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BT6270: Computational Neuroscience
Assignment 2: Stimulating the FitzHugh Nagumo Neuron Model

The simulation was done based on the following equations

$$dv/dt = f(v) - w + I$$
$$f(v) = v(a - v)(v - 1)$$
$$dw/dt = bv - rw$$

Given a = 0.5, b = 0.1, r = 0.1

The differential equations were solved using the single forward Euler integration:

$$v(t) = v(t-1) + \Delta t * (f(v(t-1)) - w(t-1) + I)$$
$$w(t) = w(t-1) + \Delta t * (bv(t-1) - rw(t-1))$$

Case I : Iext = 0
When Iext = 0, the fixed point is at (v,w) = (0,0). It is a stable fixed point. So on perturbation, the trajectory leads back to the fixed point.

i) V(0) = 0.2, W(0) = 0

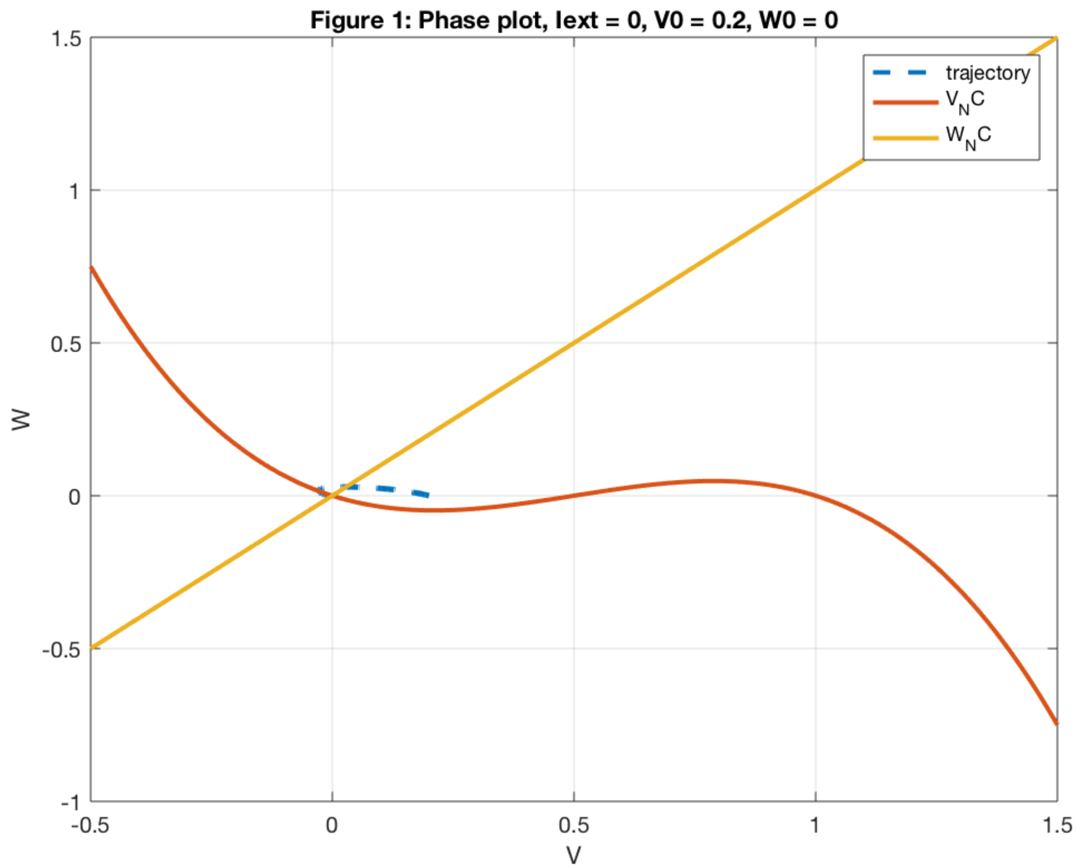


Figure 1: Phase plot, Iext = 0, V0 = 0.2, W0 = 0
trajectory
V_NC
W_NC
W
V
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5

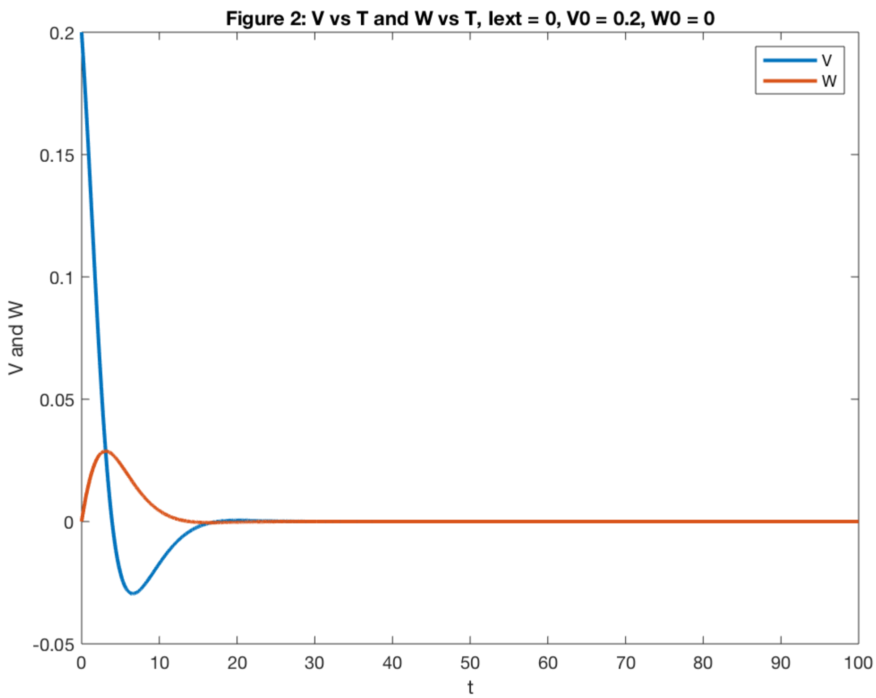


Figure 2: V vs T and W vs T, Iext = 0, V0 = 0.2, W0 = 0
V
W
V and W
t
0.2
0.15
0.1
0.05
0
-0.05
0
10
20
30
40
50
60
70
80
90
100

