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# ASSIGNMENT-1

**Q.1** Find assymptotic tight bound of given function

i) $f(n) = 5n^2+2$

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

$$5n^2 \leq 5n^2+2 \leq 5n^2+2n^2$$

$$5n^2 \leq 5n^2+2 \leq 7n^2$$

$$\Rightarrow c_1 = 5,\ c_2 = 7,\ g(n) = n^2$$

$$T(n) = \Theta(g(n))$$

$$\boxed{T(n) = \Theta(n^2)}$$

ii) $f(n) = n^2+2^n+6$

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

$$2^n \leq n^2+2^n+6 \leq 2^n + 6 \cdot 2^n$$

$$2^n \leq n^2+2^n+6 \leq 7 \cdot 2^n$$

$$c_1 = 2,\ c_2 = 7,\ g(n) = 2^n$$

$$T(n) = \Theta(g(n))$$

$$\boxed{T(n) = \Theta(2^n)}$$

iii) $f(n) = n^2 - \frac{2}{n}$

$$c_1 \cdot g(n) \le n^2 - \frac{2}{n} \le c_2 \cdot g(n)$$

$$n^2 \le n^2 - \frac{2}{n} \le n^2$$

$$c_1 = 1,\ c_2 = 1,\ g(n) = n^2$$

$$T(n) = \Theta(g(n))$$

$$\boxed{T(n) = \Theta(n^2)}$$

**Q.2** Solve the Recurrence Relation using Master Method

i) $T(n) = 7T(n/3) + n^2$

Comparing with general eq$^n$ –

$$T(n) = aT(n/b) + f(n)$$

$$a = 7,\ b = 3,\ f(n) = n^2$$

Calculating $(n^{\log_b a})$,

$$n^{\log_3 7} < f(n) = n^2 \quad [n^{1.77} \le n^2 ;\ n^{1.77 + 0.23} \le n^2]$$

$\Rightarrow$ Case 3;

$$7 \cdot \frac{n^2}{9} \le c \cdot n^2 \quad (c < 1)$$

$$T(n) = \Theta(f(n))$$

$$\boxed{T(n) = \Theta(n^2)}$$

ii) $T(n) = T(\sqrt{n}) + n$

let $m = \log n$

~~taking log both sides~~

~~logn~~

$2^m = n$

$T(2^m) = T(\sqrt{2^m}) + 2^m$

$T(2^m) = T(2^{m/2}) + 2^m$

let $P(m) = T(2^m)$, $P(m/2) = T(2^{m/2})$

$P(m) = P\left(\frac{m}{2}\right) + 2^m$

Comparing with general equation

$a = 1 \quad b = 2 \quad f(m) = 2^m$

$m^{\log_b a} = m^{\log_2 1} = m^0 < f(m)$

Case 3;

$1 \cdot 2^{m/2} \le c \cdot 2^m \quad (c < 1)$

$P(m) = \Theta(f(m))$

$P(m) = \Theta(2^m)$

Since, $P(m) = T(n)$

$2^m = n$

$\boxed{T(n) = \Theta(n)}$

i) Using Iteration

$T(n) = 2T(n-2) + 2$ —— ①

Put $n \rightarrow n-2$ in eq ①

$T(n-2) = 2T(n-2-2) + 2$

$T(n-2) = 2T(n-4) + 2$ —— ②

Put eq ② in eq ①

$T(n) = 2[2T(n-4)+2] + 2$

$T(n) = 4T(n-4) + 4 + 2$ —— ③

Put $n \rightarrow n-4$ in eq ①

$T(n-4) = 2T(n-4-2) + 2$

$T(n-4) = 2T(n-6) + 2$ —— ④

Put eq ④ in eq ③

$T(n) = 4[2T(n-6)+2] + 4 + 2$

$T(n) = 8T(n-6) + 8 + 4 + 2$ —— ⑤

General eq -

$T(n) = 2^i T(n-2i) + 2^3 + 2^2 + 2^1$

$T(n) = 2^i T(n-2i) + \sum_{k=1}^{i} 2^k$ —— ⑥

Base Case - $n - 2i = 1$

$\frac{n-1}{2} = i$

$\sum_{k=1}^{i} = 2^k \Rightarrow \frac{1(1-2^i)}{(1-2)}$

$= \frac{1-2^i}{-1} \Rightarrow 2^i - 1 = 2^i - 1$

$$T(n) = 2^{\frac{n-1}{2}} T(1) + 2^{\frac{n-1}{2}} - 1$$

$$T(n) = 2^{\frac{n+1}{2}} - 1$$

$$T(n) = 2^{\frac{n}{2}} \cdot 2^{1/2} - 1$$

$$\boxed{T(n) = O(2^n)}$$

ii) $T(n) = 5T(n-1) + n$ —— ①

Put $n \to n-1$ in eq①

$T(n-1) = 5T(n-1-1) + n$

$T(n-1) = 5T(n-2) + n$ —— ②

Put eq② in eq①

~~T(n) = 5T(~~

$T(n) = 5[5T(n-2) + n] + n$

$T(n) = 25T(n-2) + 5n + n$ —— ③

Put $n \to n-2$ in eq①

$T(n-2) = 5T(n-2-1) + n$

$T(n-2) = 5T(n-3) + n$ —— ④

Put eq④ in eq③

$T(n) = 25[5T(n-3) + n] + 5n + n$

$T(n) = 125T(n-3) + 25n + 5n + n$ —— ⑤

General eqn -

$$T(n) = 5^i T(n-i) + 5^0 n + 5^1 n + 5^2 n + 5^3 n + \cdots 5^{i-1} n \text{ —— ⑥}$$

