Consider the ~~job~~ problem of asinsing n jobs to n persons such that each jobs can be ~~assigned~~ to only one person and each person can be alloted only one job. Suppose all the n persons are capable of doing any one of n jobs but the time taken by them to perform that jobs varies from job to job. The following table represents the time taken by n persons to perform n jobs.

	Jobs			
	J_1	J_2	----	J_n
Persons P_1	t_{11}	t_{12}	----	t_{1n}
P_2	t_{21}	t_{22}	----	t_{2n}
P_n	t_{n1}	t_{n2}		t_{nn}

Our interest is to decide which job has to be assigned to which person so that the total time taken to perform all this n jobs is minimum.

This is called assignment problem.

mathematical formulation of an Assignment problem.

minimize the total cost.

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to,

(i) $\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1, 2, \ldots n$

(ii) $\sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, n$

(1) Balanced Assignment problem.

In an assignment problem no of persons is less than the number of jobs or the no of jobs is less than the no of persons then (C_{ij}) will not be a square matrix. Such an assignment problem is called an unbalanced assignment problem.

Balanced assignment problem.

In an assignment problem no of person equals to the no of jobs the (c_{ij}) cost matrix is a square matrix. Such kind of assignment problems are called balanced assignment problem.

Procedure for Problem.

step 1: Choose an les row and sub element of th

step 2: Choose an Column a all the

step 3: Test Only and e we is Con

Procedure for Solving Assignment Problem.

Step 1: Choose an least element in each row and subtract it from all element of that row.

Step 2: Choose an least element in each column and subtract it ~~add~~ from all the elements of that column.

Step 3: Test whether we can choose only one zero in each row and each column if such thing we obtained then the problem is completed. Suppose if not Connect all zeroes by minimum no of lines choose a minimum element which does not passes through the line and subtract such element in all element which does not passes through the line and add such element wherever lines are intersected and remind

same only the line passes elements. Then you have check steps again. you have to do this process untill upto getting our step 3.

① Find the optimal solution for the assignment problem with the following cost matrix.

Salesman \ Area	W	X	Y	Z
A	11	17	8	16
B	9	7	12	6
C	13	16	15	12
D	14	10	12	11

Soln

$$\begin{pmatrix} 11 & 17 & 8 & 16 \\ 9 & 7 & 12 & 6 \\ 13 & 16 & 15 & 12 \\ 14 & 10 & 12 & 11 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 9 & 0 & 8 \\ 3 & 1 & 6 & 0 \\ 1 & 4 & 3 & 0 \\ 4 & 0 & 2 & 1 \end{pmatrix}$$

	W	X	Y	Z
A	2	9	[0]	8
B	2	1	6	[0]
C	[0]	4	3	0
D	3	[0]	2	1

So the assignment is,

$A \rightarrow Y,\ B \rightarrow Z,\ C \rightarrow W,\ D \rightarrow X$

minimum cost

$= ₹\ (8+6+13+10)$

$= ₹\ 37$ //.

. Solve the assignment followings Problems

(i)

Plant	Product 1	2	3	4
A	1	8	4	1
B	5	7	6	5
C	3	5	4	12
D	3	1	6	3

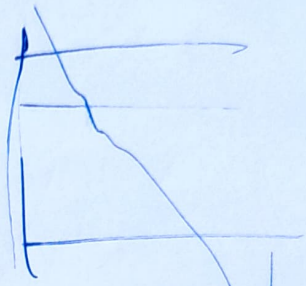
(ii)

methods	Work centre A	B	C	D
1	10	7	8	0
2	8	9	7	0
3	7	12	6	0
4	10	10	8	0

i)

$$\begin{pmatrix} 1 & 8 & 4 & 1 \\ 5 & 7 & 6 & 5 \\ 3 & 5 & 4 & 12 \\ 3 & 1 & 6 & 3 \end{pmatrix}$$

$$\begin{pmatrix} \cancel{} & 7 & 3 & \boxed{0} \\ \cancel{} & 2 & 1 & \cancel{} \\ \boxed{0} & 2 & 1 & 9 \\ 2 & \boxed{0} & 3 & 2 \end{pmatrix}$$



$$\begin{pmatrix} \boxed{0} & 7 & 2 & \cancel{} \\ \cancel{} & 2 & \cancel{} & \boxed{0} \\ \cancel{} & 2 & \boxed{0} & 9 \\ 2 & \boxed{0} & 2 & 2 \end{pmatrix}$$

Find the optimal solution of a given assignment problem.

