1/2/2020

Q.1 Consider a multimode fiber that has a core refractive index of 1.48 and core cladding index difference 2%. Find the (i) numerical aperture NA
(ii) acceptance angle
(iii) critical angle.

why $n = 1$ what is $n = ?$

Given: $n_1 = 1.48$, $\Delta = 0.02$

$NA = n_1\sqrt{2\Delta} = 0.296$

$\theta_A = n.\sin^{-1}(NA) = (1)\sin^{-1}(0.296) = \underline{17.2}$

$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(1.48)^2 - n_2^2} = 0.296$

$\boxed{n_2 = 1.45}$

$\sin\phi_c = \frac{n_2}{n_1} \Rightarrow \boxed{\phi_c = 78.44°}$

Q2. Consider the interface b/w the fiber core and cladding materials that have a refractive indices of n_1 and n_2 respectively. If $n_2 < n_1$ by ~~0~~ 1% and $n_1 = 1.45$. Show that: $n_2 = 1.435$
Also show that the critical angle is
$\phi_c = 81.9°$

$\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$\boxed{\phi_c = 81.90°}$

$n_2 = n_1(1-\Delta)$
$= 1.45(1-0.01)$
$= 1.45(0.99)$

$\boxed{n_2 = 1.435}$

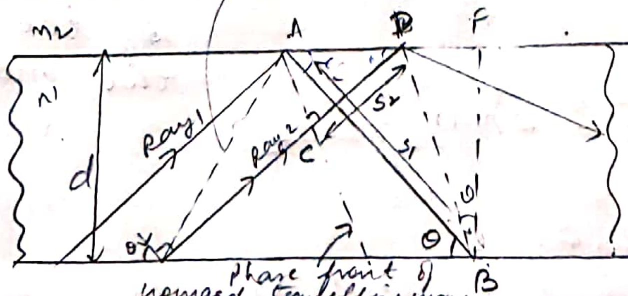
* Ray Representation :-

(P1)

phase front of downward travelling wave

Ray rep diagram

180 - 90 - θ
90 - θ



Light wave propagating along the fibre waveguide

Objective: The angle θ must satisfy the condⁿ to propagate in the dielectric slab waveguide

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Let us consider ... the rays are incident on the material interface at an angle $\theta < \theta_c = \frac{\pi}{2} - \phi_c$.

$$S_1 = d/\sin\theta$$

$S_2 =$

$$\tan\theta = \frac{FB}{AF} = \frac{d}{AF}$$

$$\boxed{AF = \frac{d}{\tan\theta}}$$

$$\overline{AD} = \overline{AF} - \overline{DF}$$

$$\overline{AD} = \frac{d}{\tan\theta} - d\tan\theta.$$

$$\tan\theta = \frac{DF}{d} \Rightarrow \boxed{DF = d\tan\theta}$$

$$S_2 = \overline{AD}\cos\theta$$

$$\boxed{S_2 = \frac{(\cos^2\theta - \sin^2\theta)d}{\sin\theta}}$$

As the rays travelling inside the dielectric slab from point A to point B and point C to point D must have phase difference by an integer multiple of 2π. Thus the waves travel through the material, it undergoes a phaseshift "δ" given by

K_1 = propagation in n_1

K = propagation (free space) constant

Q.** IM Prob

$$\boxed{\Delta = K_1 S = n_1 K S}$$

$$\boxed{\Delta = n_1 \frac{2\pi}{\lambda} S}$$

$$K = \frac{2\pi}{\lambda}$$

n_1 = core refractive index

S = distance travelled by the wave in the material.

The requirement for wave propagation can be written as:

$$\frac{2\pi n_1}{\lambda}(S_1 - S_2) + 2\delta = 2\pi m \quad \text{--- (1)}$$

where $m = 0, 1, 2, 3, \ldots$

Substituting value of S_1 and S_2 in eq^n (1)

$$\frac{2\pi n_1}{\lambda}\left(\left(d/\sin\theta\right) - \left(\frac{\cos^2\theta - \sin^2\theta}{\sin\theta}\right)d\right) + 2\delta = 2\pi m$$

$$\frac{2\pi n_1 d}{\lambda \sin\theta}\left[\frac{1}{m} - \cos^2\theta + \sin^2\theta\right] + 2\delta = 2\pi m$$

$$\frac{2\pi n_1 d}{\lambda \sin\theta}\left[\underbrace{\sin^2\theta + \cos^2\theta}_{} - \cancel{\cos^2\theta} + \sin^2\theta\right] + 2\delta = 2\pi m$$

$$\frac{\cancel{4}^{2}\pi n_1 d}{\lambda}\sin\theta + \cancel{2}\delta = \cancel{2}\pi m$$

$$\boxed{\frac{2\pi n_1 d}{\lambda}\sin\theta + \delta = \pi m} \quad \text{—— (2)}$$

$$\tan\frac{\delta_N}{2} = \frac{\sqrt{n^2\cos^2\theta - 1}}{n\sin\theta}$$

where $n = n_1/n_2$

$$\delta = -2\tan^{-1}\left(\frac{\sqrt{\cos^2\theta - (n_2^2/n_1^2)}}{\sin\theta}\right) \text{—}$$

