UPH013

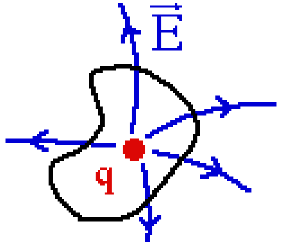
Electromagnetic waves

School of Physics and Material Science
Thapar Institute of Engineering and Technology

Electromagnetic waves

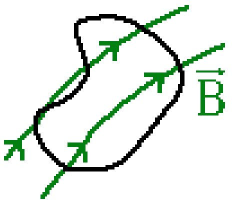
$$\oiint \vec{E} \bullet \hat{n}\, dS = \frac{q}{\varepsilon_o}$$

Gauss's Law

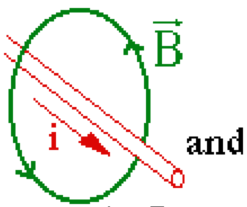


$$\oiint \vec{B} \bullet \hat{n}\, dS = 0$$

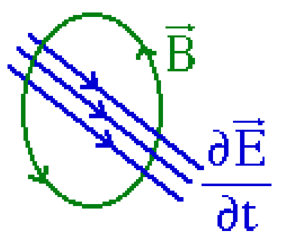
(no monopoles)



$$\oint \vec{B} \bullet d\vec{l} = \mu_{\circ}\left(i + \varepsilon_{\circ}\frac{d}{dt}\Phi_E\right)$$

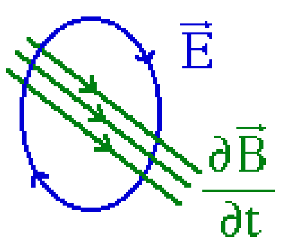


Ampère's Law



$$\oint \vec{E} \bullet d\vec{l} = -\frac{d}{dt}\Phi_B$$

Faraday's Law



Maxwell's equations

$$\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

Helmholtz theorem says that, provided the field vanishes far from its source, the curl and divergence of it are sufficient to determine that field at any point of question.

Maxwell's equations in vacuum

$$\rho = 0$$

$$\vec{J} = 0$$

$$\nabla \bullet \vec{E} = 0$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Solution of Maxwell's equations in vacuum

Problem: to determine the electric and magnetic field at a point in space at an instant of time.

Two variables   Four equations???

Lets find a solution…The Curl of third Maxwell equation gives..

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\nabla \times \left(\frac{\partial \vec{B}}{\partial t}\right)$$

$$\nabla\left(\nabla . \vec{E}\right) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right) = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Solution of Maxwell's equations in vacuum

Similarly the Curl of fourth Maxwell equation

$$\nabla \times \left(\nabla \times \vec{B}\right) = \nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\nabla\left(\nabla . \vec{B}\right) - \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\nabla \times \vec{E}\right)$$

$$-\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial \vec{B}}{\partial t} \right)$$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Governing 2nd order differential equations for **E** and **B**.

Solution of Maxwell's equations in vacuum

$$\boxed{\begin{aligned}\nabla^2\vec{E} &= \mu_0\varepsilon_0 \frac{\partial^2\vec{E}}{\partial t^2} \\ \nabla^2\vec{B} &= \mu_0\varepsilon_0 \frac{\partial^2\vec{B}}{\partial t^2}\end{aligned}}$$

<u>Conclusion</u> :

- **E** and **B** vary in space and time in form of a wave
- Equation of a propagating wave

$$\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

- The waves of **E** and **B** propagate in a similar fashion simultaneously in space and time.
- The propagation of **E** and **B** in space and time forms a wave known as ELECTROMAGNETIC WAVE.

Solution of Maxwell's equations in vacuum

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

<u>Conclusion</u> :

➢A plane wave solution of above equations can be written as

$$\boxed{\vec{E}(r,t) = \vec{E}_0 e^{i\left(\vec{k}.\vec{r}-\omega t\right)} \text{ and } \quad \vec{B}(r,t) = \vec{B}_0 e^{i\left(\vec{k}.\vec{r}-\omega t\right)}}$$

➢$\mathbf{E}_0$ and $\mathbf{B}_0$ are the complex amplitudes and the physical field are the real part of **E**(**r**,t) and **B**(**r**,t), and **k** is wave propagation vector given by

$$\vec{k} = \frac{2\pi}{\lambda}\hat{n}$$

➢The direction of **k** vector is along the propagation of EM wave.

$$\mu_0 \varepsilon_0 = \frac{1}{v^2} \qquad v = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = \sqrt{\frac{4\pi}{\mu_0} . \frac{1}{4\pi\varepsilon_0}} = \sqrt{10^7 \times 9 \times 10^9} = 3 \times 10^8 m/s$$

➢Light is an electromagnetic (EM) wave.

