

17/8/2021

Transportation Problem.

North West Corner rule.

	Supply					
P	2	11	10	3	7	4
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
Demand	3	3	4	5	6	21/21

Here demand = supply

It is a Balanced Transportation Problem

32	11	10	3	7	4(1)
1	4	7	2	1	8
3	9	4	8	12	9
3	3	4	5	6	

(0)

11	10	3	7	1(0)
4	7	2	1	8
9	4	8	12	9
3	4	5	6	

(2)

8	4	7	2	1	8(6)
9	4	8	12	9	
2	4	5	6		
(0)					

7	12	11	6(2)
4	8	12	9
4	5	6	

(0)

2	1	1	2(0)
8	12		9

(3) 5 6

3	12	9(6)
3	6	

(0)

6	12	6(0)
6		

(0)

$$\begin{aligned}
 \text{Total Cost} &= 3 \times 2 + 1 \times 1 + 2 \times 4 + 1 \times 7 + 1 \times 2 \\
 &\quad + 3 \times 8 + 6 \times 12 \\
 &= 6 + 1 + 8 + 28 + 4 + 24 + 72 \\
 &= 153.
 \end{aligned}$$

The no. of allocation is

$$(\text{Row} + \text{Column} - 1) = 3 + 5 - 1 = 7$$

\therefore It is a non degenerate basic feasible solution.

				Supply
1	2	1	4	30
3	3	2	1	50
4	2	5	9	20
		30	10	100/100

demand 20 40

Here demand = supply.

It is a balanced transportation problem.

20	1	2	1	4	3d(10)
3	3	2	1		50
4	2	5	9		20
20 (0)	40	30	10		

10	1	1	4	10(0)
3		2	1	50
2	5		9	20
	40	30	10	
(30)				

30	2	1	50(20)	
2	5	9	20	
30	30	10		
(0)				

20	1	1	20(0)	
5		9	20	
30		10		
(10)				
10	9	20(10)		
(0)				

9	10(0)
---	-------

10

(6)

$$\begin{aligned}
 \text{Total Cost} &= 20 \times 1 + 10 \times 2 + 40 \times 3 + 20 \times 2 \\
 &\quad + 10 \times 5 + 10 \times 9 \\
 &= 20 + 20 + 90 + 40 + 50 + 90 \\
 &= 310.
 \end{aligned}$$

The no of allocation is ..

$$m+n-1 = 3+4-1$$

$$= 6$$

\therefore It is a non-degenerate basic feasible solution.

(3)

			Supply
			8
			7
			9
Demand	10	10	25/25
	12	14	8
	10	10	

Here, demand = supply

\therefore It is an balanced transportation problem.

8/8	1/9	1/8	8(0)
10	7	10	7
11	9	7	9
T2	14	10	4
10	10	8	

(2)

13	10	7	10	7(5)
11	9	7	7	9
12	14	10	10	4
2	10	8		
(0)				

15	10	5(0)
9	7	9
14	10	4
10	8	
(5)		

15	9	7	9(4)
14	10	7	9
5	8		
(0)			

A/7	4(0)
10	9

8
(4)

14/10	4(0)
-------	------

4
(6)

$$\begin{aligned}
 \text{Total Cost} &= 8 \times 8 + 2 \times 10 + 5 \times 7 + 8 \times 9 + 1 \times 7 + \\
 &\quad 4 \times 10 \\
 &= 64 + 20 + 35 + 45 + 28 + 46 \\
 &= 232
 \end{aligned}$$

$$\text{No of allocation } m+n-1 = 4+3-1=6$$

\therefore It is a non-degenerate basic feasible solution.

Row minimum method.

					Supply
	5	2	4	3	22
	4	8	1	6	15
	4	6	7	5	8
demand	7	12	7	19	45/45

Here Demand = Supply

\therefore It is an Balanced T.P

5	12/2	4	3	22(10)
4	8	1	6	15
4	6	7	5	8
7	12	7	19	
(0)				

5	4	3	16(0)
4	1	6	15
4	7	5	8
	7	19	
7		(9)	

4	1	6	15(8)
4	7	5	8
	7	9	
7		(0)	

4	6	8(1)
4	5	8
	9	
7		(0)

6	1(0)
5	8

9
(8)

8	5	8(0)
---	---	------

8
(0)

$$\text{Total Cost} = 12x_2 + 10x_3 + 7x_1 + 7x_4 + 1x_5$$

$$8x_5 = 24 + 30 + 7 + 28 + 6 + 9$$

$$= 135$$

$$\text{no of allocation} = m+n-1 = 3+4-1 = 6$$

\therefore It is an non-degenerate basic feasible solution.

Q.

			Supply:
8	9	8	8
10	7	10	7
11	9	7	9
12	14	10	4
Demand	10	10	8 $\frac{28}{28}$

Here Demand = Supply.
It is an balanced T.P.

8	9	8	(80)
10	7	10	7
11	9	7	9
12	14	10	4
(2)	10	10	8

10	9	10	7(0)
11	9	7	9
12	14	10	4
2	10	8	
(3)			

11	19	87	9(1)
12	14	10	4
2	3	8	
		(0)	

11	19	1(0)
12	14	4
2	3	
	(2)	

2	14	4(2)
2	2	
(0)		

2	14	2(0)

2(0)

$$\begin{aligned}
 \text{Total Cost} &= 8 \times 8 + 7 \times 7 + 8 \times 7 + 1 \times 9 \\
 &\quad + 2 \times 12 + 2 \times 14 \\
 &= 64 + 49 + 56 + 9 + 24 + 28 \\
 &= 230
 \end{aligned}$$

No of allocation $m+n-1$

$$= 4+3-1 = 6.$$

\therefore It is an non-degenerate basic feasible solution.

16/8/2021 Column minimum method.

5	12	14	3	22
4	8	1	6	15
4	6	7	5	8
7	12	7	19	45/45

It is a balanced P-Q.
Demand = Supply.

15	2	4	3	22
7/4	8	1	6	5(8)
4	6	7	5	8

7 12 7 19

(6)

10/2	4	3	22 (10)
7/8	1	6	8
16	7	5	8

12 7 19

(8)

4	3	10
7/1	6	8 (1)
7	5	8

7 19

(0)

$$\begin{array}{r}
 10 \\
 \boxed{3} \\
 \times 6 \\
 \hline
 18 \\
 5 \\
 \hline
 19 \\
 \end{array}
 \quad 10(0)$$

(9)

$$\begin{array}{r}
 18 \\
 \boxed{6} \\
 \times 8 \\
 \hline
 48 \\
 5 \\
 \hline
 9 \\
 \end{array}
 \quad 8(0)$$

(1)

$$\begin{array}{r}
 10 \\
 \boxed{6} \\
 \end{array}
 \quad 1(0)$$

(6)

Least Cost method.

5	4	3	6
4	7	6	8
2	5	8	12
8	6	7	4
8	10	12	30/30.

* In the overall cell choose the least no.

* Suppose if we have a least no we can choose any one.

15	4	3	6
14	7	6	8
8/2	5	8	12(4)
18	6	7	4
8	10	12	(6)

4	6	3	(0)
7	6		8
5	8		4
6	7		4
10	12	6	

7	6	8	
4	5	8	(0)
6	7		4
10	6		
(6)			

7	6	8(2)
6	7	4
6	6	
(0)		

7	2
6	4(0)
6	(2)

2	2(0)
(0)	

$$\begin{aligned}
 \text{Total cost} &= (8 \times 2) + (6 \times 3) + (4 \times 1) \\
 &\quad + (6 \times 6) + (4 \times 6) \\
 &= 16 + 18 + 20 + 36 + 24 \\
 &= 128
 \end{aligned}$$

$$\text{no of allocation} = m+n-1$$

$$= 4+3-1 = 6$$

\therefore It is an non-degenerate BPS.

Vogel's Approximation Method (VAM)

Unit cost penalty method.