If we consider only electric waves with component normal to the plane of incidence, the phase shift of reflection is

$$\delta = -2\tan^{-1}\left(\frac{\sqrt{\cos^2\theta - (n_2^2/n_1^2)}}{\sin\theta}\right) \text{—— (3)}$$

note
-ve sign because wave is decaying

Put (3) in (2)

$$\frac{2\pi n_1 d}{\lambda}\sin\theta - \pi m = 2\tan^{-1}\left(\frac{\sqrt{\cos^2\theta - (n_2^2/n_1^2)}}{\sin\theta}\right)$$

$$\tan\left[\frac{\pi n_1 d\sin\theta}{\lambda} - \frac{\pi m}{2}\right] = \frac{\sqrt{\cos^2\theta - n_2^2/n_1^2}}{\sin\theta}$$

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Mode theory for circular waveguides

overview circular waveguides (optical domain)

TE_{lm} mode } X

He_{11} → fundamental mode of TE comp

TE ~~mode~~ component > TM comp }
TM component > TE } X

TE_{lm} modes, TM_{lm} modes are the electric field and magnetic field vector lying in transverse plane.

T → Transverse
H → Hybrid

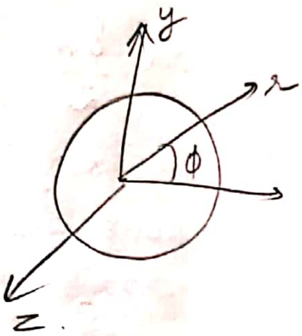
HE_{lm} mode

TE comp > TM component.

whereas;

EH_{lm} mode,

TM component > TE component



where

l = no. of variation cycles or zeroes in φ direction
m = no. of circles or zeroes in 'r' direction

LP (Linear polarised modes) in weakly guided fibres

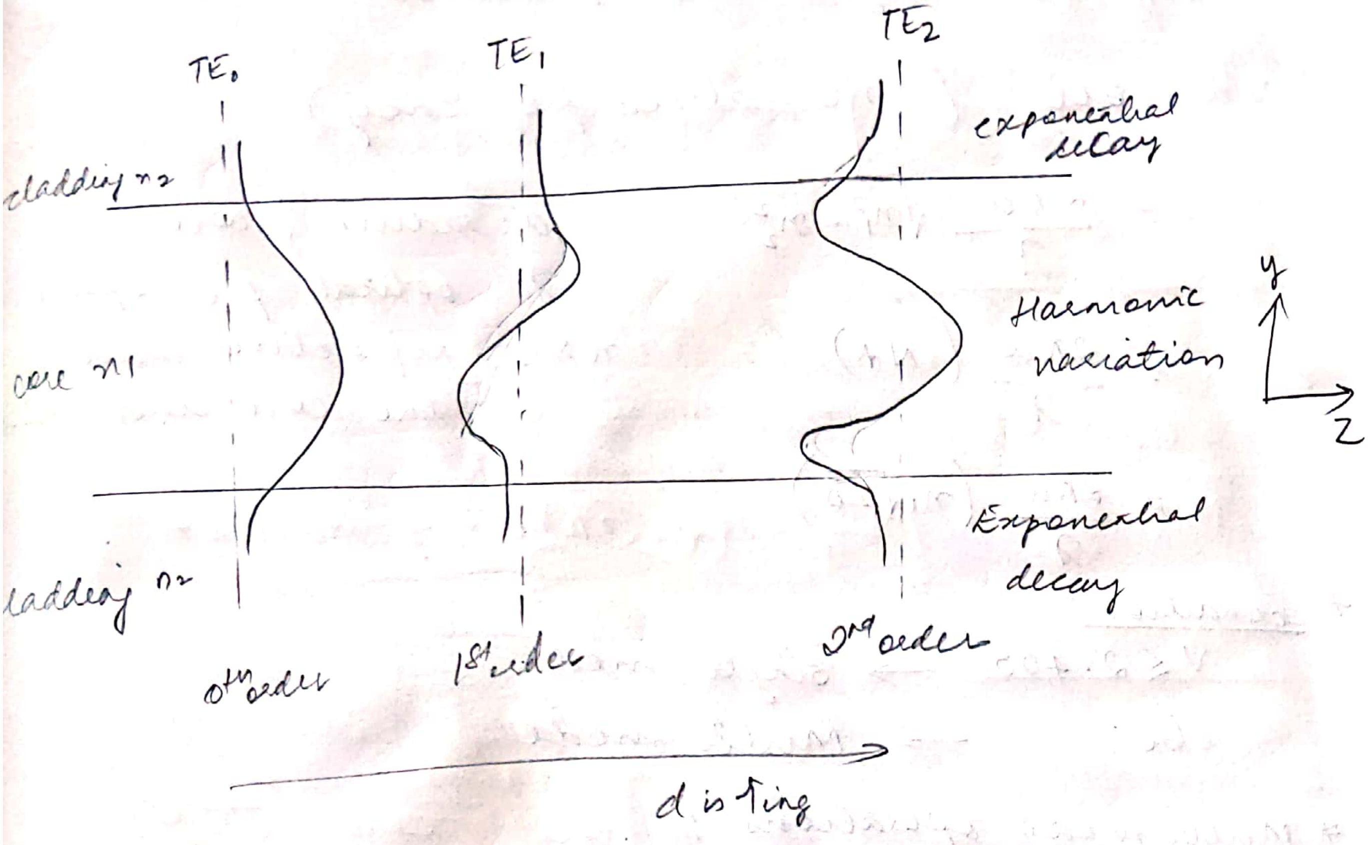
The weakly guided fibres to be considered when $n_1 - n_2 << 1$