ii) V(0) = 0.8, W(0) = 0

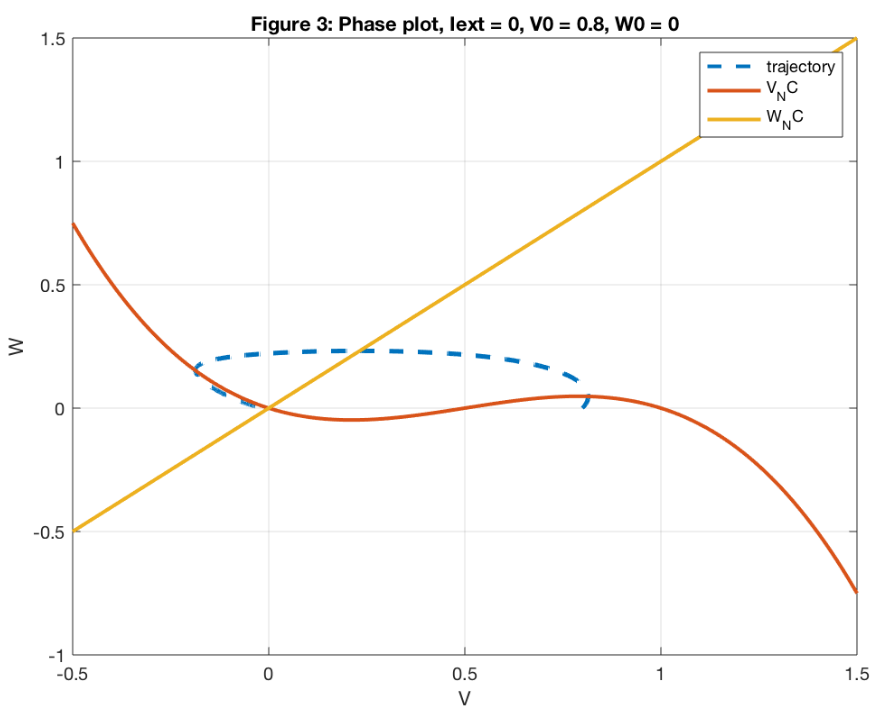


Figure 3: Phase plot, Iext = 0, V0 = 0.8, W0 = 0
trajectory
V_NC
W_NC
W
V
1.5
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
1.5

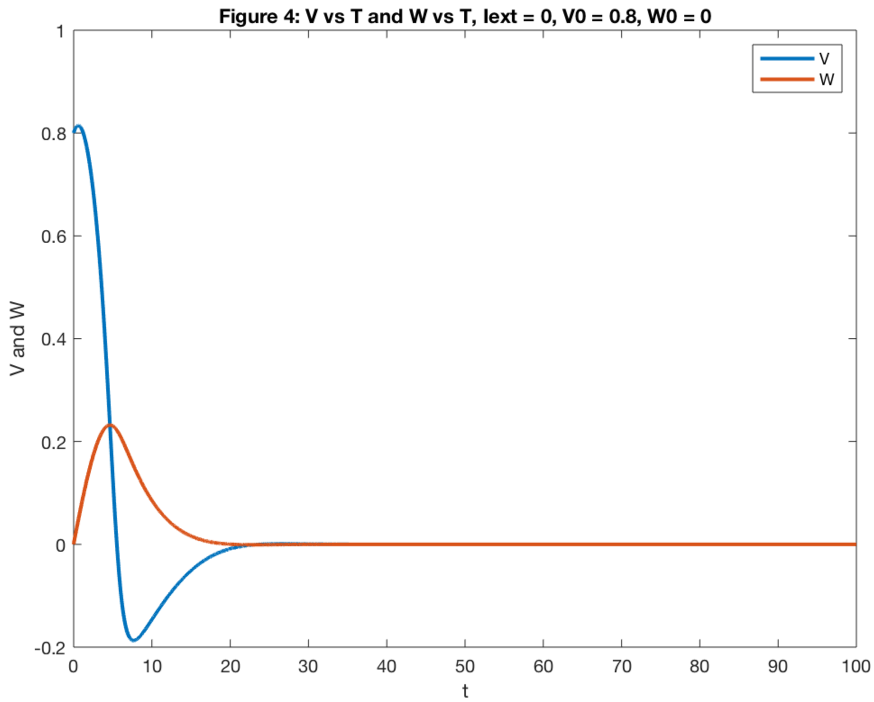


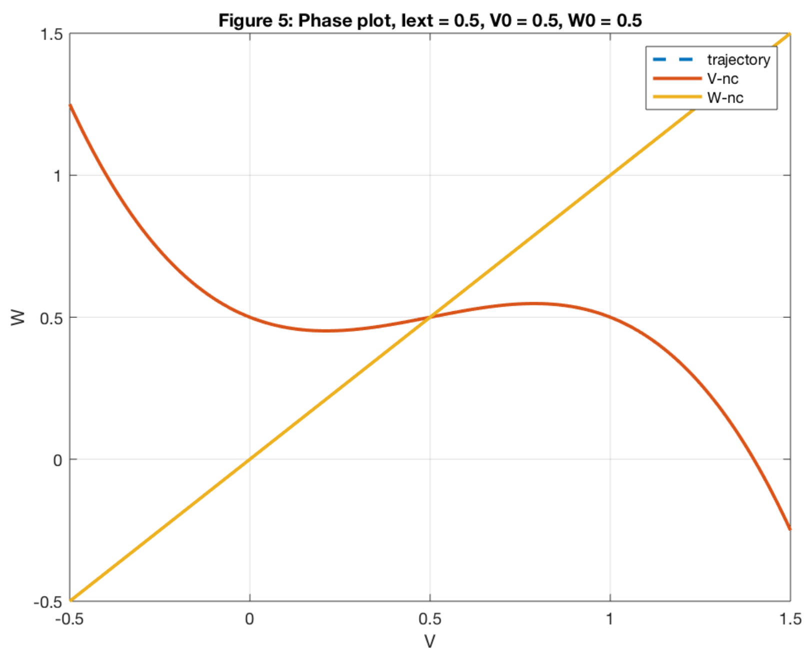
Figure 4: V vs T and W vs T, Iext = 0, V0 = 0.8, W0 = 0
V and W
t
V
W
1
0.8
0.6
0.4
0.2
0
-0.2
0
10
20
30
40
50
60
70
80
90
100

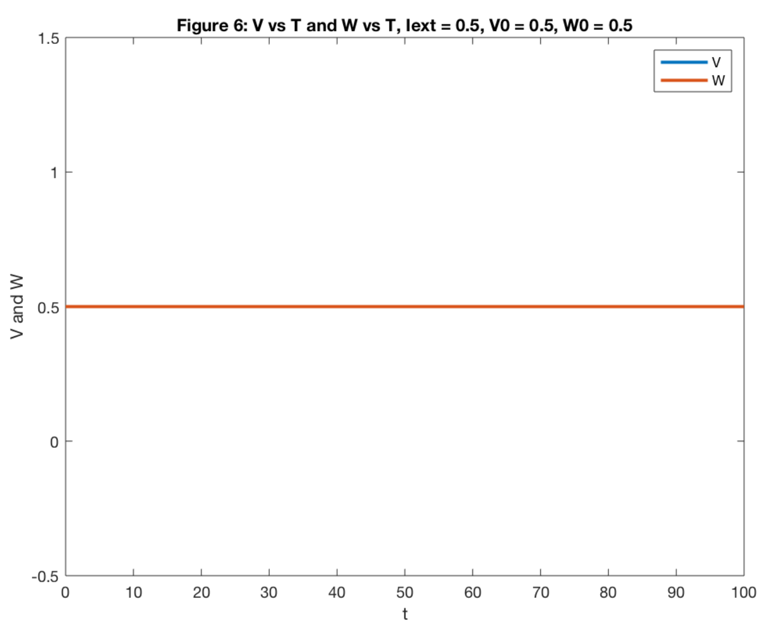
Case II : I1< Iext< I2

Here the values for I1 and I2 were found through stability analysis - When $\Delta > 0$ and $\tau > 0$, and $\tau^2 - 4\Delta < 0$, we get unstable focus

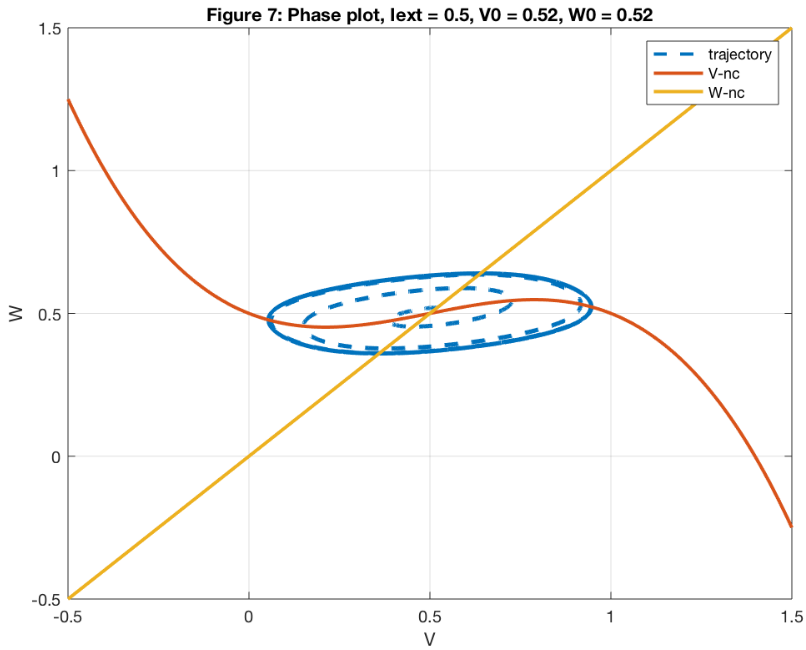
At Iext = 0.5, the model exhibits oscillations

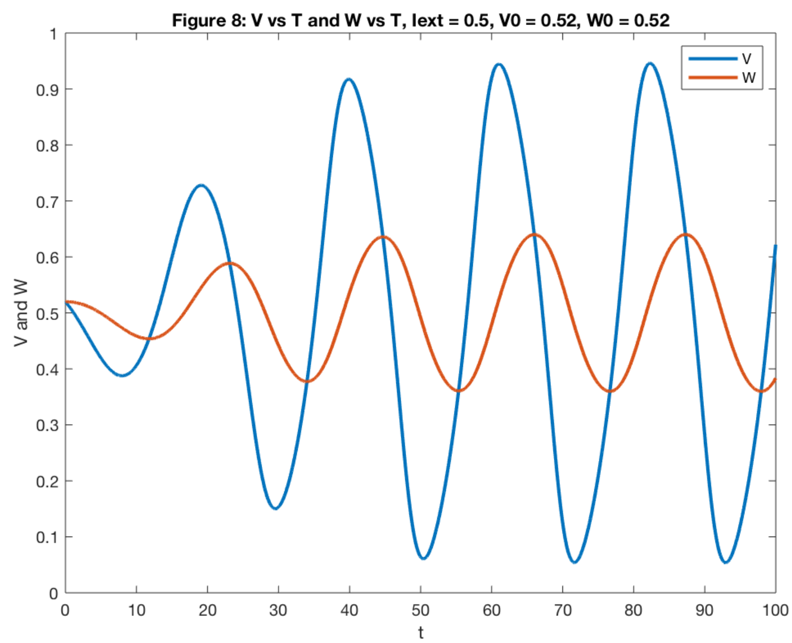
V0=0.5, W0= 0.5 is an unstable focus. At V0=0.5 and W0 = 0.5 we dont see any trajectory.- no flow at fixed points



