

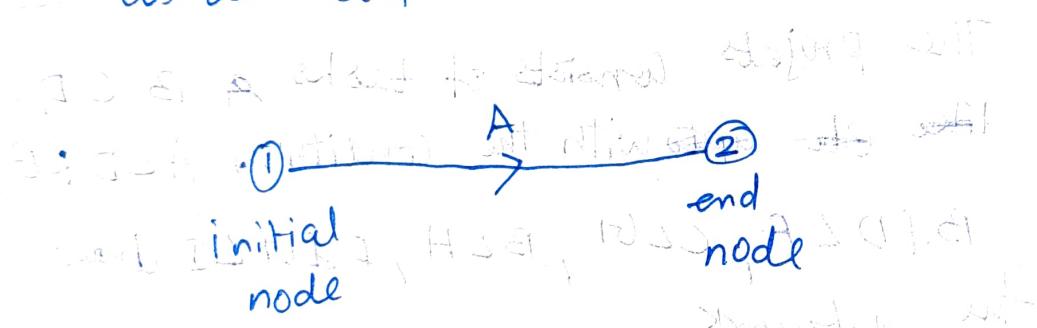
## PERT

Project evolution review Technique  
Critical Path method

Activity - It is represented by an arrow diagram with a node at the beginning and ~~the~~ a node at the end

Showing start and finishing of a activity

example : If A is an activity which is denoted by an arrow diagram.



immediate predecessor and immediate successor.



A ~~is~~ and B are Activities

A is a immediate predecessor of B  
denoted as  $A \rightarrow B$

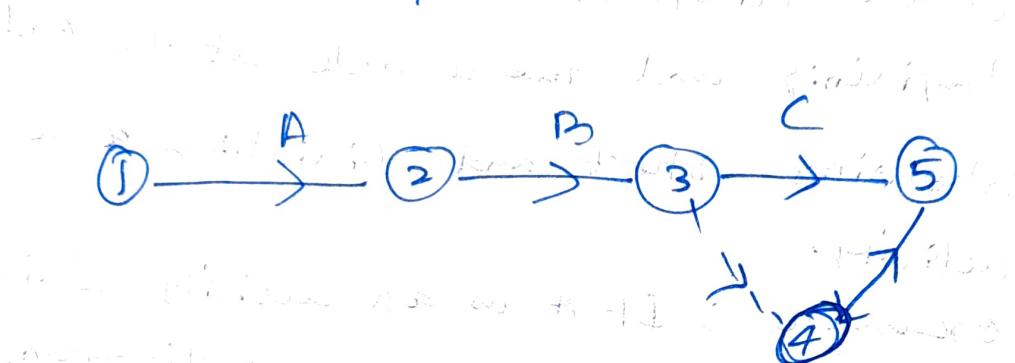
B is a immediate successor of A

## Dummy Activity.

It is an imaginary activity which does not consumes any resources

For example, C → D is a dummy activity

such as A → B, B → C, B → D



The project consists of tasks A B C D E F G H I draw the network

B & D & F, C & G, B & H, F & G & I draw the network

With the condition A → B, A → C, B → D, B → E, C → D, C → E, D → F, D → G, E → F, E → G, F → H, F → I, G → H, G → I, H → I

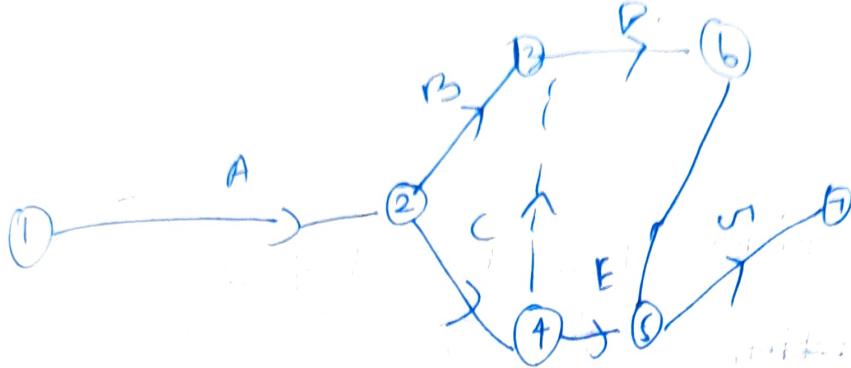
A → B, A → C, B → D, B → E, C → D, C → E, D → F, D → G, E → F, E → G, F → H, F → I, G → H, G → I, H → I

Such activities are called dummy activities

For example, A → B, A → C, B → D, B → E, C → D, C → E, D → F, D → G, E → F, E → G, F → H, F → I, G → H, G → I, H → I

Activity A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Predecessors none A A B, C C D, F, F



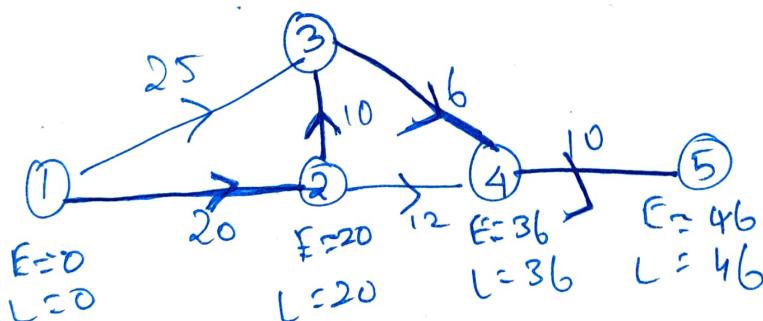
① The following Table

gives the activities in a construction project.

Activity	1-2	1-3	2-3	2-4	3-4	4-5
Duration	20	25	10	12	6	10

a) Draw the network

b) find Critical path & project duration?



