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An Improvement of Ensemble Kalman Filter for OOSM Tracking

Sirichai Pornsarayouth * **Manop Wongsaisuwan** ** **and Masaki Yamakita** *

* *Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo Japan 152-8550 (e-mail: yamakita@ctrl.titech.ac.jp).*
** *Chulalongkorn University, 254 Phayathai Road, Pathumwan, Bangkok Thailand 10330 (e-mail: wmanop@chula.ac.th).*

Abstract: In multisensor target tracking system, the measurements of the same target are always delayed, come at different rates, and arrive out of sequence. Such measurements are called "out-of sequence" measurements (OOSMs). Examples of such systems are a mobile robot or an unmanned aerial vehicle (UAV) which is observed by both inertial sensors and visual sensors, and delay caused by transmitting or processing time. Solutions via Extended Kalman Filter (EKF), