

University of Trento

Part IV: Motion Tracking

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Introduction and motivations



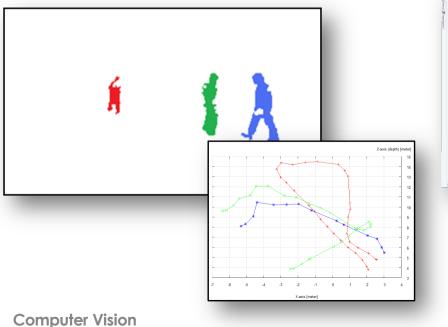
- Follow one or more moving objects for:
 - People monitoring
 - Traffic monitoring and analysis
 - Biological applications (cells tracking)
- High Level analysis for
 - Discovery of activities
 - Behavior understanding
 - Detection of threats
- More in general
 - Understand WHAT is moving in a scene
 - Understand HOW it moves/interacts with the environment

Object Tracking



- On the basis of the applications requirements
 - Track the 2D coordinates (centroid)
 - Track in 3D (more cameras are required)
 - Determine the position of complex objects (human body articulations)

Examples:







Object Tracking: applications



- Monitoring and surveillance
 - Motion classification
 - Identification of anomalous/suspicious behaviors
 - Follow a trajectory
- Human Machine Interfaces
 - Interact with a device removing physical barrier (mouse, keyboard)
 - Natural language understanding
- Virtual Reality
 - Immersive presence
 - Animation of virtual characters
- Mining and retrieval
 - Browse databases containing specific motion patterns

Benefits



- In HCI, control PC (or systems in general) → no need for additional tools
- In surveillance, Automated / Semi-automated systems → reduce the stress of human operators
- Virtual reality, computer animation → animate and drive the avatar
- But also
 - Training of athletes,
 - Gait disorders detection
 - Medical applications
 - **...**

2D Tracking



- Motion in the image plane
- Sometimes it is enough
- Different approaches
 - Region-based → set of pixels that share similar features (color)
 - Contour-based → determine position and shape of an object over time. Useful to track deformable objects.
 - Feature-based → select meaningful points (contours, corners)
 - Template-based → use specific models (hands, faces, eyes)



- Tracking regions with uniform appearance is a good method for real-time applications:
 - Fast (> 30 fps)
 - Good tradeoff quality/speed
- A region can be represented by the projection of an area with similar color on the image plane
- For example obtained from segmentation (e.g. after background suppression)



- Colors of regions must be different in order to be distinguished
- It is unstable in presence of variable illumination
- If applied to situations with variable illumination, appropriate compensation techniques have to be applied, i.e.:
 - Use HS of HSV
 - Use a normalized RGB space
- Ok indoor, troubles outdoor



- What do I want to track?
 - Any type of moving object
 - Skin vs non-skin (hand and face tracking)
 - Areas with certain colors
- Hows
 - Color thresholding if uniform
 - Color histograms
- Problems
 - Color changes over time (illumination, posture)
 - Acquired models of objects need to be updated



- A possible approach:
 - Divide the object to be tracked into regions
 - Each region is associated to a color vector (average for all pixels in the region) or histogram
 - Compute the color at each frame
 - If the ratio between the reference and actual values is close to 1, the match is good



- How to use histograms
 - For each moving object compute the histogram
 - At each step evaluate the histogram of the tracked region O[†] and compare it with the reference model Or for each region i
 - Similarity can be evaluated using:
 - Bin-by-bin comparison (intersection)

$$\bigcap (O_i^t, O_i^r) = \sum_{n=1}^{U} \min \{O_{i,n}^r, O_{i,n}^t\}$$

$$\bigcap (O_i^t, O_i^r) = \sum_{n=1}^{U} \min \{O_{i,n}^r, O_{i,n}^t\}$$

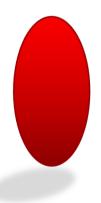
$$SSD(O_i^t, O_i^r) = \sum_{n=1}^{U} (O_{i,n}^r - O_{i,n}^t)^2$$

Bins should be neither to few, nor too many

Note: Shadows



- Shadows are source of noise → false positives
- A shadow does not correspond to the motion of a real object



- Variation of the luminance
- Chrominance remains (ideally) unaltered
- for a proper tracking shadows should be removed before tracking using a suitable algorithm

Blobs extraction



- Aggregation of a set of pixels that share common features
- An object can be made up by several blobs (head, torso, legs, ...)
- Features include also position
 - → pixels with similar color but far (in x,y) from the object must be discarded
- Typical application in combination with background suppression