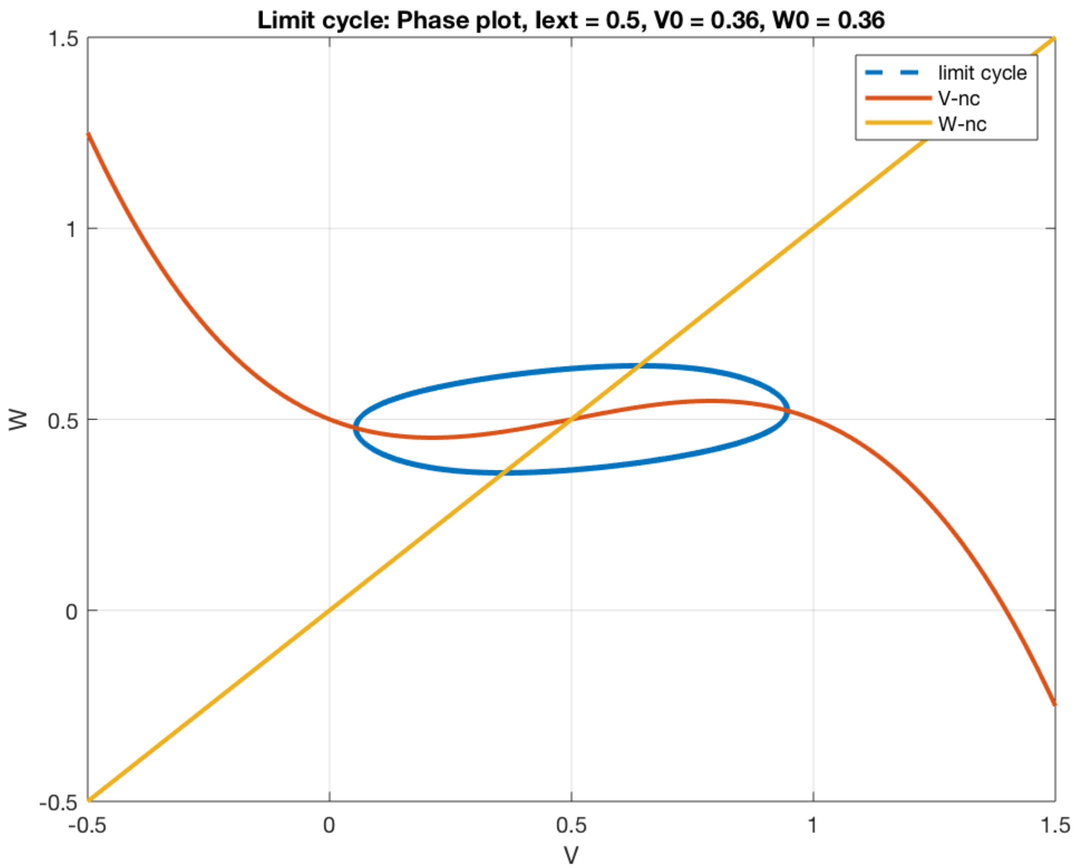


But when there is a small perturbation, the trajectory will be spiralling away until it reaches the limit cycle and it will continue to oscillate.





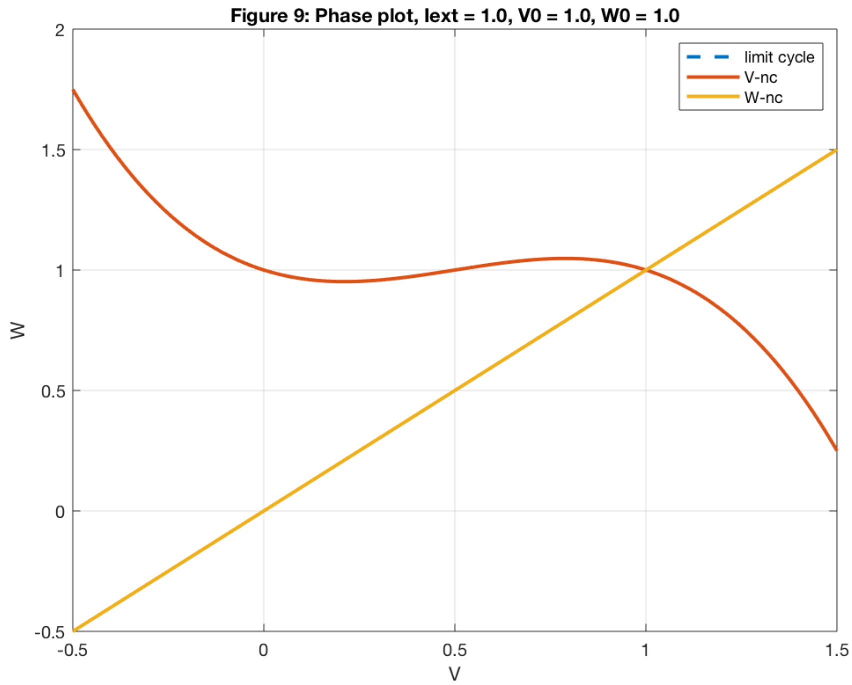
The following plot shows the limit cycle

