared value is larger than this value, there is no significant difference between our expected and observed values $1/r$ for this calculation.

In the second half, the value of current is varied, and delta V is kept constant (Appendix 1, Table 2). The graph for $1/r$ by $(1/\sqrt{\Delta V})$ is plotted (Figure 5). From this, the charge per mass ratio value determined is $e/m = 2.3 \times 10^{11} \pm 0.2 \times 10^{11}\, C/kg$ (Appendix 2).

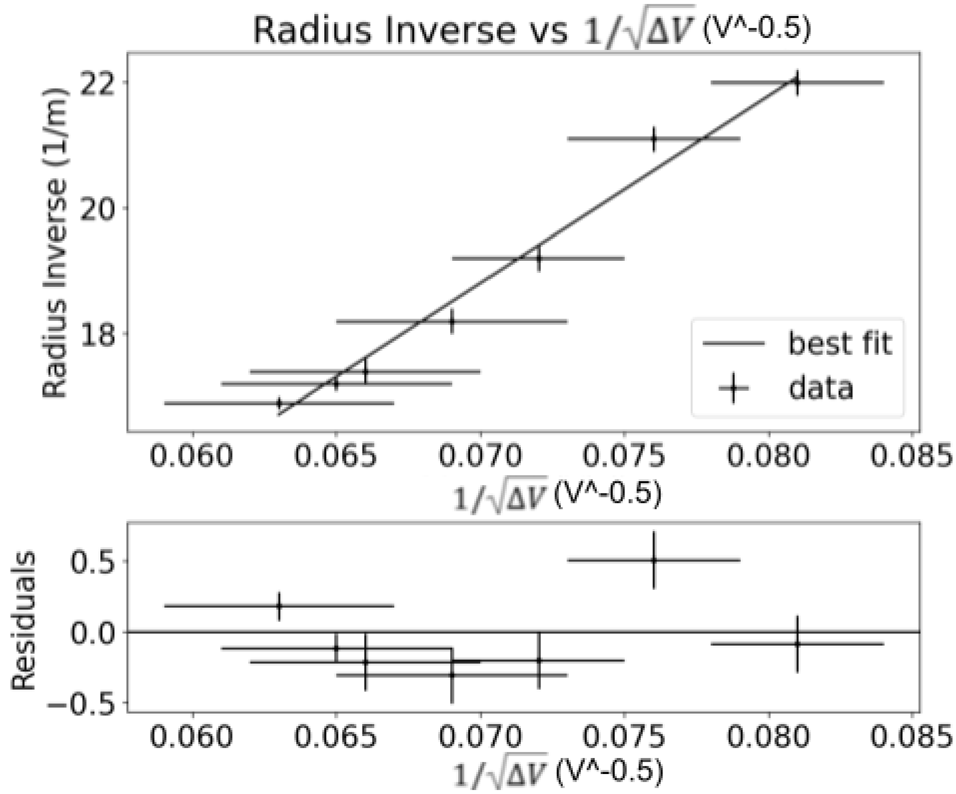

Figure 5: The graph for for holding Variation of $1/r$ *as a function of* $(1/\sqrt{\Delta V})$ *. The fitting gives a gradient value of* $300 \pm 10\ (V^{0.5}/m)$ *, and a y-intercept value of* $-\ 2.1 \pm 0.7\ (m)^{-1}$ *. The specific charge determined from this is* $e/m = 2.3 \times 10^{11} \pm 0.2 \times 10^{11}\, C/kg$ *(Appendix 2). From the Residual plot, most data points are within 2 error bars. Hence, this is a good fit.*

The Chi-squared goodness of fit value for Figure 5 is 8.753. With 8 degrees of freedom, and a P value of 0.99, the critical value is 1.647. Since our Chi-squared value is larger than this value, there is no significant difference between our expected and observed values of $1/r$ for this calculation.

The average of the two e/m values is $e/m = 2.25 \times 10^{11} \pm 0.2 \times 10^{11}\, C/kg.$

4 Discussion and Conclusion

The final value of the specific charge an electron determined is $e/m = (2.21 \pm 0.01) \times 10^{11}\, C/kg$ in the first dataset and $e/m = (2.3 \pm 0.2) \times 10^{11}\, C/kg$ in the second dataset. These values have 25.6% and 27.7% (Appendix 2) error when compared to the accepted value of $1.759 \times 10^{11}\, C/kg$ respectively. The average is

$e/m = 2.3 \times 10^{11} \pm 0.2 \times 10^{11}\,C/kg$ (27.7% error). While this is a large percentage error relative to most physical experimental values, the formula used to calculate specific charge (equation 1) has been simplified to only account for the magnetic field along the coil axis so this was expected.