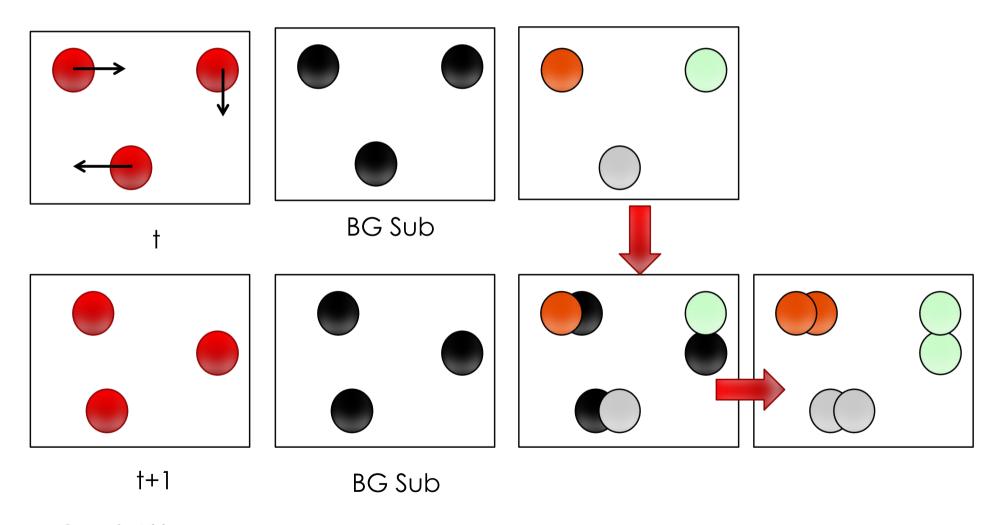
Target association



- Procedure common to all trackers, not only region-based
- In general it is worth noting that detection is not carried out on a frame-basis
- This could be too demanding in terms of computational resources
- ONCE detected, targets are followed on a proximity basis
- Example:
 - 1. Background subtraction informs about the presence of motion
 - 2. Histograms characterize each moving objects
 - 3. Unless occlusions occur, the target in the next frame should be the closest blob

Target association





Computer Vision

Target association

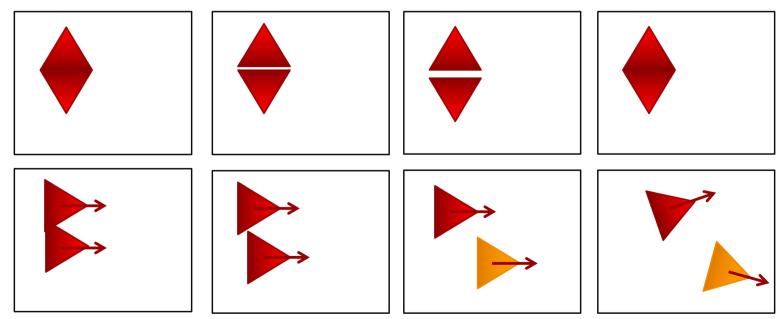


- Association can be performed as:
 - Overlapping blob (issues with scale → depends on objects size)
 - Centroid with minimum distance (as above)
 - Overlapping bounding boxes (may fail due to perspective)
 - Bounding box with minimum distance
 - Bounding box to centroid distance
- When association has been completed
 - →For each object/blob update the appearance model to account for small variations
 - →In presence of occlusions, the last saved model can be used to disambiguate

Splitting



- If objects are identified as single blobs, no problem!
- However, background suppression may return ambiguous results
 - Objects are split into several small blobs (not enough separation btw BG and FG)
 - 2. Two objects enter the scene together, then separate



Computer Vision

Splitting



- Before saying it is case 1. or 2. evidence needs to be accumulated
 - →Impossible to understand what's going on immediately
 - → A temporal interval is needed to tell if:
 - →It is necessary to keep the object merged even though it is fragmented
 - → A new object has to be created

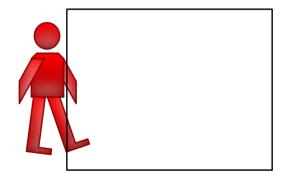
Merging



When two objects move together consistently, then perhaps they're the same object.

Example:

- A person's arm and foot enter the scene first and are detected as two well-separated FG blobs
- Then also the rest of the body enters and a single blob is created
- Blobs are close and they might share similar properties



Criteria for splitting and merging

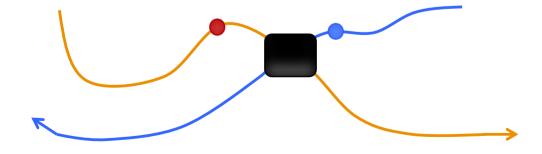


- Observation is the key
- Need to monitor the regions of interest and evaluate consistency in terms of:
 - Direction of motion
 - Distance between centroids/bounding boxes
 - Temporal range in which the phenomenon is observed
 - Velocity
 - Matching in features

Occlusions



- Moving objects occlude each other while crossing
 - One or more objects disappear from the scene
 - Bigger blobs appear as a result of the occlusion, with properties that do not belong to any of the models acquired previously
- How to resolve the occlusion?
 - Need to re-associate "A to A" and "B to B"
 - Histograms are a good way out



Occlusions



- It's an "anomalous" (though very frequent) situation
- Objects overlap and the acquired models are not reliable anymore
- Model update should be avoided during occlusions





Tracking: Feature-based



- The objective is to retrieve the motion information of a set of features
- Considering:
 - \blacksquare $A = \{A(0), A(1), ..., A(j-1)\}$ is a set of images
 - $m_i(x_i, y_i)$ i=[0, j-1] the position of the feature in the image plane in each frame
- Objective:
 - determine the displacement vector $d_i = (dx_i, dy_i)$ that best estimates the position of the feature in the next frame $m_{i+1}(x_{i+1}, y_{i+1}) = m_i + d_i$
- If needed, points can be grouped and objects can be represented using the bounding box or the convex hull



What features?



- A good feature point has distinctive characteristics:
 - Brightness
 - Contrast
 - Texture
 - Edges
 - Corners
 - Points with high curvature

Good features to track



For each candidate point, compute:

$$Z = \begin{bmatrix} \sum_{w} J_x^2 & \sum_{w} J_x J_y \\ \sum_{w} J_y J_x & \sum_{w} J_y^2 \end{bmatrix}$$

- J_x and J_y are the gradients evaluated on the point in x and y direction within W (nxn window)
- ullet A good feature point is where the smallest eigenvalue of Z is larger than a specified threshold
- In practice, it highlights corner points and textures

Good features to track



- Eigenvalues should be above the image noise
- Small eigenvalues imply strong similarity within the window
- A large and a small eigenvalue means unidirectional patterns
- If both eigenvalues are large, the point is of high interest (salt&pepper texture, corners)

How to track them?



- Must ensure that the same points are tracked throughout the video!
- Ideally we would expect that

$$A_i(Dm - d) = A_{i+1}(m)$$

- where
 - A_i , A_{i+1} successive frames
 - m is the 2D position of the feature point
 - D is the deformation matrix (affine transformation model)
 - d is the displacement vector (translational model)

How to track them?



- However:
 - Due to noise the equality does usually not hold
 - Motion across successive frames is assumed to be small → a translational model is a good approximation
- Feature dissimilarity measure is used to quantify the change of appearance between the first and current frame

$$\varepsilon = \iint_{W} \left[A_{i}(m-d) - A_{i+1}(m) \right]^{2} \omega(m) dm$$

- \bullet ω is a weighting function (e.g., Gaussian to emphasize the center of the window)
- When the feature dissimilarity grows too large, the feature point should be abandoned

How to track them?



To minimize the residual we differentiate w.r.t. unknowns (d)

$$e = 2 \iint\limits_W [A_i(m) - A_{i+1}(m)] g(m) \omega(m) dm$$

and

$$g(m) = \begin{bmatrix} \frac{\partial (A_i(m) - A_{i+1}(m))}{\partial x} \\ \frac{\partial (A_i(m) - A_{i+1}(m))}{\partial y} \end{bmatrix}$$

■ In this case the solution for the displacement vector can be expressed by the 2x2 linear system of equations (see paper for details):

$$Zd = e$$

The Lucas-Kanade optical flow

- Two-frame differential method for optical flow estimation developed by Bruce D. Lucas and Takeo Kanade (1981)
- Consider $u=[u_x, u_y]$ in frame I and $v=[v_x, v_y]$ in frame J
- The goal is to find **d** that satisfies v=u+d such as I and J are similar (translational model)
- Because of the aperture problem, similarity must be defined in
- **d** is the vector that minimizes

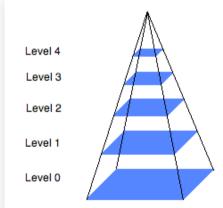
$$\epsilon(\mathbf{d}) = \epsilon(d_x, d_y) = \sum_{x=u_x - \omega_x}^{u_x + \omega_x} \sum_{y=u_y - \omega_y}^{u_y + \omega_y} (I(x, y) - J(x + d_x, y + d_y))^2.$$

We use an integration window

Pyramidal implementation



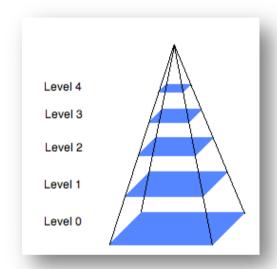
- The two key components to any feature tracker are accuracy and robustness
- Accuracy relates to the local sub-pixel accuracy attached to tracking → small integration window preferable to limit smoothness and preserve detail information (two image patches moving rapidly in different directions)
- Robustness relates to the sensitivity of tracking with respect to changes of light and big motions → a large window is preferable
- → Pyramidal implementation



Pyramidal implementation



- Level 0 is the image at original resolution
- Level 4 (in this example) is the image at lowest resolution
- The L-th level is defined as a linear combination of the elements in the previous level



$$\begin{split} I^L(x,y) &= \frac{1}{4}I^{L-1}(2x,2y) + \\ &= \frac{1}{8}\left(I^{L-1}(2x-1,2y) + I^{L-1}(2x+1,2y) + I^{L-1}(2x,2y-1) + I^{L-1}(2x,2y+1)\right) + \\ &= \frac{1}{16}\left(I^{L-1}(2x-1,2y-1) + I^{L-1}(2x+1,2y+1) + I^{L-1}(2x-1,2y+1) + I^{L-1}(2x+1,2y+1)\right). \end{split}$$

Pyramidal implementation



At each level of the pyramid an initial guess g of the flow is computed at the lower level, which is then refined at the current level.

$$\epsilon^{L}(\mathbf{d}^{L}) = \epsilon^{L}(d_{x}^{L}, d_{y}^{L}) = \sum_{x=u_{x}^{L} - \omega_{x}}^{u_{x}^{L} + \omega_{x}} \sum_{y=u_{y}^{L} - \omega_{y}}^{u_{y}^{L} + \omega_{y}} \left(I^{L}(x, y) - J^{L}(x + g_{x}^{L} + d_{x}^{L}, y + g_{y}^{L} + d_{y}^{L}) \right)^{2}$$

- g is used to pre-translate the image patch in the second image
- **d** should be small
- The information is then propagated at the upper level

$$\mathbf{g}^{L-1} = 2\left(\mathbf{g}^{\mathbf{L}} + \mathbf{d}^{L}\right).$$

Overall the displacement becomes $\mathbf{d} = \sum_{L=0}^{L_m} 2^L \mathbf{d}^L$.

Bayesian tracking



■ Idea:

- Estimate the state of a system over discrete time steps
- At each step, noisy measurements
- The state is represented by the x-y coordinates, velocity and acceleration along each dimension
- 6D vector
- Noise is typically smaller than the information about the state

Bayesian tracking



The state can be defined as

$$x_k = f_k(x_{k-1}, w_{k-1})$$

Linking the measurement with the state vector:

$$z_k = h_k(x_k, v_k)$$

- w_k is the process noise
- v_k is the measurement noise
- f_k and z_k are in general nonlinear functions defined in

$$f_k: \mathfrak{R}^{n_x} \times \mathfrak{R}^{n_w} \to \mathfrak{R}^{n_x} \text{ and } h_k: \mathfrak{R}^{n_x} \times \mathfrak{R}^{n_v} \to \mathfrak{R}^{n_z}$$

Bayesian tracking



- System and measurement should be available in probabilistic form
- Every time the measurement is available, the estimation can be computed
- It is an online method
 - \rightarrow for every step k an estimate can be computed based on the previous observations z_k up to instant k

Bayesian tracking



- The initial pdf of the state vector is given, $p(x_0|z_0)$
- \blacksquare z_0 contains no measurement
- Goal is to compute $p(x_k|z_k)$ at time k
- The process consists of two steps
 - Prediction
 - Update

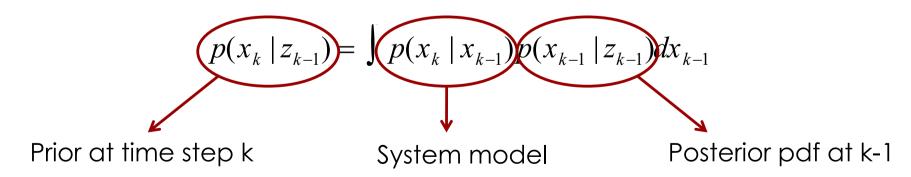
Bayesian tracking - Prediction



■ From the previous representation:

$$x_{k} = f_{k}(x_{k-1}, w_{k-1}) \to p(x_{k} \mid x_{k-1})$$
$$z_{k} = h_{k}(x_{k}, v_{k}) \to p(z_{k} \mid x_{k})$$