EM wave in vacuum

$$\nabla \bullet \vec{E} = 0$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\boxed{\begin{aligned} \vec{E}(r,t) &= \vec{E}_0 e^{i\left(\vec{k}.\vec{r} - \omega t\right)} \\ \vec{B}(r,t) &= \vec{B}_0 e^{i\left(\vec{k}.\vec{r} - \omega t\right)} \end{aligned}}$$

Lets find the relationship of **E** and **B** in an EM wave

$$\vec{k} = \hat{\imath}k_x + \hat{\jmath}k_y + \hat{k}k_z \qquad \vec{B}_0 = \hat{\imath}\vec{B}_{0x} + \hat{\jmath}\vec{B}_{0y} + \hat{k}\vec{B}_{0z}$$

$$\vec{E}_0 = \hat{\imath}\vec{E}_{0x} + \hat{\jmath}\vec{E}_{0y} + \hat{k}\vec{E}_{0z}$$

$$\vec{r} = \hat{\imath}x + \hat{\jmath}y + \hat{k}z$$

$$\vec{E}(\vec{r},t) = \left(\hat{\imath}\vec{E}_{0x} + \hat{\jmath}\vec{E}_{0y} + \hat{k}\vec{E}_{0z}\right)e^{i\left(k_x x + k_y y + k_z z - \omega t\right)}$$

$$\vec{B}(\vec{r},t) = \left(\hat{\imath}\vec{B}_{0x} + \hat{\jmath}\vec{B}_{0y} + \hat{k}\vec{B}_{0z}\right)e^{i\left(k_x x + k_y y + k_z z - \omega t\right)}$$

EM wave in vacuum

$$\vec{E}(r,t) = \vec{E}_0 e^{i(\vec{k}.\vec{r}-\omega t)}$$
$$\vec{B}(r,t) = \vec{B}_0 e^{i(\vec{k}.\vec{r}-\omega t)}$$

$$\nabla \bullet \vec{E} = 0$$
$$\nabla \bullet \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Lets find the relationship of **E** and **B** in and EM wave

$$\nabla \bullet \vec{E}(\vec{r},t) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \right) \bullet \left(\hat{i}\vec{E}_{0x} + \hat{j}\vec{E}_{0y} + \hat{k}\vec{E}_{0z} \right) e^{i(k_x x + k_y y + k_z z - \omega t)}$$
$$0 = \left(\vec{E}_{0x} i k_x + \vec{E}_{0y} i k_y + \vec{E}_{0z} i k_z \right) e^{i(k_x x + k_y y + k_z z - \omega t)}$$
$$= i\left(\hat{i}k_x + \hat{j}k_y + \hat{k}k_z \right) \bullet \left(\hat{i}\vec{E}_{0x} + \hat{j}\vec{E}_{0y} + \hat{k}\vec{E}_{0z} \right) e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$\nabla \bullet \vec{E}(\vec{r},t) = i\vec{k} \bullet \vec{E}(\vec{r},t) = 0$$

Similarly for **B**

$$\nabla \bullet \vec{B}(\vec{r},t) = i\vec{k} \bullet \vec{B}(\vec{r},t) = 0$$

k ⊥ E,

k ⊥ B

EM wave in vacuum

$$\nabla \bullet \vec{E} = 0$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\boxed{\begin{aligned} \vec{E}(r,t) &= \vec{E}_0 e^{i\left(\vec{k}.\vec{r}-\omega t\right)} \\ \vec{B}(r,t) &= \vec{B}_0 e^{i\left(\vec{k}.\vec{r}-\omega t\right)} \end{aligned}}$$