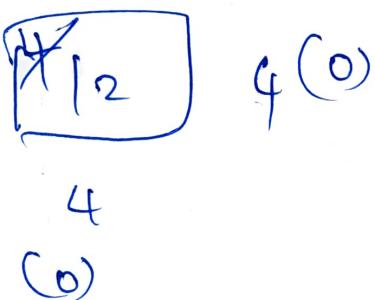
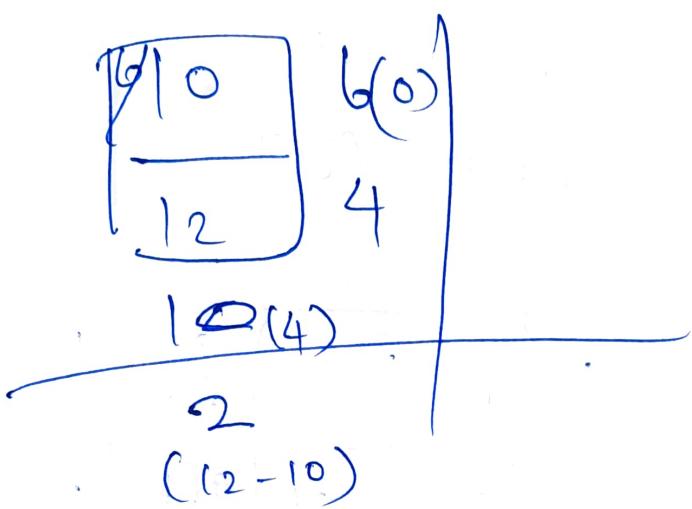
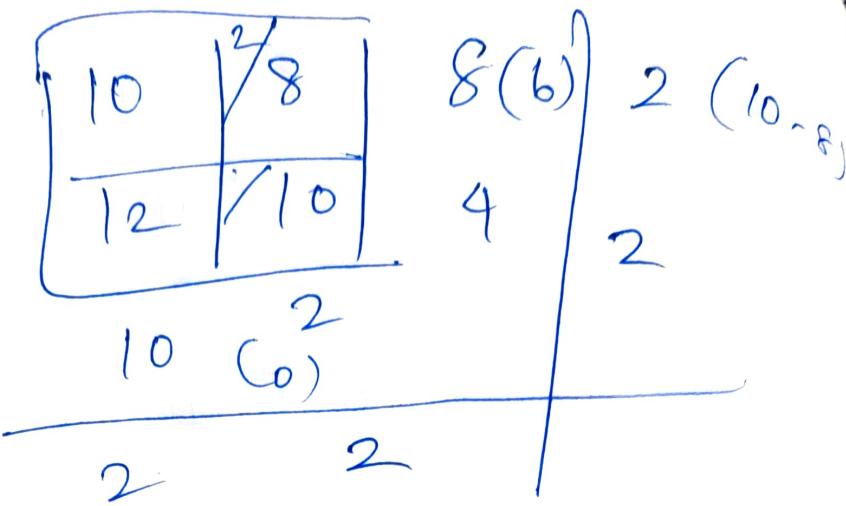
10	9	8	8
10		10	7
11	9	7	9
T ₂	14	10	4
10	10	8	

$$\begin{array}{l}
 4(9+5) \\
 4(2+7) \\
 24+14
 \end{array}$$

10	9	8	8	1 (9-8)
10	7	10	7(0)	③ (10-7)
11	9	9	9	2 (9-7)
12	14	16	4	2 (12-10)
10	10	8		
1	2	1		
(11-10)	(9-7)	(8-7)		

10	9	8	8	1
11	9	7	9(6)	2
12	14	10	4	2
10	3(0)			
1	5			
	14-9			

10	8	8	1
11	6	6(0)	4
12	10	4	2
10	8		
1	1	1	
	11-7		



$$\begin{aligned}
 \text{Total lost} &= (7 \times 7) + (3 \times 9) + (6 \times 7) + \\
 &\quad (2 \times 8) + (6 \times 10) + (4 \times 12) \\
 &= 49 + 27 + 42 + 16 + 60 + 48 \\
 &= 242.
 \end{aligned}$$

$$\text{no of allocation} = m+n-1$$

$$= 4+3-1$$

$$= 6$$

\therefore It is an non-degenerate BF.