Contractors	J_1	J_2	J_3	J_4
	Jobs			
C_1	15	27	35	20
C_2	21	29	33	17
C_3	17	25	37	15
C_4	14	31	39	21

Soln :-

$$\begin{pmatrix} 15 & 27 & 35 & 20 \\ 21 & 29 & 33 & 17 \\ 17 & 25 & 37 & 15 \\ 14 & 31 & 39 & 21 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 12 & 20 & 5 \\ 4 & 12 & 16 & 0 \\ 2 & 10 & 22 & 0 \\ 0 & 17 & 25 & 7 \end{pmatrix}$$

	[0]	2	1	~~0~~
	1	2	[0]	~~0~~
	2	[0]	6	~~0~~
	~~0~~	7	9	7

	0	2	4	5
4	2	0	0	
2	0	6	0	
0	7	9	7	

	J_1	J_2	J_3	J_4
C_1	0	[0]	2	3
C_2	6	2	[0]	~~0~~
C_3	4	~~0~~	6	[0]
C_4	[0]	5	7	5

The assignment is

$C_1 \rightarrow J_2$, $C_2 \rightarrow J_3$, $C_3 \rightarrow J_4$, $C_4 \rightarrow J_1$

The minimum cost is.

$= ₹ (27 + 33 + 15 + 14)$

$= ₹ 89$ //.

Restricted.... Assignment problem.....

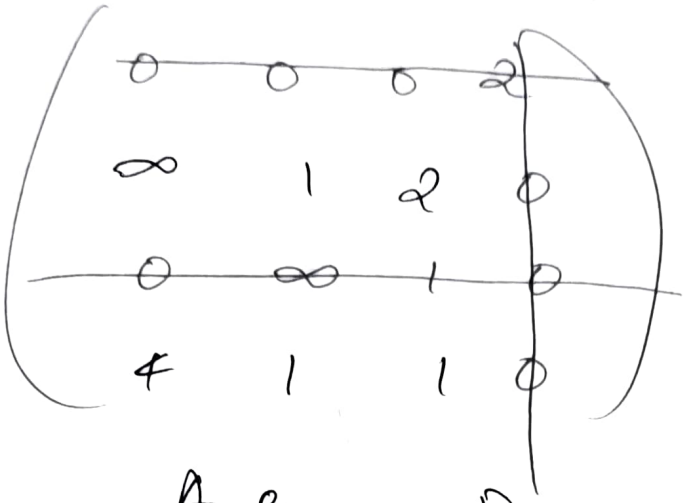
Solve the assignment problem

Clerks \ Tasks	A	B	C	D
1	4	7	5	6
2	-	8	7	4
3	3	-	5	3
4	6	6	4	2

Sol^n

$$\begin{pmatrix} 4 & 7 & 5 & 6 \\ \infty & 8 & 7 & 4 \\ 3 & \infty & 5 & 3 \\ 6 & 6 & 4 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 3 & 1 & 2 \\ \infty & 4 & 3 & 0 \\ 0 & \infty & 2 & 0 \\ 4 & 4 & 2 & 0 \end{pmatrix}$$