From IIIrd eq^{n}

$$\nabla \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$$

$$\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left[\left(\hat{i}\vec{E}_{0x} + \hat{j}\vec{E}_{0y} + \hat{k}\vec{E}_{0z}\right)e^{i\left(k_x x + k_y y + k_z z - \omega t\right)}\right] = -\frac{\partial}{\partial t}\left[\left(\hat{i}\vec{B}_{0x} + \hat{j}\vec{B}_{0y} + \hat{k}\vec{B}_{0z}\right)e^{i\left(k_x x + k_y y + k_z z - \omega t\right)}\right]$$

$$\left[\hat{i}\left(\vec{E}_{0z} ik_y - \vec{E}_{0y} ik_z\right) + \hat{j}\left(\vec{E}_{0x} ik_z - \vec{E}_{0z} ik_x\right) + \hat{k}\left(\vec{E}_{0y} ik_x - \vec{E}_{0x} ik_y\right)\right]e^{i\left(k_x x + k_y y + k_z - \omega t\right)} = -\left(-i\omega\right)\left[\left(\hat{i}\vec{B}_{0x} + \hat{j}\vec{B}_{0y} + \hat{k}\vec{B}_{0z}\right)e^{i\left(k_x x + k_y y + k_z - \omega t\right)}\right]$$

$$i\left(\hat{i}k_x + \hat{j}k_y + \hat{k}k_z\right) \times \left(\hat{i}\vec{E}_{0x} + \hat{j}\vec{E}_{0y} + \hat{k}\vec{E}_{0z}\right)e^{i\left(k_x x + k_y y + k_z - \omega t\right)} = i\omega\left[\left(\hat{i}\vec{B}_{0x} + \hat{j}\vec{B}_{0y} + \hat{k}\vec{B}_{0z}\right)e^{i\left(k_x x + k_y y + k_z - \omega t\right)}\right]$$

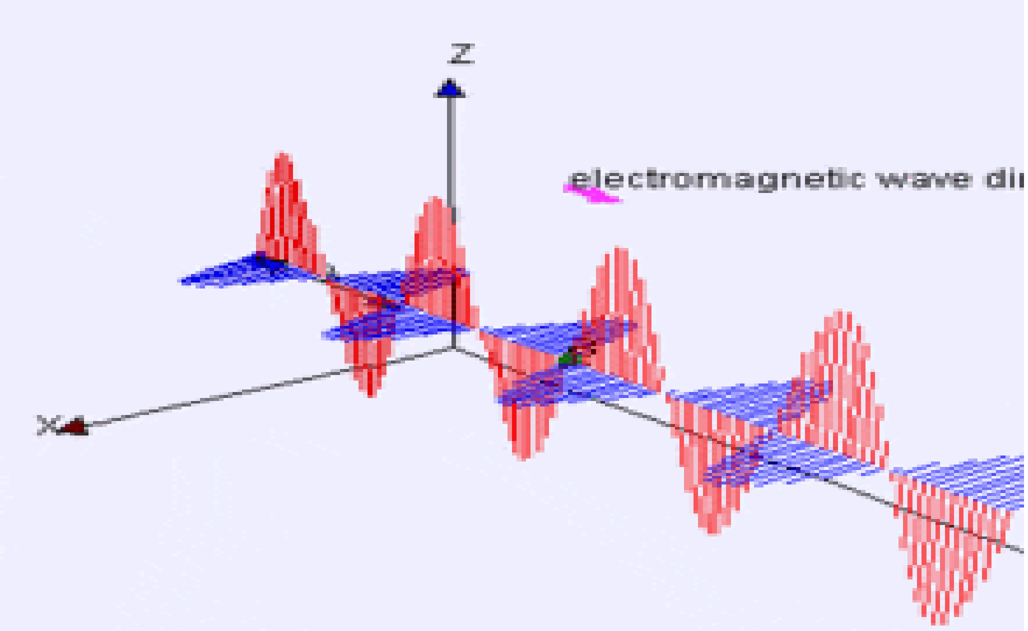
$$\vec{k} \times \vec{E}(\vec{r},t) = \omega \vec{B}(\vec{r},t)$$