* LP_{0m} (HE_{lm}) – single mode
LP_{lm} (TE_{0m} + TM_{0m} + HE_{0m}) - multimode

Fundamental mode LP_{01} (HE_{11})

BCN

TE modes in slab waveguide:-



Linear Polarized

$$E_m(x, y, z, t) = e_x f_m(y) \cos(\omega t - \beta_m z)$$

$m = 0, 1, 2, 3$ (mode number)

* Important condition:

* A mode remains guided as long as

$$n_2 k < \beta \;,\; \beta < n_1 k.$$

$$n_2 k < \beta < n_1 k$$

The order of the mode = the no of field zeroes across the guide

The order the mode is also related to the angle in which ray congruence corresponding to this mode makes within the plane of waveguide / plane of ~~the~~ the fibre axis.

The steeper ~~of~~ the angle, higher the ~~the~~ order of the mode.

8/2/2020

cut off wavelength and V-number:-

V number (normalised frequency)

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

a: radius of core

λ: optical free space λ

n_1 & n_2: refractive indices of the core and cladding

$$= \frac{2\pi a}{\lambda}(NA)$$

$$= \frac{2\pi a}{\lambda}(n_1\sqrt{2\Delta})$$

* condition:

$V \leq 2.405 \longrightarrow$ single mode

else $\longrightarrow$ Multi mode.

* Multi mode operations :-

$$M \simeq \frac{V^2}{2}$$

* M = total number of modes, supported by a multi mode fibre is approximately (when V is large) is given by:

* Power distribution in the core and the cladding:

Another quantity of interest is the is the ratio of the mode power in the cladding P_{clad} to the total optical power in the fibre P, which at the wavelengths (or frequencies) far from the cut off is given by:

$$\boxed{\frac{P_{clad}}{P} \approx \frac{4}{3\sqrt{M}}} \leftarrow \text{Power distribution}$$

Since $M \propto V^2$, the power flow in the cladding decreases as V increase. However this increase in no of modes in the fiber is not desirable for high BW capability.

problems :

1. Consider a multi mode step index fibre with a 62.5 μm core diameter and core cladding index difference of 1.5 %. If the core refractive index is 1.48 Estimate the normalised freq of the fibre and total no of modes supported in the fibre at a λ of 850 nm

Given :

$a = \frac{62.5}{2}\ \mu m = \underline{31.25\ \mu m}$

$\lambda = 850\ nm$

$n_1 = 1.48$

$\Delta = 1.5\ \% = 0.015$

To find : V , M

$V = \frac{2\pi a}{\lambda}(NA)$; $NA = n_1\sqrt{2\Delta} = \underline{0.256}$

$= \frac{(2 \times 3.14 \times 31.25 \times 10^{-6})(0.256)}{850 \times 10^{-9}}$

$\boxed{V = 59.185}$

$M = \frac{V^2}{2} = \frac{(59.185)^2}{2} = 1751.43$

$\boxed{M = 1752}$

Q2. Consider a multi mode step index fibre that has a core radius of 25 μm of core index of 1.48 and index difference $\Delta = 0.01$. what are the no of modes in the fibre at a $\lambda = 860\ nm$, ~~1300 130~~ 1310 nm and 1550 nm.

a) : Given : $a = 25\ \mu m$
$n_1 = 1.48$
$\Delta = 0.01$
$\lambda = 860\ nm$.

