Ax=b: An Old Problem Through a New Lens

Karthik P. N.

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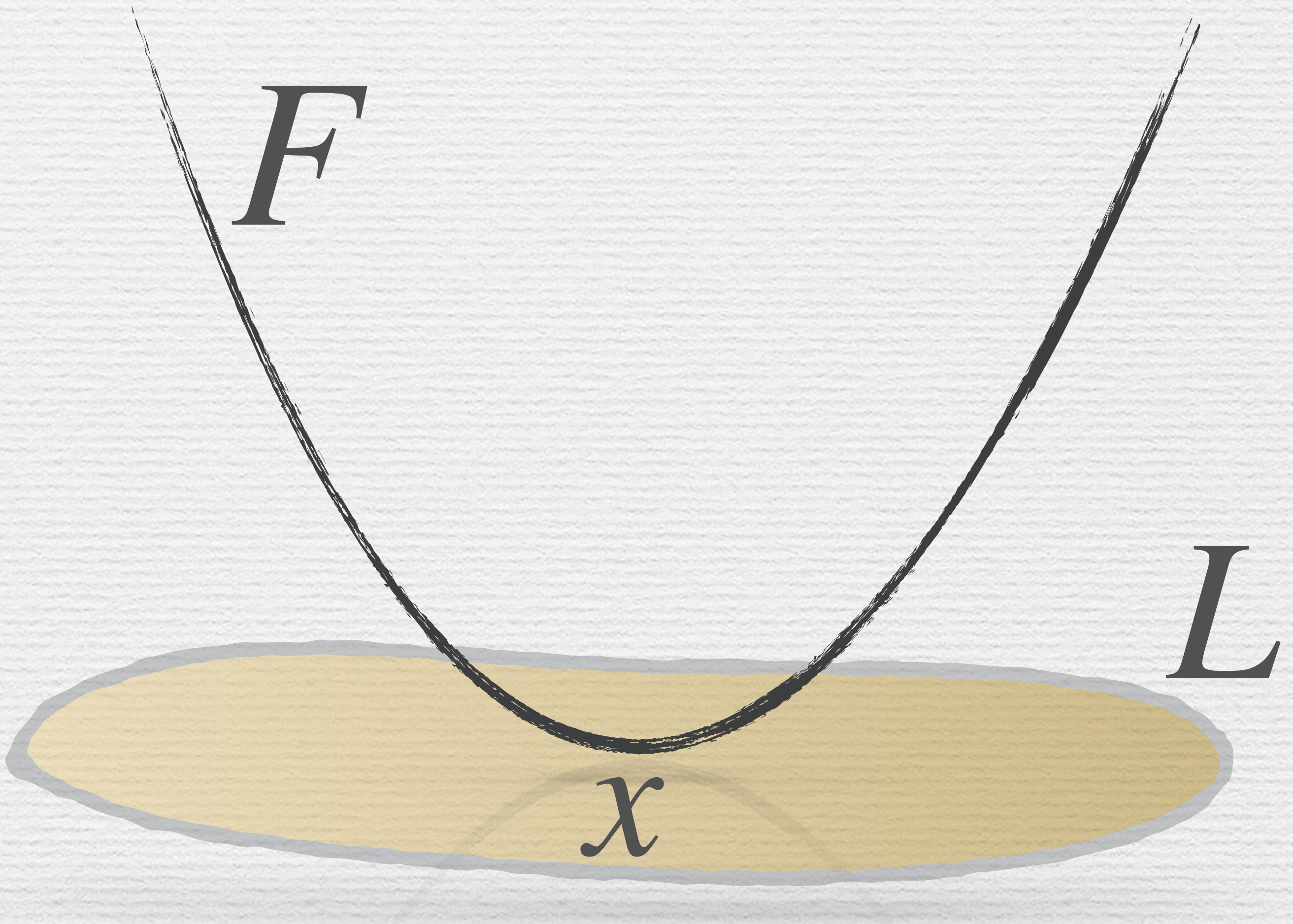