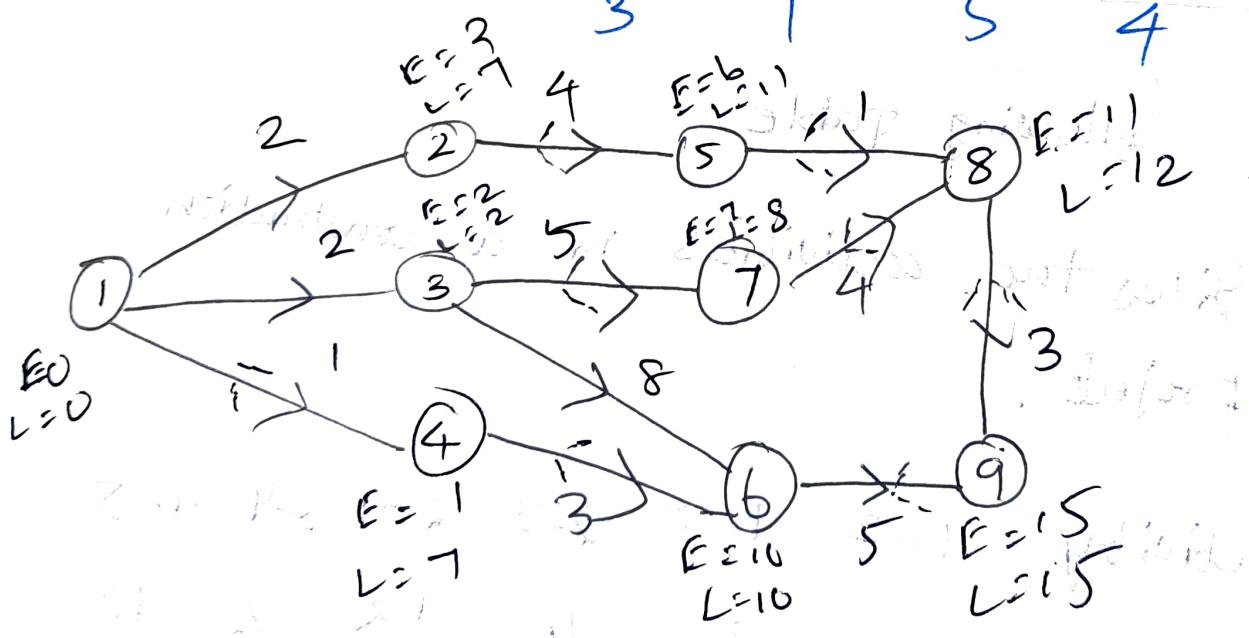
The critical Path is

1-2-3-4-5

and project duration is 46

Activities

	1-2	1-3	1-4	2-5	3-6	3-7
Duration	2	2	1	4	8	5
	4-6	5-8	6-9	7-8	8-9	
	3	1	5	4	3	

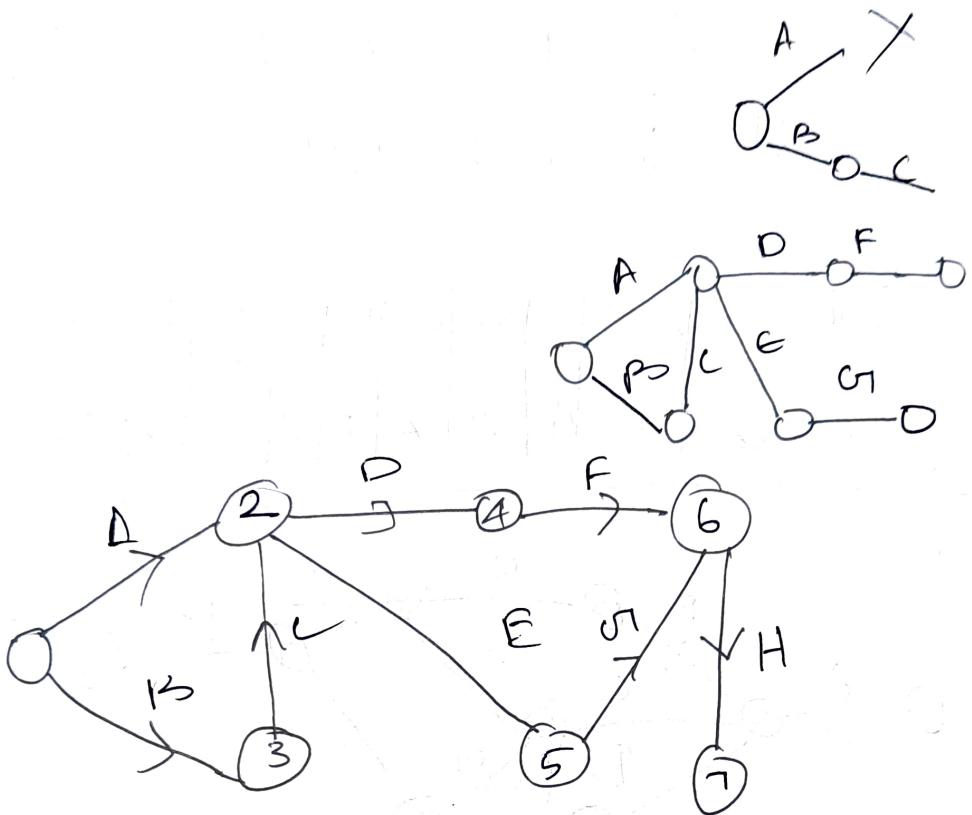


Critical path is 1-3-6-9

project duration is 15

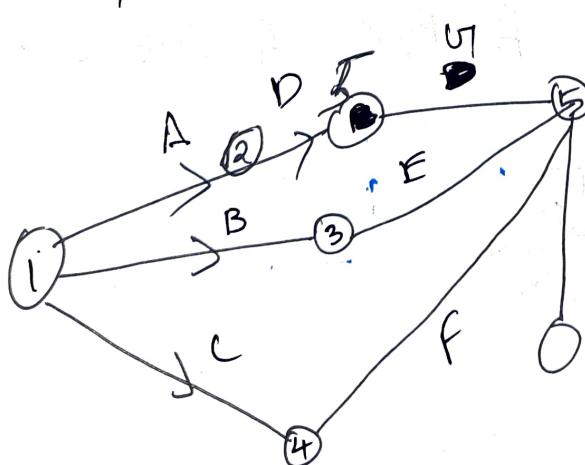
Draw network diagram.

Job A B C D E F G H  
Predecessor - - B A, C A, C D E F, G



Draw the network diagram

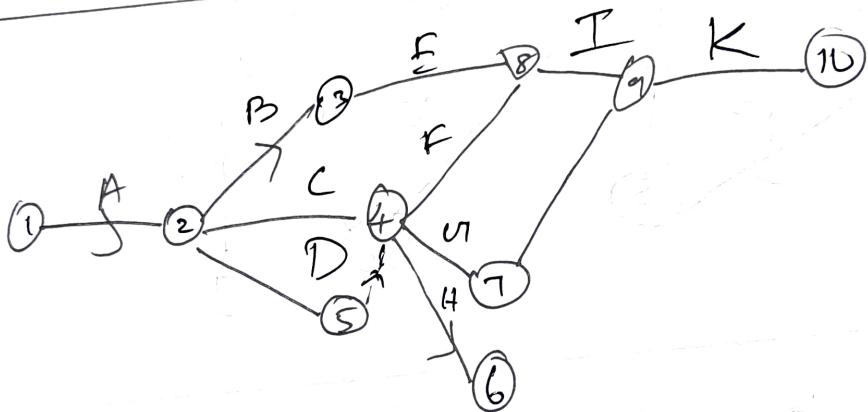
ALD, BLE, CLF, D, ELF, F, GLH



Draw the network

$A \leq B, C, D$ ,  $B \leq E$ ,  $C \leq F, U, H$   
 $D \leq H$ ,  $E \leq I$ ,  $F \leq T$ ,  $G \leq J$ ,  
 $H \leq J$ ,  $I \leq K$ ,  $J \leq K$

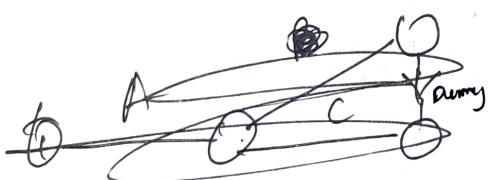
Jobs	A	B	C	D	E	F	U	H	J	K
Predessecor		-A	A	A	B	C	C	C, F, F, U, H	E	

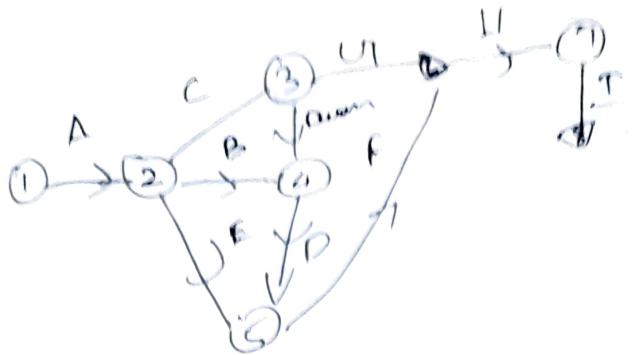


Draw the networks

Jobs A B C D E F U H T

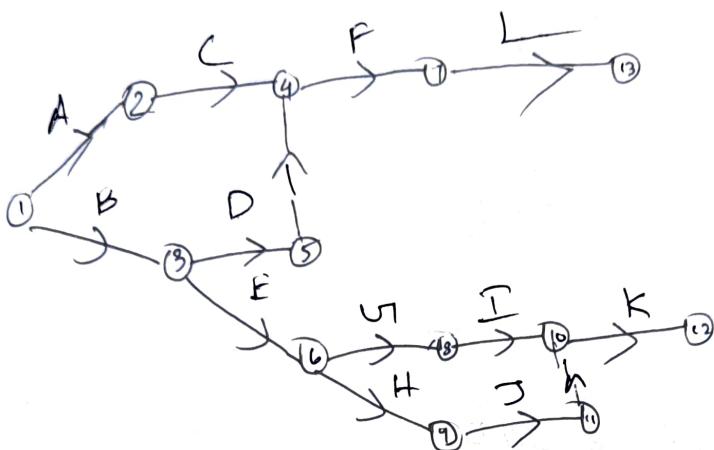
Predessecor - A A B, C A D/E C F, G H





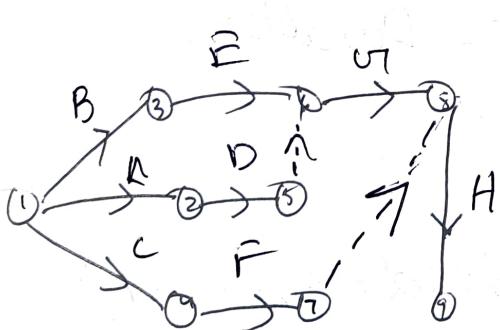
Homework.

