Signal Processing Project Record

Fractional Fourier Transform (FRFT) and Watermarking in FRFT

Introduction:

We know how to analyze a signal which varies with time in time domain. That is by plotting values of the signal with respect to time and we are also aware of signals varying with frequency where We can analyze them by plotting the signal with respect to frequency. This can be done by using Fourier Transformation. We usually interpret Fourier Transformation as a transformation of a time domain signal into a frequency domain signal and Inverse Fourier transformation as a transformation of a frequency domain signal into a time domain signal. Apart from this we come across so many signals which vary with time and frequency like speech, music, images and signals related to medical etc. To analyze these signals, we use time frequency representations. One of the basic forms of Time frequency representation is Short-time Fourier Transform. Fractional Fourier Transform is also one of the important time frequency representations which can think of rotation in time-frequency domain. Digital watermarking of signals and images, which can be defined as "imperceptible insertion of information into multimedia data".

Problem and Solution:

A signal with function of time considered as a representation with perfect time resolution and Fourier transform is considered as a representation with perfect frequency resolution. But in real-life we face many situations where signal frequencies are varying with time. The practical motivation for time-frequency analysis is that classical Fourier analysis assumes that signals are infinite in time, while in practice many signals are of short duration and change substantially over their duration.

How to analyze those signals? Here is the solution. The representation which allows a signal to analyze in both time and frequency simultaneously is **Time-Frequency representation**. The analysis we do here is **Time-Frequency analysis.**

Short-Time Fourier Transformation:

As the name suggests, short-time Fourier Transform means dividing longer time signal into shorter segments of equal length and then computing the

Fourier transform separately. We can also say it is a sequence of Fourier Transformation of a windowed signal. Window having Narrow-width results in a better resolution in the time domain and it generates a poor resolution in the frequency domain, and vice versa.

Formula to compute short-time Fourier Transform of x(t) is,

$$X(t,w) = \int_{-\infty}^{\infty} x(\tau)\, w(t-\tau)\, e^{-jw\tau} \cdot d\tau$$

NOTE: From this we can say that classical Fourier Transform is nothing but short-time Fourier Transform with window of infinite length.

Apart from short-time Fourier Transform, there are so many time-frequency representations. One of them is Fractional Fourier Transform(FRFT).

Fractional Fourier Transform:

Fractional Fourier transformation can transform a signal into the domain between time and frequency. FRFT can be interpreted as rotation in time-frequency domain with some angle α. In mathematics, Fractional Fourier Transform(FRFT) is a generalization of classical Fourier Transform (FT). Let x(t) be the time domain signal and X(w) be the Fourier transform of x(t). Then FRFT of that signal id represented by X_α (u). If angle is 0 then X_α (u) will be time domain signal which is x(t). If the angle is **90°** then the X_α (u) will be X(w). If angle is π, X_α (u) be x(-t).

The formula for X_α (u) is given by

$$X_\alpha(u) = \int_{-\infty}^{\infty} x(t)\, K_\alpha(t,u)\, dt$$

Kernal K_α (t,u) is given by,

$$K_\alpha(t,u) = \begin{cases} \sqrt{\dfrac{1-j\cot\alpha}{2\pi}}\; e^{j\frac{t^2+u^2}{2}\cdot\cot\alpha - jut\csc\alpha} & \text{if } \alpha \text{ is not multiple of } \pi \\ \delta(t-u) & \text{if } \alpha \text{ is a multiple of } 2\pi \\ \delta(t+u) & \text{if } \alpha+\pi \text{ is a multiple of } 2\pi \end{cases}$$

After substituting the kernel into the formula, we get

$$
X_\alpha(u) = \begin{cases} \sqrt{\frac{1-j\cot\alpha}{2\pi}}\, e^{j\frac{u^2}{2}\cot\alpha} \int_{-\infty}^{\infty} x(t)\, e^{j\frac{t^2}{2}\cot\alpha}\, e^{jut\csc\alpha} \cdot dt & \text{if } \alpha \text{ is not multiple of } \pi \\ x(t) & \text{if } \alpha \text{ is a multiple of } 2\pi \\ x(-t) & \text{if } \alpha+\pi \text{ is a multiple of } 2\pi \end{cases}
$$

Properties of FRFT:

Frft satisfies additivity, commutativity and Associativity.

