Game Theory.

A Game is essentially a situation of conflict between two or more parties. The party termed as players.

Types of games.

1. Two person game - two parties play the game
2. N person game - N parties playing a game.
3. 0-sum game - number of winners is equals to the number of losers.
4. non-zero-sum-game - number of winners is not equal to number of losers.
5. Constant sum game - ~~[illegible]~~ Sum of loser and winner are constant.
6. pure strategy game - In a game eache player employes one particular strategy.

has the best strategy throughout the game. This is called as pure strategy game.

7. mixed-strategy games — By situtation to change the strategy to win the game.

pure-strategy

A type of strategy used to maximize the profit of ~~your~~ a player and correspondigly minimize the loss of another player is called Pure Strategy.

A pure strategy exists where the maximin ($\underline{v}$) of the game is equal to the minimax ($\overline{v}$) value of the game.

Mixed Strategy

A mixed strategy exists where the maximin ($\underline{v}$) of the game is not equal to the minimax ($\overline{v}$) of the game.

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Saddle point.

Saddle point on your pay-off matrix is that point of intersection between maximum value of row minimum and the minimum value of column maximum. A pure strategy occurs at the Saddle point. Saddle point is denoted by S_0.

Pay-off matrix.

A set of values obtained by players in a game and presented in rows and columns is called pay-off matrix.

Fair game

A game is said to be fair if maximin $(\underline{V})$ = minimax $(\overline{V})$ = value of the game (V)

Solve the following game the pay-off Matrix will given below.

Player B

$$
\begin{array}{c|ccccc|c}
 & & & & & & \text{Row-minimum} \\
\text{Player A} & \multicolumn{5}{c|}{\begin{pmatrix} 9 & 3 & 1 & 8 & 0 \\ 6 & 5 & 4 & 6 & 7 \\ 2 & 4 & 3 & 3 & 8 \\ 5 & 6 & 2 & 2 & 1 \end{pmatrix}} & \begin{matrix} 0 \\ 4 \\ 2 \\ 1 \end{matrix} \\
\text{Column maximum} & 9 & 6 & 4 & 8 & 8 &
\end{array}
$$

maximin $(\underline{v})$ = 4

minimax $(\bar{v}) = 4$

maximin $(\underline{v})$ = minimax $(\bar{v})$ = value of the game $(v) = 4$

Saddle point $= a_{23}$

Dominance properties

solving of game whose pay-off matrix

$$\begin{array}{ccc} & B_1 & B_2 \\ A_1 & \multicolumn{2}{c}{\multirow{2}{*}{}} \end{array}$$

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & a & b \\ A_2 & c & d \end{array}$$

Strategy of player A

$$P_1 = \frac{d-c}{a-b-c+d}$$

$$P_2 = \cancel{\frac{d-b}{a-b-c+d}} \quad 1-P_1$$

Strategy of player A $[P_1 \; P_2]$

Strategy for player B

$$q_1 = \frac{d-b}{a-b-c+d}$$

$$q_2 = 1-q_1$$

Strategy of player $B = \frac{a-c}{a-b-c+d}$

The value of the game is

$$= \frac{ad-bc}{a-b-c+d}$$

1)

	B_1	B_2	B_3	B_4	Row minimum
A_1	0.4	0.5	0.9	0.3	0.3
A_2	0.8	0.4	0.3	0.7	0.3
A_3	0.7	0.6	0.8	0.9	0.6
A_4	0.7	0.2	0.4	0.6	0.2
Column maximum	0.8	0.6	0.9	0.9	

$(\underline{v})$ maximin $= 0.6$

minimax $(\bar{v}) = 0.6$

maximin $(\underline{v}) =$ minimax $(\bar{v}) = 0.6\,(v)$

Saddle point $= A_3 B_2$

2)

$$\begin{array}{c c c}
 & \text{Player B} & \text{Row minimum} \\
\text{Player A} & \begin{bmatrix} 20 & 12 & 15 \\ 11 & 10 & 12 \\ 15 & 11 & 10 \end{bmatrix} & \begin{array}{c} 12 \\ 10 \\ 11 \end{array} \\
\text{Column maximum} & \begin{array}{ccc} 20 & 12 & 15 \end{array} &
\end{array}$$

maximin $= 12$

minimax $= 12$

maximin $=$ minimax $= 12\,(v)$

Saddle point $= A_1 B_2$

By Row minimum

.3 0.3

.7 0.3

.9 0.6

ab 0.2

.9

)=0.6(V)