- 1) Jobs
- |                     |        |           |           |     |     |     |        |     |     |
|---------------------|--------|-----------|-----------|-----|-----|-----|--------|-----|-----|
| $\underline{\quad}$ | $A, B$ | $C$       | $D, E, F$ | $G$ | $H$ | $I$ | $J$    | $K$ | $L$ |
| $\underline{\quad}$ | $- A$  | $B, C, D$ | $E$       | $F$ | $G$ | $H$ | $I, J$ | $K$ | $L$ |



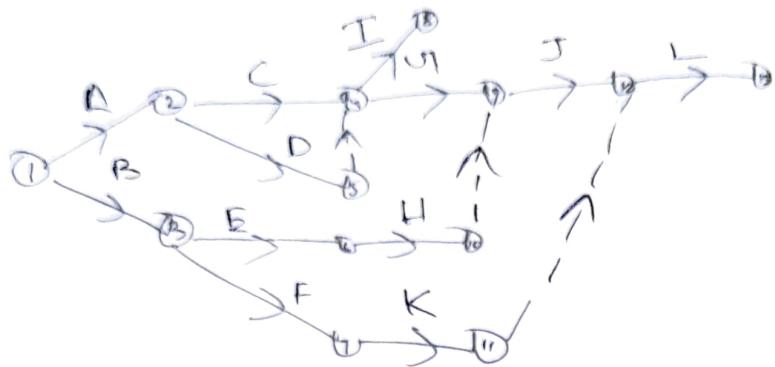
$T_{\text{EA}} - T_{\text{EL}} = 7$

- 2) Jobs
- |                     |                     |                     |        |     |        |        |     |
|---------------------|---------------------|---------------------|--------|-----|--------|--------|-----|
| $A$                 | $B$                 | $C$                 | $D$    | $E$ | $F$    | $G$    | $H$ |
| $\underline{\quad}$ | $\underline{\quad}$ | $\underline{\quad}$ | $A, B$ | $C$ | $D, E$ | $F, G$ | $H$ |



③ Inter A B C D E F G H J  
 A A B B C D E L, K

J K L  
 M, H F J, K



①  
 2 → H  
 1 →

Activity 1-2

Duration 20

① total float

$$TF = LST - EST$$

- (i) Draw the network diagram
- (ii) Find the ES, EF, LS, LF values
- (iii) find the float values

②  $EFT = EST + \text{Duration}$

③  $LST = LFT - \text{Duration}$

④ free float

$$FF = TF - \text{Head event (L-E)}$$

⑤  
 $b < 0$   
 $c > 0$

⑤ Independent float

$$IF = RF - \text{Tail event (L-F)}$$



$\alpha \rightarrow$  Head event

$\backslash \rightarrow$  Tail event

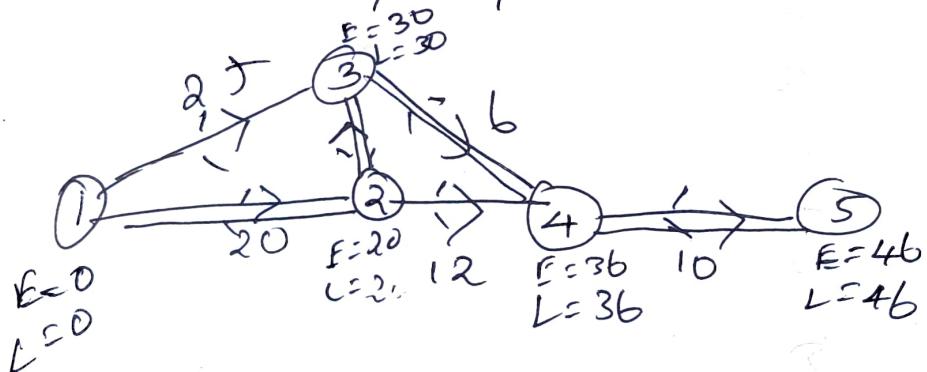
Activity 1-2 1-3 2-3 2-4 3-4 4-5

Duration 20 25 10 12 6 10

(i) Draw the project network

(ii) Find the critical Path & project duration

(iii) Find FF, TF, IF



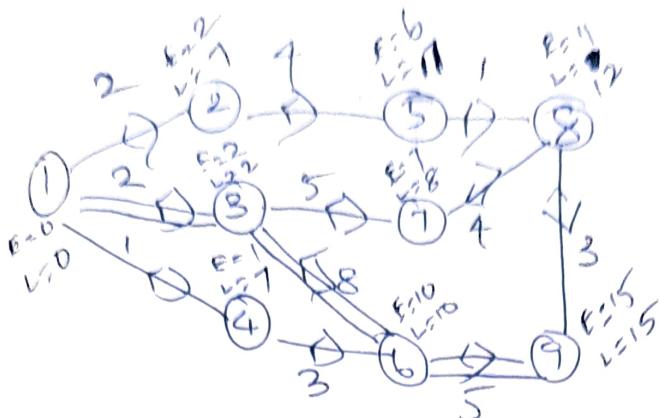
critical path is 1-2-3-4-5  
project duration is 46

Activity	duration	EST	EFT	LST	LFT	TF	FF	If
1-2	20	0	20	0	20	0	0	0
1-3	25	0	25	5	30	5	5	5
2-3	10	20	30	20	30	0	0	0
2-4	12	20	32	24	36	4	4	4
3-4	6	30	36	30	36	0	0	0
4-5	10	36	(46)	36	(46)	0	0	0

### Homework

① Activity. Time in months

1-2	2
1-3	2
1-4	1
2-5	4
3-6	8
3-7	5
4-6	3
5-8	1
6-9	5
7-8	4
8-9	3



Activity	duration	EST	EFT	LST	LF	TF
1-2	2	0	2	5	7	5
1-3	2	0	2	0	2	0
1-4	1	0	1	6	7	6
2-5	4	2	6	6	7	1
3-6	8	2	10	7	15	5
3-7	5	2	7	3	8	1
4-6	3	1	4	7	10	6
5-8	1	6	7	11	12	5
6-9	5	10	15	10	15	0
7-8	4	7	11	8	12	1
8-9	3	11	14	12	15	1

Critical path 1 - 3 - 6 - 9

Project duration. 15 months.