To find : V = ?
M = ?

$M = \frac{V^2}{2} = \left[\frac{2\pi a^{\times 10^{-6}}}{\lambda}(n_1\sqrt{2\Delta})\right]^2 \cdot \frac{1}{2} = \underline{730}$

b) $a = 25\,\mu m$ $\quad n_1 = 1.48$
$\Delta = 0.01$ $\quad \lambda = 1310\,nm$

$$M = \frac{V^2}{2} = \left[\frac{2\pi a}{\lambda}\left(n_1\sqrt{2\Delta}\right)\right]^2 \cdot \frac{1}{2} = \underline{\underline{314.62}}$$

$$\boxed{M = 315}$$

c) $a = 25\,\mu m$ $\quad n_1 = 1.48$
$\Delta = 0.01$ $\quad \lambda = 1550\,nm$.

$$M = \frac{V^2}{2} = \left(\frac{2\pi a}{\lambda}\left(n_1\sqrt{2\Delta}\right)\right)^2 \cdot \frac{1}{2} = 224.73$$

$$\boxed{M = 225}$$

Q3. Consider a multimode step index fibre that has a core radius of $\underline{25\,\mu m}$ (a), core index of $\underline{1.48}$ (n_1) and index difference $\underline{= 0.01}$ (Δ). Find the % of optical power that operates in the cladding at 840 nm.

A: $$M = \frac{V^2}{2} = \left(\frac{2\pi a}{\lambda}\left(n_1\sqrt{2\Delta}\right)\right)^2 \cdot \frac{1}{2} = 765.181$$

* ~~Single mode fibres~~ operation :-

→ the cut off λ or freq for each mode is obtained from :-

$$\beta_{lm}(\omega_c) = n_2 k = \frac{2\pi n_2}{\lambda_c} = \frac{\omega_c n_2}{c}$$

→ Single mode operation is possible (single mode fibre) when :-

$V \leq 2.405$

→ Only HE_{11} can propagate faithfully along optical fibre

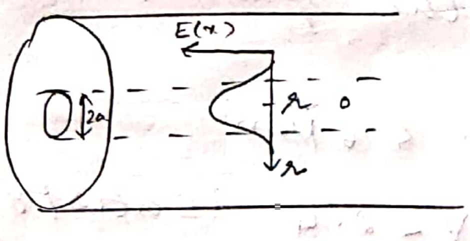
single mode fibres

Δ = 0.1% to 1% ; a = 6 to 12 μm ;

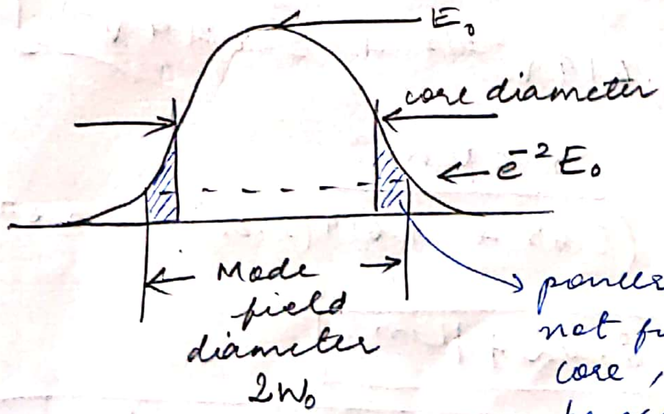
V = 2.3 to 2.4 @ max freq or min λ

* Example. A fibre with radius of 4 μm and $n_1 = 1.5$ and $n_2 = 1.498$ has a normalised freq $V = 2.38$ at a λ 1 μm. The fibre is single mode for all λ ...

* Mode field diameter :-



<peak is obtained at $r = 0$>
<centre of the fibre>



note
($2W_0 > 2a$.)

