

[CSE537] Tracking – HMM (Hidden Markov Model)

# **REPORT for TEAM PROJECT #6**

**by Oleksii Starov, Hyungjoon Koo**

**Dec. 16, 2013**

## A. General Information

- (1) Team-up (Group 41): Oleksii Starov ([ostarov@cs.stonybrook.edu](mailto:ostarov@cs.stonybrook.edu)), Hyungjoon Koo ([hykoo@cs.stonybrook.edu](mailto:hykoo@cs.stonybrook.edu))
- (2) Project information: Tracking (PJT#6), <http://www.cs.sunysb.edu/~ram/cse537/project06-2013.html> (Due date: Dec 16, 2013)
- (3) Implementation: Tracking with Ghostbuster – Hidden Markov Model or HMM (Extra)

## B. Implementation Note

### (Q1) The *observe* function

What we have changed is to compute observation (or evidence) update according to the following formula.  $P(x_t|e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1})P(x_t|x_{t-1})$

```
def observe(self, observation, gameState):  
  
    """ YOUR CODE HERE """  
    allPossible = util.Counter()  
    for p in self.legalPositions:  
        trueDistance = util.manhattanDistance(p, pacmanPosition)  
        if emissionModel[trueDistance] > 0:  
            allPossible[p] += self.beliefs[p] * emissionModel[trueDistance]  
  
    allPossible.normalize()  
    self.beliefs = allPossible  
    print "UPDATE FOR EVIDENCE: ", self.beliefs
```

### (Q2) The *elapsedTime* function

This part is the implementation of updating online belief over time according to the following formula.  $P(x_t|e_{1:t}) = \alpha P(x_t|e_{1:t-1})P(x_t|e_t)$

```
def elapsedTime(self, gameState):  
  
    """ YOUR CODE HERE """  
    newBeliefs = util.Counter()  
  
    for eachPosition in self.legalPositions:  
        newPosDist = self.getPositionDistribution(self.setGhostPosition(gameState, eachPosition))  
        for newPos, prob in newPosDist.items():  
            newBeliefs[newPos] += prob * self.beliefs[eachPosition]  
  
    self.beliefs = newBeliefs  
    print "UPDATE FOR TIME: ", self.beliefs
```

### C. The results and the description on *analysis.py*

The outcome below indicates the belief updates over time and observation for each position, which shows the probability of the ghost position.

#### Question 1(a)

```
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```

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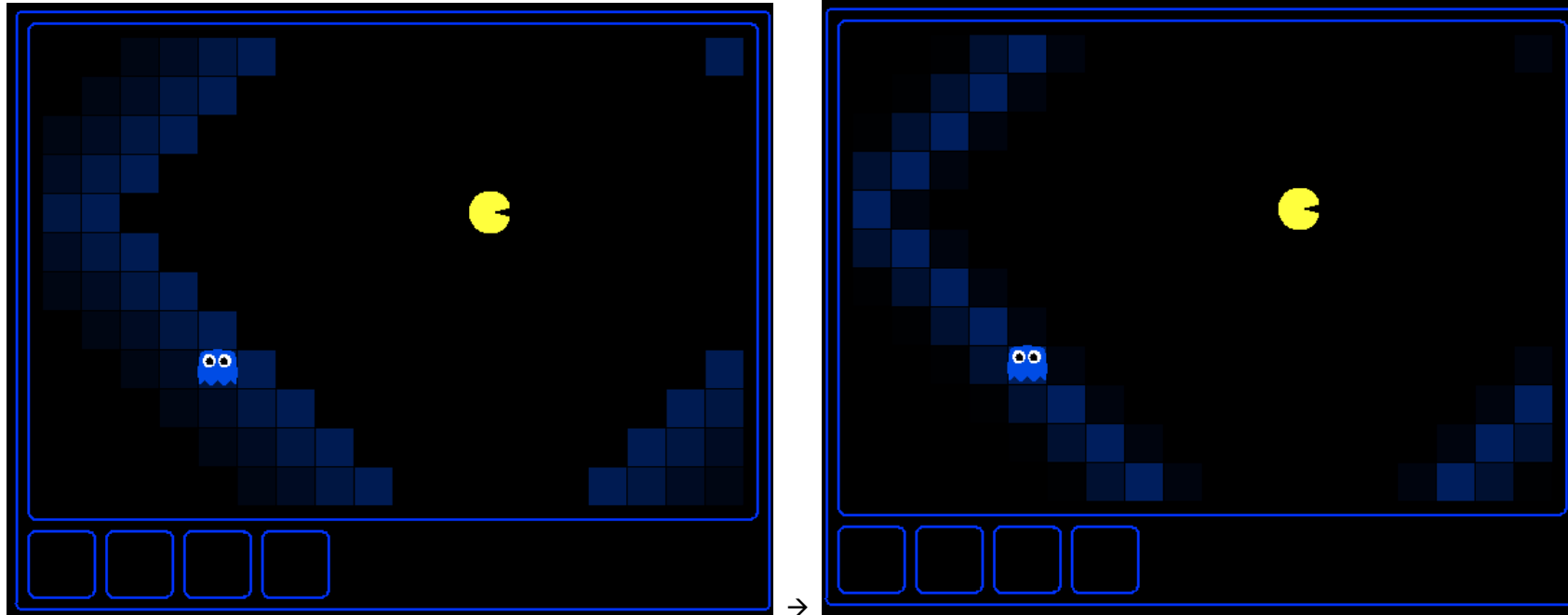
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The beliefs are initialized as the same value, 0.0046296 for all positions. After updating the beliefs over evidence or observation as time goes by, the Pacman gets to detect the position of the stationary ghost with high probability.

### Question 1(b)