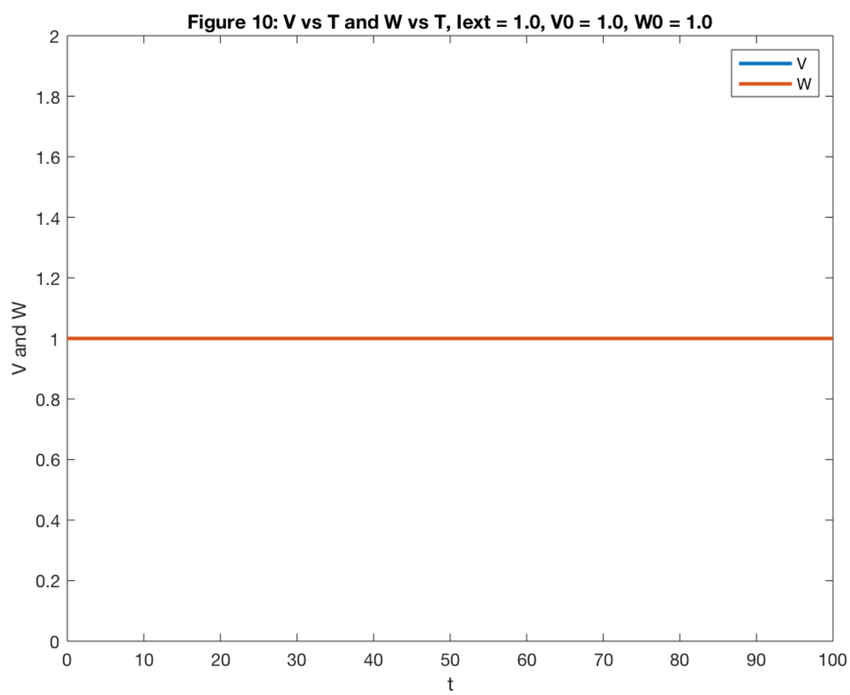


Case III : I > I2

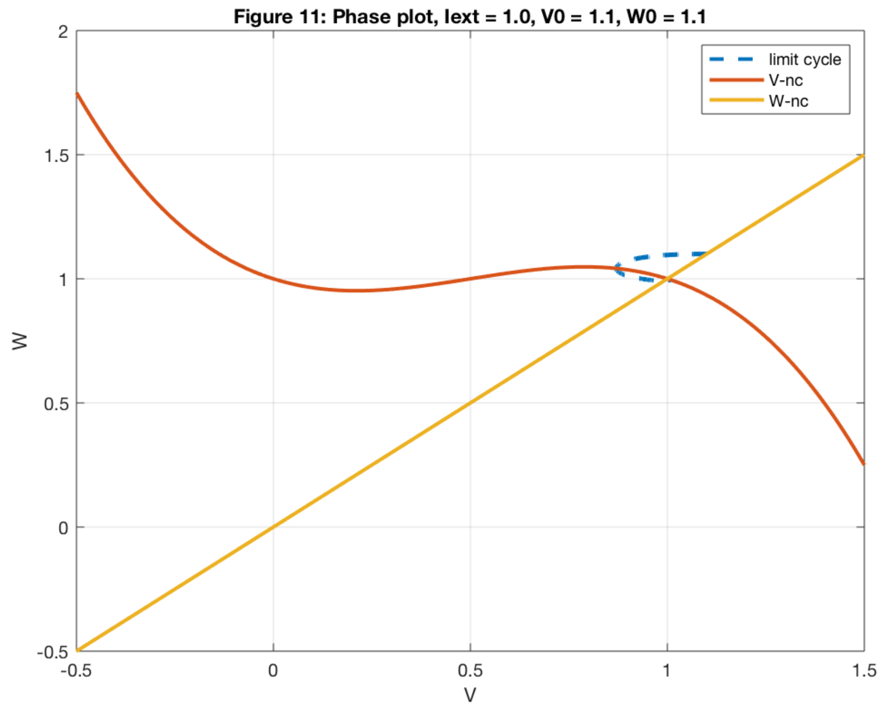
Iext = 1.0

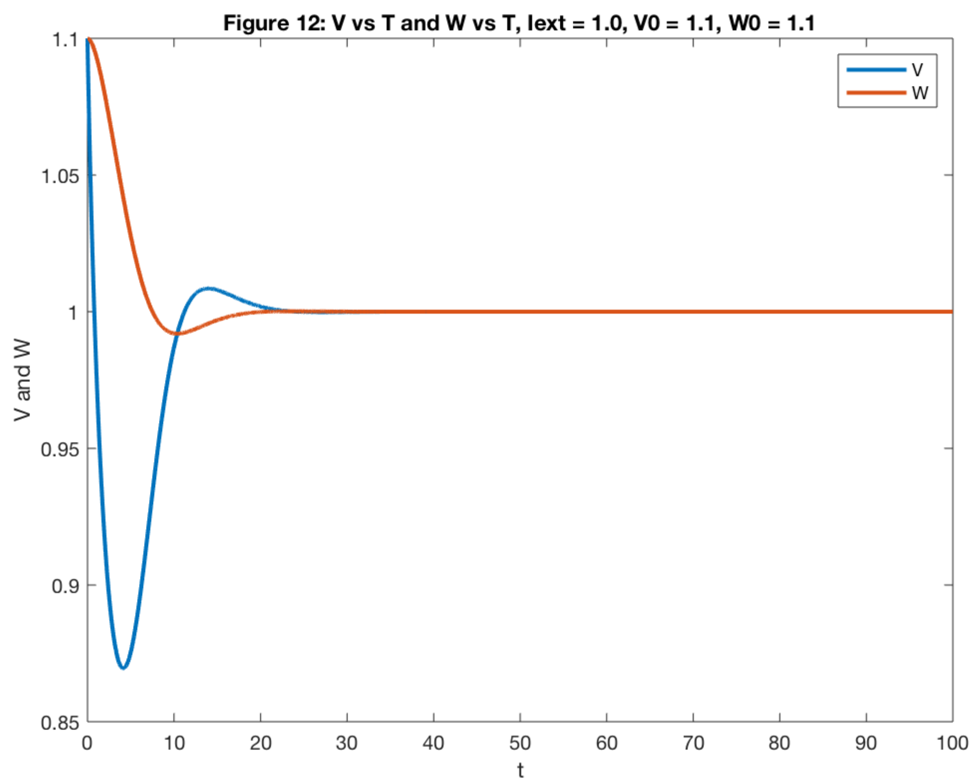
The fixed point is at V0 = 1.0 and W0 = 1.0. At the fixed point, there is no trajectory - no flow at fixed points





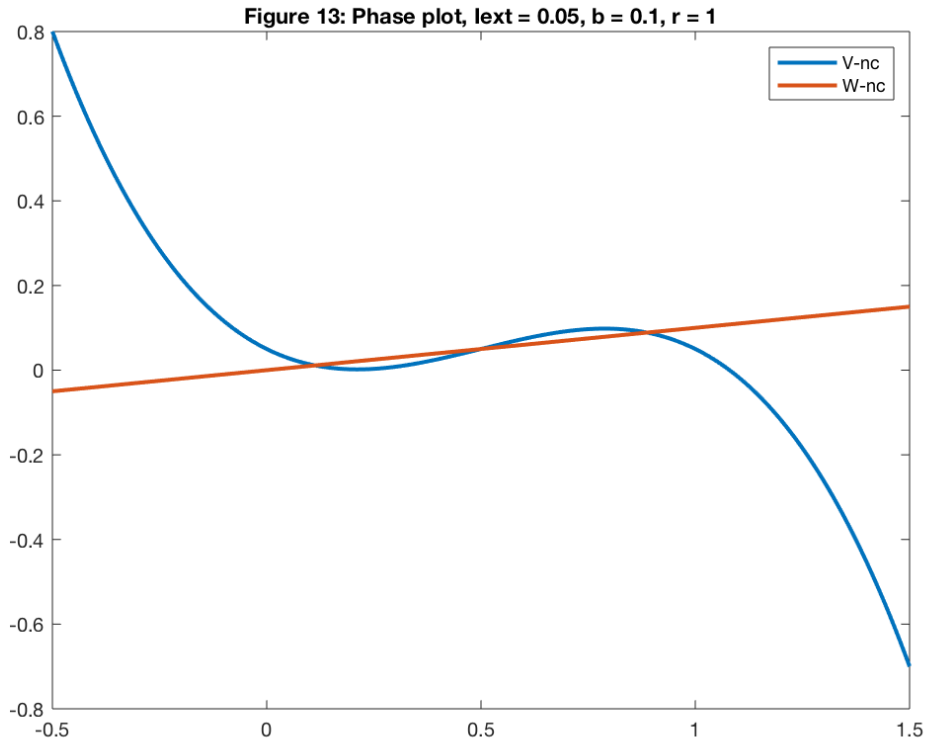
On perturbation, we can see that the trajectory returns to the fixed point. SO the fixed point is stable





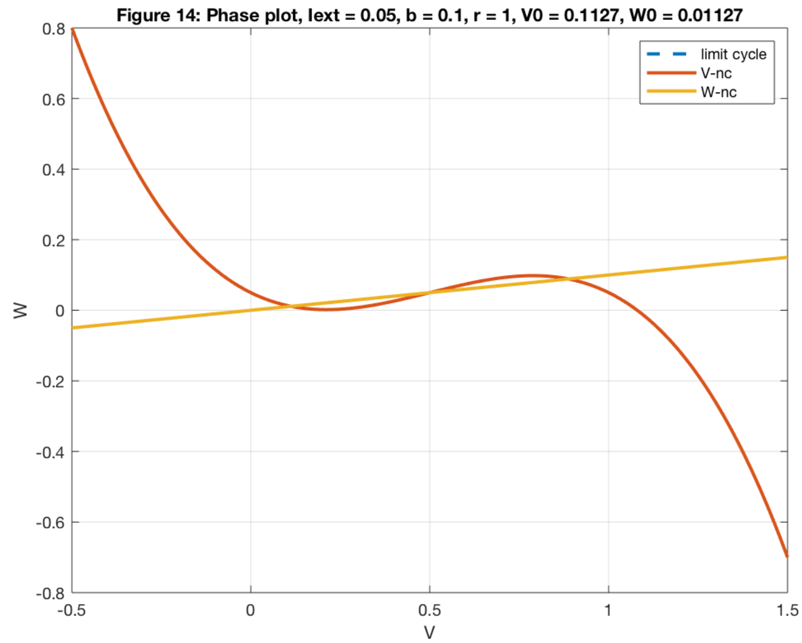
Case IV: selecting Iext and b/r such that there are 3 intersection points between the V and W nullclines