To find MFD, is to measure the far field intensity distribution, $E^2(r)$ and then calculate the MFD using Peterman II equation.

$$MFD = 2w_0$$
$$= 2\left[\frac{2\int_0^\infty E^2(r)\, r^3\, dr}{\int_0^\infty E^2(r)\, r\, dr}\right]^{1/2}$$

, where the parameter w_0 is the full width of the far field distribution,
w_0 is also called as spot size or MFRadius

@ $r=0$, we will get peak value.

$$E(r) = E_0 \exp\left(-r^2/w_0^2\right)$$

Approximation, relative spot size $\left(\frac{w_0}{a}\right)$ for $1.2 < V < 2.4$

$$\boxed{\left(\frac{w_0}{a}\right) = 0.65 + 1.619\, V^{-3/2} + 2.879\, V^{-1}}$$

if $V = 2.405$, $(w_0/a) = 1.1005$

Q1. A certain single mode step index fiber has MFD $= 11.2\ \mu m$ and $V = 2.25$. What is the core diameter of this fiber?

A: $w_0 = \frac{MFD}{2} = \frac{11.2}{2}\ \mu m = 5.6\ \mu m$.

$V = 2.25 \longrightarrow 1.2 < V < 2.4$ (satisfying)

$$\frac{w_0}{a} = \left[0.65 + 1.619\, V^{-3/2} + 2.879\, V^{-1}\right] = x. \quad \text{(say)}$$

$$a = \frac{w_0}{x} = \frac{5.6\ \mu m}{\left(0.65 + 1.619(2.25)^{-3/2} + 2.879(2.25)^{-1}\right)}$$

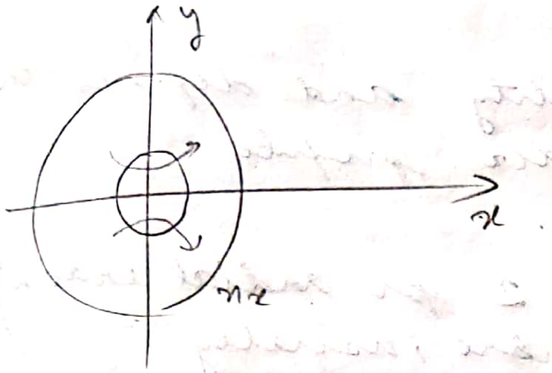
$$\boxed{a = 2.324\ \mu m}$$

$$\boxed{\text{core diameter} = 2a = 4.648\ \mu m}$$

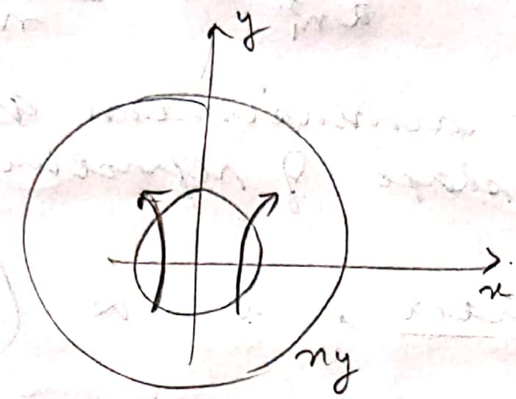
* Birefringence in single-mode fibre (SMF) :-

Because of asymmetric the refractive indices for the 2 degenerate modes (vertical & horizontal polarization) are different. This difference is referred to as birefringence, B_f :

$$\boxed{B_f = n_y - n_x}$$



horizontal mode.



vertical mode

$$\boxed{\beta = k_0 (n_y - x_x)} \quad ; \quad \boxed{B_f = \frac{\beta}{k_0}}$$

where k_0 = free space propagation constant $\frac{2\pi}{\lambda}$

* Fibre beat length :-

In general, a linearly polarized mode is a comb of both of degenerate modes. As the modal wave travels along the fibre, the diff. in refractive indices would change the phase diff b/w these 2 components & thereby the state of polarization of the mode. However after certain length referred to as FBL, the modal wave will produce its original state of polarization. This length is simply given by

$$\boxed{L_p = \frac{2\pi}{K B_f}}$$

1/2/2020

Graded Index fibre : (Refractive index Profile)

$$n(r) = \begin{cases} n_1 \left[1 - 2\Delta\left(\frac{r}{a}\right)^{\alpha}\right] \\ n_1(1-2\Delta)^{1/2} \simeq n_1(1-\Delta) = n_2 \text{ for } r > a \end{cases}$$

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \simeq \frac{n_1 - n_2}{n_1}$$