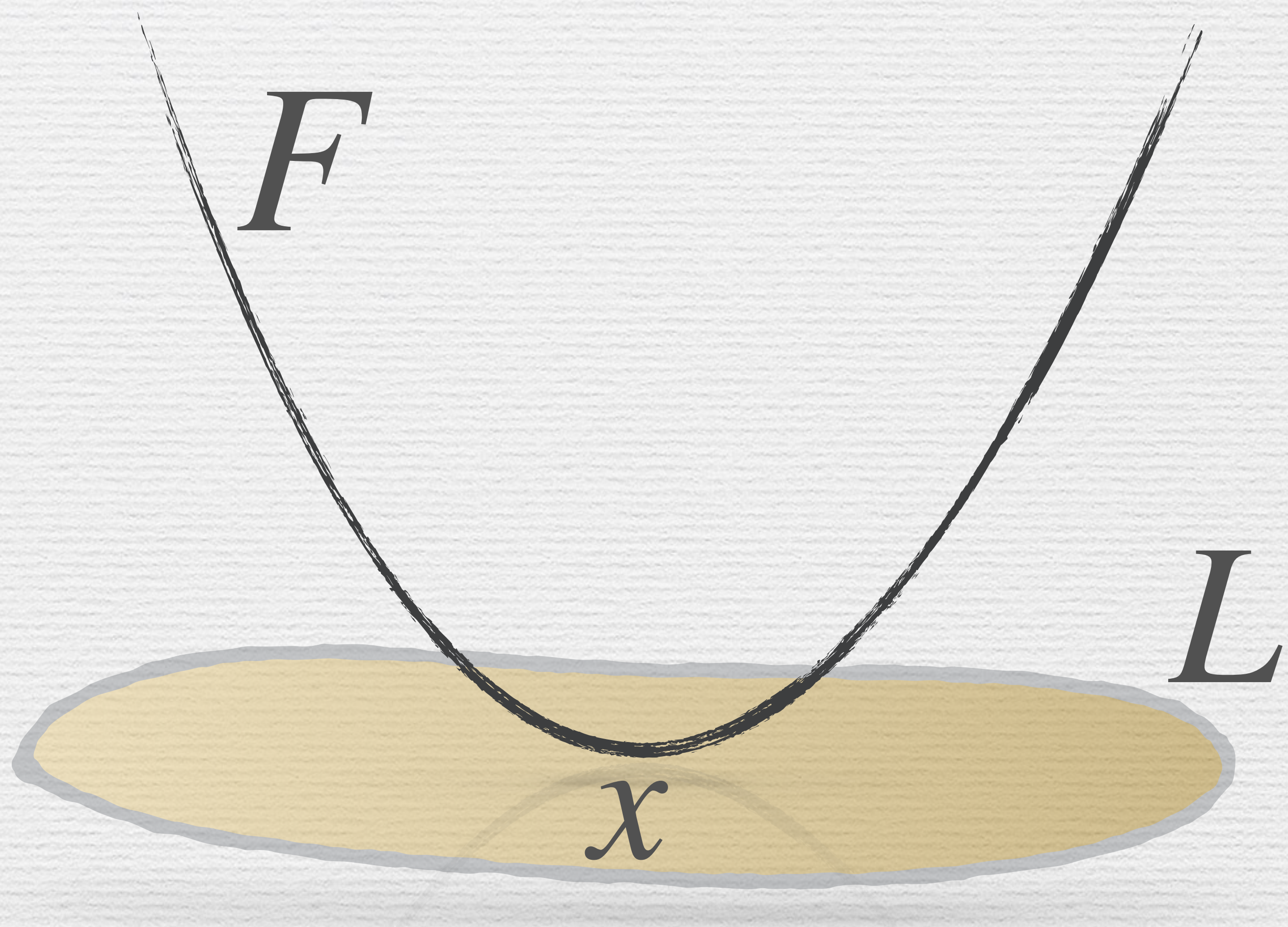


$$x = \arg\min_{w \in L} F(w)$$

The Typical Approach

- Define a cost function
- Choose an element that minimises the value of the cost function



