EE6506 – COMPUTATIONAL ELECTROMAGNETICS

ASSIGNMENT – 1

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EE18B038

I, Srivenkat A, state that the submitted work is my original work.
I have discussed with these people: Sreekar Sai R, Chandan Bhat
A. Srivenkat – 23/02/21

1) a) Green's 1^{ST} identity: $\oint_S \varphi \frac{\partial \psi}{\partial n} ds = \int_V \varphi \nabla^2 \psi \, dV + \int_V \nabla\varphi \cdot \nabla\psi \, dV$

proof:
V: volume of a bound region with surface S.
at any point on S, define outward normal: $\hat{n}$ $(\therefore \hat{n}\cdot\hat{t} = 0)$.
tangent to surface: $\hat{t}$

$\therefore \nabla = \frac{\partial}{\partial n}\hat{n} + \frac{\partial}{\partial t}\hat{t}$: del operator

$LHS = \oint \varphi \frac{\partial \psi}{\partial n} \cdot ds = \oint \varphi \left(\frac{\partial \psi}{\partial n} \hat{n} \cdot d\hat{s} \right)$

$d\hat{s}$: small area element $= ds\hat{n}$. $\therefore \frac{\partial \psi}{\partial n} \hat{n} \cdot d\hat{s} = \left(\frac{\partial \psi}{\partial n}\hat{n} + \frac{\partial \psi}{\partial t}\hat{t} \right) \cdot d\hat{s}$

as the 2^{ND} dot product gives 0.

$\therefore$ LHS $= \oint_S \varphi \frac{\partial \psi}{\partial n} \hat{n} \cdot d\hat{s} = \oint_S (\varphi \nabla \psi) \cdot d\hat{s}$

from Gauss' Divergence theorem, $\oint_S \vec{A} \cdot d\hat{s} = \int_V (\nabla \cdot \vec{A}) \, dV$

for any vector field $\vec{A}$. V: vol. enclosed by S.

here, the vector field $\vec{A} \triangleq \varphi \nabla \psi$

$\therefore \oint_S (\varphi \nabla \psi) \cdot d\hat{s} = \int_V \nabla \cdot (\varphi \nabla \psi) \, dV$.

from std. theorems of vector calculus,

$\rightarrow \nabla \cdot (\varphi \vec{A}) = \varphi (\nabla \cdot \vec{A}) + \vec{A} \cdot \nabla \varphi$ for a vector field $\vec{A}$.

here, $\vec{A} \triangleq \nabla \psi$. $\therefore \nabla \cdot (\varphi \nabla \psi) = \varphi (\nabla \cdot \nabla \psi) + (\nabla \psi) \cdot (\nabla \varphi)$.

$\rightarrow \nabla \cdot (\nabla \psi) = \nabla^2 \psi$, by defn. of scalar Laplacian operator.

$\therefore \int_V \nabla \cdot (\varphi \nabla \psi) \, dV = \int_V \left[\varphi \nabla^2 \psi + (\nabla \varphi) \cdot (\nabla \psi) \right] dV$

$=$ RHS

$\therefore \oint_S \varphi \frac{\partial \psi}{\partial n} ds = \int_V (\varphi \nabla^2 \psi) \, dV + \int_V (\nabla \varphi) \cdot (\nabla \psi) \, dV$. proved

b) Green's 2ND identity: $\oint_S \left(\varphi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \varphi}{\partial n} \right) ds = \int_V (\varphi \nabla^2 \psi - \psi \nabla^2 \varphi) \, dV$

proof: as a corollary of the 1^{st} identity,

$$\oint_S \varphi \frac{\partial \psi}{\partial n} ds = \int_V [\varphi \nabla^2 \psi + (\nabla\varphi \cdot \nabla\psi)] dV.$$

$$\oint_S \psi \frac{\partial \varphi}{\partial n} ds = \int_V [\psi \nabla^2 \varphi + (\nabla\psi \cdot \nabla\varphi)] dV.$$

on subtracting the above two eqns.,

$$\oint_S \left(\varphi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \varphi}{\partial n}\right) ds = \int_V (\varphi \nabla^2 \psi - \psi \nabla^2 \varphi) dV$$. proved

2) given conditions about the medium : source free, homogenous, istropic & linear.

