

Number System (Binary & Decimal)

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→ Number System - A method to represent numeric values or quantities using different digits.

→ Types of number system

i) Binary number system (Base = 2)

ii) Octal " " (" = 8)

iii) Decimal " " (" = 10)

iv) Hexadecimal " " (" = 16)

→ Binary number system

It uses only 2 digits (i.e. 0 & 1)

It has base 2

{ 0 = OFF (False) }
{ 1 = ON (True) }

→ Octal number system

It uses 8 digits from 0 - 7.

It has base 8

→ Decimal number system

It uses 10 digits from 0 - 9

It has base 10.

→ Hexadecimal number system

It uses digits from 0 - 9 & alphabet from A - F

It uses total 16 digits & it has base 16.

→ Base - It is the no. of symbols (digits) that a number system uses.

→ **Decimal To Binary conversion**
 if
Division Method.

a) Divide decimal number by 2.

b) Store remainder. (That will be a bit in binary number)

c) Repeat above steps with the quotient, until quotient is less than 2.

d) Reverse the bits so obtained.

$$\text{Eg:- } (10)_{10} \rightarrow (?)_2$$

2	10	Remainder	
2	5 - 0	↑	
2	2 - 1	Reverse	
2	1 - 0	0	
0 -	1		

$$\therefore (10)_{10} \rightarrow (1010)_2$$

→ To represent the binary representation of 10 $(1010)_2$ in integer. we multiply each binary bits by 10 to the power of bits place value.

$$\begin{array}{r}
 1 \ 0 \ 1 \ 0 \\
 | \quad | \quad | \quad | \\
 \text{→ one place (1)} = 10^0 \\
 \text{→ Ten place (10)} = 10^1 \\
 \text{→ Hundred place (100)} = 10^2 \\
 \text{→ Thousand place (1000)} = 10^3
 \end{array}$$

$$\begin{aligned}
 1010 &= 1 \times 10^3 + 0 \times 10^2 + 1 \times 10^1 + 0 \times 10^0 \\
 &= 1000 + 0 + 10 + 0
 \end{aligned}$$

= 1010. (Now it is a Integer number, not a bits of binary representation)

→ Code of Division Method To convert Decimal To Binary

```
# include <iostream>
```

```
# include <cmath> (To use power function)
```

```
using namespace std;
```

```
int decimalToBinary (int num) {
```

```
    int binaryNum = 0; (store binary numbers (bits))
```

```
    int power = 0; (Indicate Power of 10, which is  
    initialize with 0)
```

```
    while (num > 0) {
```

```
        int bit = num % 2; (Getting last digit or remainder)
```

```
        binaryNum += bit * pow(10, power);
```

```
        num = num / 2; (Remove last digit (quotient))
```

```
        power++; (Increase 'power' after each iteration)
```

```
}
```

```
return binaryNum;
```

```
}
```

→

DRY RUN

num = 10, Binary representation = 1010

convert binary representation in Integer.

```
integer = 0; power = 0; (Power of 10)
```

1st Iteration :-

$$\text{bit} = \text{num} \% 2 \Rightarrow 10 \% 2 = 0$$

$$\begin{aligned}\text{integer} &= \text{integer} + (\text{bit} * 10^{\text{power}}) \\ &= 0 + (0 * 10^0)\end{aligned}$$

$$= 0 + 0 = 0$$

$$\text{num} = \text{num} / 2 \Rightarrow 10 / 2 = 5$$

$$\begin{aligned}\text{power}++ &= 0 + 1 \\ &= 1\end{aligned}$$

2nd iteration :-

$$\text{bit} = \text{num} \% 2 \Rightarrow 5 \% 2 = 1 \quad (\text{Remainder})$$

$$\begin{aligned}\text{integer} &= \text{integer} + (\text{bit} * 10^{\text{power}}) \\ &= 0 + (1 * 10^1) \\ &= \textcircled{10}\end{aligned}$$

$$\text{num} = \text{num}/2 \Rightarrow 5/2 = \textcircled{2}$$

$$\text{Power}++ = 1+1 = \textcircled{2}$$

3rd iteration :-

$$\text{bit} = \text{num} \% 2 \Rightarrow 2 \% 2 = 0$$

$$\begin{aligned}\text{integer} &= \text{integer} + (0 * 10^2) \\ &= 10 + 0 = \textcircled{10}\end{aligned}$$

$$\text{num} = 2/2 = \textcircled{1}$$

$$\text{Power}++ = 2+1 = \textcircled{3}$$

4th iteration :-

$$\text{bit} = \text{num} \% 2 \Rightarrow 1 \% 2 = 1$$

$$\begin{aligned}\text{integer} &= \text{integer} + (1 * 10^3) \\ &= 10 + 1000 \\ &= \textcircled{1010}\end{aligned}$$

$$\text{num} = 1/2 = 0$$

$$\text{Power}++ = 3+1 = 4.$$

5th iteration :-

while ($\text{num} > 0$) condition become false, & control exist from loop & return integer (i.e. 1010).

ii) Bitwise Method. (This method is fast)

a) obtain bit with bitwise AND (&) operator (i.e. $N \& 1$)

b) Right shift Number (N) by 1. ($N = N \gg 1$) \rightarrow To remove last digit of N.

c) Repeat above step till $N > 0$.

d) Reverse the bits so obtained.