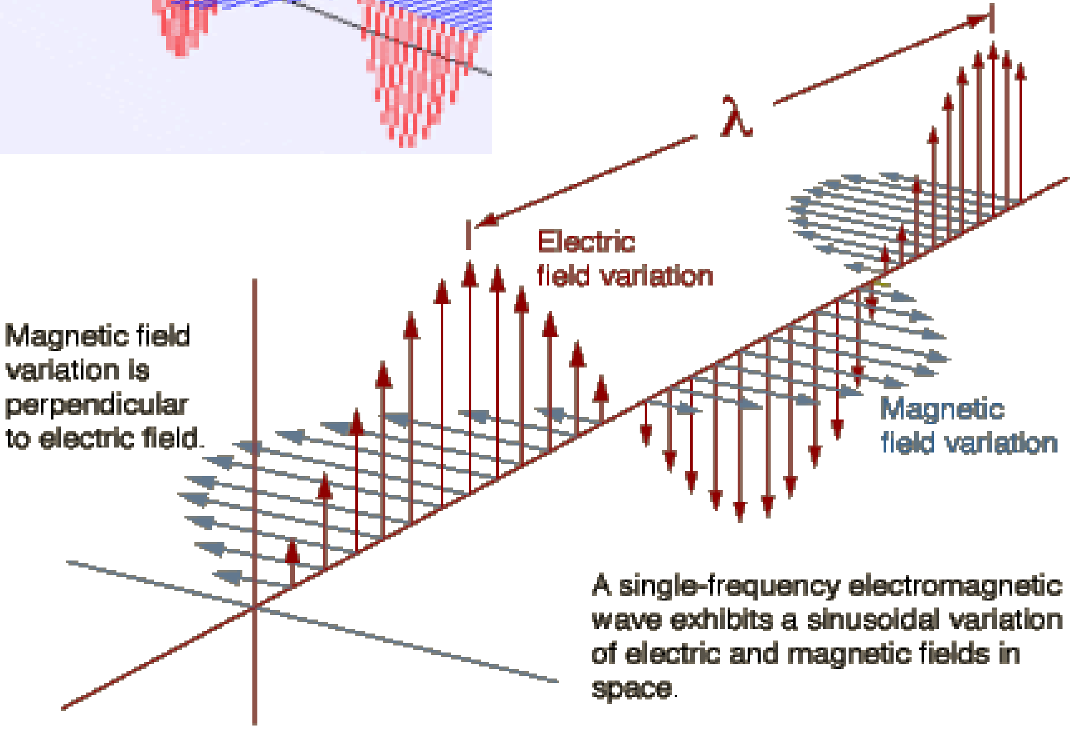
Similarly from IVth eq^{n}.

$$\vec{k} \times \vec{B}(\vec{r},t) = -\omega \vec{E}(\vec{r},t)$$

B ⊥ E, k

E ⊥ B, k


z
electromagnetic wave di
x


λ
Electric
field variation
Magnetic field
variation is
perpendicular
to electric field.
Magnetic
field variation
A single-frequency electromagnetic
wave exhibits a sinusoidal variation
of electric and magnetic fields in
space.

Maxwell's equations in a dielectric medium

$$\begin{aligned}\varepsilon_0 &\rightarrow \varepsilon = \varepsilon_r \varepsilon_0 \\ \mu_0 &\rightarrow \mu = \mu_r \mu_0\end{aligned} \quad , \qquad \rho = 0, \qquad \vec{J} = 0$$

$$\nabla \bullet \vec{E} = 0$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$$

Solution of Maxwell's equations in dielectric medium

Curl of third Maxwell equation gives..

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\nabla \times \left(\frac{\partial \vec{B}}{\partial t}\right)$$

$$\nabla\left(\nabla . \vec{E}\right) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right) = -\frac{\partial}{\partial t}\left(\mu\varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$-\nabla^2 \vec{E} = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Similarly the Curl of fourth Maxwell equation gives

$$\nabla^2 \vec{B} = \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

Velocity of EM waves in dielectric medium

$$\mu\varepsilon = \frac{1}{v^2}$$

$$v = \sqrt{\frac{1}{\mu\varepsilon}} = \sqrt{\frac{1}{\mu_r\mu_0\varepsilon_r\varepsilon_0}} = \sqrt{\frac{1}{\mu_r\varepsilon_r}\frac{4\pi}{\mu_0}.\frac{1}{4\pi\varepsilon_0}}$$

$$= \sqrt{\frac{10^7.9\times10^9}{\mu_r\varepsilon_r}} = \frac{3\times10^8 m/s}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

<u>Conclusion</u> :

➢Velocity of EM wave in a dielectric medium is less then that in vacuum.

$$n = \frac{c}{v} = \sqrt{\mu_r\varepsilon_r} \approx \sqrt{\varepsilon_r} > 1$$

➢Refractive index is always greater then 1.