α = dimensionless quantity and defines the shape of refractive index profile

For Step index, $\alpha = \infty$ ($\alpha = 2$ for radial index fibre, usually)

$$NA(r) = \begin{cases} \left[n^2(r) - n_2^2\right]^{1/2} \simeq NA(0)\sqrt{1-\left(\frac{r}{a}\right)^{\alpha}} \ ; \ r \le a \\ 0 \quad \text{for } r > a \end{cases}$$

where axis NA defined as

$$NA(0) = \left(n^2(0) - n_2^2\right)^{1/2} = \left(n_1^2 - n_2^2\right)^{1/2}$$
$$= n_1\sqrt{2\Delta}$$

The no of bound modes in a graded index fibre is

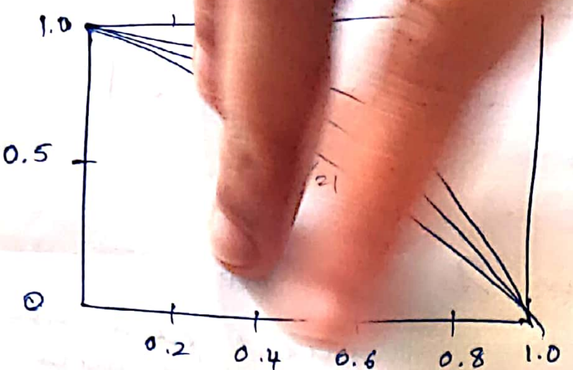
$$M_g = \frac{\alpha}{\alpha+2} a^2 k^2 n_1^2 \Delta \simeq \frac{\alpha}{\alpha+2} \cdot \frac{V^2}{2}$$

where $k = \frac{2\pi}{\lambda}$

---

if $\alpha = 2$,

If $\alpha = 2$, M_g = [illegible] (step index f[illegible]



$\alpha = 2$ preferred commercially though both 1 and 4 are also available

M (mode) of GIFS

V is given by

$$\boxed{V = 2.405\sqrt{1+\frac{2}{\alpha}}}$$

* Problems :-

Q1. If we have 50 μm dia GIF that has a parabolic reflective index of $\alpha = \sqrt{2}$. If the fibre has a NA = 0.22, what is the total length of modes at 1310 nm?

A: $$V = \frac{2\pi a}{\lambda}(NA)$$

$$= \frac{2\pi \times 25\times10^{-6}}{1310\times10^{-9}}[0.22]$$

$$\boxed{V = 26.38}$$

$$M = \frac{V^2}{2} = 347.95$$

$$M_g = \frac{V^2}{4} = \frac{\alpha}{\alpha+2}\times\frac{V^2}{2}.$$

$$\boxed{M_g = 173.975}$$

Q2. Calculate the no of modes at 820 nm and 1.3 μm in a GIF, having a parabolic index profile $\alpha = 2$. A 25 μm core radius $n_1 = 1.48$ and $n_2 = 1.46$. Then how does this compare to a SIF?

5/2/2020

Absorption :-

caused by 3 different mechanisms :-

1. Impurities in fibre material
2. extrinsic absorption → fibre composition
3. Atomic defects

glass fibre
↓
usually adopted
↓

*Absorption is caused by 3 diff mechanisms :-

(1) Absortion by atomic defects in the glass composition

(2) Extrinsic absorption by impurity atoms in the glass material

(3) Intrinsic absorption by the basic constituent atoms of the fibre material.

Absorption → J/Kg

Mathematics :- → PROBLEMS

GeO2 doped fibre material

$$\alpha_{UV} = C e^{E/E_0} \text{ dB/km} \rightarrow \text{Urbach's rule}$$

E → photon energy; C, E_0 → emphirical constant

$$\alpha_{UV} = \frac{154.2x}{46.6x+60} \times 10^{-2} \exp\left(\frac{4.63}{\lambda}\right) \text{ dB/km}$$

$$\alpha_{IR} = 7.18 \times 10^{11} \times \exp\left(-\frac{48.48}{\lambda}\right) \text{ dB/km.}$$

emphirical formula need to use chart → emphirical expressions observed using experimental setup.

here x → % of doping
if 6% doping is done ⇒ x = 0.06

* Overall attenuation consists of
  - → absorption
  - → scattering
  - → Bending

Problem :-

Q1. Consider 2 silica fibres that are doped with 6% and 18% ~~wholt~~ mole fraction of GeO_2 respectively compare the UV absorptions at λ of 0.7 μm and 1.3 μm.