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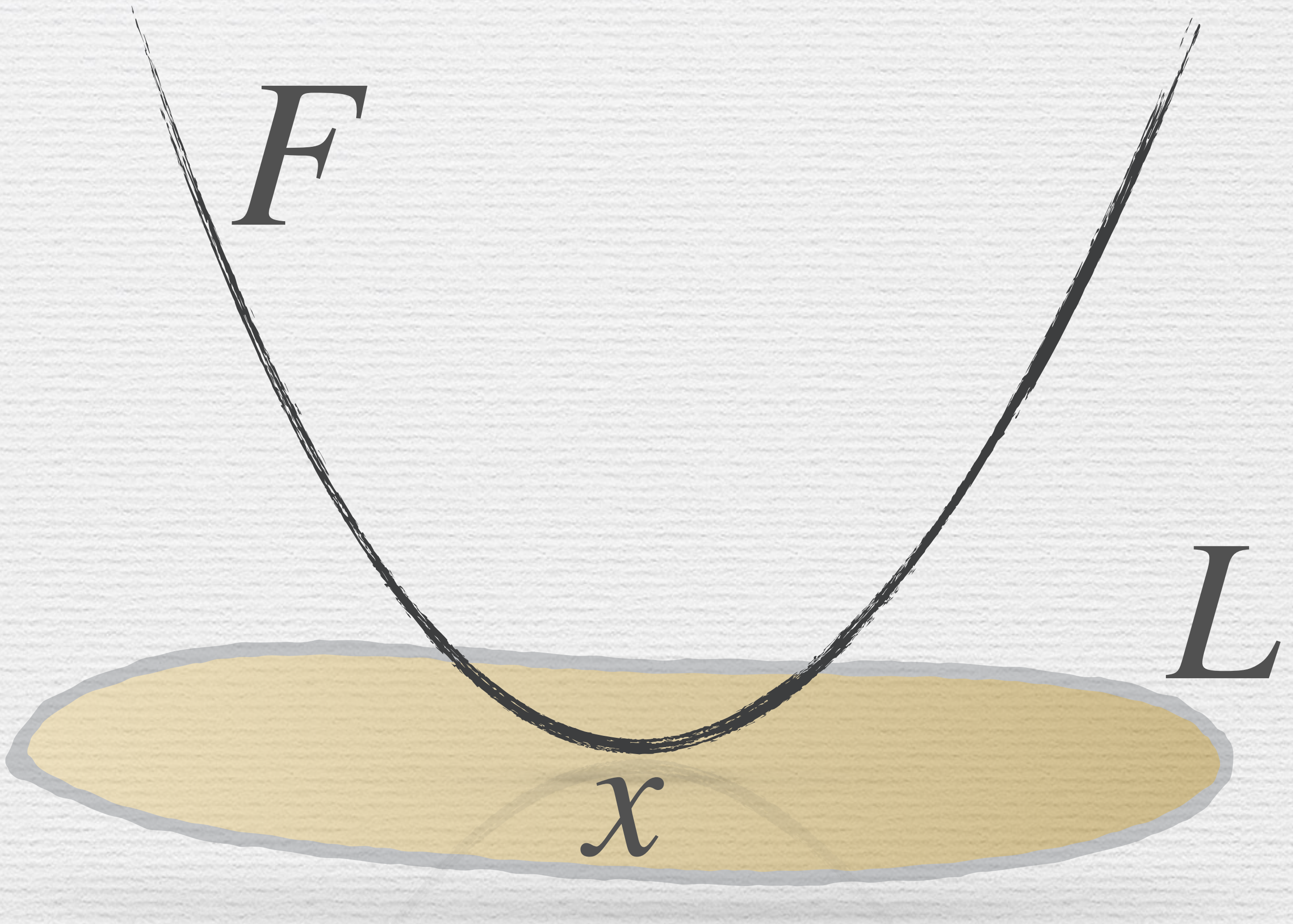

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Examples: least squares, maximum entropy (max-ent)

Why Least Squares and Maximum Entropy? Why not xyz?

