AP Topic Questions 5.2

1. Which of the following functions of x is guaranteed by the Extreme Value Theorem to have an absolute maximum on the interval $[0, 2\pi]$?

    (A) $y = \frac{1}{1+\sin x}$

    (B) $y = \frac{1}{x^2+\pi}$

    (C) $y = \frac{x^2-2\pi x+\pi^2}{x-\pi}$

    (D) $y = \frac{|x-\pi|}{x-\pi}$

2.

x	0	1	2	3
$f(x)$	15	14	12	9

Let f be a function with selected values given in the table above. Which of the following statements must be true?

    I. By the Intermediate Value Theorem, there is a value c in the interval $(0, 3)$ such that $f(c) = 10$.
    II. By the Mean Value Theorem, there is a value c in the interval $(0, 3)$ such that $f'(c) = -2$.
    III. By the Extreme Value Theorem, there is a value c in the interval $[0, 3]$ such that $f(c) \le f(x)$ for all x in the interval $[0, 3]$.

(A) None

(B) I only

(C) II only

(D) I, II, and III

3. Let g be the function given by $g(x) = \sqrt{1 - \sin^2 x}$. Which of the following statements could be false on the interval $0 \le x \le \pi$?

    (A) By the Extreme Value Theorem, there is a value c such that $g(c) \le g(x)$ for $0 \le x \le \pi$.

    (B) By the Extreme Value Theorem, there is a value c such that $g(c) \ge g(x)$ for $0 \le x \le \pi$.

    (C) By the Intermediate Value Theorem, there is a value c such that $g(c) = \frac{g(0)+g(\pi)}{2}$.

    (D) By the Mean Value Theorem, there is a value c such that $g'(c) = \frac{g(\pi)-g(0)}{\pi-0}$.

4.

x	0	1	2	3
$f(x)$	0	4	7	6

Let f be a function with selected values given in the table above. Which of the following statements must be true?

    I. By the Intermediate Value Theorem, there is a value c in the interval $(0, 3)$ such that $f(c) = 2$.
    II. By the Mean Value Theorem, there is a value c in the interval $(0, 3)$ such that $f'(c) = 2$.
    III. By the Extreme Value Theorem, there is a value c in the interval $[0, 3]$ such that $f(c) \le f(x)$ for all x in the interval $[0, 3]$.

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    (A)   None

    (B)   I only

    (C)   II only

    (D)   I, II, and III

5.   Let g be the function given by $g(x) = \sqrt{1 + \cos x}$. Which of the following statements could be false on the interval $\frac{\pi}{2} \leq x \leq \frac{7\pi}{4}$?

    (A)   By the Extreme Value Theorem, there is a value c such that $g(c) \leq g(x)$ for $\frac{\pi}{2} \leq x \leq \frac{7\pi}{4}$.

    (B)   By the Extreme Value Theorem, there is a value c such that $g(c) \geq g(x)$ for $\frac{\pi}{2} \leq x \leq \frac{7\pi}{4}$.

    (C)   By the Intermediate Value Theorem, there is a value c such that $g(c) = \frac{g\left(\frac{\pi}{2}\right) + g\left(\frac{7\pi}{4}\right)}{2}$.

    (D)   By the Mean Value Theorem, there is a value c such that $g'(c) = \frac{g\left(\frac{7\pi}{4}\right) - g\left(\frac{\pi}{2}\right)}{\frac{7\pi}{4} - \frac{\pi}{2}}$.