EE6506 Computational Electromagnetics: Jan - May 2021, Uday Khankhoje

Assignment 1, 10 Feb 2021, due 25 Feb 2021 (before class on Moodle)

Clearly state the CONCEPT involved in solving each problem. Do NOT write lengthy answers. Make reasonable assumptions and STATE them. You are free to discuss with your classmates and look at reference books. But, you must write your solutions yourself and mention who you collaborated with for your solution. Looking for solutions online is not allowed. You may LaTeX your solutions, or scan your handwritten solutions for upload.

1. For a volume V with a surface S and scalar functions ψ, ϕ, prove:
   (a) $\oint_S \phi \frac{\partial \psi}{\partial n} ds = \int_V (\phi \nabla^2 \psi) dv + \int_V (\nabla \phi \cdot \nabla \psi) dv$: Green's first identity
   (b) $\oint_S (\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}) ds = \int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dv$: Green's second identity
2. Given that the instantaneous electric field inside a source-free, homogeneous, isotropic, and linear medium is given by $\mathcal{E} = [\alpha(x+y)\hat{x} + \beta(x-y)\hat{y}] \sin(\omega t)$, find a relation between α, β.
3. An electric line source of infinite length and constant current along the z axis radiates in free space, and at large distances from the source the expression for the magnetic field is as follows: $\vec{H} = \hat{\phi} H_0 \exp(-j\beta_0 \rho)/\sqrt{\rho}$, ($H_0$ is a constant). Find the corresponding electric field.
4. (MATLAB) Construct a hill using a 2D gaussian function, e.g. $z = \exp(-(x-5)^2 - 4(y-3)^2)$. Write a function that creates an outward normal to the hill's surface given a point on it as input. Now, construct a contour around the hill whose height is half the maximum height. At equispaced points on this contour, plot the outward normals. In the main body of the submission, your plot must show the hill and the chosen contour as well. Hint: use the `quiver3` function. You must include your code (with comments) in the submission as an appendix, as well as a stand-alone .m file.
5. (MATLAB) Assume the time harmonic form of Maxwell's equations (i.e. time dependence is as $\exp(j\omega t)$). Write a code that takes as input an electric field and returns the magnetic field. Note that you must write a *symbolic* code. For e.g. your code should be able to take an electric field of the following form as input: $\vec{E} = (x\sin(z), y^2, z^3 x)$. Hint: Look up the symbolic toolbox. This problem will prepare you to learn a very powerful way of doing math and scientific computing. You must include the output of the code to the above example in the main body of the submission. You must include your code (with comments) in the submission as an appendix, as well as a stand-alone .m file.
6. Give a short proof of the uniqueness theorem, being sure to point out why the medium needs to be lossy.
7. Derive a continuity relation for the normal components of $\vec{D}$ at an interface between two media in the presence of a surface charge.
8. Derive (but do not solve) the form of the differential equation obeyed by a time-harmonic wave travelling along the z axis of a waveguide at a point away from the exciting current. The cross-section of the waveguide is not a function of the z co-ordinate, but an arbitrary $\epsilon(x, y)$. Hint: What is the logical and simplest form of the z-dependence of the field? Make simplifying assumptions like the choice of polarization, etc., and state them.

At the start of your submission, please write the following text:

I, ____________________(name), state that the submitted work is my original work.

I have discussed with these people: ____

______________________________(sign and date).