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- Is there a way to classify rules as “naturally appealing” or “good”?

Least squares / maximum entropy rule

Rule xyz

An Axiomatic Approach

The Annals of Statistics
1991, Vol. 19, No. 4, 2032–2066

WHY LEAST SQUARES AND MAXIMUM ENTROPY? AN AXIOMATIC APPROACH TO INFERENCE FOR LINEAR INVERSE PROBLEMS[1]

BY IMRE CSISZÁR

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An attempt is made to determine the logically consistent rules for selecting a vector from any feasible set defined by linear constraints, when either all n-vectors or those with positive components or the probability vectors are permissible. Some basic postulates are satisfied if and only if the selection rule is to minimize a certain function which, if a "prior guess" is available, is a measure of distance from the prior guess. Two further natural postulates restrict the permissible distances to the author's f-divergences and Bregman's divergences, respectively. As corollaries, axiomatic characterizations of the methods of least squares and minimum discrimination information are arrived at. Alternatively, the latter are also characterized by a postulate of composition consistency. As a special case, a derivation of the method of maximum entropy from a small set of natural axioms is obtained.

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- Those rules that satisfy one or more axioms are classified as good

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An Axiomatic Approach

- Csiszár provides a method for classifying rules as good by defining a set of naturally appealing axioms
- Those rules that satisfy one or more axioms are classified as good
- Csiszár demonstrates that **<u>least squares and max-ent methods arise naturally</u>** as a result of satisfying some axioms

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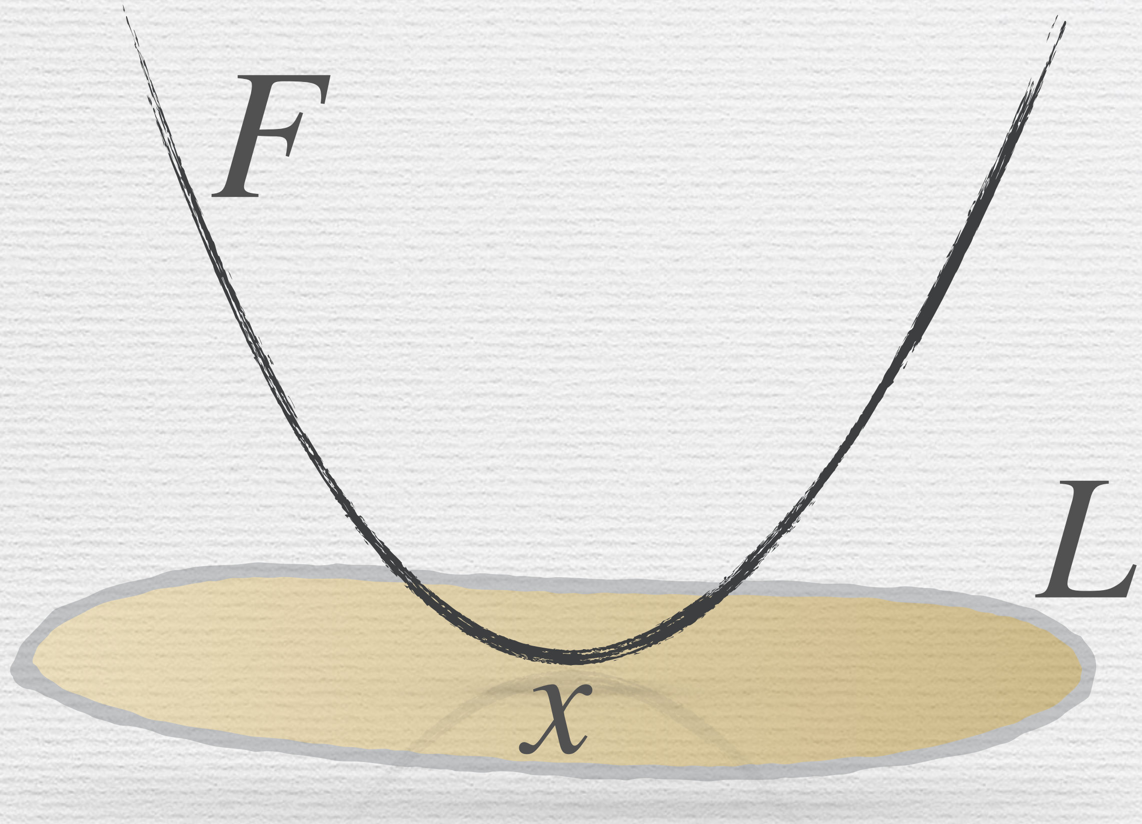