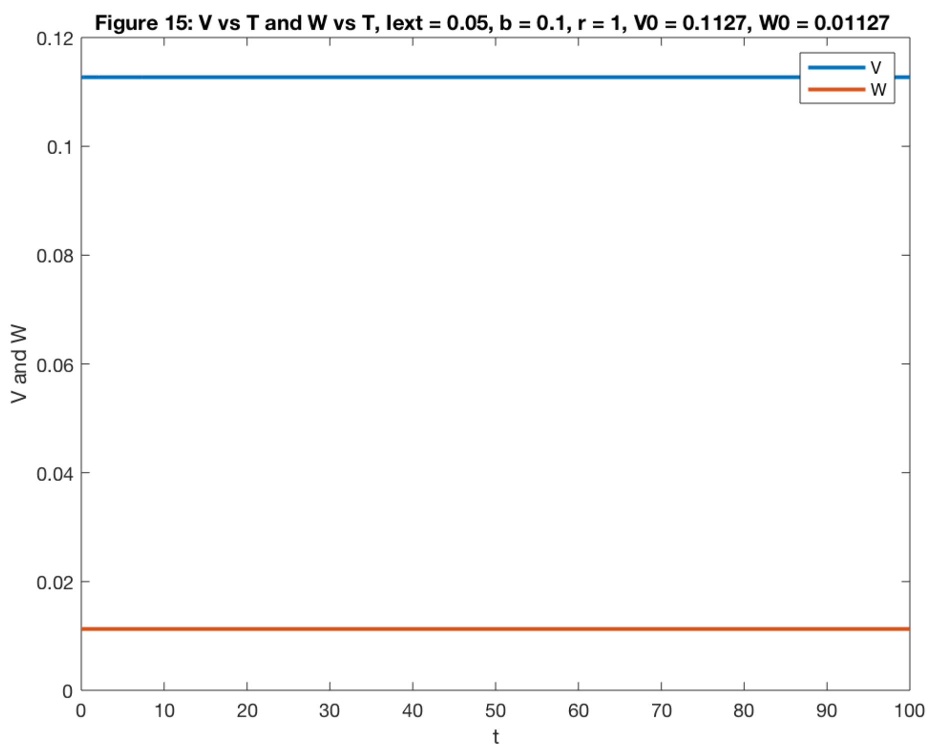
b = 0.1, r = 1.0, Iext = 0.05



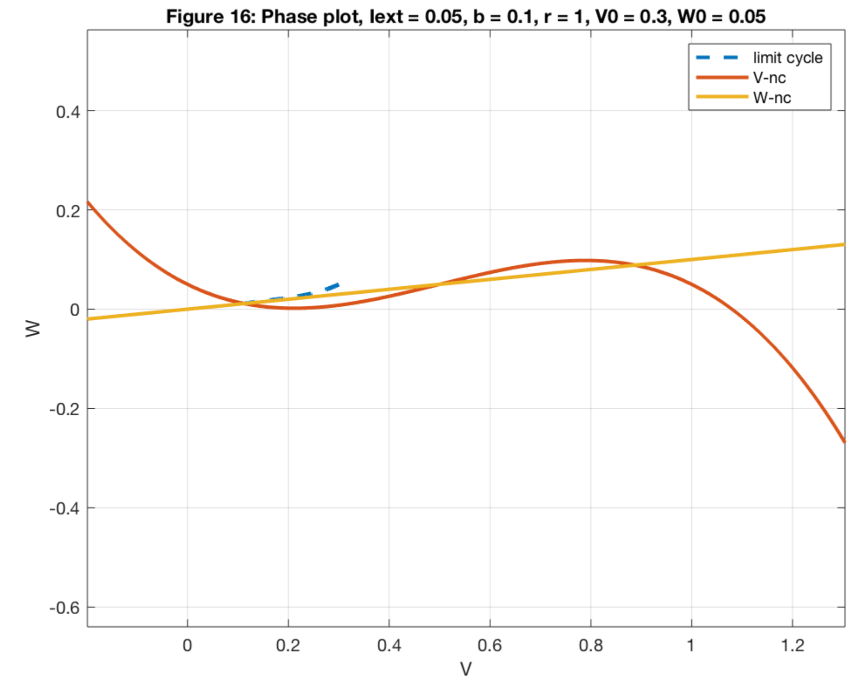
There are fixed points at p1, p2, p3- the points where the v and w nullclines intersect. At the fixed point, we don't observe any trajectory- no flow at the fixed points. Through the following graphs, we'll observe the stability of the fixed points by observing their trajectories for perturbations

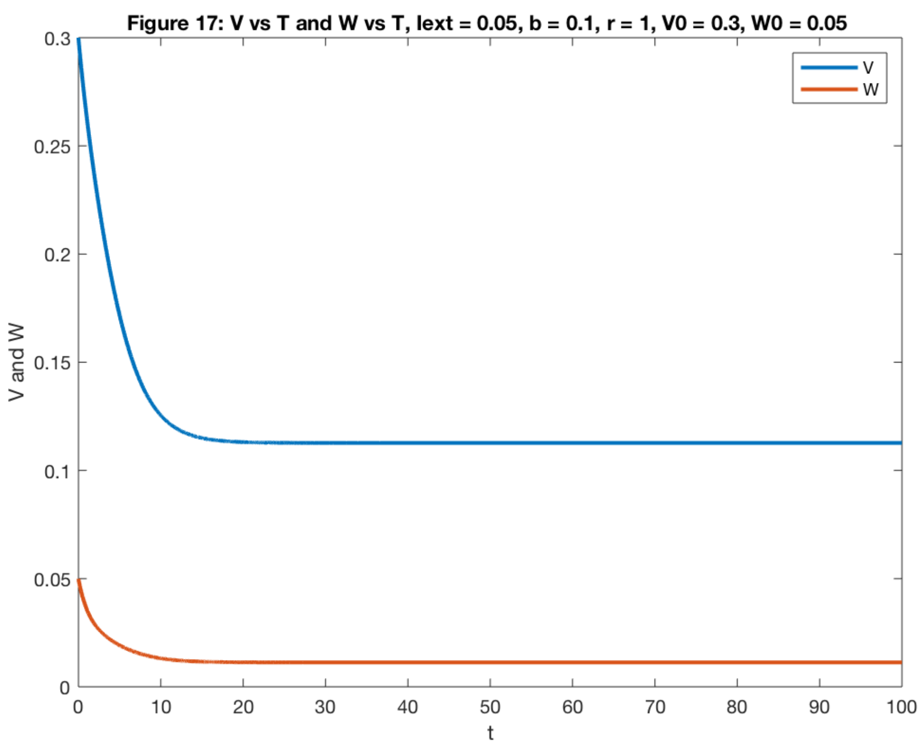
P1 : (V0 ,W0) = (0.1127, 0.01227)