Maxwell's equations in a conducting medium

$$\begin{matrix} \varepsilon_0 \to \varepsilon = \varepsilon_r \varepsilon_0 \\ \mu_0 \to \mu = \mu_r \mu_0 \end{matrix} \quad , \qquad \rho = 0, \qquad \vec{J} \neq 0$$

$$\nabla \bullet \vec{E} = 0$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu \left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right)$$

A conducting medium

1. **Ohm's law:**

The current density in a conductor is directly proportional to the electric field applied to flow the current.

$$\vec{J} \propto \vec{E}$$

$$\vec{J} = \sigma \ \vec{E}$$

σ is called conductivity of the conducting material.

$$\frac{I \ \vec{L}}{\vec{A}} = \sigma \ \vec{E}.\vec{L}$$

$$\left(\frac{1}{\sigma}\frac{L}{A}\right) I = V \qquad \Rightarrow \qquad R = \frac{1}{\sigma}\frac{L}{A} = \rho \frac{L}{A}$$

A conducting medium

2. **Charge in conductor:**

Lets start with continuity equation.

$$\frac{\partial \rho}{\partial t} = -\nabla.J = -\nabla.(\sigma E) = -\sigma(\nabla.E) = -\sigma\frac{\rho}{\varepsilon} \qquad \Rightarrow \int_{\rho_0}^{\rho}\frac{1}{\rho}\partial\rho = -\frac{\sigma}{\varepsilon}\int_0^t \partial t$$

$$\rho = \rho_0 \exp\left(-\frac{\sigma}{\varepsilon}t\right)$$

The time after which ρ_0 reduces to ρ_0/e is known as the characteristic time t = τ.

τ depends upon the conductivity and permittivity of the medium.

$$\rho = \frac{\rho_0}{e} = \rho_0 \exp\left(-\frac{\sigma}{\varepsilon}\tau\right)$$

$$\Rightarrow \tau = \frac{\varepsilon}{\sigma}$$

Solution of Maxwell's equations in conducting medium

$$\nabla \bullet \vec{E} = 0$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu\left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

Curl of third Maxwell equation gives..

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\nabla \times \left(\frac{\partial \vec{B}}{\partial t}\right)$$

$$\nabla\left(\nabla . \vec{E}\right) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right) = -\frac{\partial}{\partial t}\left[\mu\left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)\right]$$

$$-\nabla^2 \vec{E} = -\mu\left[\frac{\partial \vec{J}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}\right]$$

$$\nabla^2 \vec{E} = \mu \frac{\partial \vec{J}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Similarly fourth Maxwell eq^{n}. gives..

$$\nabla^2 \vec{B} = \mu\sigma \frac{\partial \vec{B}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

Solution of Maxwell's equations in conducting medium

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu\sigma \frac{\partial \vec{B}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

Above equations represent the wave equation with an extra first order differential term at right hand side.

The solution can be written as :

$$\vec{E}(r,t) = \vec{E}_0 e^{i\left(\vec{k}.\vec{r} - \omega t\right)}$$

$$\vec{B}(r,t) = \vec{B}_0 e^{i\left(\vec{k}.\vec{r} - \omega t\right)}$$

Consequence of extra term at R.H.S.

The solutions for **E** and **B** give on inserting in the compact equations:

$$(ik)^2\,\vec{E} = \mu\sigma(-i\omega)\vec{E} + \mu\varepsilon(-i\omega)^2\,\vec{E}$$

$$(ik)^2\,\vec{B} = \mu\sigma(-i\omega)\vec{B} + \mu\varepsilon(-i\omega)^2\,\vec{B}$$

or

$$k^2 = i\mu\sigma\omega + \mu\varepsilon\omega^2$$

This represents that k vector is a complex quantity.

Since k vector is related with **E** and **B**, a complex k vector indicates a phase lag between **E** and **B** in conducting medium.

Complex wave vector

The magnitude of a complex k vector can be written as.

$$k = a + ib$$

$$k^2 = \left(a^2 - b^2\right) + i\left(2ab\right) \qquad = i\mu\sigma\omega + \mu\varepsilon\omega^2$$

Which on comparison gives: $a^2 - b^2 = \mu\varepsilon\omega^2$, $2ab = \mu\sigma\omega$

$$a = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}+1\right]^{1/2} \quad and$$

$$b = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}-1\right]^{1/2}$$

Complex wave vector

For good conductors σ >> ωε

$$a = b = \omega\sqrt{\frac{\varepsilon\mu}{2}}\sqrt{\frac{\sigma}{\varepsilon\omega}} = \sqrt{\frac{\mu\sigma\omega}{2}}$$

For poor conductors σ << ωε

$$a = \omega\sqrt{\varepsilon\mu} \quad and$$

$$b = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

Consequence of complex wave vector

Maxwell's third equation reads:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$i\vec{k} \times \vec{E}_0 = -(-i\omega)\vec{B}_0$$

$$\vec{B}_0 = \frac{1}{\omega}(a + ib)\hat{k} \times \vec{E}_0$$

The vectors **E** and **B** of electromagnetic waves in conducting medium are not in phase.