It also follows the following properties:

Unitary:

$$\int x^*(\omega)\, g(\omega)\, d\omega = \int x_\alpha^*(\omega)\, g_\alpha(\omega)\, d\omega$$

Time Reversal:

$$X_\alpha P = P X_\alpha$$

$$X_\alpha[x(-\omega)] = x_\alpha(-\omega)$$

Applications of FRFT:

Watermarking

We use this watermarking in the field of multimedia like copyright protection. Here we basically create a large number of watermarks and embed into the image to protect the actual owner's data. So, this embedding and detecting of watermarking can be done by using FRFT.

Filtering

The idea of FRFT can be applied to the fundamental signal processing procedures such as filtering and for applications involving optical information processing.

Communications

Usually multicarrier systems (MC) are designed with the DFT based schemes and the techniques which is used here is the orthogonal frequency-division multiplexing (OFDM). But this can also be done using discrete FRFT instead DFT in multicarrier systems.

Pattern Recognition

Pattern recognition can be done by using Fourier transform but FRFT provides an extra parameter (e.g., the rotation angle) in comparison with the ordinary Fourier transform and it also provides additional degrees of freedom in pattern recognition systems.

Cryptography

The FRFT can also be used in the field of cryptography.

Relation with Short-time Fourier Transform (STFT):

The short-time Fourier transform is the time-frequency analysis tool. The STFT of a signal x(t) is defined as

$$X(t,\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(\tau)\, w^*(t-\tau)\, e^{-j\omega\tau}\, d\tau$$

where w(t) is a suitably chosen as window. The STFT can also be simplified in a simple way from the Fourier transform of x(t) :

$$X(t,\omega) = \frac{1}{\sqrt{2\pi}} e^{-j\omega t} \int_{-\infty}^{\infty} X(\nu)\, W^*(\omega-\nu)\, e^{j\nu t}\, d\nu$$

where X and W are the Fourier transforms of x and w respectively. This equation is similar to (35), except for the presence of the exponential factor e-jwt . This is an asymmetry between time and frequency. So we wish to avoid since we want to deal with rotations in the time-frequency plane. We will therefore define a modified short-time Fourier transform (MSTFT)

$$\vec{X}(t,\omega) = \frac{1}{\sqrt{2\pi}} e^{-i\omega t/2} \int_{-\infty}^{\infty} x(v)\, w^*(\omega-v)\, e^{jvt}\, dv$$

The relation between the FRFT and MSTFT can be derived as

$$\vec{X}(t,\omega) = \frac{1}{\sqrt{2\pi}} e^{\frac{j\omega v}{2}} \int_{-\infty}^{\infty} X_\alpha(z)\, W_\alpha^*(u-z)\, e^{-jzv}\, dz$$

What is digital watermarking?

Watermarking is a technique of adding information to files like audio, video, image, or document files to protect the ownership of the data. The information added to it is known as **Watermark.** This includes 3-step process which includes Embedding, Modifying and Extraction. There are different types of watermarks includes visible and invisible. This ensures tamper-resistance which is hard to remove by unauthorized persons.

Here are the some of the Examples written below

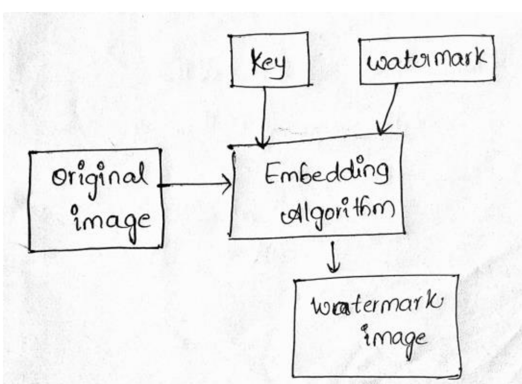
The goal is to detect the unauthorized distribution of image, audio, video and other documents. owners can copyright their work by inserting a digital water marking within a file which allows identification of true owner of the file.

Watermarking in other domains:

The following are some of the domains in which watermarking can be done.

Name of Technique	Domain	Resistance against Attacks	Host Image	Advantages of Using this technique	Disadvantages of using this technique
Spatial domain Algorithm for Gray Scale Images Watermarking[2]	Spatial	Gaussian noise, compression, Weiner filtering	Gray Scale	Simple and easy to implement algorithm	Cannot be used for colored images, less robust against Gaussian noise attack
Novel Multiple Spatial Watermarking Technique in Color Images[3]	Spatial	Median filtering, compression, Image cropping, scaling, rotation	Color Image	Can be used for Color images. Robust against a variety of attacks	Complex algorithm
A DWT Domain Visible Watermarking Techniques for Digital Images[4]	DWT	Compression	Gray Scale	Using DWT it is more accurately model aspects of HVS as compared to FFT and DCT	Not robust against various attacks
Collusion Attack Resistant Watermarking Scheme for Colored Images using DCT[5]	DCT	Collusion attack	Color Image	Specially designed for collusion attack	Finding suitable color channel for watermark embedding is not easy
A new Lossless Watermarking Scheme Based on Fuzzy integral and DCT[6]	DCT	Compression, Gaussian noise, Median filtering	Gray Scale	A blind watermarking technique which find usage in real applications	Usage of fuzzy integral adds to more computation overhead

Digital Watermarking in FRFT:



watermark embedding.