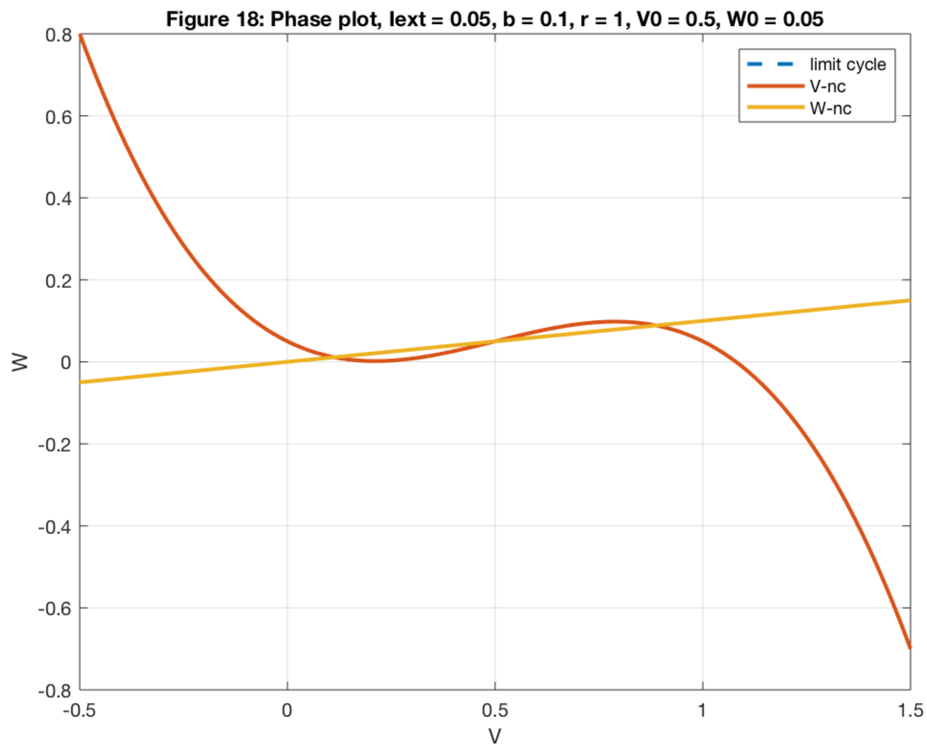


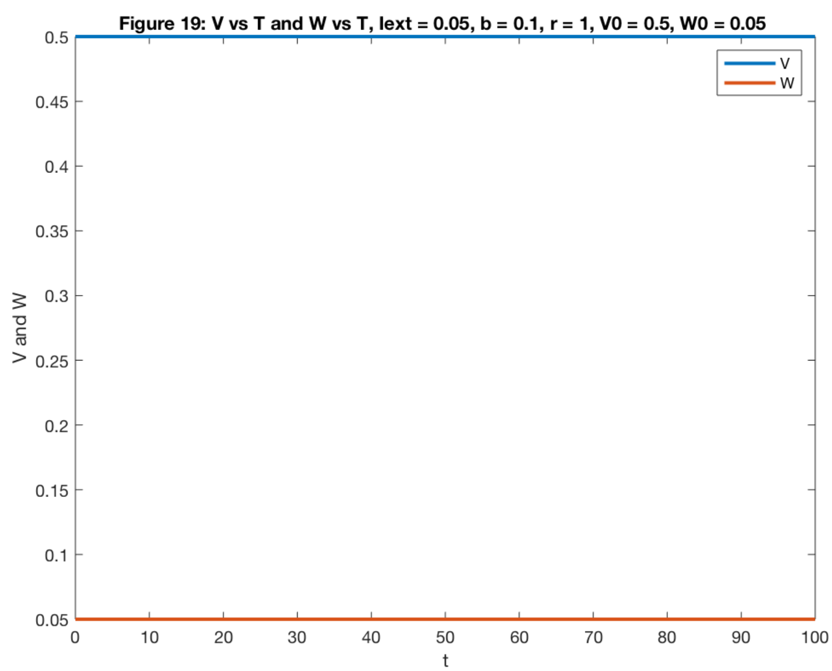
On perturbation - The trajectory returns to the fixed point.
∴ It is a stable fixed point





P2 : (V0 ,W0) = (0.5, 0.05)





On perturbation - the trajectory starts going away from p2
∴ It is an unstable fixed point. P2 is a saddle point- we can see from the graph that the trajectory kind of represents a saddle. The trajectory first moves closer to p2 and then starts moving away