	A	B	C	D
C_1	~~∞~~	[0]	~~0~~	3
C_2	∞	~~∞~~	1	[0]
C_3	[0]	∞	1	1
C_4	3	~~∞~~	[0]	~~0~~

The assignment is,

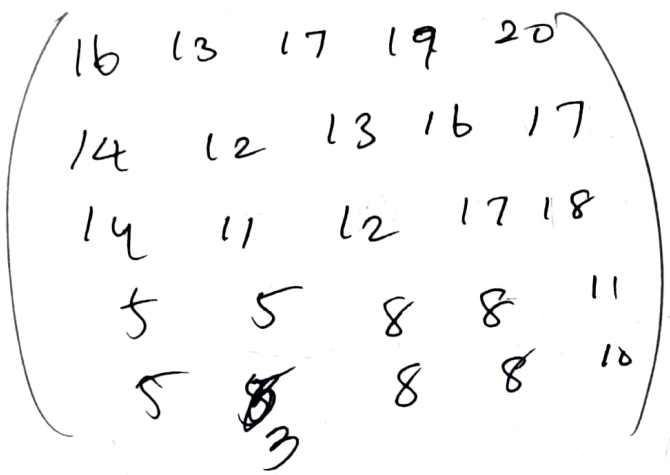
$C_1 \rightarrow B$, $C_2 \rightarrow D$, $C_3 \rightarrow A$, $C_4 \rightarrow C$

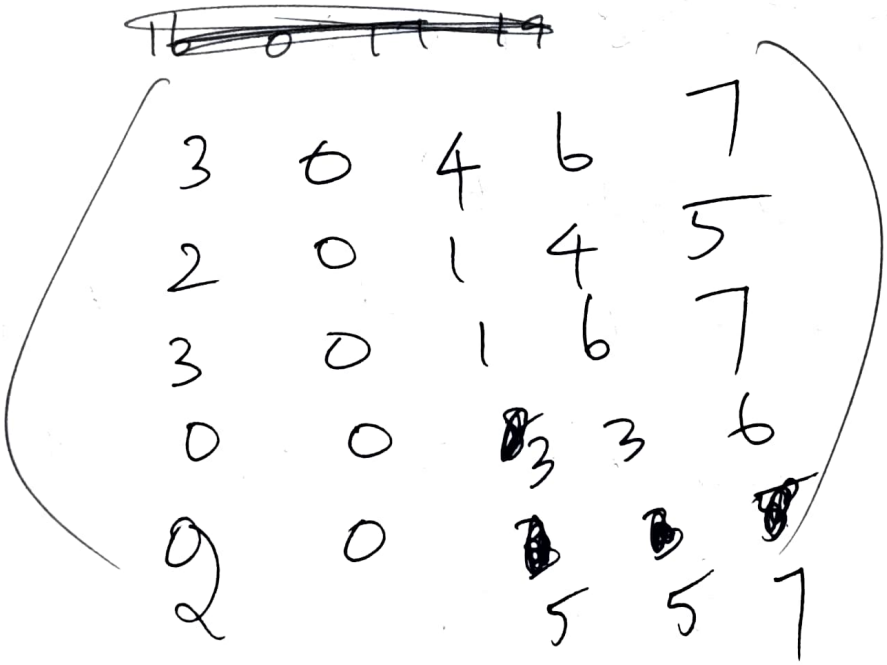
minimum cost = ₹(7+4+3+4)

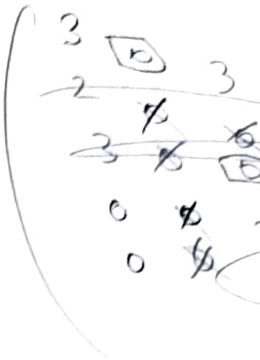
= ₹(18).

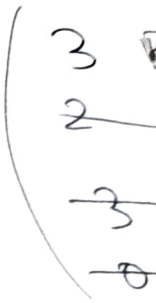
1.