Given :

(DOUBT)

$\lambda = 0.7\ \mu m,\ 1.3\ \mu m$

$n = 6\%,\ 18\%$

0.06 0.18

note:

to convert to dB.
10 log ( )

CASE I $\lambda = 0.7\ \mu m$ $x = 0.06$

$$\boxed{\alpha_{UV} = \frac{154.2\,x}{46.6x + 60} \times 10^{-2} \exp\left(\frac{4.63}{\lambda}\right)}$$

→ leave it in exp form

$= 1.47 \times 10^{-3} \exp(6.6 \times 10^{6})$

$= 1.10\ dB/km$

CASE II : $\lambda = 0.7\ \mu m$ $x = 0.18$.

CASE III : $\lambda = 1.3\ \mu m$, $x = 0.06 \rightarrow 0.07$ (dB/km)

CASE IV : $\lambda = 1.3\ \mu m$, $x = 0.18$

* Scattering Loss :- (ppt)

→ Scattering losses are due to microscopic variations in the material density, compositional fluctuations (inclusion of GeO_2, P_2O_5, etc) structural inhomogeneities and defects

→ The 1st 2 effects give rise to refractive index variations

→ The refractive index variation causes Rayleigh scattering, which is the same ~~as~~ phenomenon as the scattering of light in the atmosphere giving rise to blue sky.

→ Rayleigh scattering is proportional to λ^{-4}

→ The structural inhomogeneities and defects (created during fabrication) can cause scattering of light out of fibre. These defect include trapped gas bubbles, unreacted starting materials and crystallised regions.

→ These defect are minimized by improving perform manufacturing methods and this scattering is negligible compared to the Rayleigh scattering.

$$\boxed{\alpha_{scattering} = \frac{8\pi^3}{3\lambda^4}(n^2-1)\,K_B\,T_f\,\beta_T}$$

Base 'e' units nepers/km $\Rightarrow X$

Note: To get dB/km, $(X * 4.343) \Rightarrow$ dB/km

(or)

$$\boxed{\alpha_{scattering} = \frac{8\pi^3}{3\lambda^4}\,n^3 p^2 K_B\,T_f\,\beta_T}$$ nepers/km.

where

β_T = isothermal compressibility of the material

T_f = Fictive temperature (temp at which the density fluctuations are frozen into the glass as it solidifies.)

p = photo elastic coefficient

K_B = Boltzmann constant

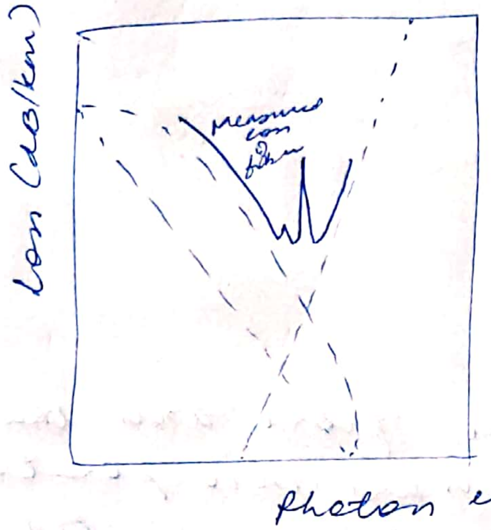
(ppt)

1) Impurities in fibre materials : from transition metal ion (must be in order of parts per billion (ppb)) and particularly from OH ions with absorption peaks at wavelengths 2700 nm, 1400 nm, 950 nm and 725 nm.

2. Intrinsic absorption (fundamental lower limit): Electronic absorption band (UV region) and atomic bond ~~vai~~ vibration band (IR region) in basic SO_2.

3. Atomic defects : typically relatively small except exposed to nuclear radiation level (eg, inside a nuclear reactor, nuclear explosion).

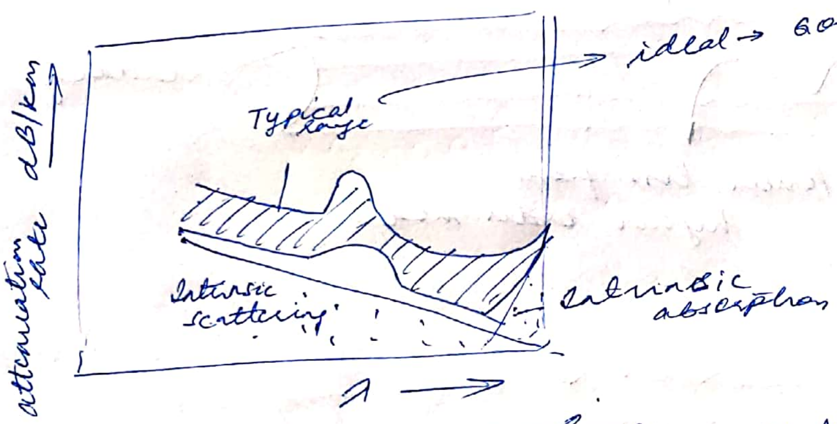
Absorption and scattering losses in fibres :-