$\therefore \rho_e = 0$ all through the medium. (source free)
ε : constant all through the medium (istropic)

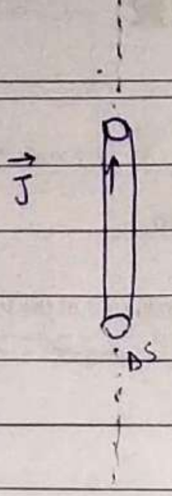
$\therefore$ as $\nabla \cdot \bar{D} = \rho_e$, $\varepsilon \nabla \cdot \bar{E} = 0$. $\therefore \underline{\nabla \cdot \bar{E} = 0}$.

$\vec{E} = [\alpha(x+y)\hat{x} + \beta(x-y)\hat{y}] \sin(\omega t)$. $\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$

$\therefore \nabla \cdot \bar{E} = \alpha \frac{\partial}{\partial x}(x+y) + \beta \frac{\partial}{\partial y}(x-y) = 0.$

$\therefore \alpha \cdot 1 + \beta(-1) = 0$ $\therefore \underline{\underline{\alpha = \beta}}$: reln. between α & β

for $\alpha = \beta$, even $\nabla \times \bar{E} = 0$. $\therefore$ consistent.

3) 

current is z- invariant

implicit time dependence of $e^{j\omega t}$ is assumed.

in the far field region, medium is free space.

$\therefore \varepsilon = \varepsilon_0, \ \mu = \mu_0$.

$J = 0$ in free space.

$\therefore$ from Maxwell's equations, $\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = j\omega \varepsilon_0 \vec{E}$.

$$\vec{E} = \frac{-j}{\omega \varepsilon_0} (\nabla \times \vec{H})$$

$$\nabla \times \vec{H} = \frac{H_0}{\rho} \begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ 0 & \rho \frac{e^{-j\beta_0\rho}}{\sqrt{\rho}} & 0 \end{vmatrix}$$

$$\therefore \nabla \times \vec{H} = \frac{H_0}{\rho} \cdot \hat{z} \frac{\partial}{\partial \rho}\left(\sqrt{\rho}\, e^{-j\beta_0\rho}\right) = \frac{H_0}{\rho} \hat{z} \cdot \left[\sqrt{\rho}(-j\beta_0) e^{-j\beta_0\rho} + \frac{e^{-j\beta_0\rho}}{2\sqrt{\rho}}\right]$$

$$= \frac{H_0 e^{-j\beta_0\rho}}{2\rho^{3/2}} (1 - 2j\beta_0\rho)\, \hat{z}.$$

$$\therefore \vec{E} = \frac{-jH_0 (1-2j\beta_0\rho) e^{-j\beta_0\rho}}{2\rho^{3/2}\omega\varepsilon_0} \hat{z}$$

4) the MATLAB code is added in the appendix.
the plots are also added in the appendix along with the code
the meshgrid function is used to form the 2D array of x,y coordinates.
hill is constructed using the gaussian function: $z = \exp(-(x-5)^2 - 4(y-3)^2)$

the outward normal is calculated as $\nabla(g)$ where $g(x,y,z) = z - f(x,y)$.
$f(x,y) = \exp(-(x-5)^2 - 4(y-3)^2)$

the results are validated against the inbuilt MATLAB function, surfnorm

5) The medium is assumed to be isotropic & homogenous. $\therefore$ μ : constant throughout the medium

$\therefore$ from Maxwell's equations, $\nabla \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t}$

as the fields are time-harmonic, $\dfrac{\partial \bar{B}}{\partial t} = \mu \dfrac{\partial \bar{H}}{\partial t} = j\omega\mu \bar{H}$

$\therefore \nabla \times \bar{E} = -j\omega\mu \bar{H}$. $\quad \therefore \vec{H} = \dfrac{j}{\omega\mu} (\bar{\nabla} \times \bar{E})$.

$\nabla \times \bar{E}$ is computed symbolically in terms of x, y, z.

for $\vec{E} = (x \sin z, y^2, z^3 x)$, obtained $\vec{H} = \dfrac{j}{\omega\mu} (0, x\cos(z) - z^3, 0)$

6) To prove Uniqueness Theorem, consider a region of volume V, bounded by a surface S.

Say, we have 2 possible sets of soln.: E^a, H^a, E^b, H^b.