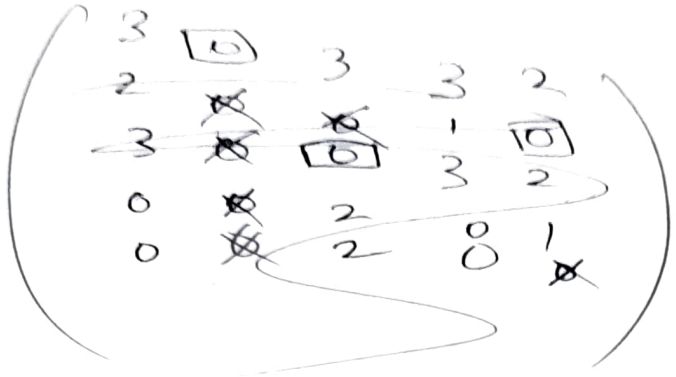
	1	2	3	4	5
A	16	13	17	19	20
B	14	12	13	16	17
C	14	11	12	17	18
D	5	5	8	8	11
E	5	3	8	8	10

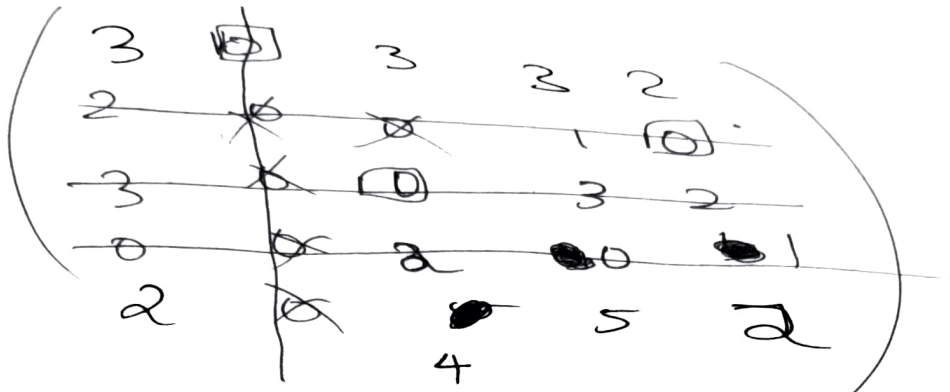


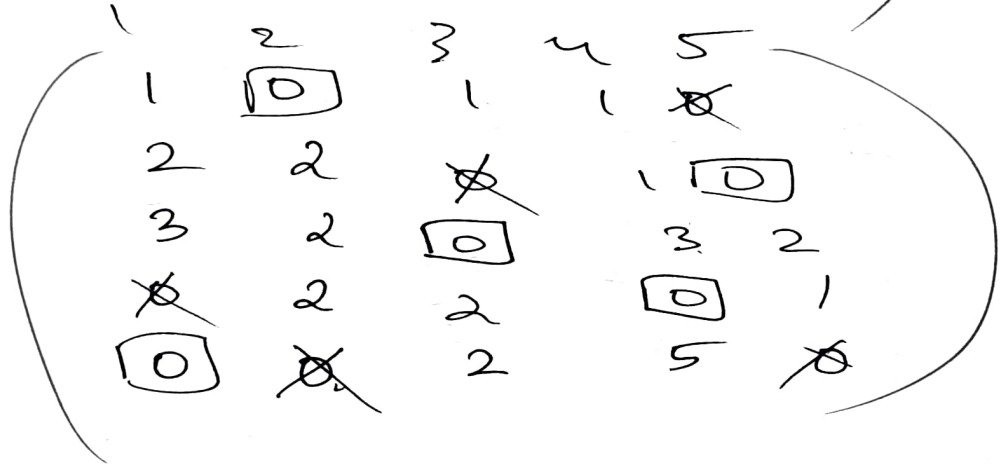












A
B
C
D
E

A → 2, B → 5, C → 3, D → 1, E → 4

$= \frac{1}{2}(13+17+12+8+5)$

$= 55$ minutes

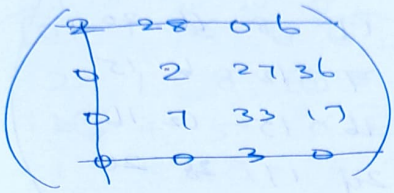
2.