Consequence of complex wave vector

A complex k-vector can be expressed also as

$$k = |k| \exp(i\phi) = \sqrt{a^2 + b^2}\ e^{i\phi}$$

Where ϕ is the phase shift between **E** and **B**. It is given by

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\phi = \begin{cases} \pi/4 & for\ \ good\ \ conductors \\ \tan^{-1}\left(\dfrac{\sigma}{2\varepsilon\omega}\right) & for\ \ poor\ \ conductors \end{cases}$$

Effect of complex wave vector in a conductor

$$\vec{E}(r) = \vec{E}_0 \exp i\{(a+ib).r - \omega t\}$$
$$\vec{B}(r) = \vec{B}_0 \exp i\{(a+ib).r - \omega t\}$$

This can be written also as:

$$\left.\begin{array}{l}\vec{E}(r,t) = \vec{E}_0 \exp(-b.r)\exp i(a.r - \omega t) \quad and \\ \vec{B}(r,t) = \vec{B}_0 \exp(-b.r)\exp i(a.r - \omega t)\end{array}\right\}$$

It is evident that the behavior of the EM wave is changed in conducting medium except an extra term multiplied at right hand side. This can be written also as :

$$\left.\begin{array}{l}\vec{E}(r,t) = \vec{E}_0^* \exp i(a.r - \omega t) \quad and \\ \vec{B}(r,t) = \vec{B}_0^* \exp i(a.r - \omega t)\end{array}\right\} \quad with \quad \left.\begin{array}{l}\vec{E}_0^*(r) = \vec{E}_0 \exp(-b.r) \quad and \\ \vec{B}_0^*(r) = \vec{B}_0 \exp(-b.r)\end{array}\right\}$$

Effect of complex wave vector in a conductor

The amplitudes of **E** and **B** are:

$$\left.\begin{aligned}\vec{E}_0^*(r) &= \vec{E}_0 \exp(-b.r) \quad and \\ \vec{B}_0^*(r) &= \vec{B}_0 \exp(-b.r)\end{aligned}\right\}$$

The amplitudes of **E** and **B** are not constant in a conducting material. Here 'r' is the depth inside the material:

r = 0 at the surface of conductor

r ≠ 0 inside the conductor

The amplitudes of **E** and **B** decrease exponentially, as EM waves propagates inside a conductor.

Effect of complex wave vector in a conductor

$$\left.\begin{array}{l}\vec{E}_0^*(r) = \vec{E}_0 \exp(-b.r) \quad \mathit{and} \\ \vec{B}_0^*(r) = \vec{B}_0 \exp(-b.r)\end{array}\right\}$$

The decrease in amplitudes is governed by b

Put $\quad r = 1/b$

$$\left.\begin{array}{l}\vec{E}_0^*(r) = \vec{E}_0 \exp\left[-b.\left(\dfrac{1}{b}\right)\right] = \vec{E}_0 / e \quad \mathit{and} \\ \vec{B}_0^*(r) = \vec{B}_0 \exp\left[-b.\left(\dfrac{1}{b}\right)\right] = \vec{B}_0 / e\end{array}\right\}$$

Skin depth in conducting medium

At a depth equal to *1/b* the amplitude of EM wave will remain *1/e* part of that outside the medium.
Amplitudes are spatially attenuated as b and/or *r* increases.

The distance it takes to reduce the amplitude by a factor of *1/e* (about one third) is called the **skin depth** or **penetration depth** and is denoted by δ

$$\delta = \frac{1}{b}$$

$$\delta = \frac{1}{b} = \begin{cases} \sqrt{\dfrac{2}{\mu\sigma\omega}} & \text{for good conductors} \\[2ex] \dfrac{2}{\sigma}\sqrt{\dfrac{\varepsilon}{\mu}} & \text{for poor conductors} \end{cases}$$

Skin depth in conducting medium

$$\delta = \frac{1}{b} = \begin{cases} \sqrt{\dfrac{2}{\mu\sigma\omega}} & \textit{for good conductors} \\ \dfrac{2}{\sigma}\sqrt{\dfrac{\varepsilon}{\mu}} & \textit{for poor conductors} \end{cases}$$

In good conductors the skin depth is inversely proportional to the squire root of the frequency of electromagnetic wave.
Typical values of δ for copper at 60 HZ A.C. is 8.6 mm and that at 1 GHZ is 67 μm.

Therefore a direct current flows through the whole volume of a conductor and an alternating current flows only on the surface of a conductor.