②

Job      Duration

1 - 2      2

2 - 3      7

2 - 4      3

3 - 4      3

3 - 5      5

4 - 6      3

5 - 8      8

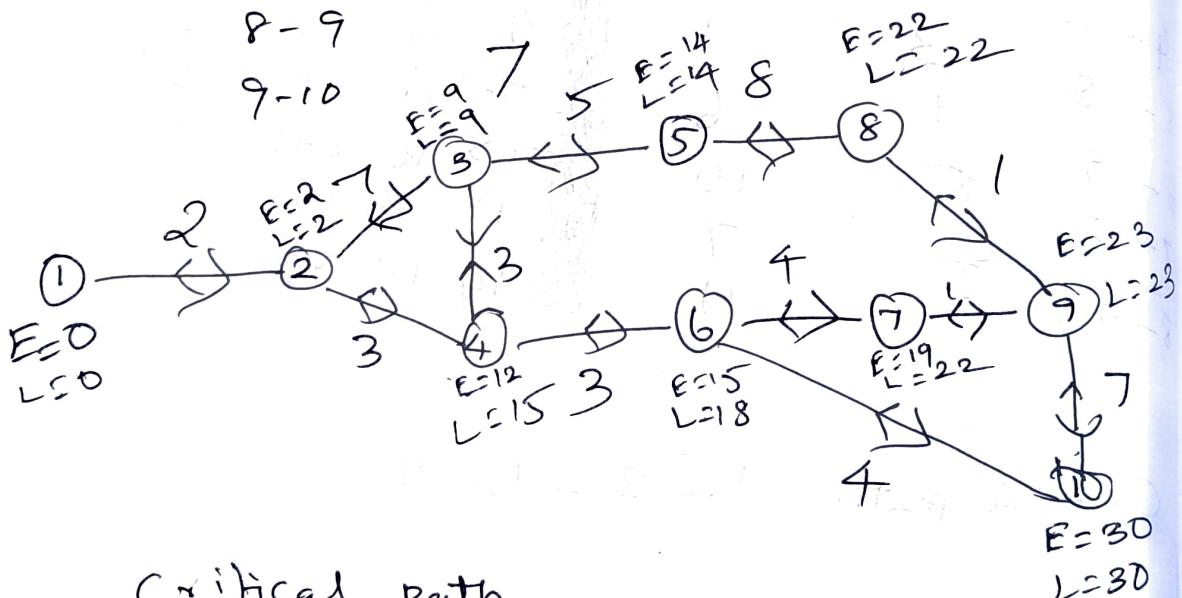
6 - 7      4

6 - 10      9

7 - 9      1

8 - 9      1

9 - 10      1

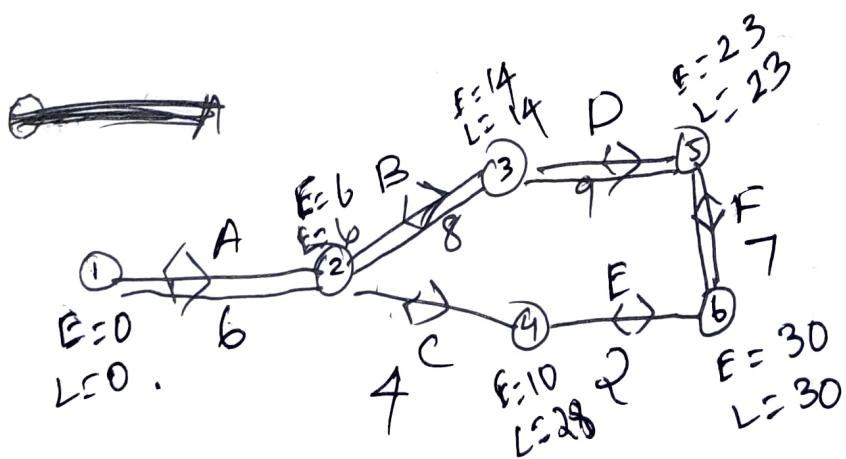
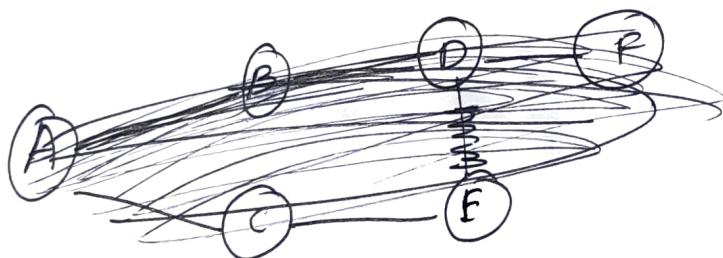


Critical path

~~Project duration~~ 1 - 2 - 3 - 5 - 8 - 9 - 10

project duration - 30

Activity	Duration	Preceding Activity
A	6	-
B	8	A
C	4	A
D	9	B
E	2	C
F	7	D



activity	duration	EST	FFT	LST	LFT	TF	<del>ES</del>
A	6	0	6	0	6	0	-
B	8	6	14	6	14	0	-
C	4	6	10	24	28	18	-
D	9	14	23	14	23	0	-
E	2	10	12	28	30	18	-
F	7	23	30	23	30	0	-

FF IF

0

0

0

0

0

0

0

critical Path is

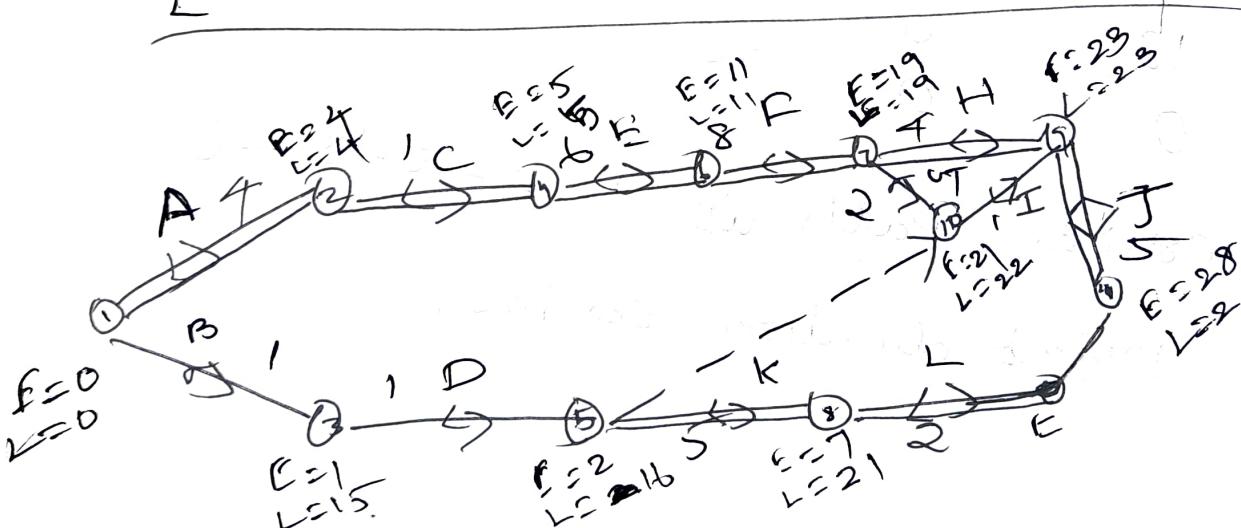
project duration ~~30 hrs.~~

1-2-3-5-6

5,6,7 10,16,17

## Example 5

Activity	Duration in days	Predecessors
A	4	none
B	1	none
C	1	A
D	6	B
E	8	C
F	3	E
G	4	F
H	1	G
I	5	H, I
J	5	I
K	2	J
L		K



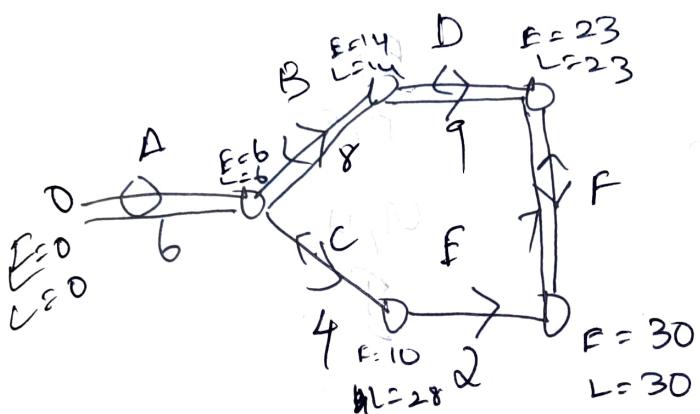
Critical path is A - C - E - F - G - H - I - J - K - L

A - C - E - F - G - H - I - J - K - L

Project duration 28 days.

### Example - 6

Activity	Duration (weeks)	Predessor
A	6	none
B	8	A
C	4	A
D	9	B
E	2	C
F	7	D



Critical path A - B - D - F

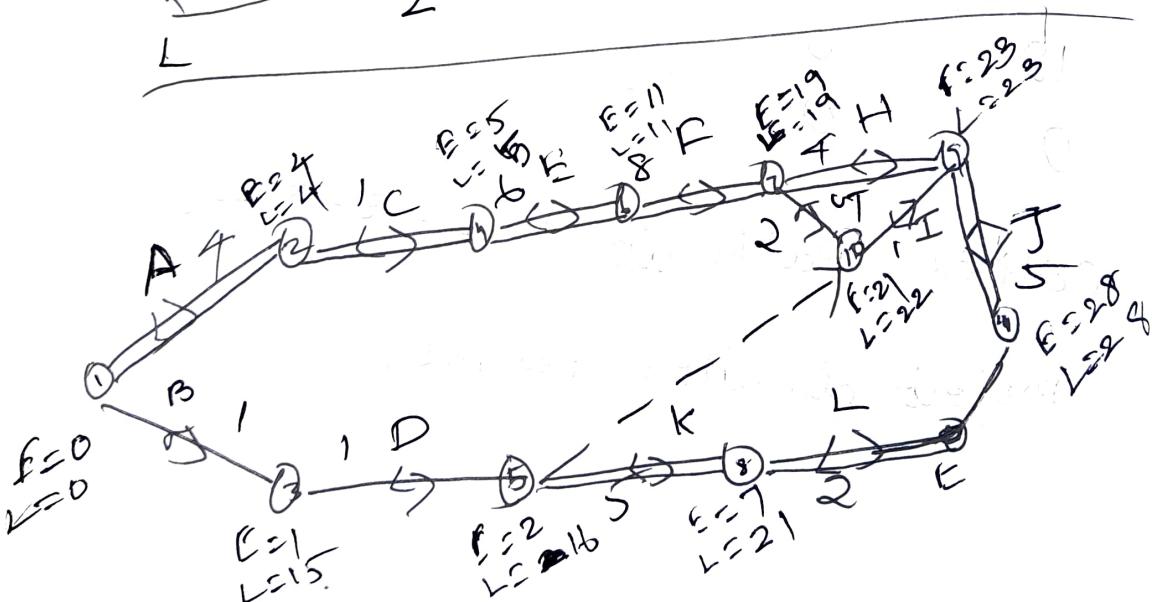
Project duration 30 weeks.