Q.

8	7	3	60
3	8	9	70
11	3	5	80
50	80	80	

\therefore It is a balanced TP.
Demand \approx Supply.

8	7	3	60	4	(7-3)
50/3	8	9	70(20)	5	(8-3)
11	3	5	80	2	(5-3)
50.	80	80			
5	4	2			

8-3 7-3 5-3

$$\begin{array}{c|cc}
 & 60 & \\
 \hline
 7 & 13 & \\
 \hline
 8 & 9 & \\
 \hline
 3 & 5 & \\
 \hline
 \end{array} \quad \left| \begin{array}{c} 60(0) \\ 20 \\ 80 \\ \hline 80 \quad 80 \\ (20) \end{array} \right. \quad \left| \begin{array}{c} 4 \\ 2 \\ 1 \\ 2 \\ 3 \\ \hline 4 \quad 2 \\ 7-3 \quad 5-3 \end{array} \right. \quad \begin{array}{l} (7-3) \\ 9-8 \\ 5-3 \end{array}$$

$$\begin{array}{c|cc}
 & 20 & \\
 \hline
 8 & 19 & \\
 \hline
 3 & 5 & \\
 \hline
 \end{array} \quad \left| \begin{array}{c} 1 \\ 2 \\ 80(0) \end{array} \right. \quad \left| \begin{array}{c} 1 \\ 2 \\ 2 \\ 2 \\ \hline 5 \\ 2 \\ (8-3) \end{array} \right. \quad \begin{array}{l} 1 \\ 2 \\ 2 \\ 2 \end{array}$$

$$\begin{array}{c|c}
 20 & 9 \\ \hline
 20 & \\ \hline
 \end{array} \quad \begin{array}{c} 20(0) \\ 20 \\ (0) \end{array}$$

$$\text{Total Cost} = (50 \times 3) + (60 \times 3) + (80 \times 3) + (20 \times 9)$$

$$= 150 + 180 + 240 + 180$$

$$\text{No of allocation} = m+n-1 = 3+3-1 = 5.$$

It follows a degenerate BFS.

Unbalanced Transportation Problem:

25	17	25	300
15	10	18	500

300 300 800 1100/800

Here demand ≠ supply.

25	17	25	300
15	10	18	500
0	0	0	300

300 300 500 1100/1000

Maximization problems on transportation
Problem.

145 143 140 125 185

152 154 151 148 146

160 161 162 160 155

Demand 3000 3000 10000 5000 4000

Solve maximization transportation
problem.

(step 1 : Convert maximization problem
into minimization problem
by multiplying - in all the
cost entries.)

	-145	-143	-140	-125	-135	5000
	-152	-154	-151	-148	-146	10000
	-160	-161	-162	+160	-155	12500
Supply 8000	3000	3000	10000	5000	4000	
12500						

1) North West Corner rule:

-145	-143	-140	-125
-152	-154	-151	-148
-160	-161	-162	+160

-145	-143	-140	-125	-125	0
-152	-154	-151	-148	-146	0
-160	-161	-162	+160	-155	0

i) North West corner rule.

$\frac{5000}{-145}$	-143	-140	-125	-135
-152	-154	-151	-148	-146
-160	-161	-162	-160	-155
2000	3000	10000	5000	4000
(0)				

$\frac{2000}{-143}$	-140	-125	-135	0
-154	-151	-148	-146	0
-161	-162	-160	-155	0
8000	10000	5000	4000	2500
(1000)				

$\frac{1000}{-154}$	-151	-148	-146	0
-161	-162	-160	-155	0
1000	10000	5000	4000	2500
(0)				

35	0
46	0
55	0
00	2500

5000 (0000)
10,000
10,5000

9000	-151	-148	-196	0	0
-162	-160	-155	0	0	12500
10000	5000	4000	2500		
(1000)					

~~(11,500)~~

10000	-162	-160	-155	0	12,500
10000	5000	4000	2500		
(0)					