 The posterior pdf at k-1 is propagated forward in time using the system model



Bayesian tracking - Update



■ Using the Bayes theorem it is possible to obtain the desired pdf $p(x_k|z_k)$

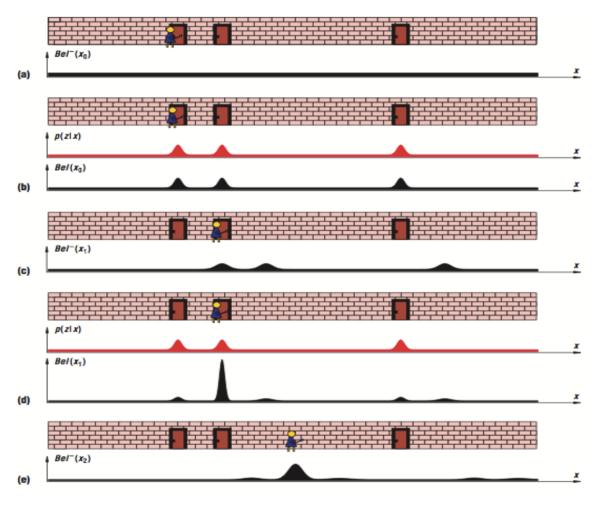
$$p(x_k | z_k) = \frac{p(z_k | x_k)p(x_k | z_{k-1})}{p(z_k | z_{k-1})}$$

• Where $p(z_k|z_{k-1})$ is used for normalization and computed as

$$p(z_k | z_{k-1}) = \int p(z_k | x_k) p(x_k | z_{k-1}) dx_k$$

Bayesian tracking – Toy Example





- User has a door "sensing device"
- User walks at typical walking speed
- (a) position is unknown
- (b) the sensor senses a door
- User can be in any of the three positions
- (c) User moves
- (d) another door is sensed
- (e) user moves again

D. Fox, J. Hightower, L. Liao, D. Schulz, G. Borriello, "Bayesian Filters for Location Estimation", IEEE Pervasive Computing, 2003.

Computer Vision



The Kalman filter in a nutshell



- In line with Bayesian tracking:
 - Take a measurement
 - Measurement is subject to error
 - Derive the state of the system from the measurement
- We start from:

$$z_1, \sigma_{z1}^2$$

$$\hat{x}_1 = z_1$$

$$\sigma_1^2 = \sigma_{z1}^2$$

The Kalman filter in a nutshell



- Add a second measurement
- Combine the two
- Iterate...

$$\hat{x}_{2}, \sigma_{z2}^{2} \\
\hat{x}_{2} = ?$$

$$\hat{x}_{2} = \hat{x}_{1} + K_{2}(z_{2} - \hat{x}_{1})$$

$$K_{2} = \frac{\sigma_{z1}^{2}}{\sigma_{z1}^{2} + \sigma_{z2}^{2}}$$

$$\frac{1}{\sigma_{2}^{2}} = \frac{1}{\sigma_{z1}^{2}} + \frac{1}{\sigma_{z2}^{2}}$$

- The algorithm works online
- It's a weighted average

$$\sigma_2^2 = \frac{\sigma_{z1}^2 \sigma_{z2}^2}{\sigma_{z1}^2 + \sigma_{z2}^2}$$

The Kalman filter in a nutshell



- Not all the difference between two measurements is noise
- Motion occurs
- Need to include a motion model, taking into account for example position and velocity
- The process:

```
Loop {
```

- Predict the new state and the uncertainty
- 2. Correct using the new measurement

}

KF in practice



- KF provides a computationally efficient solution to the least squares method
- Assuming that w_k and v_k are normal distributions (zero-mean, Q_k and R_k covariance), then
- The state and measurement model can be written as:

$$x_k = A_k x_{k-1} + B_k u_k + w_{k-1}$$
$$z_k = H_k x_k + v_k$$

KF in practice



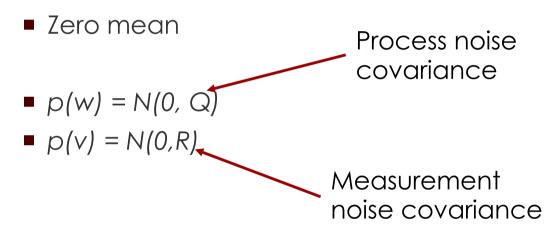
$$x_k = A_k x_{k-1} + B_k u_k + w_{k-1}$$
$$z_k = H_k x_k + v_k$$