3)

$$\begin{array}{c|cccc|c}
 & \multicolumn{4}{c|}{\text{Player B}} & \text{Row minimum} \\
\hline
\text{Player A} & 8 & 2 & 9 & 5 & 2 \\
 & 6 & 5 & 7 & 18 & 5 \\
 & 7 & 3 & -4 & 10 & -4 \\
\hline
\text{Column maximum} & 8 & 5 & 9 & 18 &
\end{array}$$

maximin = 5

minimax = 5

maximin = minimax = 5 (V)

Saddle point = A_2B_2

4)

$$\begin{array}{c|ccc|c}
 & X & Y & Z & \text{Row minimum} \\
\hline
A & 5 & 7 & 5 & 5 \\
B & 4 & 0 & 1 & 0 \\
C & 5 & 9 & 3 & 3 \\
\hline
\text{Column maximum} & 5 & 9 & 5 &
\end{array}$$

maximin = 5

minimax = 5

maximin = minimax = 5 (V)

Saddle point = AX

5)

$$
\begin{array}{c|c|c}
 & \text{Firm B} & \text{Row minimum} \\
\hline
\text{Firm A} & \begin{bmatrix} 3 & -1 & 4 & 6 & 7 \\ -1 & 8 & 2 & 4 & 12 \\ 16 & 8 & 6 & 14 & 12 \\ 1 & 11 & -4 & 2 & 1 \end{bmatrix} & \begin{matrix} -1 \\ -1 \\ 6 \\ -4 \end{matrix} \\
\hline
\text{Column maximum} & \begin{matrix} 16 & 11 & 6 & 14 & 12 \end{matrix} &
\end{array}
$$

maximin = 6

minimax = 6

maximin = minimax = 6 (v)

saddlepoint = A_3, B_3.

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(7) Solve the following game $\begin{bmatrix} 3 & -2 \\ -2 & 5 \end{bmatrix}$

$$\begin{bmatrix} 3 & -2 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$P_1 = \frac{d-c}{a-b-c+d}$$

$$= \frac{5+2}{3+2+2+5} = \frac{7}{12}$$

$$\boxed{P_1 = 7/12}$$

$P_2 = 1 - P_1$

$P_2 = 1 - \frac{7}{12}$

$\boxed{P_2 = \frac{5}{12}}$

$q_1 = \frac{d-b}{a-b-c+d}$

$= \frac{5+2}{3+2+2+5} = \frac{7}{12}$

$\boxed{q_1 = \frac{7}{12}}$

$q_2 = 1 - q_1$

$q_2 = 1 - \frac{7}{12}$

$\boxed{q_2 = \frac{5}{12}}$

$V = \frac{ad - bc}{a-b-c+d}$

$V = \frac{(3\times5) - ((-2)\times(-2))}{3+2+2+5}$

$= \frac{15 - 4}{12} = \frac{11}{12}$

$\boxed{V = \frac{11}{12}}$

Answer.

The str for player A

$$[p_1, p_2] = \left[\tfrac{7}{12} \quad \tfrac{5}{12}\right]$$

The str for Player B

$$[q_1, q_2] = \left[\tfrac{7}{12} \quad \tfrac{5}{12}\right]$$

The value of the game is

$$v = \frac{11}{12}$$

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1) Solve the following game.