2000 (0)
10,000
12,5000

10,0 00 (9000)
12,5 00

-160	-155	0	2500
5000	4000	2500	
(6,500)			

6 -160	-155	0	11,500
5000	4000	2500	
(0)			

(155)	0	6,500 (2500)
4000	2500	
(0)		

2500 0
2500
(0)

$$= \cancel{3000} \times (-145) + \cancel{(-2000)} \rightarrow$$

$$= 3000 \times (-145) + 2000 \times (-143) + 1000 \times (-154) \\ + 9000 \times (-151) + 1000 \times (-162) + \\ 5000 \times (-160) + 4000 \times (-155)$$

$$= \cancel{-435000} - 286000 \cancel{-} 1,54,000 \cancel{-} 162,000 \\ - 1359000 \cancel{-} 800000 \cancel{-} 620000$$

$$\boxed{\min z = -3,81,16,000}$$

$$\max z = \cancel{3,81,16,000} .$$

ii) Row minimum method.

-145	-143	-140	-125	-135	0	(3000)
-152	-154	-151	-148	-146	0	10,000
-160	-161	-162	-160	-155	0	12,500
3000	3000	10,000	5000	4000	2500	

(0)

-143	-140	-125	-135	0	2000(0)
-154	-151	-148	-146	0	10,000
-161	-162	-160	-155	0	12,500
3000	10,000	5000	4000	2500	

(- 000)

-154	-151	-148	-146	0	10,000
-161	-162	-160	-155	0	12,500
1000	10,000	5000	4000	2500	

(0)

-151	-148	-146	0	9000
-162	-160	-155	0	12,500

(- 1000) 10,000 5000 4000 2500

$$10 \boxed{0} - 160 \boxed{1} - 155 \boxed{1} \quad | \quad 10 \boxed{1} \quad | \quad 12,500$$

1000 5000 4000 2800

(o) $(11,500)$

$$500 \boxed{0} - 160 \boxed{1} - 155 \boxed{1} \quad | \quad 0 \quad | \quad 11,500$$

5000 4000 2800

(o)

$(6,500)$

$$4000 \boxed{-155} \boxed{1} \quad | \quad 0 \quad | \quad 6,500$$

4000 2800

(o) $(2,500)$

(o)

(0)

$\boxed{200} \boxed{0} \quad | \quad 0 \quad | \quad 2,500$

$\boxed{250} \quad | \quad 0 \quad | \quad 2,500$

(o)

$\boxed{250} \quad | \quad 0 \quad | \quad 2,500$

(o) $(2,500)$

$$\begin{aligned} \text{Total Cost} &= (3000x - 145) + (2000x - 143) + \\ &\quad (1000x - 154) + (9000x - 151) + (1000x \\ &\quad - 162) + (5000x - 160) + (4000x - 155) \\ &\quad + (2500x) \end{aligned}$$

$$\approx -435000 - 286000 - 157000$$

$$-1359000 - 162000 = 800000$$

$$-620000 \text{ to}$$

$$\approx -3,816,1000$$

(11,500)
12,500

00)

iii) Column minimum method.

-145	-143	-140	-125	-135	0	5000
-152	-154	-151	-148	-146	0	10,000
0/-160	-161	-162	-160	-155	0	12,500
3000	3000	10,000	5000	4000	2500	(9,500)

(0)

-143	-140	-125	-135	0	5000
-154	-151	-148	-146	0	10,000
161	-162	-160	-155	0	12,500

(0)

-140	-125	-135	0	5000
-151	-148	-146	0	10,000
162	-160	-155	0	12,500

(31500)

$\boxed{140}$	125	135	0	5000
$\boxed{-181}$	-148	-146	0	$10,000$
3500	5000	4000	2500	$(6,500)$

(e)

$\boxed{125}$	135	0	5000	
$\boxed{-148}$	-146	0	61000	(11500)
8000	4000	2500		

(o)

$\boxed{135}$	0	5000		
$\boxed{+46}$	0	$1500(0)$		
4000	2500			
(2500)				

$\boxed{135}$	0	$5000(2500)$		
2500	2500			

(o)