- x_k is the current state
- x_{k-1} is the previous state
- A_k is the state transition matrix
- w_k is the process noise
- z_k is the actual measurement
- H_k is the measurement matrix
- v_k is the measurement noise
- B_k and u_k refer to additional and optional control input

KF in practice



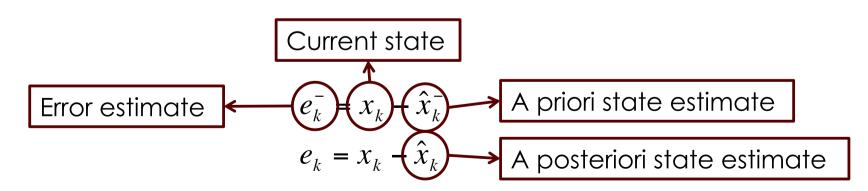
- We assume that the process noise and measurement noise are not changing over time, plus
 - Gaussian



Predict-and-correct stages



- During the first phase the current state estimate together with the error estimate are propagated forward in time
- In the second stage a new measurement is taken to modify the two estimations
- Evaluation of
 - a priori estimate (based on the past measurements)
 - \blacksquare a posteriori estimate (as soon as measurement z_k is available)



Predict-and-correct stages



Once the error estimates have been computed, determine the error covariance by:

$$P_k^- = E[e_k^- e_k^{-T}]$$

$$P_k^- = E[e_k^- e_k^{-T}]$$

 For the prediction stage we have then (discarding the optional control parameter):

$$\hat{x}_{k}^{-} = A_{k} \hat{x}_{k-1}$$

$$P_{k}^{-} = A_{k} P_{k-1} A_{k}^{T} + Q_{k-1}$$

Predict-and-correct stages



 The update stage begins with the computation of the "gain" of the KF, which minimizes the a posteriori error covariance

$$K_{k} = P_{k}^{-}H_{k}^{T}(H_{k}P_{k}^{-}H_{k}^{T} + R_{k})^{-1}$$

The gain is used to modify the a priori estimate and to compute the a posteriori state estimate:

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H_k \hat{x}_k^-)$$

And to compute the a posteriori error covariance:

$$P_k = (I - K_k H_k) P_k^-$$

How to:



 Predict the position of a point following a certain trajectory in (x,y) plane

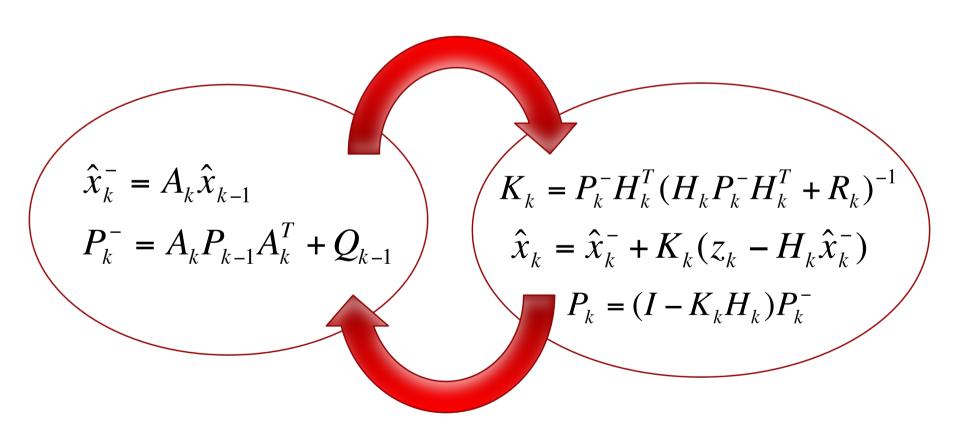
$$x_k = A_k x_{k-1} + w_{k-1}$$
$$z_k = H_k x_k + v_k$$

- x, w, z, and v are 2x1 vectors
- A and H are 2x2
- Given w and v compute Q and R (in some cases determined empirically)
- During initialization
 - $x_0 = Hz_0$
 - P₀ diagonal matrix with reasonable values for covariance (maximum allowed displacement?)