EST    EFT    LST    LFT    TF.

0	6	0	6	0
6	14	6	14	0
6	10	24	28	18
14	23	14	23	0
10	12	28	30	18
23	30	23	30	0

Example 5

Activity	Duration in days	Predecessors
A	4	none
B	1	none
C	1	A
D	6	B
E	8	C
F	2	E
G	4	F
H	1	F
I	5	G, H
J	5	I
K	2	J
L	1	K



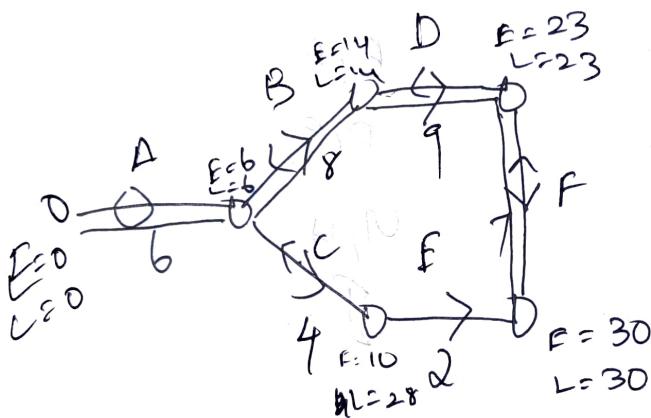
Critical path is ~~A - C - E - F - G - K - L~~

A - C - E - F - G - K - L

Project duration  
A - C - F - F - H - J  
7 days.

## Example - 6

Activity	Duration (weeks)	Predecessor
A	6	none
B	8	A
C	4	A
D	9	B
E	2	C
F	7	D



Critical path A-B-D-F

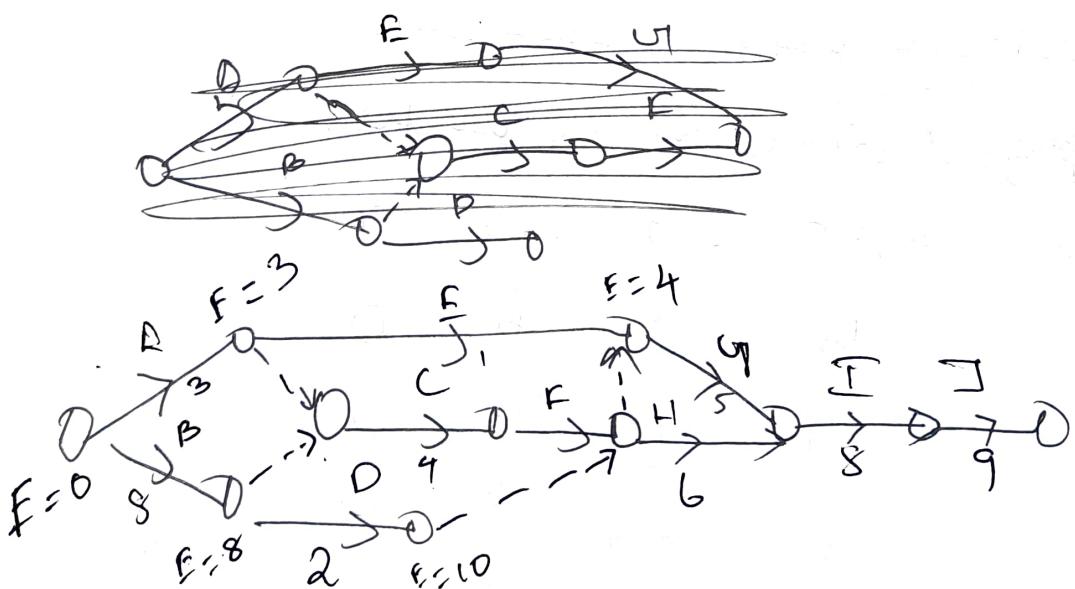
Project duration 30 weeks.

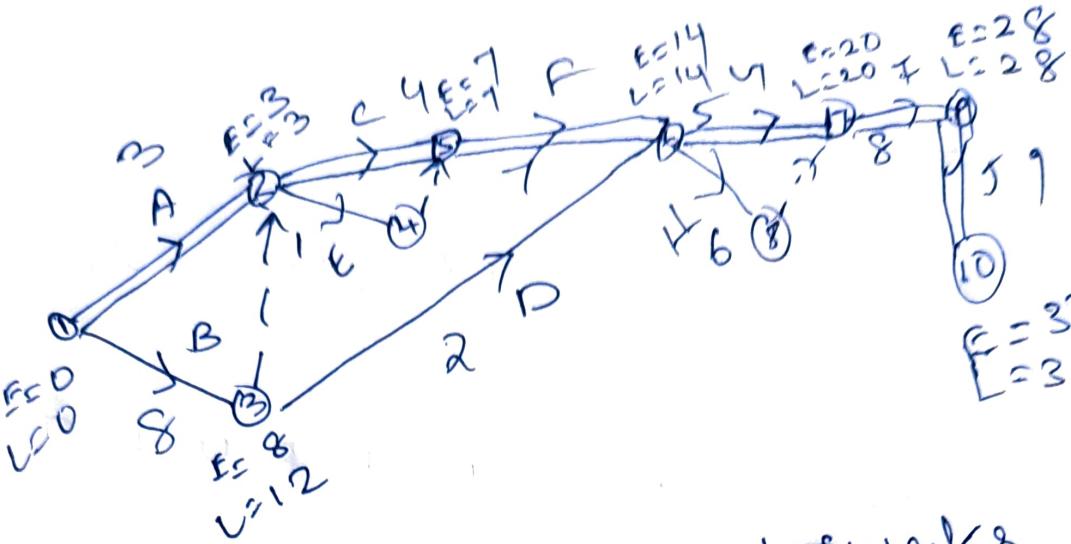
EST    EFT    LST    LFT    TF.

0	6	0	6	0
6	14	6	14	0
6	10	24	28	18
14	23	14	23	0
10	12	28	30	18
23	28	23	30	0

Example 7.

Activity	Duration (weeks)	Predcessors
A	3	none
B	8	none
C	4	A, B
D	2	B
E	1	A
F	7	C
G	5	E, F
H	6	D, F
I	8	G, H
J	9	I





Project duration

37 weeks.

Critical path is A - C - F - G - I - J

Example-10.

$A \leftarrow D \leftarrow E; B, D \leftarrow F; C \leftarrow U; B \leftarrow H; F, U \leftarrow I$