$\therefore$ for uniqueness theorem to be valid, we need to prove $E^a = E^b$, $H^a = H^b$

proof:

both soln. satisfy Maxwell's theorem

$$\therefore \nabla \times E^a = -j\omega\mu H^a - M \qquad \nabla \times H^a = j\omega\varepsilon E^a + J$$

$$\nabla \times E^b = -j\omega\mu H^b - M \qquad \nabla \times H^b = j\omega\varepsilon E^b + J$$

let $\delta E \triangleq E^a - E^b$, $\delta H = H^a - H^b$

$$\therefore \nabla \times \delta E = -j\omega\mu\, \delta H \qquad \nabla \times \delta H = j\omega\varepsilon\, \delta E \quad \rightarrow \text{(i)}$$

here, μ, ε are complex and functions of position (need not be isotropic medium)

energy carried by the diff. of fields $= \int_S (\delta\bar{E} \times \delta\bar{H}^*) \cdot d\bar{s}$

$$= \int_V \nabla \cdot (\delta\bar{E} \times \delta\bar{H}^*)\, dV \quad \text{[from divergence theorem]}$$

$\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot (\nabla \times \bar{A}) - \bar{A} \cdot (\bar{\nabla} \times \bar{B})$: std. theorem of vector calculus

$$\therefore \nabla \cdot (\delta\bar{E} \times \delta\bar{H}^*) = \delta\bar{H}^* \cdot (\nabla \times \delta\bar{E}) - \delta\bar{E} \cdot (\nabla \times \delta\bar{H}^*)$$

from eqn (i), $\delta\bar{H}^* \cdot (\nabla \times \delta\bar{E}) = \delta\bar{H}^* \cdot (-j\omega\mu\, \delta\bar{H}) = -j\omega\mu\, |\delta H|^2$

$\nabla \times \delta\bar{H}^* = (\nabla \times \delta\bar{H})^* = -j\omega\varepsilon^* \delta\bar{E}^*$.

$$\therefore \delta\bar{E} \cdot (\nabla \times \delta\bar{H}^*) = -j\omega\varepsilon^* |\delta E|^2$$

$$\therefore \int_S (\delta\bar{E} \times \delta\bar{H}^*) \cdot d\bar{s} = \int_V \left(-j\omega\mu\, |\delta H|^2 + j\omega\varepsilon^* |\delta E|^2\right) dV \quad \rightarrow \text{(ii)}$$

as μ, ε are complex, $\varepsilon = \varepsilon_R + j\varepsilon_i$, $\mu = \mu_R + j\mu_i$

$\therefore$ RHS of (ii) : $\int_V -|\delta H|^2 j\omega(\mu_R + j\mu_i) + |\delta E|^2 j\omega(\varepsilon_R - j\varepsilon_i) \, dV$

$$= \underbrace{\int_V \left(\omega\mu_i |\delta H|^2 + \omega\varepsilon_i |\delta E|^2\right) dV}_{\text{real part}} + \underbrace{\int_V \left(-j\omega\mu_R |\delta H|^2 + j\omega\varepsilon_R |\delta E|^2\right) dV}_{\text{imaginary part}}.$$

in LHS of (ii), $d\vec{s} = \hat{n}\, ds$, $\hat{n}$: outward normal

$\therefore (\delta\bar{E} \times \delta\bar{H}^*) \cdot \hat{n} = (\hat{n} \times \delta\bar{E}) \cdot \delta\bar{H}^* = -(\hat{n} \times \delta\bar{H}^*) \cdot \delta\bar{E}$

- if tangential part of $\bar{E}$ is specified at boundary, $E_t^a = E_t^b = E_t^0$ (specified)

$\therefore \delta\bar{E} = \delta E_t \hat{t} + \delta E_n \hat{n}$. $\delta E_t = 0$ as $E_t^a = E_t^b$.

$\therefore \delta\bar{E} = \delta E_n \hat{n}$. $\therefore \hat{n} \times \delta\bar{E} = \delta E_n (\hat{n} \times \hat{n}) = \underline{\underline{0}}$

- similarly, if tangential part of $\bar{H}$ is specified at boundary, $\delta\bar{H}_t = 0$.

$\therefore \delta\bar{H} = \delta H_n \hat{n}$. $\therefore \hat{n} \times \delta\bar{H}^* = (\hat{n} \times \delta\bar{H})^* = \delta H_n^* (\hat{n} \times \hat{n}) = \underline{\underline{0}}$

- if either $\bar{H}_{tan}$ or $\bar{E}_{tan}$ is specified at all pts of the boundary, wherever $\bar{E}_{tan}$ specified, $\hat{n} \times \delta\bar{E} = 0$. wherever $\bar{H}_{tan}$ is specified, $\hat{n} \times \delta\bar{H} = 0$

$\therefore$ at all pts on bnd., $\underline{(\delta\bar{E} \times \delta\bar{H}^*) \cdot \hat{n} = 0}$

if any of the condn. are true, LHS = 0.