How to:



Now predict ... and correct:



Implementation of the KF



Estimate a constant function y=c (voltage)



- Measurements we take are corrupted by white noise (quantization error due to conversion)
- Problem statement is easy, assuming A and H are both =1

$$X_k = X_{k-1} + W_k$$

$$z_k = x_k + v_k$$

Example taken from "An introduction to the Kalman Filter" by Greg Welch and Gary Bishop

Filter equations



Our a priori state estimate is given by the previous state

$$\hat{x}_k^- = \hat{x}_{k-1}$$

$$P_k^- = P_{k-1} + Q$$

And the measurement update

$$K_{k} = P_{k}^{-} (P_{k}^{-} + R)^{-1}$$

$$\hat{x}_{k} = \hat{x}_{k}^{-} + K_{k} (z_{k} - \hat{x}_{k}^{-})$$

$$P_{k} = (1 - K_{k}) P_{k}^{-}$$

- We assume the process noise is small
- To make the algorithm converge we have to set a "wrong" initial value
- P≠0 must be chosen as well

Matlab code

- % State transition matrix A and measurement matrix H are unitary
- % Assume the measurement error is normal (white) with sigma=0.1
- % Assume the process error variance is small 1e-5

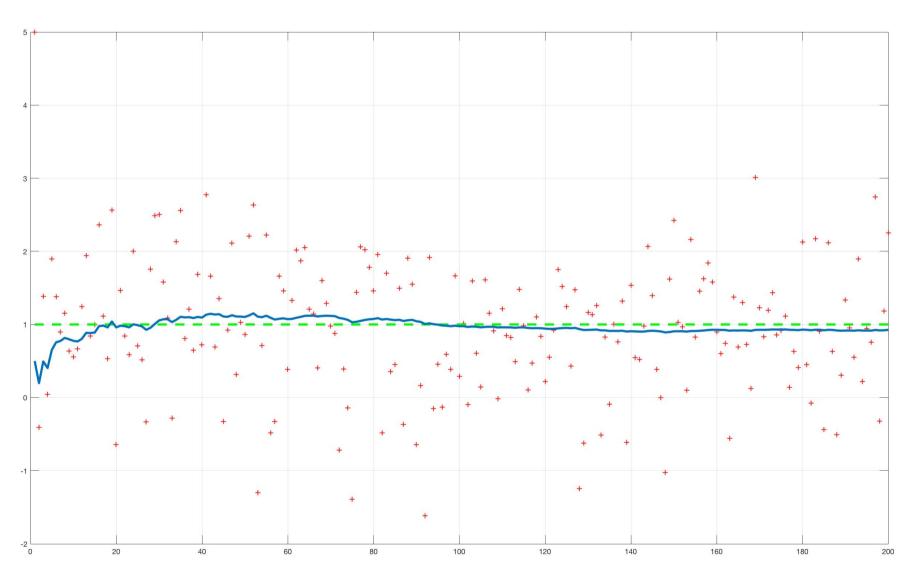
```
x=1*ones(1,200); % the constant value to predict is 1
R=1: Q=1e-5:
N=size(x); % Let's take a number of measurements N
% Set priors
x_{est} = zeros(1,size(x,2));
P_{est} = zeros(1, size(x, 2));
x_{est}(1) = 0.5;
P_{est}(1) = 0.5;
z = zeros(1,size(x,2)); % Set vector for measurements
z(1)=5;
for n=2:size(x,2)
  x_{prior} = x_{est(n-1)};
  P_{prior} = P_{est}(n-1) + randn*Q;
  z(n) = x(n) + randn*sqrt(R);
  K = P_{prior} / (P_{prior} + R);
  x_{est}(n) = x_{prior} + K^*(z(n) - x_{prior});
  P_{est}(n) = (1-K)*P_{prior};
end
plot(1:size(x,2),z, 'r+');
hold on;
grid on;
plot(1:size(x,2),[x], 'g--'); plot(1:size(x,2),x_est, '-');
```

Computer Vision



Matlab code





The extended KF (EKF)



- Assumption in KF: state and measurement are linear
- In general wrong in object tracking
- "Adapt" the filter to linearize mean and covariance
- The process is still defined through f_k and h_k , which are in this case not linear

$$x_k = f_k(x_{k-1}, w_{k-1})$$
$$z_k = h_k(x_k, v_k)$$

The EKF



- Use partial derivatives to linearize the estimation
- Partial derivatives of:
 - Process (A)
 - Measurements (H)
- Since we don't know the values of noise we can assume here for simplicity that state and measurement are

$$\tilde{x}_k = f(\hat{x}_{k-1}, 0)$$

$$\tilde{z}_k = h(\tilde{x}_k, 0)$$

EKF prediction



Using the same notation as for KF, the prediction stage is given by:

$$\tilde{x}_{k} = f(\hat{x}_{k-1}, 0)$$

$$P_{k}^{-} = A_{k-1}P_{k-1}A_{k-1}^{T} + W_{k-1}Q_{k-1}W_{k-1}^{T}$$

- A_k is the Jacobian matrix of partial derivatives of f with respect to x_k
- W_k is the Jacobian matrix of partial derivatives of f with respect to W_k
- Q_k is the process noise covariance matrix