However, several precautions were taken to reduce procedural and reading errors while performing this experiment. Using the metre rule, the self-illuminated scale was adjusted to be parallel to the table surface, and the plastic reflector was adjusted to be parallel to the Helmholtz coils. During measurement, the measurement-taker centred themselves using the centre of the illuminated scale as guidance to avoid parallax error. The chair that the measurement-taker sat on was lowered to align their eyes with the apparatus. The same person took both sets of data to ensure that the method was consistent between trials.

For the case of high current in the coils I ($3.0150 \pm 0.0005\,A$) and low accelerating voltage ΔV ($124.012 \pm 0.0005\,V$), the Helmholtz Coils produce a strong magnetic field and the electrons have low kinetic energies (equation 3), and hence have low velocities. As a result, a circular column of electron beam was observed along the coil axis. The different parts of the trajectory are not equally affected, the electron beam is most focused towards the center and most diffused away from the center of the column which makes it hard to measure the radius of the charge orbit.

To eliminate the effect of high I and low ΔV, both the sets of data were taken for low I (between $1.0350 \pm 0.0005\,A$ and $1.6050 \pm 0.0005\,A$) and high ΔV (between $141.036 \pm 0.0005\,V$ and $251.027 \pm 0.0005\,V$). Additionally, to account for the decrease in the z-component of the magnetic field $B(\rho)$ as the off-axis distance ρ increased, a correction factor was calculated using (6) (Appendix 1, Table 3) if the radius of the charge orbit was in the range $0.2R < \rho < 0.5R$ where R is the HC radius. For $\rho < 0.2R$, no correction factor was applied as $B(\rho)$ is smaller than $B(\rho)$ (magnetic field at the axis) by less than 0.075% and hence, approximately the same.

The effects of other external, stable and significant magnetic fields, such as those from the earth, the building and other laboratory equipment (such as the magnetic fields of neighboring Helmholtz coils), is accounted for in B_e. Apart from these stable external fields, this experiment was performed in the presence of ferromagnetic materials and other sources of magnetic fields, such as laptops and cellphones. The effect of these magnetic fields on the measurements of radius, when brought very close to the glass bulb, was found to be visually negligible. However, their effects are accounted for in B_e.

To reduce the effects of experimental error in the future, photographs of the charge orbit at different radii may be taken by a camera on a steady tripod at a fixed position in space. These photographs can be used to determine the radius much more accurately using software such as Tracker [1]. Furthermore, doing more trials at different constant values of voltage and current, and taking an average between the specific charge results can reduce the effects of anomalies in the final calculated value.

5 References

[1] Tracker video analysis and Modelling Tool. ComPADRE.org. (n.d.). https://www.compadre.org/osp/items/detail.cf m?ID=7365

6 Appendix

1. **Appendix 1: Data tables**

Voltage V (constant) [V]	Uncertainty in Voltage ΔV [V]	Current I (varying) [A]	Uncertainty in Current ΔI [A]	Charge Orbit Diameter d [cm]	Uncertainty in Diameter Δd [cm]
150.0350	0.0005	1.0350	0.0005	10.4	0.1
150.0350	0.0005	1.0870	0.0005	10.3	0.1
150.0350	0.0005	1.1090	0.0005	9.5	0.1
150.0350	0.0005	1.1590	0.0005	9.0	0.1
150.0350	0.0005	1.2450	0.0005	8.4	0.1
150.0350	0.0005	1.3050	0.0005	8.1	0.1
150.0350	0.0005	1.4060	0.0005	7.4	0.1
150.0350	0.0005	1.5070	0.0005	6.8	0.1
150.0350	0.0005	1.6050	0.0005	6.6	0.1

Table 1. Raw data collected for varying current and constant voltage.

Voltage V (varying) [V]	Uncertainty in Voltage ΔV [V]	Current I (constant) [A]	Uncertainty in Current ΔI [A]	Charge Orbit Diameter d [cm]	Uncertainty in Diameter Δd [cm]
150.9	0.0005	1.0970	0.0005	9.1	0.1
170.9	0.0005	1.0990	0.0005	9.5	0.1
190.9	0.0005	1.0980	0.0005	10.4	0.1
209.9	0.0005	1.0970	0.0005	11.0	0.1
230.9	0.0005	1.0970	0.0005	11.5	0.1
240.0	0.0005	1.0970	0.0005	11.6	0.1
250.9	0.0005	1.0970	0.0005	11.8	0.1

Table 2. Raw data collected for varying voltage and constant current.

Radius r (m)	Uncertainty in Radius r Δr (m)	Correctional Factor CF	Uncertainty for Correctional Factor ΔCF
0.052	0.0005	0.971	0.001
0.0515	0.0005	0.972	0.001
0.0475	0.0005	0.979	0.001
0.045	0.0005	0.983	0.001
0.042	0.0005	0.987	0.001
0.0405	0.0005	0.989	0.001
0.037	0.0005	0.992	0.0005
0.034	0.0005	0.9943	0.0004
0.033	0.0005	0.9950	0.0003

Table 3 Correctional factors with their corresponding radii.