$$T(n) = 5^i T(n-i) + n \sum_{k=0}^{i-1} 5^k$$

Base Case - $n - i = 1$

$i = n - 1$

$$\sum_{k=0}^{i-1} 5^k = \frac{1(1-5^i)}{1-5}$$

$$\Rightarrow \frac{5^i - 1}{4}$$

$$T(n) = 5^{n-1} T(1) + \frac{5^{n-1} - 1}{4}$$

$$T(n) = \frac{4 \cdot 5^{n-1} + 5^{n-1} - 1}{4}$$

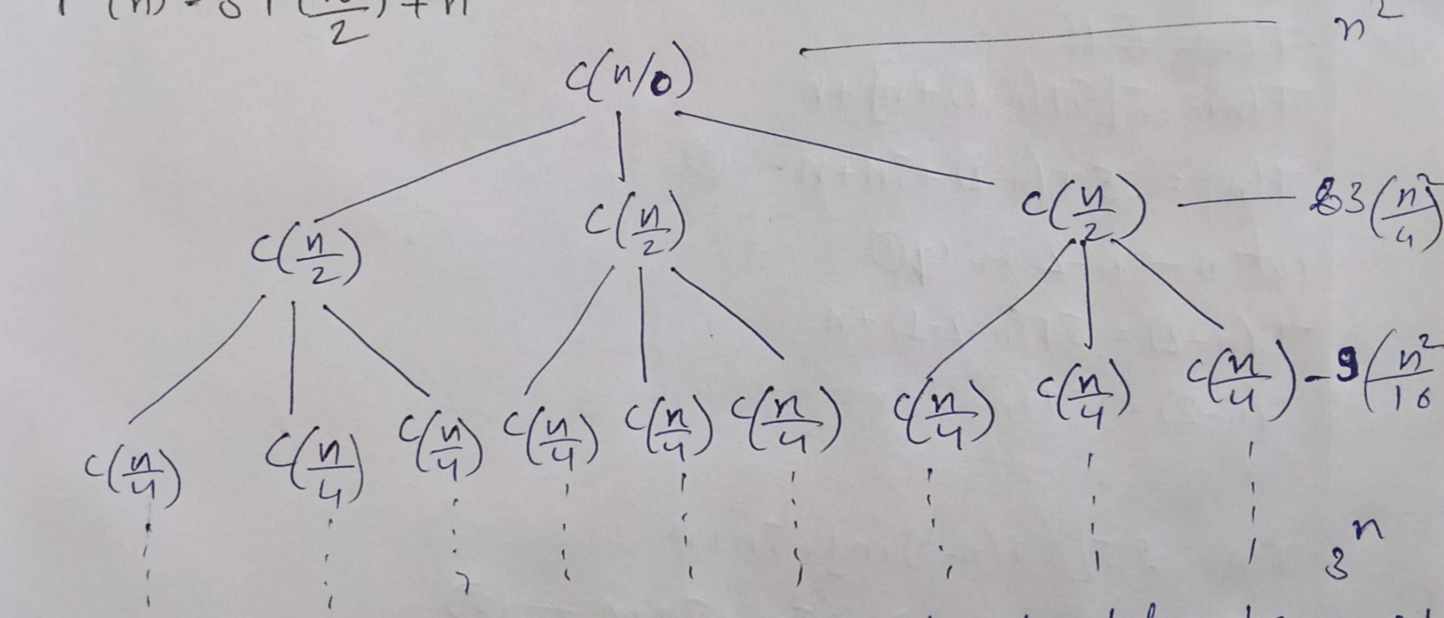
$$T(n) = \frac{5 \cdot 5^{n-1} - 1}{4}$$

$$T(n) = 5^n$$

$$\boxed{T(n) = O(5^n)}$$

**Q.3** Solve using Recursion Tree

i) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$




The recursion stops when the size of subproblem becomes 1
let depth of tree be $i$.

Base Case — $\frac{n}{2^i} = 1$

$i = \log n$ (height of tree)

Cost of last level $= 3^i$. (No. of subproblem)

Cost of each other level $= \left(\frac{n}{2^i}\right)^2 \times$ no. of subprolem at that level.

Total cost $= 3^i \cdot \left(\frac{n}{2^i}\right)^2 \Rightarrow 3^{\log n} \cdot \frac{n^2}{(2^{\log n})^2} =$ $\boxed{T(n) = n^2}$