	A	B	C	D
W	41	72	39	52
X	22	29	49	65
Y	27	39	60	51
Z	45	50	48	52

$$\begin{pmatrix} 41 & 72 & 39 & 52 \\ 22 & 29 & 49 & 65 \\ 27 & 39 & 60 & 51 \\ 45 & 50 & 48 & 52 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 33 & 0 & 13 \\ 0 & 7 & 27 & 43 \\ 0 & 12 & 33 & 24 \\ 0 & 5 & 3 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 28 & 0 & 6 \\ 0 & 2 & 27 & 36 \\ 0 & 7 & 33 & 17 \\ 0 & 0 & 3 & 0 \end{pmatrix}$$



	A	B	C	D
W	4	28	0	6
X	~~0~~	0	25	34
Y	0	5	31	15
Z	2	~~0~~	3	0

$W \to C, X \to B, Y \to A, Z \to D$

$= (39 + 29 + 27 + 52)$

$= 147$ hours

3)

11	17	8	16	20
9	7	12	6	15
13	16	15	12	16
21	24	17	28	26
14	10	12	11	15

~~3 9 8 12~~

~~3 1 6 9~~

3	9	0	8	12
3	1	6	0	9
1	4	3	0	4
4	7	0	11	9
4	0	2	1	5

2	9	0	8	8
~~2~~	1	6	0	5
0	4	3	0	0
3	7	0	11	5
3	2	2	1	1

	I	II	III
1	10	7	
2	2	1	
3			
4			
5			

1 → I,

=

=

4.

	I	II	III	IV	V
1	0	7	~~8~~	6	6
2	2	1	8	0	5
3	~~0~~	4	5	~~8~~ 0	
4	1	5	0	9	3
5	~~3~~	0	4	1	1

1 → I, 2 → IV, 3 → V, 4 → III, 5 → II

$= (11 + 6 + 16 + 17 + 10)$

$= 60$

4.

$$\begin{pmatrix} 12 & 10 & 10 & 8 \\ 14 & 12 & 15 & 11 \\ 6 & 10 & 16 & 4 \\ 8 & 10 & 9 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 & 2 & 0 \\ 3 & 1 & 4 & 0 \\ 2 & 6 & 12 & 0 \\ 1 & 3 & 2 & 0 \end{pmatrix}$$

	J_1	J_2	J_3	J_4
O_1	3	8	[10]	~~0~~
O_2	2	[0]	2	~~0~~
O_3	1	5	10	[0]
O_4	[10]	2	~~0~~	~~0~~

$O_1 \rightarrow J_3$, $O_2 \rightarrow J_2$, $O_3 \rightarrow J_4$, $O_4 \rightarrow J_1$