$\therefore$ RHS = 0. $\quad R_e(\text{RHS}) = 0, \quad I_m(\text{RHS}) = 0.$

in the $I_m(\text{RHS})$, $\left(\varepsilon_R |\delta E|^2 - \mu_R |\delta H|^2\right)\omega = 0.$

can be true for positive ε_R, μ_R.

but, in the $R_e(\text{RHS})$, $\left(\varepsilon_i |\delta E|^2 + \mu_i |\delta H|^2\right)\omega = 0.$

if the medium is lossless, $\varepsilon_i = \mu_i = 0$ $\therefore$ inconclusive about δE & δH.

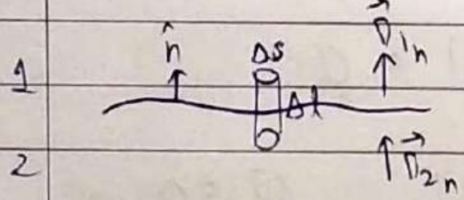
but, if the medium is lossy, $\varepsilon_i < 0$ & $\mu_i < 0$.

$\therefore$ both terms ≤ 0 $\therefore$ for $\varepsilon_i |\delta E|^2 + \mu_i |\delta H|^2 = 0$

$\Rightarrow \underline{|\delta E| = |\delta H| = 0}$

$\therefore$ for a lossy medium, the soln. obtained for the fields in a region is unique.

7) at the interface between two regions 1 and 2, analysing continuity of $\vec{D}_n$



assume a gaussian pillbox at the interface. $\Delta S, \Delta l \to 0.$

from maxwell's eqns, $\nabla \cdot \bar{D} = \rho_e$ at any point.

integrate over vol. of cylinder. $\int_V \nabla \cdot \bar{D}\, dV = \int_V \rho_e\, dV$

$\int_V \nabla \cdot \bar{D}\, dV = \int_S \bar{D} \cdot d\bar{S}$ from divergence theorem

$\int_V \rho_e \, dV$ = Q enclosed in the cylinder = $\rho_{e_s} \cdot \Delta S$, ρ_{e_s} : surface charge density

$$\int_S \bar{D} \cdot d\bar{S} = D_{1_n} \Delta S - D_{2_n} \Delta S + D_{planar} \cdot 2\pi r \Delta l$$

make $\Delta l \to 0$. $\therefore \rho_{e_s} \Delta S = (D_{1_n} - D_{2_n}) \Delta S$.
(property of assumed cylinder)

$\therefore D_{1_n} - D_{2_n} = \rho_{e_s}$. $\qquad D_{1_n} - D_{2_n} = (\bar{D}_1 - \bar{D}_2) \cdot \hat{n}$.

$\therefore \hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_{es}$

↳ surface charge

↳ difference in normal components of $\vec{D}$

8) The waveguide is known to have a z-invariant cross section + permittivity ($\varepsilon(x,y)$)

note: $\varepsilon(x,y)$ is the permittivity of the bulk of the waveguide.

$\therefore$ maxwell's equations : $\nabla \cdot \bar{B} = 0 \Rightarrow \nabla \cdot \bar{H} = 0$

$\nabla \cdot \bar{D} = 0$ (as there are no free charges). $\Rightarrow \nabla \cdot (\varepsilon \bar{E}) = 0$

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} = \varepsilon \frac{\partial \bar{E}}{\partial t}$$

Analysing for time-harmonic waves propagating along the z direction,

time dependence : $e^{+j\omega t}$

z dependence : $e^{-\gamma z}$, $\gamma = \alpha + j\beta$

$\therefore$ all the field terms have an implicit $e^{j\omega t - \gamma z}$ term attached.