→ Code of Bitwise Method.

```
int decimalToBinary (int num) {
    int binaryNum = 0;
    int power = 0;
    while (num > 0) {
        int bit = (num & 1); Getting last digit
        binaryNum += bit * pow(10, power);
        num = num >> 1; Removing last digit
        power++;
    }
    return binaryNum;
}
```

→ Binary To Decimal conversion.

i) Multiply each digits by its place value

i.e. 1 0 1

$$\begin{array}{r} \xrightarrow{\text{one}} 1 \times 10^0 \\ \xrightarrow{\text{Ten}} 0 \times 10^1 \\ \xrightarrow{\text{Hundred}} 1 \times 10^2 \end{array} = 1 \times 10^2 + 0 \times 10^1 + 1 \times 10^0 = 10 + 0 + 1 = 11$$

ii) Add up all place values.

iii) Sum is the decimal number

Eg:- $(1010)_2 \rightarrow (?)_{10}$

$$\begin{array}{r} 1 0 1 0 \\ \xrightarrow{\text{One place}} 0 \times 2^0 = 0 \\ \xrightarrow{\text{Ten }} 1 \times 2^1 = 2 \\ \xrightarrow{\text{Hundred }} 0 \times 2^2 = 0 \\ \xrightarrow{\text{Thousand }} 1 \times 2^3 = 8 \end{array}$$

Power = 8
Base = 2

Add up all place values

$$= 0 + 2 + 0 + 8 = 10$$

Digit

$$\text{Ex:- } (10111)_2 = (?)_{10}$$

$$\begin{aligned} &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 16 + 0 + 4 + 2 + 1 \\ &= 23 \end{aligned}$$

$$\therefore (10111)_2 = (23)_{10}$$

→ code of Binary To Decimal conversion.

```
#include <iostream>
```

```
#include <cmath>
```

```
using namespace std;
```

```
int binaryToDecimal (int num) {
    int decimalNum = 0;
    int power = 0;
    while (num != 0) {
        int bit = num % 10;
        decimalNum += bit * pow (2, power);
        num /= 10;
        power++;
    }
    return decimalNum;
}
```

→ DRY RUN.

$$(1010)_2 \rightarrow (?)_{10}$$

1st iteration :-

$\text{num} = 1010, \text{power} = 0, \text{decimalNum} = 0$

$\text{bit} = \text{num} \% 10 \Rightarrow 1010 \% 10 = 0 \text{ (Remainder)}$

$\text{decimalNum} += \text{bit} \times \text{pow} (2, \text{power})$

$$= 0 + 0 \times 2^0 = 0$$

$$\text{num} = \text{num}/10 \Rightarrow 1010/10 = 101 \text{ (quotient)}$$

$$\text{Power}++ \Rightarrow 0+1 = 1.$$

2nd iteration :-

$$\text{num} = 101, \text{ decimalNum} = 0, \text{ power} = 1$$

$$\text{bit} = 101 \% 10 = 1$$

$$\begin{aligned}\text{decimalNum} &= 0 + 1 \times 2^1 \\ &= 2\end{aligned}$$

$$\text{num} = 101 / 10 = 10$$

$$\text{Power}++ = 1+1 = 2$$

3rd iteration :-

$$\text{num} = 10, \text{ decimalNum} = 2, \text{ power} = 2$$

$$\text{bit} = 10 \% 10 = 0$$

$$\begin{aligned}\text{decimalNum} &= 2 + 0 \times 2^2 \\ &= 2\end{aligned}$$

$$\text{num} = 10 / 10 = 1$$

$$\text{Power}++ = 2+1 = 3$$

4th iteration :-

$$\text{num} = 1, \text{ decimalNum} = 2, \text{ power} = 3$$

$$\text{bit} = 1 \% 10 = 1$$

$$\begin{aligned}\text{decimalNum} &= 2 + 1 \times 2^3 \\ &= 2 + 8 = 10\end{aligned}$$

$$\text{num} = 1 / 10 = 0$$

$$\text{Power}++ = 3+1 = 4$$

5th iteration :-

(~~while true~~) while ($\text{num} \neq 0$) condition become false
& control exist from loop & return decimalNum (10).

$$\therefore (1010)_2 = (10)_{10}$$