$$\begin{bmatrix} 8 & 1 \\ 4 & 6 \end{bmatrix}$$

2) Solve the following game and determine the Strategies

$$\begin{bmatrix} 11 & 4 \\ 7 & 9 \end{bmatrix}$$

3) Solve the follo

determine the

$$\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$P_1 = \frac{d-c}{a-}$

$=$

$P_2 =$

$=$

$q_1 =$

q_2

3) solve the following game and determine the strategies

$$\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$P_1 = \frac{d-c}{a-b-c+d}$$

$$= \frac{-2-0}{-1-0-0-2} = \frac{-2}{-3} = \frac{2}{3}$$

$$P_2 = 1 - P_1$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

$$q_1 = \frac{d-b}{a-b-c+d}$$

$$= \frac{-2-0}{-1-0-0-2} = \frac{-2}{-3} = \frac{2}{3}$$

$$q_2 = 1 - q_1$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

$$\boxed{q_2 = \frac{1}{3}}$$

$$V = \frac{ad - bc}{a - b - c + d}$$

$$= \frac{(-1)\times(-2) - (0)\times(0)}{-1 - 0 - 0 - 2}$$

$$\boxed{V = \frac{2}{-3}}$$

Answer

The str for player A

$$[P_1 \; P_2] = \left[\frac{2}{3} \; \frac{1}{3}\right]$$

the Str for player B

$$[q_1 \; q_2] = \left[\frac{2}{3} \; \frac{1}{3}\right]$$

The value of the game is

$$V = -\frac{2}{3}$$

Dominance properties

Rule 1 : If the pay-off of the i^{th} row are less than or equal to those in the k^{th} row, i^{th} row is dominated by k^{th} row hence omitt the i^{th} row.

Rule 2 : If the pay-off of the i^{th} column are greater than or equal to those in the k^{th} column, i^{th} column is dominated by k^{th} column then omitt the i^{th} column.

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1) Using dominance principle solve the game.

$$\begin{bmatrix} 8 & 10 & 9 & 14 \\ 10 & 11 & 8 & 12 \\ 13 & 12 & 14 & 13 \end{bmatrix}$$

$$\begin{array}{c|cccc} & B_1 & B_2 & B_3 & B_4 \\ \hline A_1 & 8 & 10 & 9 & 14 \\ A_2 & \cancel{10} & \cancel{11} & \cancel{8} & \cancel{12} \\ A_3 & 13 & 12 & 14 & 13 \end{array}$$

Here second row is less than are equal to third row. This implies second row is dominated by third row.

So we omit 2nd row

$$\begin{array}{c|cccc} & B_1 & B_2 & B_3 & B_4 \\ \hline A_1 & 8 & 10 & \cancel{9} & 14 \\ A_3 & 13 & 12 & \cancel{14} & 13 \end{array}$$

Here third column is greater than or equal to first column. This implies third colum is domated by first column. So we omitt third column.

$$\begin{array}{c|ccc} & B_1 & B_2 & B_4 \\ A_1 & 8 & 10 & 14 \\ A_3 & 13 & 12 & 13 \end{array}$$

Here 4th column is greater than or equal to 1st column. This implies. So the 4th column is dominated 1st column So we ommit 4th Column.

$$\begin{array}{c|cc} & B_1 & B_2 \\ A_1 & \cancel{8} & \cancel{10} \\ A_3 & 13 & 12 \end{array}$$

Here 1st row is <= 3rd row this implies 1st row is dominated by 3rd row. So we ommit 1st row.

$$\begin{array}{c|cc} & B_1 & B_2 \\ A_3 & \cancel{13} & 12 \end{array}$$

Here ~~2~~ 1st column is greater than or equal to 2nd column. This implies 1st column is dominated by 2nd Column. So we omitt 1st column.

$$\begin{array}{c|c} & B_2 \\ A_3 & [12] \end{array}$$

The value of the game is 1

& the saddle point is (A_3, B_2)

The matching player is Paid rupees 8 if two coins turned both heads and rupees 1 if the coins turned both tail. The non matching player is paying ₹3 when two coins do not match. Given the choice of being the matching or non-matching player which one would you choose what would be your strategy

$$\begin{array}{c|cc} & H & T \\ \hline H & 8 & -3 \\ T & -3 & 1 \end{array} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$P_1 = \frac{d-c}{a-b-c+d} = \frac{1+3}{8+3+3+1}$$

$$\boxed{P_1 = \frac{4}{15}}$$

game is 12.
is (A_3, B_2)

Paid rupees
heads
and both
is paying
match.
matching
one
would.