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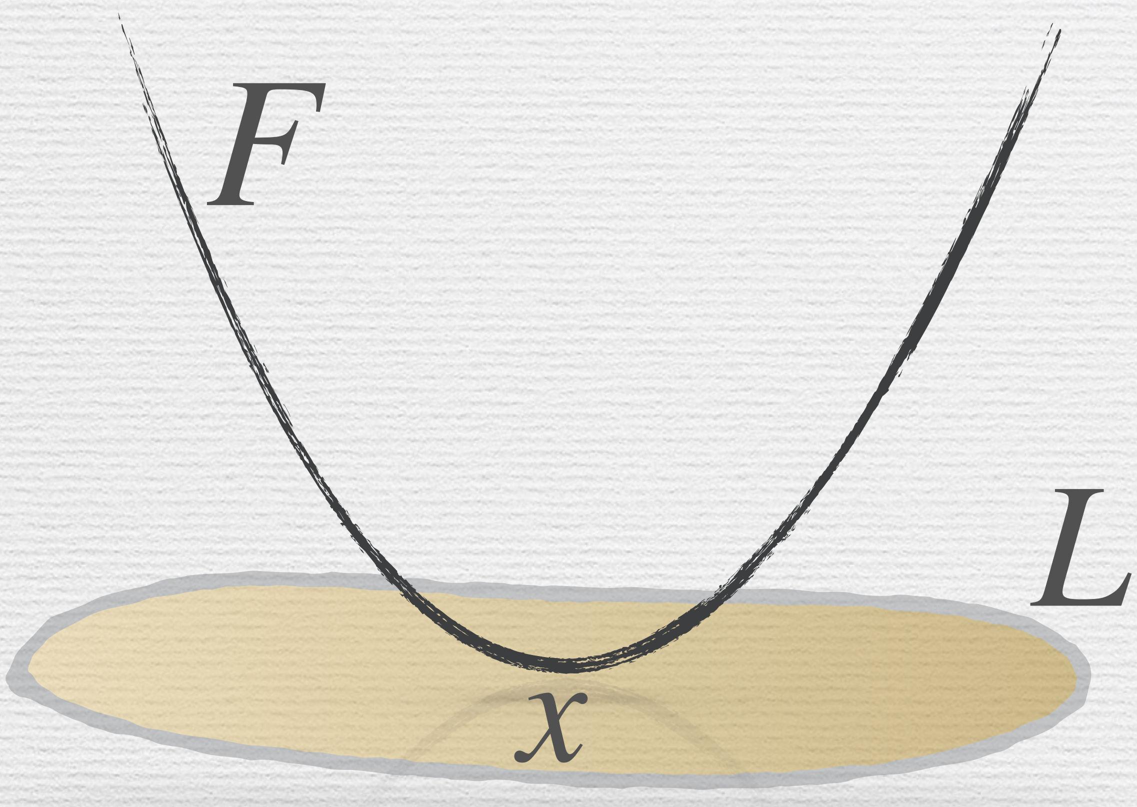