$\boxed{150\%}$	2500	(0)		

$2500(0)$

$$\begin{aligned}
 & \text{Total cost} = (3000x - 160) + (5000x - 161) + (6500x - 162) \\
 & + (3500x - 151) + (-148 \times 5000) \\
 & + (1500x - 146) + (2500x - 135) \\
 & + 2500x^0 \\
 & = -480,000 - 483,000 - 1053000 \\
 & - 528500 - 740000 - 219000 - \\
 & 337500
 \end{aligned}$$

$$\min z = -3841000$$

$$\max_2 = 3841000$$

iv) Least cost method.

-145	-143	✓ 140	-125	-135	0
-152	-154	✓ 151	-148	-146	0
-160	-161	✓ 162	-160	-155	0
3000	3000	10,000	5000	4000	2500 (25)
(0)					

-145	-143	-125	-135	0	5000
-152	-154	-148	-146	0	10,000
-160	✓ 161	-160	-155	0	2500 (0)
3000	3000	5000	4000	2500	
(500)					

-145	✓ -143	-125	-135	0	5000
-152	✓ -154	-148	-146	0	10,000
3000	3000	5000	4000	2500	
500					
(0)					

-145	-125	-135	0
✓ 152	-148	-146	0
3000	5000	4000	2500 (4000)

500	-125	-135	70	5000
0	-148	-148	0	6500
5000	4000	2500		(1000)
(0)				

1500	-135	0	5000	2500
0	-146	0	(100)	
4000	2500			
(2500)				

2500	-135	0	5000	(2500)
0	2500	2500		
2500	0	2500	(0)	
(0)				

$$\begin{aligned}
 \text{Total cost} &= (10,000x - 162) + (2500x - 161) \\
 &\quad + (500x - 154) + (3000x - 152) \\
 &\quad + (5000x - 148) + (1500x - 146) \\
 &\quad + (2500x - 135) + (2500x - 0)
 \end{aligned}$$

$$\begin{aligned}
 &= -1,620,000 - 402,500 - 17000 \\
 &\quad - 456,000 - 740,000 - 219,000
 \end{aligned}$$

$$-337500$$

$$= -3,82,000$$

$$\min z = -3,82,000$$

$$\max z = 3,82,000$$

v) VAM.

-145	-143	140	125	135	0	5000
-152	-154	-151	-148	-146	0	10,000
160	161	162	161	155	0	2500
3000	3000	10,000	5000	9000	2500	(0)
7	11	11	23	11	0	

$$0 - (-125) = 125$$

$$146$$

$$155$$

-145	-143	140	125	135	0	5000
-152	-154	-151	-148	-146	0	10,000
160	161	162	160	155	0	2500
3000	3000	10,000	5000	9000	2500	(0)
7	11	11	23	11	0	

$0 - (-125) = 125$
 146
 155

$-160 \quad -161 \quad 4000$
 $3000 \quad 3000 \quad (1000)$
 (b)

$-160 \quad -162 \quad 6000$
 $3000 \quad (6000)$

$$\begin{array}{r}
 3\overset{4}{\cancel{0}}\overset{0}{0} \\
 - 160 \\
 \hline
 3000
 \end{array}
 \quad
 \begin{array}{r}
 -161 \\
 \hline
 3000
 \end{array}
 \quad
 \left\{
 \begin{array}{l}
 6000 \\
 (3000)
 \end{array}
 \right.$$

Diagram of a trapezoid ABCD with parallel bases AB and CD. The top base AB is labeled 16. The left slanted side AD is labeled 3000'. The right slanted side BC is labeled 3000'. The bottom base CD is labeled 3000'.

Total no of cost = $2000x^0$ +

$$(5000x - 125) +$$

$$(10000x - 151) +$$

$$(4000x - 155) +$$

$$(3000x - 160) +$$

$$(3000x - 161)$$

$$= -625000 - 1510000 -$$

$$620000 - \cancel{160000}^{48}$$

$$- \cancel{23000}^{48}$$

$$\min z = -37,18600$$

$$\max z = 3718,000$$