$P_2 = 1 - P_1$

$= 1 - \frac{4}{15}$

$= \frac{15-4}{15}$

$\boxed{P_2 = \frac{11}{15}}$

$q_1 = \frac{d-b}{a-b-c+d} = \frac{1+3}{8+3+3+1}$

$= \frac{4}{15}$

$q_2 = 1 - q_1$

$= 1 - \frac{4}{15} = \frac{15-4}{15}$

$\boxed{q_2 = \frac{11}{15}}$

$V = \frac{ad-bc}{a-b-c+d} = \frac{(8)(1) - [(-3)(-3)]}{8+3+3+1}$

$= \frac{8-9}{15} = \frac{-1}{15}$

The str for Player A $[P_1, P_2] = [4/15, 11/15]$
The str for Player B $[q_1, q_2] = [4/15, 11/15]$
The value of the game is $-1/15$
~~The saddle point is.~~

Solve the game whose pay-off matrix is

$$\begin{array}{c c}
\begin{bmatrix} 4 & -1 & 5 \\ 0 & 5 & 3 \\ 5 & 3 & 7 \end{bmatrix} & \begin{array}{c} \text{Row minimum} \\ -1 \\ 0 \\ 3 \end{array} \\
\begin{array}{ccc} 5 & 5 & 7 \end{array} & \\
\text{Column maximum} &
\end{array}$$

$$\text{maximin } (\underline{v}) = 3$$

$$\text{minimax } (\bar{v}) = 5$$

So here minimax $\neq$ maximin so we can next method using dominance property.

$$\begin{bmatrix} \cancel{4} & \cancel{-1} & \cancel{5} \\ 0 & 5 & 3 \\ 5 & 3 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 5 & \cancel{3} \\ 5 & 3 & \cancel{7} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 5 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$P_1 = \frac{d-c}{a-b-c+d} = \frac{3-5}{0-5-5+3} = \frac{\cancel{-}2}{\cancel{-}7}$$

$$\boxed{P_1 = \frac{2}{7}}$$

$$P_2 = 1 - P_1$$
$$= 1 - \frac{2}{7} = \frac{5}{7}$$

$$\boxed{P_2 = \frac{5}{7}}$$

$$q_1 = \frac{d-b}{a-b-c+d} = \frac{3-5}{0-5-5+3}$$
$$= \frac{-2}{-7}$$

$$\boxed{q_1 = \frac{2}{7}}$$

$$q_2 = \boxed{1 - q_1}$$
$$= 1 - \frac{2}{7}$$

$$\boxed{q_2 = \frac{5}{7}}$$

$$V = \frac{ad-bc}{a-b-c+d} = \frac{0(3)-5(5)}{0-5-5+3}$$

$$= \frac{-25}{-7}$$

$$\boxed{V = \frac{25}{7}}$$

The Str of player A $\{p_1, p_2\} = \{$ [illegible] $\}$

The Str of player B $\{q_1, q_2, q_3\} = \{$ [illegible] $\}$

The value of the game is $\frac{25}{7}$

Graphical Method.

Method of Solving $2 \times n$ Game. Plot the ~~Paase~~ pairs of pay-off of the n Strategies of the players A and B on ~~two~~ veritical axis and Connect the pairs of points by Straightlines locate the highest Point on the line segment that from the lower boundary of the graph The lines that intersect at this point identity the Strategies player B Should adopt in its optimum Strategy.

Method of solving nx2 games.

* Plot the pairs of pay-off of the n strasies of the players A and B on two vertical axis and Connect the pairs of points by Straigh lines. Locate the lowest point on the line of segments that from the upper boundary the lines that intersect at this point identify the stratgies of the player A.

Solve the game

$$\begin{bmatrix} \cancel{3} & \cancel{4} & \cancel{4} & \cancel{0} \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix}$$

$$\begin{bmatrix} \cancel{3} & 4 & 2 & 4 \\ \cancel{4} & 2 & 4 & 0 \\ \cancel{0} & 4 & 0 & 8 \end{bmatrix}$$

$$\begin{bmatrix} \cancel{4} & \cancel{2} & \cancel{4} \\ 2 & 4 & 0 \\ 4 & 0 & 8 \end{bmatrix}$$