$B \leftarrow R \leftarrow H \leftarrow T$

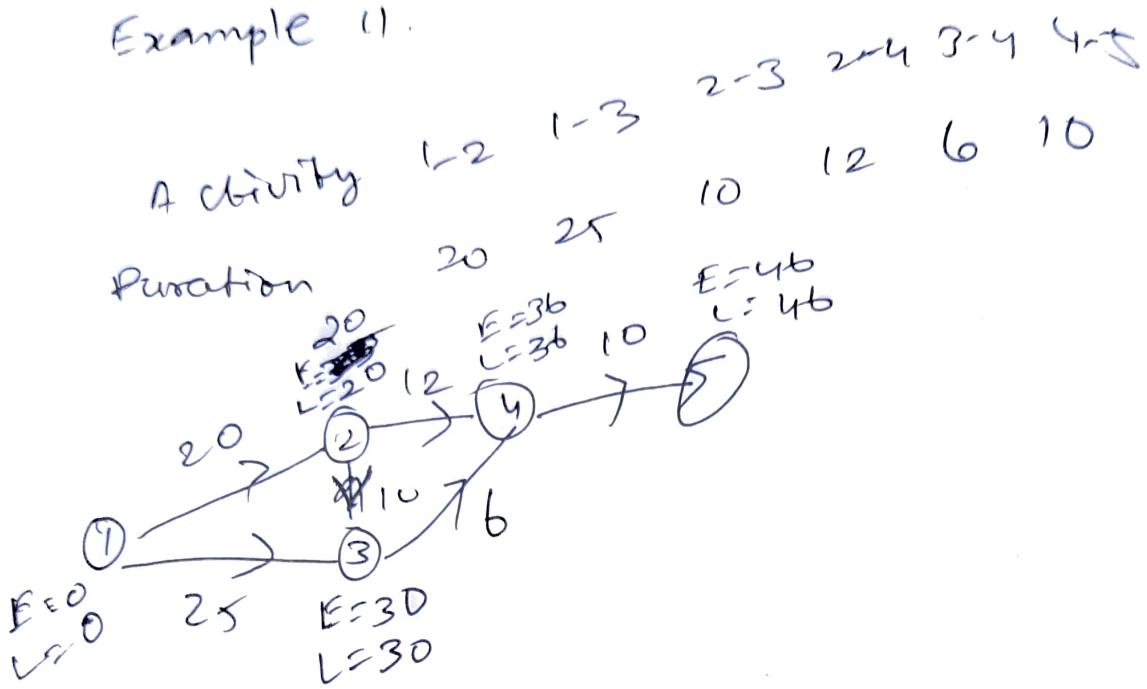
task A B C D B R H T

Time (days) 2 3 8 20 16 24 18 19 4 10

Activity Precedence Duration

A	-	23
B	-	8
C	-	20
D	A	16
E	A	24
F	B, D	18
G	C	19
H	B	4
I	F, U	10

Example 11.



The paths

1-2-4-5 42

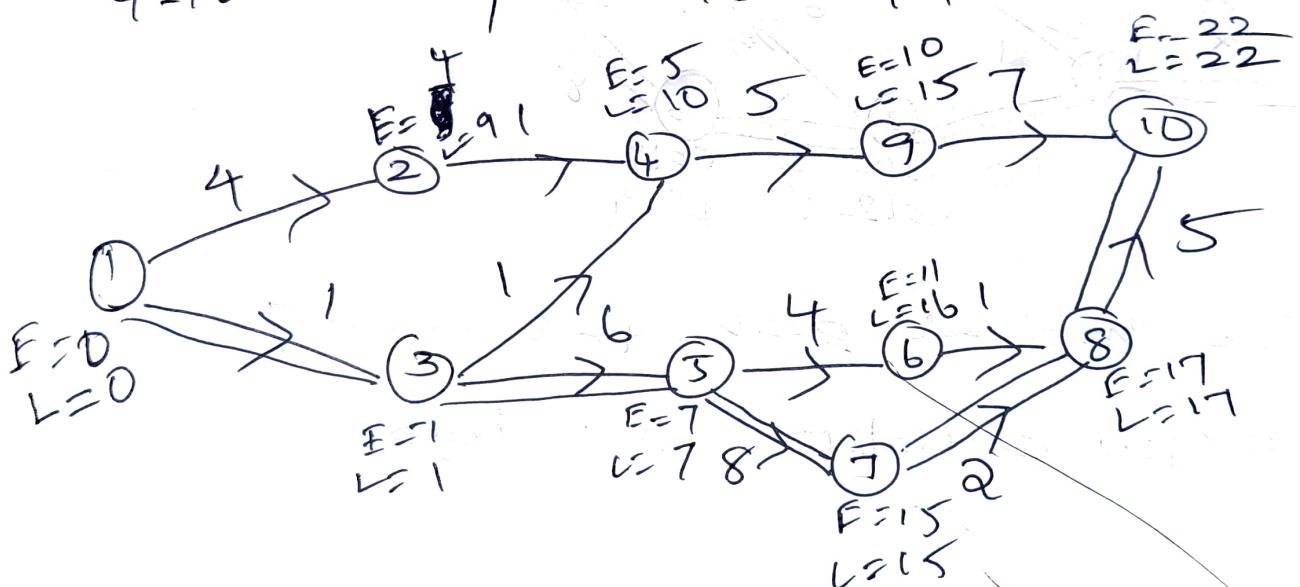
1-3-4-5 41

1-2-3-4-5 46

The critical path is 1-2-3-4-5  
Project duration 46 days.

example -16.

Activity	days	EST	EFT	LST	LFT	TF
1-2	7	0	4	5	9	5
1-3	1	0	1	0	1	0
2-4	1	4	5	9	10	5
3-4	1	1	2	9	10	3
3-5	6	1	7	1	7	0
4-9	5	5	10	10	15	5
5-6	4	7	11	7	15	0
5-7	8	7	15	7	17	5
6-8	1	11	12	16	17	0
7-8	2	15	17	17	17	0
8-10	5	17	22	17	22	5
9-10	7	10	17	15	22	5

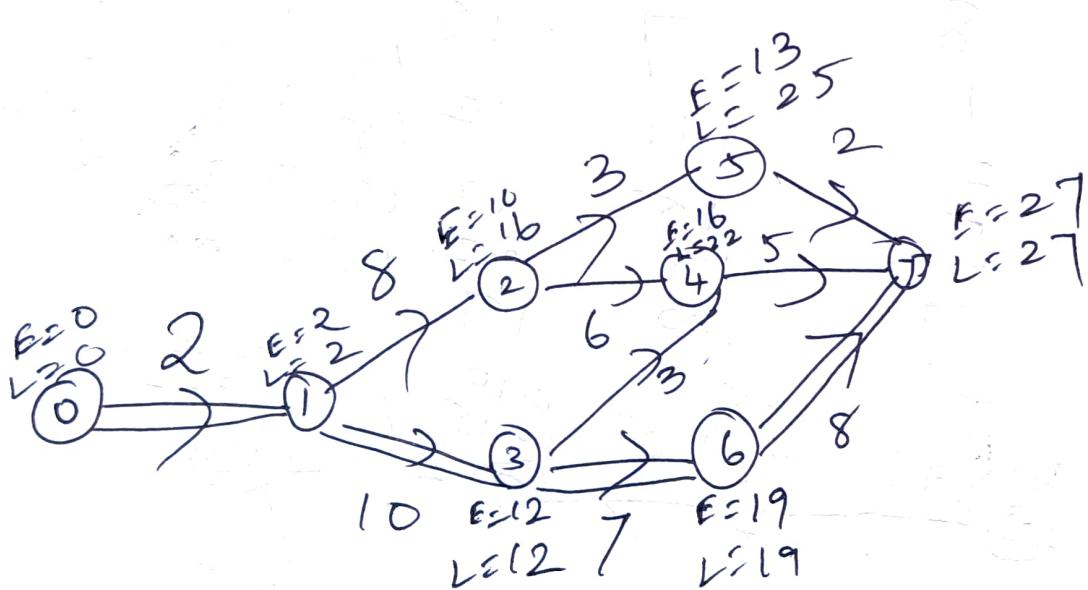


The critical path  $1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 8 \rightarrow 10$

Project duration 22 days.

### Example 17.

Activity	D	EST	EFT	LST	LFT	TF	FF	EJK
0-1	2	0	2	0	2	0	0	0
1-2	8	2	10	8	16	6	0	0
1-3	10	2	12	2	12	0	0	0
2-4	6	2	16	16	22	6	0	0
2-5	3	10	13	22	25	12	0	0
3-4	3	12	15	18	22	6	0	0
3-6	7	12	19	12	19	6	6	0
4-7	5	16	21	22	27	12	2	0
5-7	2	13	15	25	27	0	0	0
6-7	8	19	27	19	27			



The critical path 0-1-3 - 6-7

Project duration 27 days.

# Project Evaluation Review technique

(PERT)

$$Z = \frac{x - \bar{x}}{\sigma} \quad \begin{matrix} \sigma = S.D \\ \bar{x} = \text{project duration} \\ x = \text{expected value} \end{matrix}$$

- ① optimistic time ( $T_0$ ) or ~~a~~
- ② pessimistic time ( $T_p$ ) or ~~b~~
- ③ most likely time ( $T_m$ ) or ~~c~~
- ④ expected time  $T_e = \frac{T_0 + T_p + T_m}{6}$
- ⑤ Standard deviation  $S.D = \frac{T_p - T_0}{6}$

⑥ variance =  $(S.D)^2$

⑦ standard deviation of project =  $\sqrt{\frac{\text{sum of variances}}{\text{in critical path}}}$

A project has following characteristic and

time estimate

optimistic time (a), most likely time ( $T_m$ ), (c)

pessimistic time (b)

Construct ~~PERT~~ network.