EKF update



The update step becomes:

$$K_{k} = P_{k}^{-} H_{k}^{T} (H_{k} P_{k}^{-} H_{k}^{T} + V_{k} R_{k} V_{k}^{T})^{-1}$$

$$\hat{x}_{k} = \tilde{x}_{k} + K_{k} (z_{k} - h_{k} (\tilde{x}_{k}, 0))$$

$$P_{k} = (I - K_{k} H_{k}) P_{k}^{-}$$

- ullet H_k is the Jacobian matrix of partial derivatives of h with respect to x_k
- ullet V_k is the Jacobian matrix of partial derivatives of h with respect to z_k
- R_k is the measurement noise covariance matrix

EKF predict-update



Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_{k} = f(\hat{x}_{k-1}, u_{k-1}, 0)$$

(2) Project the error covariance ahead

$$P_{k}^{-} = A_{k} P_{k-1} A_{k}^{T} + W_{k} Q_{k-1} W_{k}^{T}$$



(1) Compute the Kalman gain

$$K_{k} = P_{k}^{T} H_{k}^{T} (H_{k} P_{k}^{T} H_{k}^{T} + V_{k} R_{k} V_{k}^{T})^{-1}$$

(2) Update estimate with measurement z_k

$$\hat{x}_k = \hat{x}_k + K_k(z_k - h(\hat{x}_k, 0))$$

(3) Update the error covariance

$$P_k = (I - K_k H_k) P_k$$

Initial estimates for \hat{x}_{k-1} and P_{k-1}

Particle filters



- KF and EKF assume the posterior probability to be Gaussian
- In case it is not Gaussian performances are reduced
- PFs can do the job
- Main concept:
 - Represent alternative solutions as a set of samples
 - Each sample with a weight
 - The more the samples, the closer the optimal Bayesian estimate

Properties



- While KF is an optimal solution to the estimation problem, PFs provide an approximate solution
- Can be applied to either linear and non-linear problems
- Can handle multimodal distributions
- Multiple hypothesis on the process state

How PFs work



Take a set of points associated with the corresponding weights:

$$\{x_k^i\}_{i=1}^N, \{w_k^i\}_{i=1}^N$$

The a posteriori pdf is represented by:

$$p(x_k \mid z_k) \approx \sum_{i=1}^{N} w_{k-1}^i \delta(x_k - x_{k-1}^i)$$

PFs – SIR algorithm

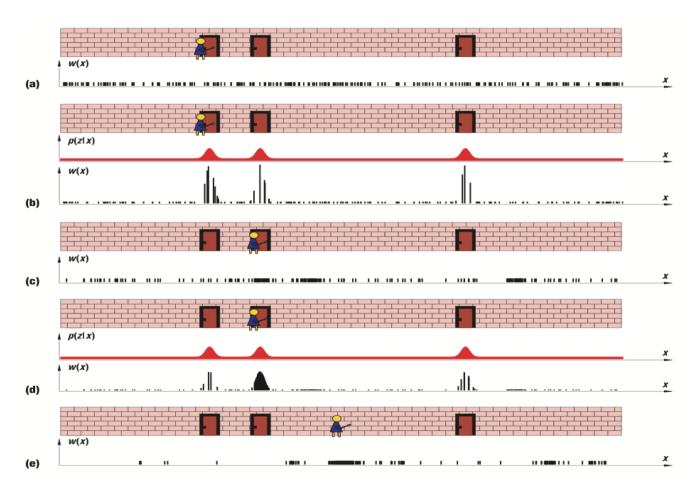


- How do we select $\{x_k^i\}_{i=1}^N$, $\{w_k^i\}_{i=1}^N$ for the pdf approximation?
- Select <u>PROPOSAL DISTRIBUTION</u> $x_k^i \approx \pi(x_k \mid x_{k-1}^i)$ with $i = 1...N_s$
- Update the weights $\hat{w}_k^i = w_{k-1}^i p(\mathbf{z}_k | \mathbf{x}_k^i)$
- Normalize $w_k^i = \hat{w}_k^i / \sum_{i=1}^{Ns} (w_k^i)^2$
- Resample:
- draw Ns particles with probability proportional to their weight;
- (ii) set uniform weight for the new set

$$w_k^i = 1/N_s$$

PFs – Toy Example





D. Fox, J. Hightower, L. Liao, D. Schulz, G. Borriello, "Bayesian Filters for Location Estimation", IEEE Pervasive Computing, 2003.