(graph not there)

neper/km → 'base e' units → don't multiply 4.343.
dB/km → ~~base 10~~ → [x 1/4 4.343 ie 10 loge] → ~~[illegible]~~

* Typical spectral absorption and scattering attenuation for single mode fibre



LOSS not (FIXED)

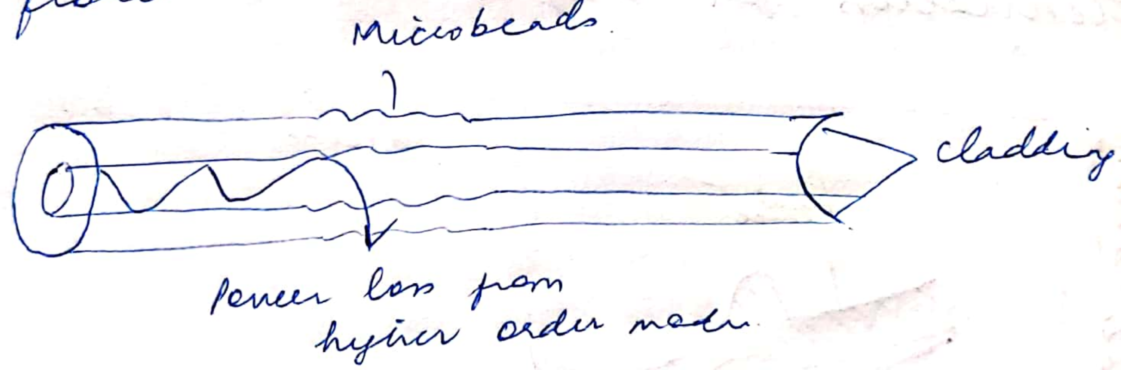
usually not called loss (dB/km)

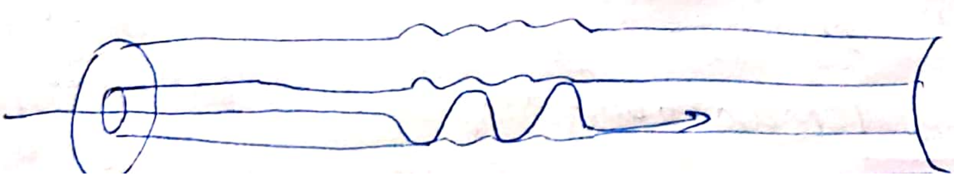
* Bending loss (Macrobending & Microbending)
↳ installation

Macrobending loss : The curvature of the bend is much larger than fibre diameter. lightwave suffers ~~se~~ sever loss due to radiation of the evanescent field in the cladding region. As the radius of the curvature decreases, the loss increases exponentially until it reaches at a certain critical radius. For any radius a bit ~~sm~~ smaller than this point, the losses suddenly becomes extremely large. Higher order modes radiate faster than lower order mode.

* Microbending loss

Microscopic bends of the fibre axis that can arise when the fibers are incorporated into the cable. The power is dissipated through the microbended fibre, because of the repetitive coupling of energy between guided modes and the leaky or radiation modes in the fibre





① a

b 1 prob → overview of mode

c

I

NA, using ray theo, how light ray propagate inside fiber? refn → refra, refle, TIR,

② a

b 1 prob

c

. 6 Index stu with mathem notations
. exp about fiber materials
. Loss → 1Q. ** VERY IMP.
. (fibre loss → about attenuation, scattering, bending loss.)

Macrobending :-

$$M_{eff} = M_{\infty}\left\{1 - \frac{\alpha+2}{2\alpha\Delta}\left[\frac{2a}{R} + \left(\frac{3}{2n_2 kR}\right)^{2/3}\right]\right\}$$

R ← Radius of curvature

$$M_{\infty} = \frac{\alpha}{\alpha+2}(n_1 ka)^2\Delta$$ = Total no of modes in a straight fiber.

α = index profile → graded index
for step index, $\alpha = \infty$

Δ = core cladding index diff

a = core radius

R = Radius of curvature of macrobending

k = wave propagation constant $= (2\pi/\lambda)$

$$\alpha_{VM} = \alpha_1 \frac{P_{core}}{P} + \alpha_2 \frac{P_{clad}}{P}$$

fractional Power

$$\boxed{\alpha_{VM} = \alpha_1 + (\alpha_2 - \alpha_1)\frac{P_{clad}}{P}}$$

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