assuming medium is non magnetic, $\mu = \mu_0$

as the dependence on t, z $\because$ uniform for all field components,

$$\frac{\partial}{\partial t}(\cdot) = j\omega(\cdot). \quad \frac{\partial}{\partial z}(\cdot) = -\gamma(\cdot)$$

as $\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$, $\Rightarrow$ $$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -j\omega\mu_0 (H_x, H_y, H_z).$$

as $\nabla \times \bar{H} = \varepsilon \frac{\partial \bar{E}}{\partial t}$, $\Rightarrow$ $$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}, \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = j\omega\varepsilon (E_x, E_y, E_z)$$

$$\therefore \frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu_0 H_x \quad -(i) \qquad \frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x \quad -(iv)$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu_0 H_y \quad -(ii) \qquad -\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \quad -(v)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z \quad -(iii) \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial H_y} = j\omega\varepsilon E_z \quad -(vi).$$

these 6 equations are valid even for any arbitrary $\varepsilon(x,y)$.

solve this system of eqns. to represent E_x, E_y, H_x, H_y in terms of E_z, H_z

(Signature of the Invigilator)

solving (i) & (v), $H_x = \frac{1}{(\gamma^2+\omega^2\mu_0\varepsilon)}\left[j\omega\varepsilon\frac{\partial E_z}{\partial y} - \gamma\frac{\partial H_z}{\partial x}\right]$

$$E_y = \frac{1}{(\gamma^2+\omega^2\mu_0\varepsilon)}\left[j\omega\mu_0\frac{\partial H_z}{\partial x} - \gamma\frac{\partial E_z}{\partial y}\right]$$

solving (ii) & (iv), $H_y = \frac{-1}{(\gamma^2+\omega^2\mu_0\varepsilon)}\left[j\omega\varepsilon\frac{\partial E_z}{\partial x} + \gamma\frac{\partial H_z}{\partial y}\right]$

$$E_x = \frac{-1}{(\gamma^2+\omega^2\mu_0\varepsilon)}\left[j\omega\mu_0\frac{\partial H_z}{\partial y} + \gamma\frac{\partial E_z}{\partial x}\right]$$

to bring all info into one eqn.,

$$\nabla\times(\nabla\times\bar{H}) = \nabla\times(j\omega\varepsilon\bar{E})$$

$$\nabla(\cancel{\nabla\cdot\bar{H}}^{\,0}) - \nabla^2 H = j\omega\ \nabla\times(\varepsilon\bar{E})$$

from std. theorems of vector calculus,

$$\nabla\times(\varepsilon\bar{E}) = \varepsilon(\nabla\times\bar{E}) + (\nabla\varepsilon\times\bar{E})$$

$$= -j\omega\mu_0\varepsilon\bar{H} + (\nabla\varepsilon\times\bar{E})$$

$$\therefore -\nabla^2\bar{H} = \omega^2\mu_0\varepsilon\bar{H} + (\nabla\varepsilon\times\bar{E})$$

$$\therefore \nabla^2\bar{H} + \omega^2\mu_0\varepsilon\bar{H} = -(\nabla\varepsilon\times\bar{E})$$

as $\frac{\partial}{\partial z}(\cdot) = -\gamma\cdot(\cdot)$, $\nabla^2(\cdot) \triangleq \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \gamma^2\right)(\cdot)$

define $h^2 = \gamma^2 + \omega^2 \mu_0 \varepsilon$. note : h^2 : not a constant.
depends on (x,y) due to ε.

→ analysing for TE polarisation $(E_z = 0)$:

$$\therefore \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right)(\bar{H}) = -\nabla\varepsilon \times \bar{E} = \bar{E} \times \nabla\varepsilon$$

$$= \frac{-1}{h^2}\left[\left(j\omega\mu_0 \frac{\partial H_z}{\partial y} + \gamma \cancelto{0}{\frac{\partial E_z}{\partial x}}\right)\hat{x} + \left(\gamma \cancelto{0}{\frac{\partial E_z}{\partial y}} - j\omega\mu_0 \frac{\partial H_z}{\partial x}\right)\hat{y}\right] \times (\nabla\varepsilon)$$

since $\varepsilon \triangleq \varepsilon(x,y)$, $\nabla\varepsilon = \frac{\partial\varepsilon}{\partial x}\hat{x} + \frac{\partial\varepsilon}{\partial y}\hat{y}$.

$\therefore$ RHS has only $\hat{z}$ terms. $\therefore$ $\hat{x}, \hat{y}$ terms on LHS : 0.