Find critical path and project duration

# Activity

	a	b	m	$t_c = \frac{a+b+m}{6}$	<del>SD</del>
1-2	1	5	1.5	2	$\frac{1}{6}$
2-3	1	3	2	2	0.666
2-4	1	5	3	3	0.333
3-5	3	5	4	4	0.666
4-5	2	4	3	3	0.333
4-6	3	7	5	5	0.333
5-7	4	6	5	5	0.333
6-7	6	8	7	7	0.666
7-8	2	6	4	4	0.666
7-9	4	8	6	6	0.666
8-10	1	3	2	2	0.333
9-10	3	7	5	5	0.666

Variance ( $SD$ )<sup>2</sup>

0.443

0.110

0.443

0.110

0.110

0.443

0.110

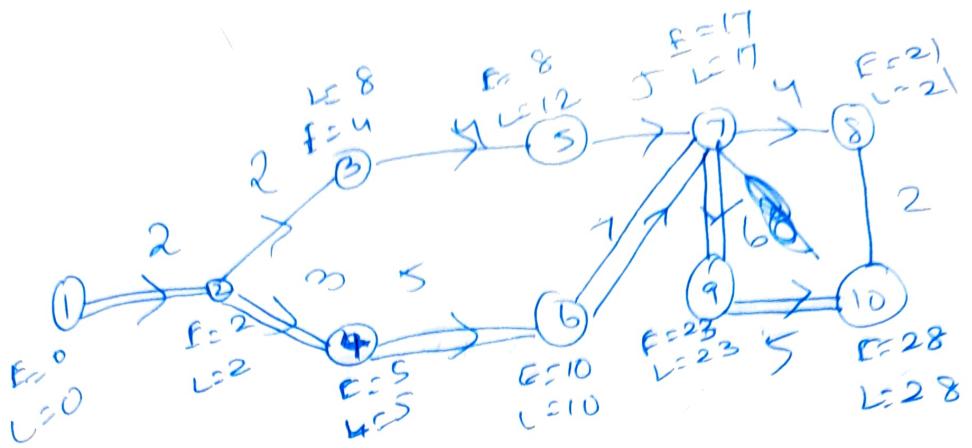
0.443

0.443

0.443

0.110

0.443



Critical path 1-2-4-6-7-9-10

Project duration 28

The activity have the following PERT

	estimate	opt	most	pes
Jobs	a	b	m	c
1-2	3	3	6	15
7-8	4	4	9	28
2-3	6	6	12	30
3-5	5	5	11	17
5-8	1	1	4	7
6-7	3	3	9	27
4-5	3	3	6	8
2-4	2	2	5	14
1-6	2	2	5	14

i) Draw network diagram.

ii) determine the critical path

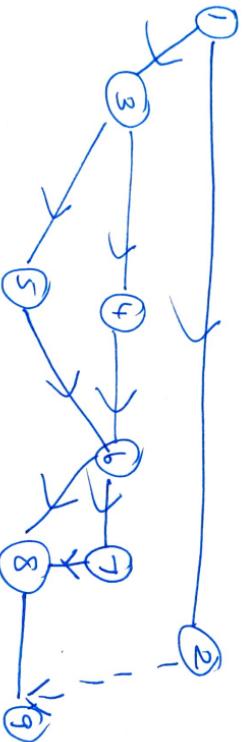
Find ~~completion~~ project duration and it's

duration.

### Example 27

$$\frac{33+56}{6}$$

Additivity	a	m	b	$\tau_c = \frac{abm}{6}$	SD.	Variance
1 - 2	12	14	21	14.833	1.5	2.25
1 - 3	7	10	16	10.5	1.5	2.25
3 - 5	4	6	10	6.333	1	0.444
3 - 4	36	40	60	42.666	4	16
4 - 6	12	15	24	16	2	4
5 - 6	6	8	12	8.333	1.5	2.25
6 - 7	9	12	18	12.5	1.5	2.25
6 - 8	6	10	15	10.166	0.5	2.25
7 - 8	4	5	7	5.166	1	0.25
8 - 9	8	10	14	10.333	1	0.25

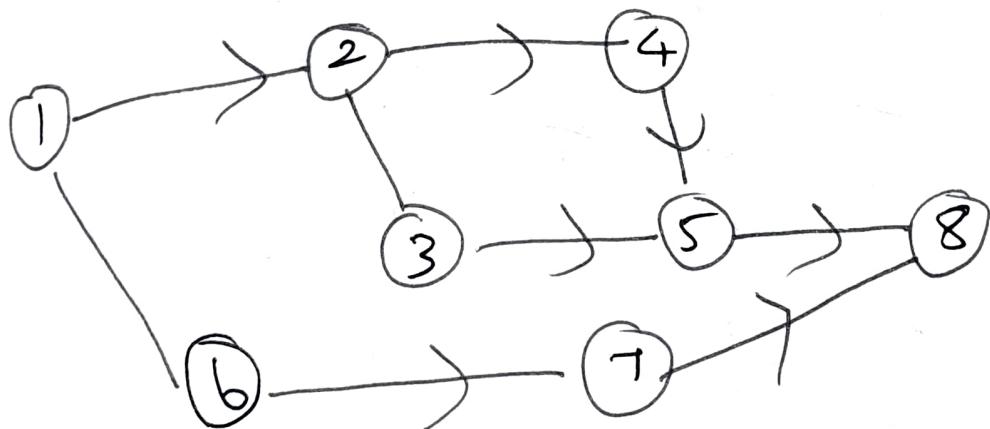


The critical path is 1 - 3 - 4 - 6 - 7 - 8 - 9

The project duration =  $\frac{583}{6} = 97.17$  days.

$$\text{Variance of the project} = \frac{8(1+5+6+14+81+74)}{36} = 25.75$$

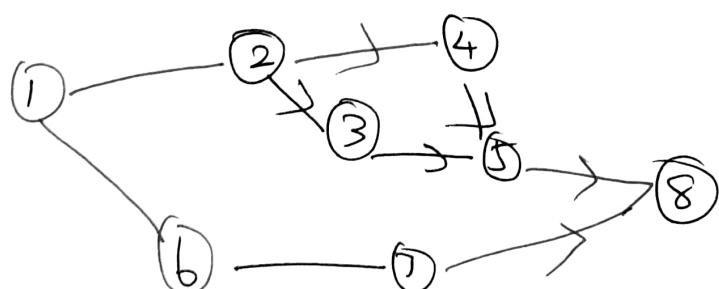
$J_{ab}$	a	m	b	$T_e =$	SD	Variance
1-2	3	6		15		
1-6	2	5		14		
2-3	6	12		30		
2-4	2	5		8		
3-5	5	11		17		
4-5	3	6		15		
6-7	3	9		27		
5-8	1	9		7		
7-8	4	19		28		



For the following table shows the jobs of the network along with their time estimates

Jobs	a	m	b	s.d	v
1-2	1	7	13	2	4
1-6	2	5	14	2	4
2-3	2	14	26	4	16
2-4	2	5	8	1	1
3-5	7	10	19	2	4
4-5	5	5	17	2	4
6-7	5	8	29	4	16
5-8	3	3	9	1	1
7-8	8	17	32	4	16

Draw the project network & ~~find~~ find the probability that project is completed 40 days.



~~The critical path~~

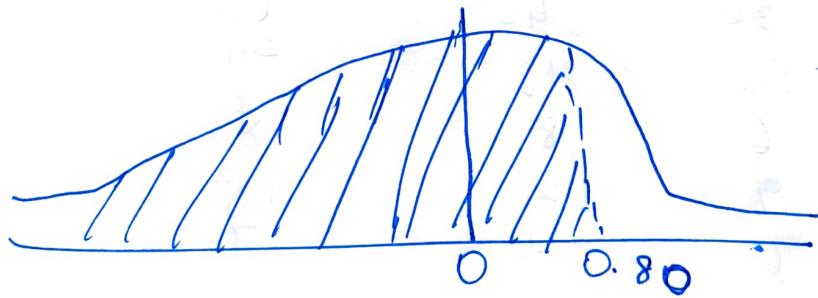
Critical path 1-2-3-5-8

project duration 36 days.

$$\text{standard deviation of project length} = \sqrt{4+16+4+1} \\ = \sqrt{25} = 5$$

$$z = \frac{x - \bar{x}}{\sigma}$$

$$z = \frac{40-36}{5} = \frac{4}{5} = 0.8$$



$$P(Z \leq 0.8) = 0.5 + 0.2881$$

$$P(Z \leq 0.8) = 0.7881$$

$$= 0.7881 \times 100$$

$$= 78.81\%$$

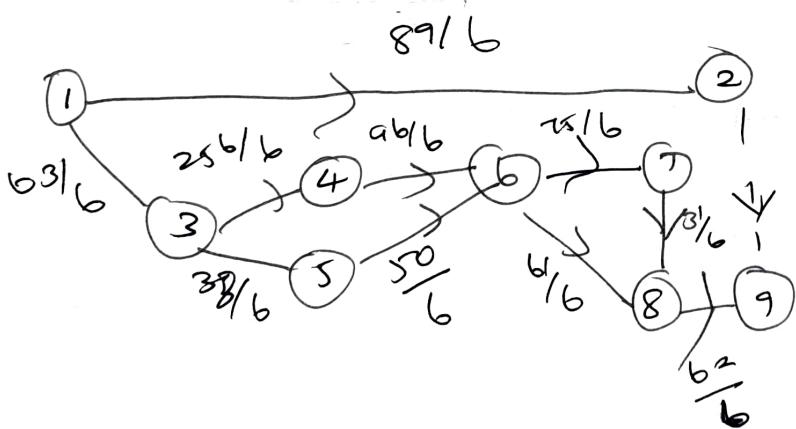
Home work.