```
# python busters.py -s -g StationaryGhost
```

```
.....
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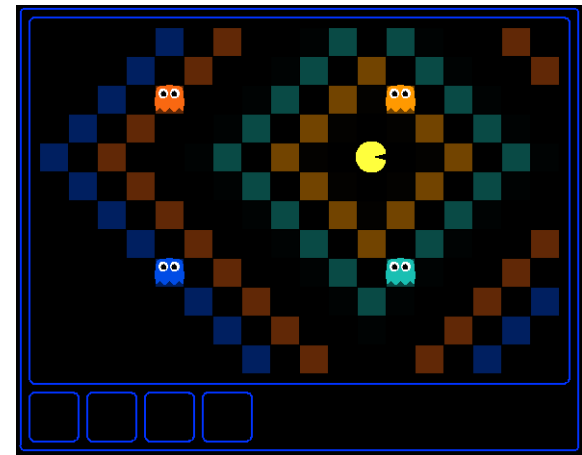
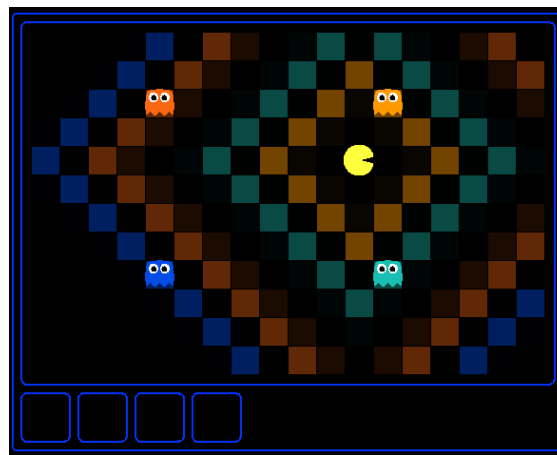
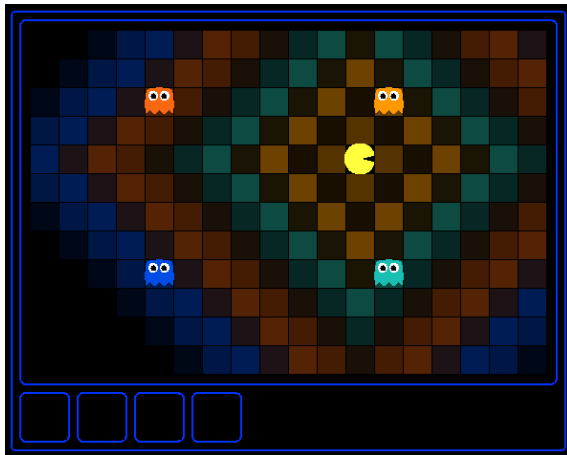
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Based upon the updated beliefs, the Pacman ends up with knowing where each ghost is located as the shown images above.