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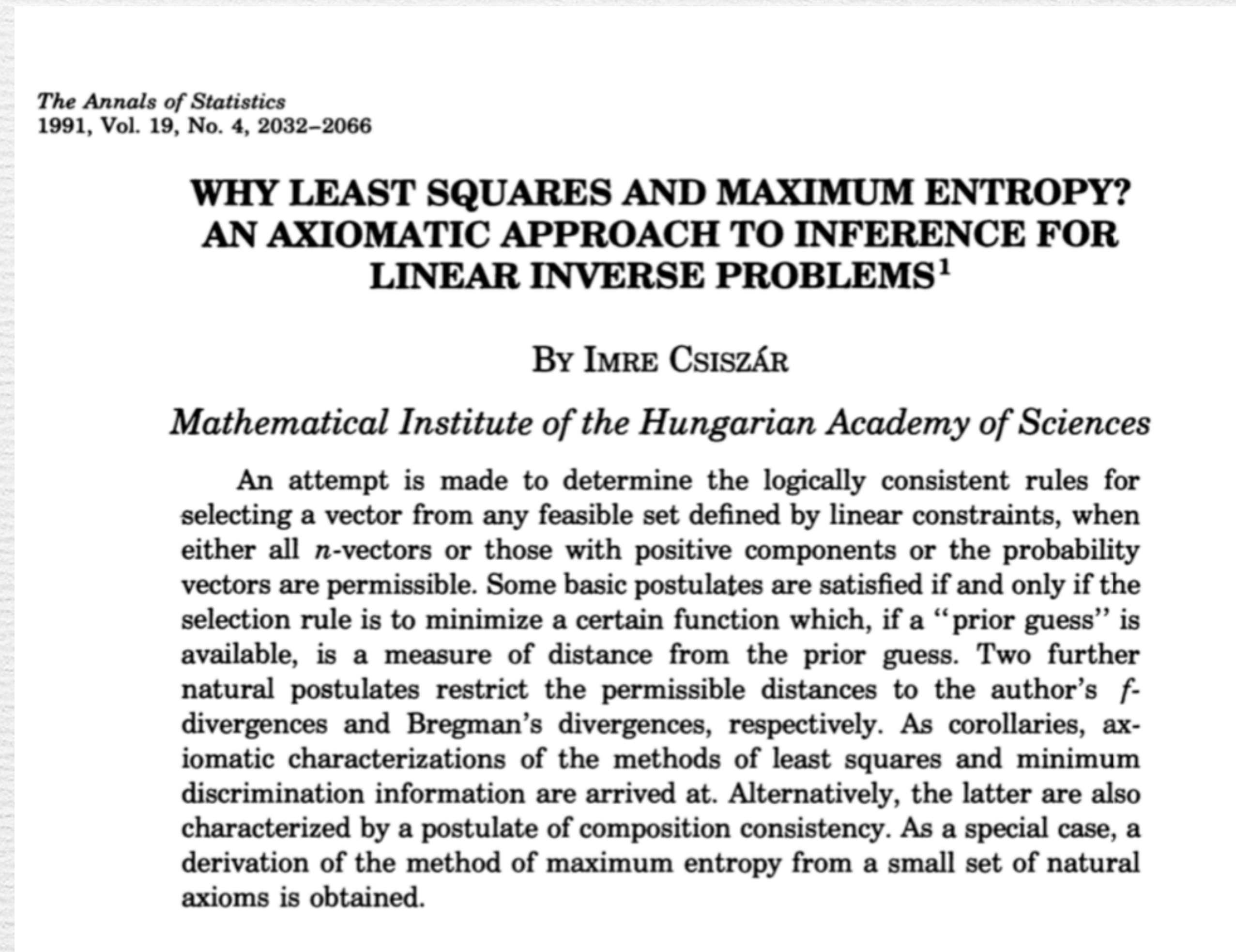

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Is there a connection between the two approaches?

What the Main Talk Will be About

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- Old patient v/s new doctor

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- Some axioms that are natural in many applications

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- Old patient v/s new doctor
- Some axioms that are natural in many applications
- Conservative fields and an interesting open question

Thank You

Our Objective

To obtain an axiomatic characterisation for a class of rules larger than that considered by Csiszár