Example 27.

Activity	a(days)	m(days)	b(days).
1-2	12	14	21
1-3	7	10	16
3-5	9	6	10
3-4	36	40	60
4-6	12	15	24
5-6	6	8	12
6-7	9	12	18
6-8	6	10	15
7-8	4	5	7
8-9	8	10	14

$$T_E = \frac{a+b+4m}{6} \quad SD = \frac{b-a}{6} \quad \text{variance}$$

$\frac{89}{6}$	$\frac{91}{6}$	$\frac{81}{6}$
$\frac{63}{6}$	$\frac{91}{6}$	$\frac{81}{36}$
$\frac{38}{6}$	$\frac{61}{6}$	$\frac{36}{81}$
$\frac{256}{6}$	$\frac{24}{6}$	$\frac{576}{36}$
$\frac{96}{6}$	$\frac{12}{6}$	$\frac{144}{36}$
$\frac{50}{6}$	$\frac{6}{6}$	$\frac{36}{36}$
$\frac{75}{6}$	$\frac{9}{6}$	$\frac{81}{36}$
$\frac{61}{6}$	$\frac{3}{6}$	$\frac{9}{36}$
$\frac{31}{6}$	$\frac{6}{6}$	$\frac{36}{36}$
$\frac{62}{6}$		



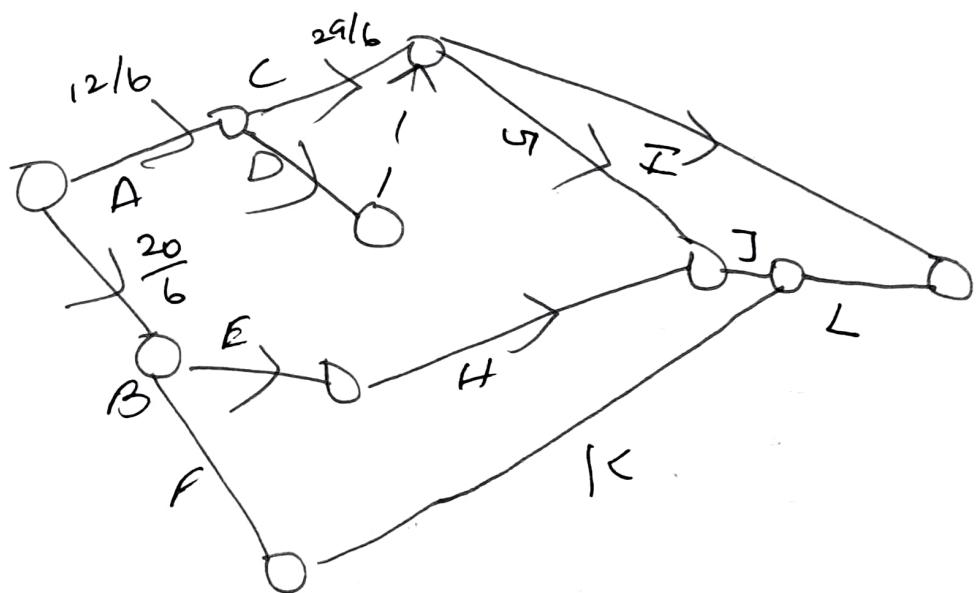
The critical path is 1 - 3 - 4 - 6 - 7 - 8 - 9

The project duration is  $\frac{583}{6} = 97.17$  days.

$$\begin{aligned} \text{Variance of the project} &= \frac{81 + 576 + 144 + 81 + 729}{36} \\ &= \frac{927}{36} = 25.75 \end{aligned}$$

Example 33.

Activity	Preceding	Optimis	Normal	Pessim
A	-	2	2	2
B	-	1	3	7
C	A	4	7	8
D	A	2	5	7
E	B	2	6	9
F	B	5	9	11
G	C, D	3	6	8
H	E	2	5	9
I	C, D	3	3	8
J	G, H	1	4	5
K	R	4	5	7
L	J, K	2		



	SD	Variance	Total slack
1/6	2	4	15/6
12/6	2	4	0
20/6	4	16	15/6
40/6	1	1	26/6
29/6	2	4	12/6
35/6	2	4	6/6
52/6	4	16	12/6
35/6	1	1	12/6
35/6	7/6	49/36	115/6
31/6	5/6	25/36	12/6
17/6	3/6	9/36	0
47/6	7/6	49/36	0
29/6	5/6	25/36	

The critical path B-F-K-L

The project duration is  $\frac{148}{6} = 24.67$  days.

The variance of critical path is  $= \frac{36}{36} + \frac{36}{36} + \frac{49}{36} + \frac{25}{36}$

$$= \frac{146}{36}$$

$$SD = \sqrt{\frac{146}{36}} = 2.01$$

$$Z = \frac{x - \bar{x}}{s} = \frac{x - 24.67}{2.01}$$

When  $Z = 0$ ,  $z = z_0$  is 0.49

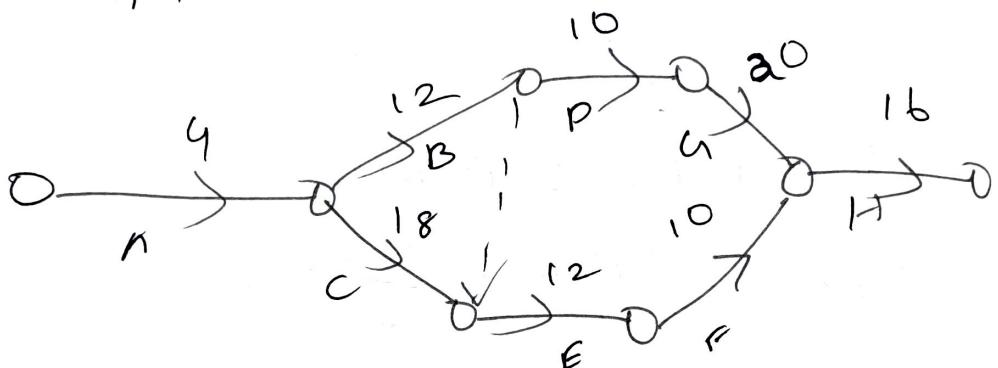
$$Z_1 = 2.33$$

$$2.33 = \frac{x - 24.67}{2.01} \quad x = 2.33 \times 2.01 + 24.67 \\ = 29.35$$

The project will be completed in 29.35 with 97% prob

## Example 4b.

	a	m	b	$T_E$	SD	Vari
A	-	3	4	4	4/6	16/
B	A	8	12	12	8/6	64/
C	A	14	16	18	16/6	256/
D	B	9	10	10	12/6	144/
E	B	6	12	12	12/6	144/
F	E	6	8	22	10	16/6
G	D	18	18	30	20	12/6
H	F, G	8	14	32	16.	256/



The critical path is A-B-D-Q-H

The project duration is 62 days.

$$\sigma = \sqrt{\frac{6+64+144+144+576}{36}}$$

$$= 5.12$$

$$\bar{x} = \frac{x - \bar{x}}{\sigma} = \frac{62 - 5.12}{5.12}$$

$$x = 75, z = \frac{13}{5.12} = 2.54$$

The prob that the project will be required at least 75 days is  $5 - 4945 = .0055$