### Question 2(a)

```
# python busters.py -s -k 1
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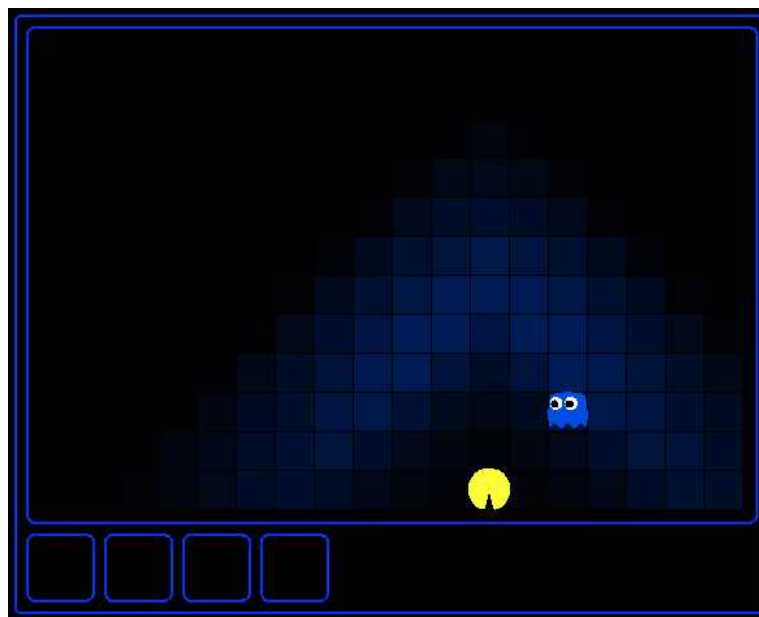
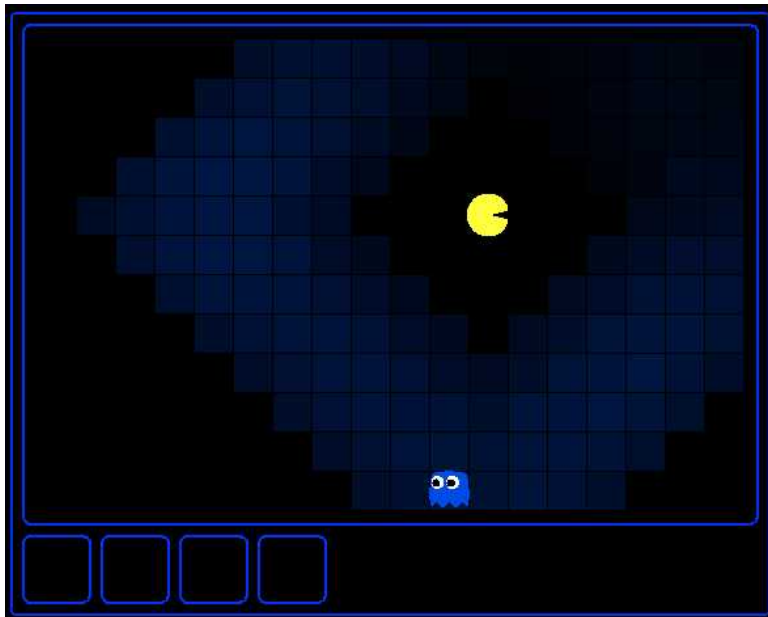
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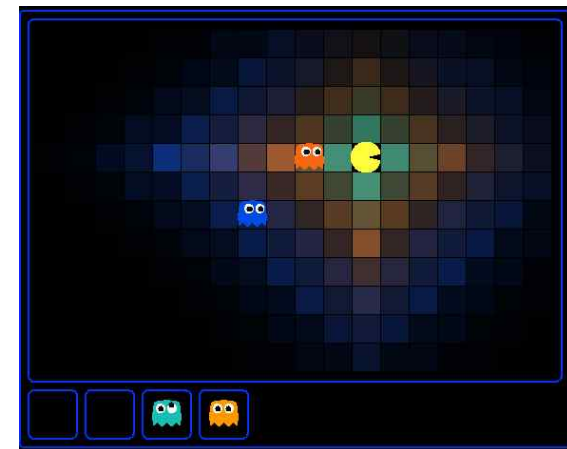
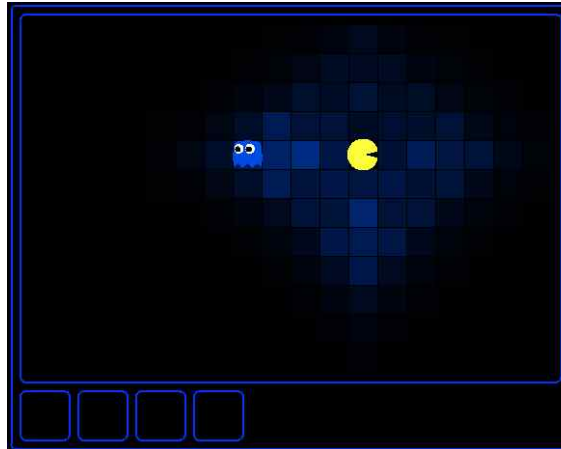


The belief distribution is consistently updated as the ghost moves over time. Based upon the current observation and previous belief, the Pacman is able to keep track of the position of the moving ghost.

## Question 2(b)

```
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```

```
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In this grid, the ghost follows the direction where the Pacman moves. We checked it when there are multiple ghosts.