LHS : $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right)(H_x\hat{x} + H_y\hat{y} + H_z\hat{z})$

$$\therefore \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right) H_x = 0.$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right) H_y = 0$$

$\therefore$ <u>H_x & H_y satisfy scalar Helmholtz equation</u>

analysing for H_z :

RHS : $\frac{-1}{h^2}\left[j\omega\mu_0 \frac{\partial H_z}{\partial y}\cdot\frac{\partial\varepsilon}{\partial y} + j\omega\mu_0 \frac{\partial H_z}{\partial x}\cdot\frac{\partial\varepsilon}{\partial x}\right]\hat{z}$.

$$\therefore \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = \frac{-j\omega\mu_0}{h^2}\left(\frac{\partial H_z}{\partial y}\frac{\partial \varepsilon}{\partial y} + \frac{\partial H_z}{\partial x}\cdot\frac{\partial \varepsilon}{\partial x}\right)$$

can be written as $$\nabla^2 H_z + k^2 H_z = \frac{-j\omega\mu_0}{h^2}\left(\nabla H_z \cdot \nabla\varepsilon\right)$$

$$\nabla^2 H_z + \omega^2\mu_0\varepsilon H_z = \frac{-j\omega\mu_0}{\gamma^2+\omega^2\mu_0\varepsilon}\left(\nabla H_z \cdot \nabla\varepsilon\right)$$

once H_z is known,

$$E_x = \frac{-j\omega\mu_0}{\gamma^2+\omega^2\mu_0\varepsilon}\frac{\partial H_z}{\partial y}$$

$$E_y = \frac{j\omega\mu_0}{\gamma^2+\omega^2\mu_0\varepsilon}\frac{\partial H_z}{\partial x}$$

Appendix

1 Question 4

The plots are attached below:

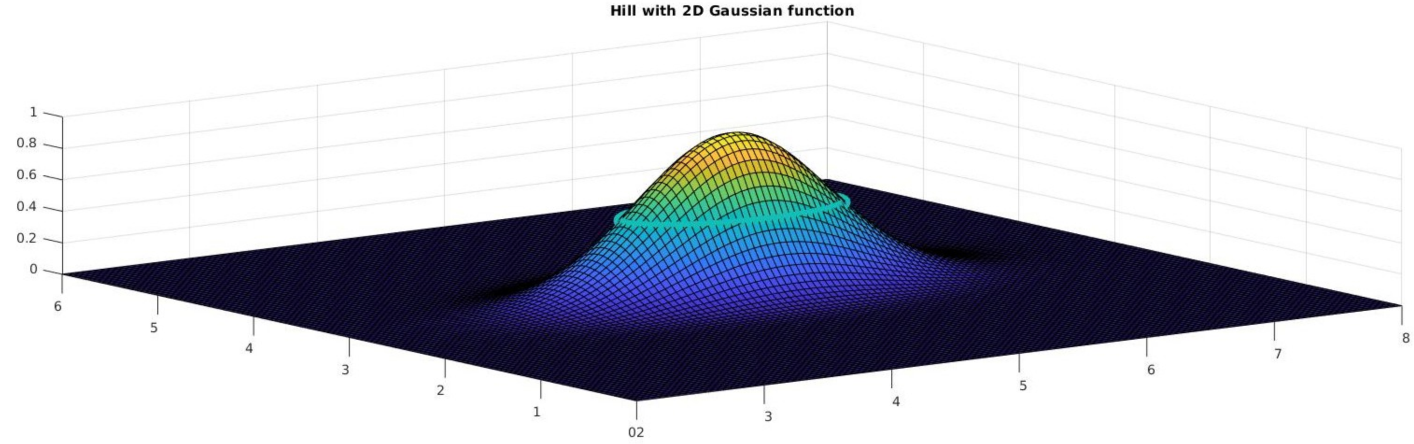


Figure 1: Hill constructed using 2D gaussian function

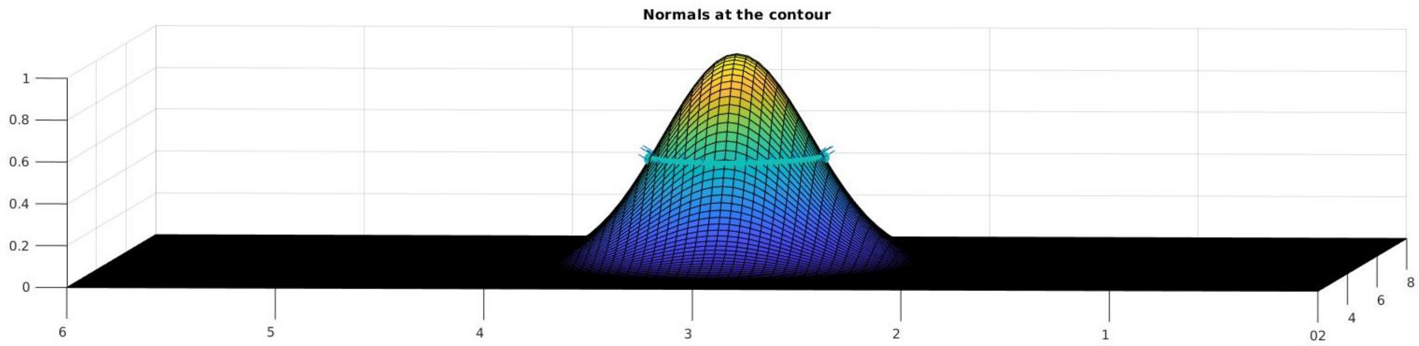


Figure 2: Normals plotted at the contour

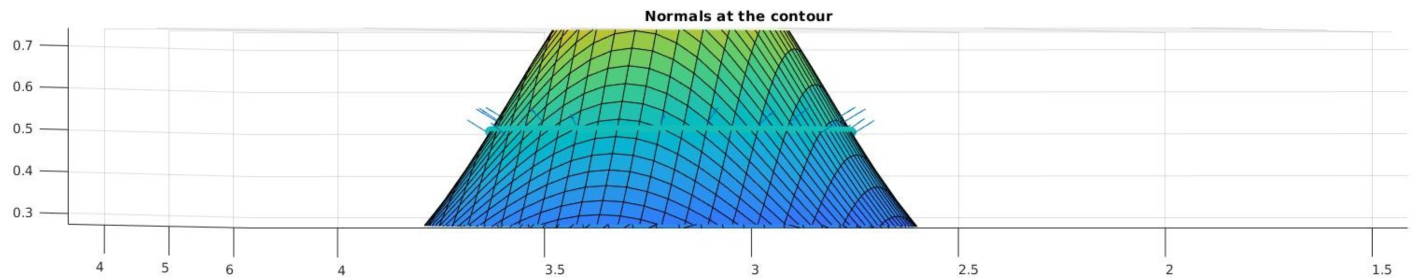


Figure 3: Zoomed in view to see the normals

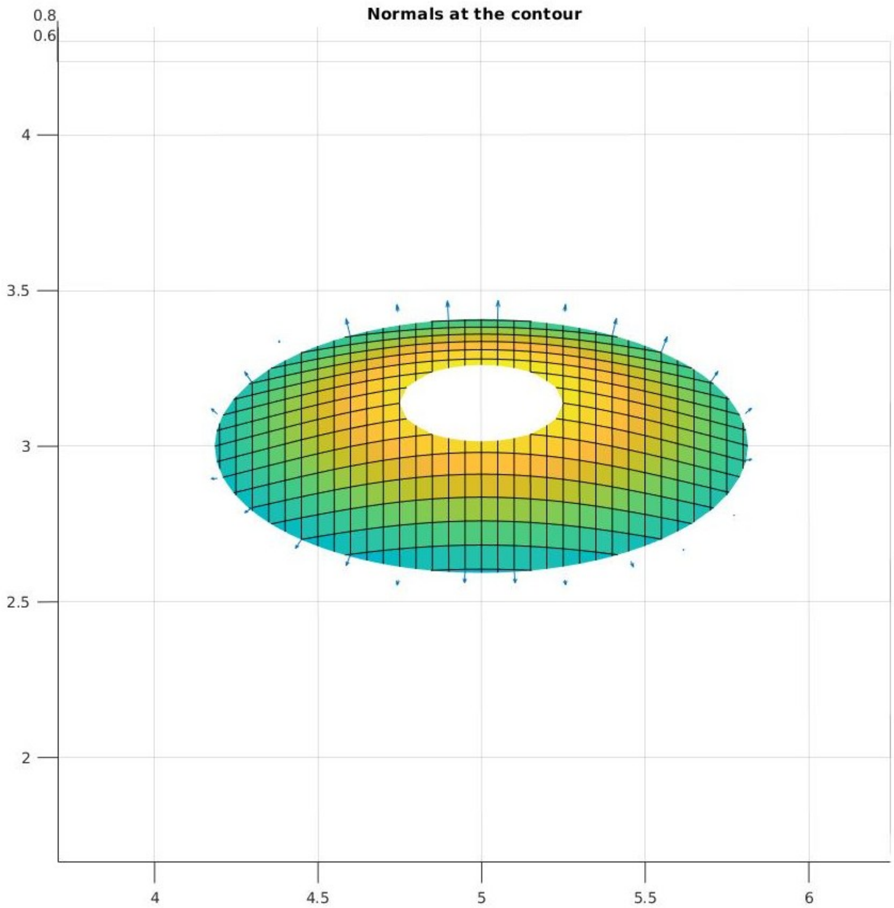


Figure 4: Top View to see the equispaced contours

Note: I have used the external function, **curvspace** in MATLAB to analytically compute equispaced points on the chosen contour and then plot normals at those points. ref: Curvspace in MATLAB file exchange

1.1 Code used

```
w = 0.05;    %discretisation parameter
x = 2:w:8;   %range for x and y
y = 0:w:6;

[X,Y] = meshgrid(x,y);

Z = exp( -(X-5).^2 - 4.*(Y-3).^2 );    %construct 2d gaussian hill
z_max = max(max(Z));
z_contour = z_max/2;     %find contour at half the height

figure(1);
surf(X,Y,Z);     %plotting the 2d gaussian hill
title("Hill with 2D Gaussian function");
hold on;
[~,H] = contour3(X,Y,Z,[z_contour,z_contour]);     %marking contour at half  the height
H.LineWidth = 8;
axis equal;
hold off;

figure(2);
surf(X,Y,Z);     %plotting the 2d gaussian hill
title("Normals at the contour");
hold on;
```

```
[C,H] = contour3(X,Y,Z,[z_contour,z_contour]);    %plot contour
H.LineWidth = 8;

P = C(:,2:101)';
Eq_P = curvspace(P,25)';    %find equispaced points on the contour

[nx,ny,nz] = find_normal(Z,w);    %defined function to calculate normal
filter = filterxy(X,Y,Eq_P);
quiver3(X, Y, Z, nx .* filter, ny .* filter, nz .* filter);    % plotting normals at equispaced points
axis equal;
hold off;

%{
% Validation for finding the normal done using the inbuilt surfnorm function
figure(3);
surf(X,Y,Z);