2. Appendix 2: Calculations

Percentage error = $\left|\frac{Accepted - Experimental}{Accepted}\right| \times 100$

Error propagation for graph 1 (B_c vs $1/r$)

- $\Delta r = \frac{\Delta d}{2}$
- $\Delta \frac{1}{r} = -1 \times r^{-2} \times \Delta r$
- $\Delta \frac{I}{R} = \frac{I}{R}\sqrt{(\frac{\Delta I}{I})^2 + (\frac{\Delta R}{R})^2}$
- $\Delta B_c = (\frac{4}{5})^{1.5}\mu_0 n \Delta(\frac{I}{R})$
- $\Delta B_{c,corrected} = B_c \times CF \times \sqrt{(\frac{\Delta B_c}{B_c})^2 + (\frac{\Delta CF}{CF})^2}$

Error propagation for graph 2 (varying current, constant voltage)

For varying current, $\frac{1}{r}$ vs $I + \frac{I_0}{\sqrt{2}}$ was plotted.

- $\Delta r = \frac{\Delta d}{2}$
- $\Delta \frac{1}{r} = -1 \times r^{-2} \times \Delta r$
- $\Delta(I + \frac{I_0}{\sqrt{2}}) = \sqrt{(\frac{\Delta I}{I})^2 + (\frac{\Delta I_0}{I_0} \times \frac{1}{\sqrt{2}})^2}$

From the graph, $m = \sqrt{\frac{e}{m}} k \frac{1}{\sqrt{\Delta V}}$. Hence, $\frac{e}{m} = (\frac{m\sqrt{\Delta V}}{k})^2$

- $\Delta \frac{e}{m} = \frac{e}{m} \times \sqrt{(\frac{\Delta m}{m})^2 + (\frac{\Delta k}{k})^2 + (\frac{0.5\Delta V^{-1.5} \times \Delta(\Delta V)}{\sqrt{\Delta V}})^2}$

Error propagation for graph 3 (varying voltage, constant current)

For varying voltage, $\frac{1}{r}$ vs $\frac{1}{\sqrt{\Delta V}}$ was plotted.

- $\Delta r = \frac{\Delta d}{2}$
- $\Delta \frac{1}{r} = -1 \times r^{-2} \times \Delta r.$
- $\Delta(\frac{1}{\sqrt{\Delta V}}) = 0.5\Delta V^{-1.5} \times \Delta(\Delta V)$

From the graph, $m = \sqrt{\frac{e}{m}} k (I + \frac{I_0}{\sqrt{2}})$. Hence, $\frac{e}{m} = (\frac{m}{k \times (I + \frac{I_0}{\sqrt{2}})})^2$

- $\Delta(I + \frac{I_0}{\sqrt{2}}) = \sqrt{(\frac{\Delta I}{I})^2 + (\frac{\Delta I_0}{I_0} \times \frac{1}{\sqrt{2}})^2}$
- $\Delta \frac{e}{m} = \frac{e}{m} \times \sqrt{(\frac{\Delta m}{m})^2 + (\frac{\Delta k}{k})^2 + (\frac{\Delta(I + \frac{I_0}{\sqrt{2}})}{(I + \frac{I_0}{\sqrt{2}})})^2}$

Error propagation for correction factor, CF

Correction factor, $\frac{B(\rho)}{B(0)} = 1 - \frac{\rho^4}{R^4(0.6583+0.29\frac{\rho^2}{R^2})^2}$.

- $\Delta\rho^4 = 4 \times \rho^3 \times \Delta\rho$
- $\Delta R^4 = 4 \times R^3 \times \Delta R$
- $\Delta(0.6583 + 0.29\frac{\rho^2}{R^2}) = 0.29\frac{\rho^2}{R^2}\sqrt{(\frac{2\rho\Delta\rho}{\rho^2})^2 + (\frac{2R\Delta R}{R^2})^2)}$
- $\Delta(0.6583 + 0.29\frac{\rho^2}{R^2})^2 = 2 \times (0.6583 + 0.29\frac{\rho^2}{R^2}) \times \Delta(0.6583 + 0.29\frac{\rho^2}{R^2})$
- $\Delta CF = \Delta(\frac{B(\rho)}{B(0)}) = \frac{B(\rho)}{B(0)}\sqrt{(\frac{\Delta\rho^4}{\rho^4})^2 + (\frac{\Delta R^4}{R^4})^2 + (\frac{\Delta(0.6583+0.29\frac{\rho^2}{R^2})^2}{(0.6583+0.29\frac{\rho^2}{R^2})^2})^2}$