$= (10 + 12 + 4 + 8)$

$= 34$

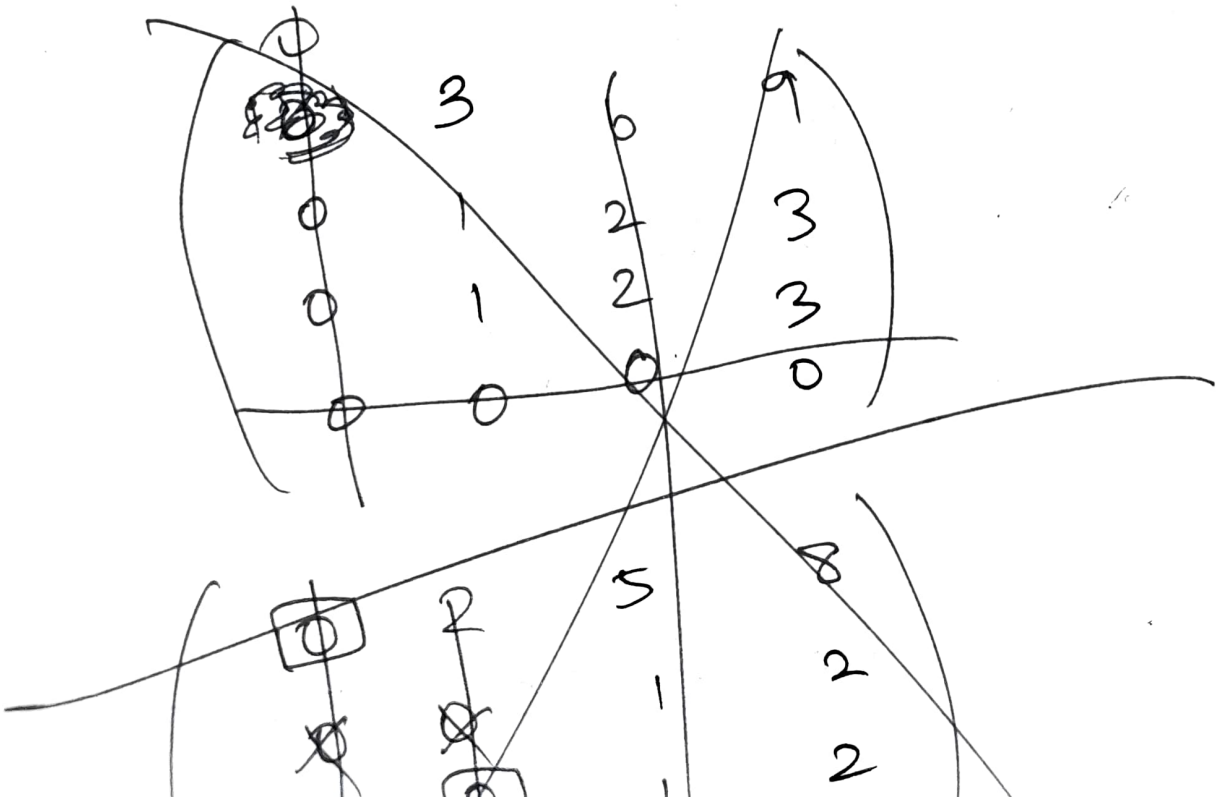
Solve the maximization of assignment problem.

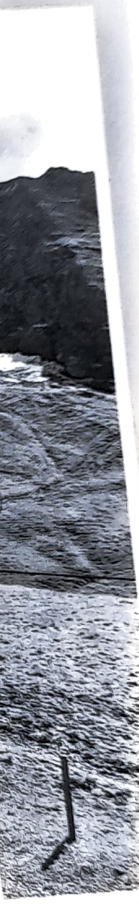
Jobs	Machines A	B	C	D
I	42	35	28	21
II	30	25	90	15
III	30	25	20	15
IV	24	20	16	12

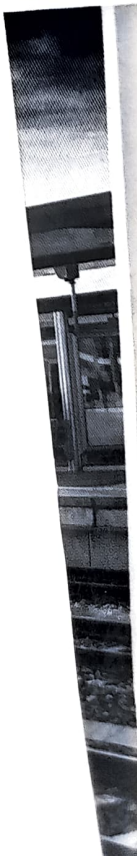
athi

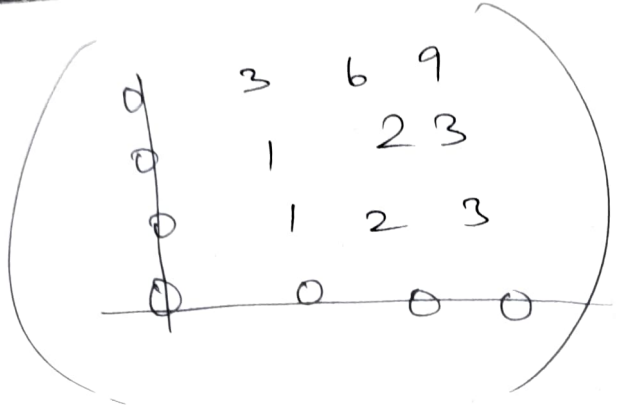
$$\begin{bmatrix} 0 & 7 & 14 & 21 \\ 12 & 17 & 22 & 27 \\ 12 & 17 & 22 & 27 \\ 18 & 22 & 26 & 30 \end{bmatrix}$$