Figure 20: Phase plot, Iext = 0.05, b = 0.1, r = 1, V0 = 0.5, W0 = 0.1

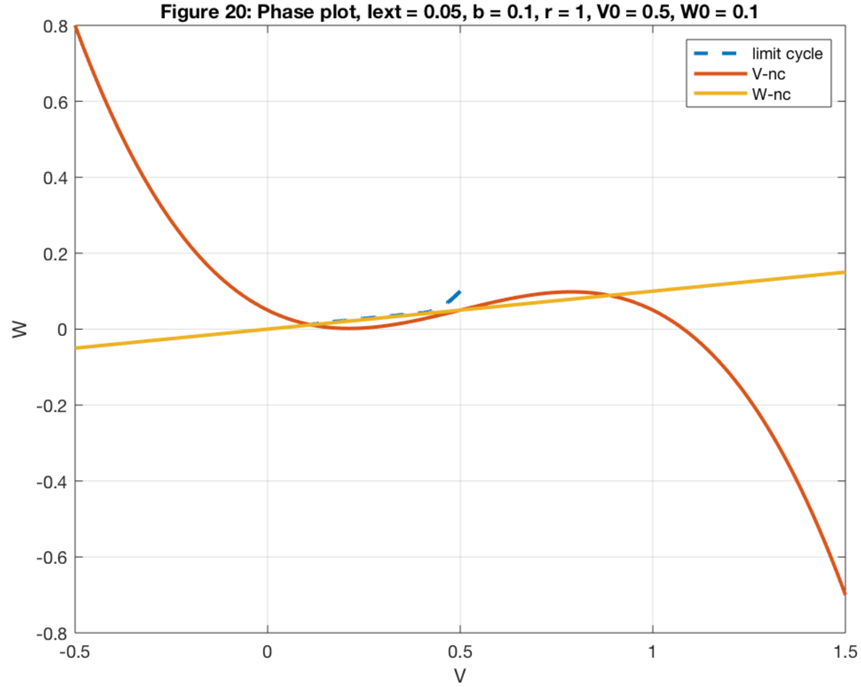
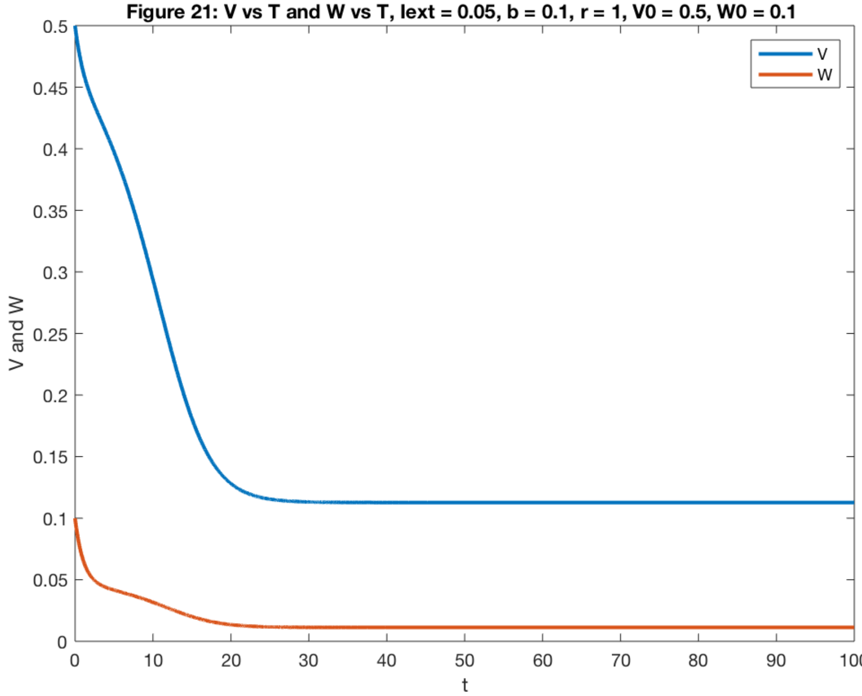
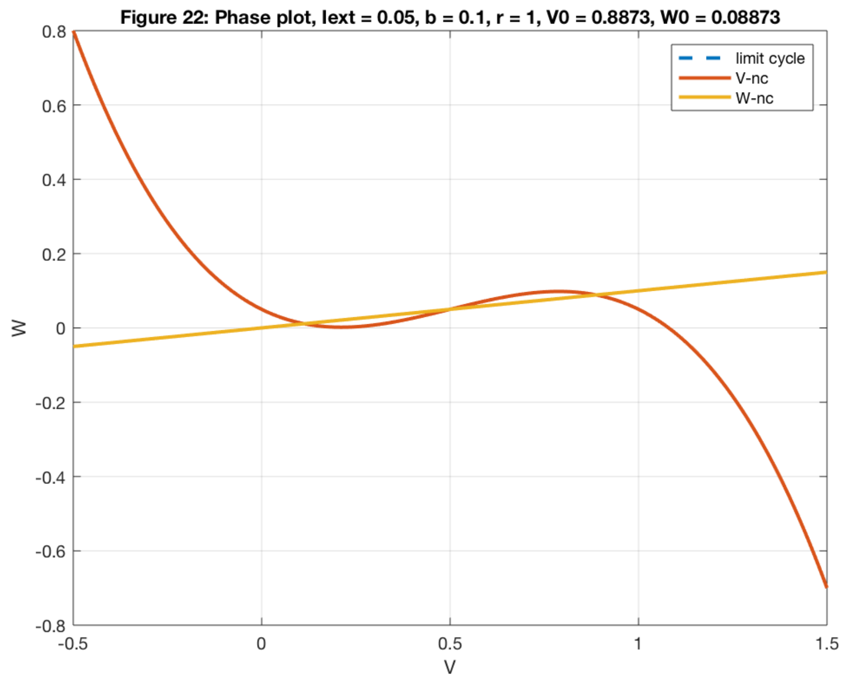
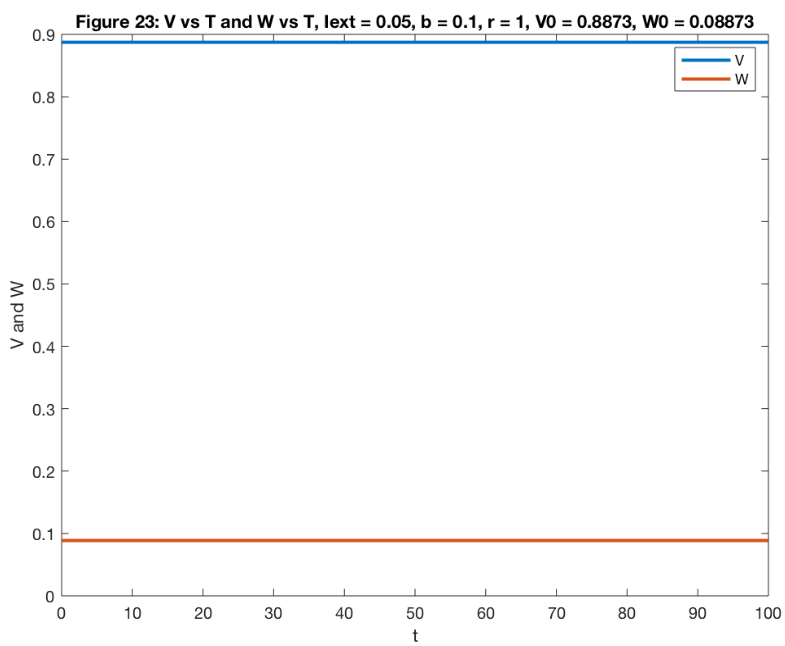


Figure 21: V vs T and W vs T, Iext = 0.05, b = 0.1, r = 1, V0 = 0.5, W0 = 0.1



P3 - (V0,W0) = (0.8873, 0.08873)





On perturbation, the trajectory moves back to P3

∴ It is a stable fixed point