title("Validation: Normals found from surfnom");
hold on;
[~,C] = contour3(X,Y,Z,[z_contour,z_contour]);
C.LineWidth = 8;
[U,V,W] = surfnorm(X,Y,Z);
quiver3(X,Y,Z,U,V,W);
axis equal;
hold off;
%}

function filter = filterxy(X,Y,q)    %to overlay equispaced points on the predefined mesh
    tol = 0.026;
    filter = zeros(size(X));
    %calculate nearest points on the mesh for each one of the equispaced
    %points. then do elementwise OR for individual filters to get net filter
    for i=1:size(q')
        temp_filter = X < (q(1,i) + tol) & X > (q(1,i) - tol) & Y < (q(2,i) + tol) & Y > (q(2,i) - tol);
        filter = filter | temp_filter;
    end
end

function [nx,ny,nz] =  find_normal(g,w)    %to find normal for a surface g
    %g = z - f(x,y),  normal(g) = 1 - grad(f)
    [nx,ny] =  gradient(g .* (-1/w));
    nz = ones([1 + (6/w), 1 + (6/w)]);
end
```

2 Question 5

Explanation mentioned in handwritten solution. Obtained expression for H is

$$\vec{H} = \frac{j}{\omega\mu}(x(cos(z) - z^3)\hat{y} \tag{1}$$

2.1 Code used

```
syms x y z w mu;     %define symbols for x,y,z, permiability

E = [x * sin(z), y^2, z^3 * x];     %give input electric field expression here
%mu = 4*pi*(10^(-7));     %give permeability value here

H = symbolic_H_finder(E, mu)

function H = symbolic_H_finder(E, mu)
    syms x y z w;
    Ex = E(1); Ey = E(2); Ez = E(3);     %define x,y,z components of E

    %curl(E) = -jwu*H,     H = (j/wu) * curl(E)
    Hx = ( diff(Ez,y) - diff(Ey,z) ) * 1i / (w * mu) ;     %assign corresponding curl elements to components of H
    Hy = ( diff(Ex,z) - diff(Ez,x) ) * 1i / (w * mu) ;
    Hz = ( diff(Ey,x) - diff(Ex,y) ) * 1i / (w * mu) ;

    H = [Hx, Hy, Hz];
end
```