Embedding of a watermark signal is an interesting research and application field in the copyright protection of multimedia signals (images, sounds, movies).

By using the concept of 2D Discrete fractional Fourier Transform (DFRFT). We have described the implementation of watermark embedding technique for images

Even though there are many techniques in various domains to do watermarking. But we want to discuss here only one specific technique which computes the FRFT of the whole image then adds a random watermark with certain deterministic properties and transform it to back into an image.

Watermark embedding

Basic idea

Compute the transform

↓

Modify coefficients of the transform

↓

Transform back

First we have to compute the DRFFT of the image. The watermark itself is a sequence of M complex numbers and it should be embedded by excluding some starting coefficients and also avoid in the larger coefficients because that would disturb the image too much.

We have to sort the DFRFT coefficients according to their magnitude in an increasing order and denote the sorted array as given below

$$S_i = V_i + jW_i : |S_i| \leq |S_{i+1}|$$

$$i = 1, 2, \ldots$$

The real and imaginary parts are drawn from a normal distribution with mean 0 and variance σ^2/2, Let us denote the watermark as

$$S_i = V_i + jW_i : |S_i| \le |S_{i+1}|$$

$$i = 1, 2, \ldots$$

As we have to embed the watermark, so we have to make some modifications to the sorted vector Si as stated below

$$S_i^{\omega} = S_i + v_i|V_i| + j\omega_i|W_i|$$

$$i = L+1, \ldots, L+M$$

After the modification It has been rearranged in the original two-dimensional array where the original Si come from, and finally the watermarked image is then obtained by computing the inverse DFRFT.

Detection of the watermark

It can be detected by first transforming with the same DFRFT angles and the result which is obtained is kept in the same order as used for the embedding because the image may have undergone modifications due to some accidental attacks to the image, we don't get the same coefficients $S_w{}^i$ as we obtained before instead, we get some other coefficients $S_a{}^{i.}$

Next the detection value d is computed as

$$d = \sum_{i=L+1}^{L+M} [v_i - j\omega_i] S_i^a$$

The expected value of d (assuming that the watermark and the image are uncorrelated and assuming that $S_a{}^i = S_i{}^w$) is given by

$$E[d] = \frac{1}{2}\sigma^{2}\sum_{i=L+1}^{L+M}\left(|V_i| + |W_i|\right)$$

The computed value of d is larger than a threshold, which is set to E[d]/2.

As we already mentioned above that there may be a possibility of getting modifications in the image due to some attacks, this will lead to a change in values.

If there is a false conclusion in detecting watermark, there are many ways to do correct detection using some extra computations.

Our implementation is on 2d image which is gray scaled image. For this We used 2d-Discrete Fourier Transform (ref-4).

Conclusion:

We have discussed briefly about time-frequency analysis and its importance in real-life. We also discussed one of the important time- frequency representations i.e., Short-time Fourier Transform. We have gone through Fractional Fourier Transformation and properties of FRFT. Fractional Fourier Transform can be used in many applications like filtering and watermarking etc. We have discussed them briefly.

We found watermarking is very much feasible in embedding the watermark. Which is just finding the Discrete FRFT coefficients and adding watermark to the coefficients in the way suggested in algorithm and using inverse Discrete FRFT to convert it to image.

There are many domains in which watermarking can be done. To have a better comparison we mentioned all the advantages and disadvantages of watermarking in other domains in one table. We can conclude watermarking embedding in discrete FRFT is very simpler to implement, even if there is a false detection there because of attacks and noise, we can do some extra computations and can-do correct detection.

List of References

1.I. Djurović, S. Stanković, and I. Pitas, "Digital watermarking in the fractional Fourier transformation domain," Journal of Network and Computer Applications, vol. 24, no.2,pp.167-173,Apr.2001.

2. L. B. Almeida 1994. The fractional Fourier transform and time-frequency representations. IEEE

Trans. Sig. Proc., 42(11), 3084–3091.

3.Fractional Fourier transform as a **signal processing** tool: An overview of recent developments

4.The Discrete Fractional Fourier Transform

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