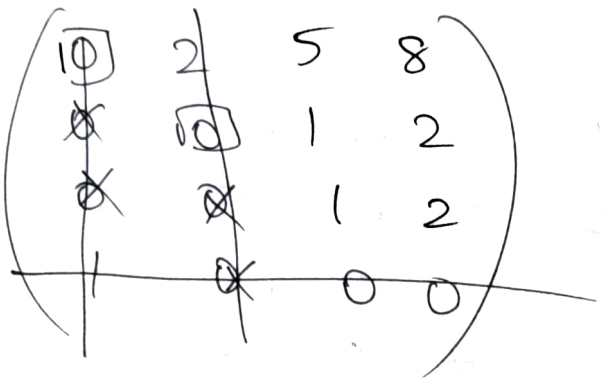
$$\begin{pmatrix} 0 & 7 & 14 & 21 \\ 0 & 5 & 10 & 15 \\ 0 & 5 & 10 & 15 \\ 0 & 4 & 8 & 12 \end{pmatrix}$$





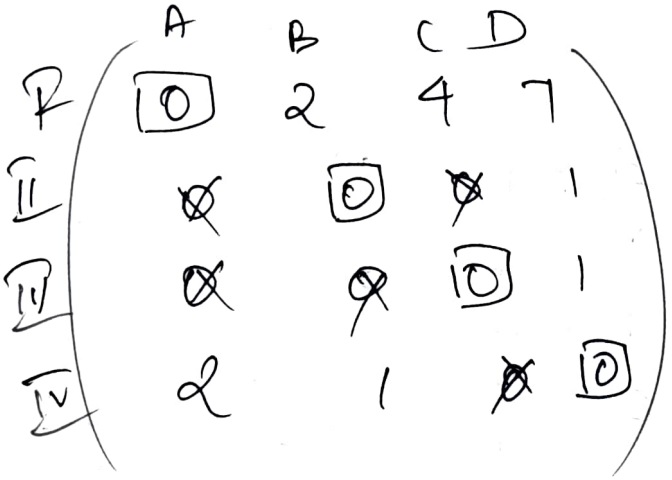


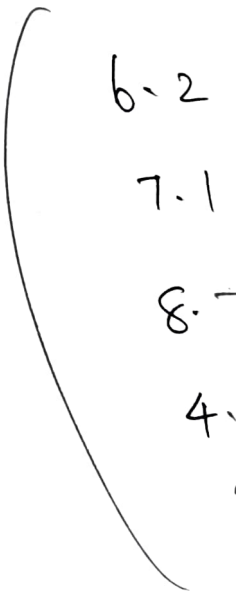




So we the maximization

machines	Jobs A	B
1	6.2	7.8
2	7.1	8.
3	8.9	9
4	4.8	





$I \rightarrow A, \underline{II} \rightarrow B, \underline{III} \rightarrow C, \underline{IV} \rightarrow D$

$= 42 + 25 + 20 + 12$

$= 99.$

Solve the maximization assignment problem

machines \ Jobs	A	B	C	D	E
1	6.2	7.8	5.0	10.1	8.1
2	7.1	8.4	6.1	7.3	5.9
3	8.7	9.2	11.1	7.1	8.1
4	4.8	6.4	8.7	7.7	8.0

$$\begin{pmatrix} 6.2 & 7.8 & 5.0 & 10.1 & 8.1 \\ 7.1 & 8.4 & 6.1 & 7.3 & 5.9 \\ 8.7 & 9.2 & 11.1 & 7.1 & 8.1 \\ 4.8 & 6.4 & 8.7 & 7.7 & 8.0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4.9 & 3.3 & 6.1 & 1.0 & 3.0 \\ 4.0 & 2.7 & 5.0 & 3.8 & 5.2 \\ 2.4 & 1.9 & 0 & 4.0 & 3.0 \\ 6.3 & 4.7 & 2.4 & 3.4 & 3.1 \\ 11.1 & 11.1 & 11.1 & 11.1 & 11.1 \end{pmatrix}$$

37.70

3.9	2.3	5.1	0	2.0
1.3	0	2.3	1.1	2.5
2.4	1.9	0	4.0	3.0
3.9	2.3	0	1	0.7
0	0	0	0	0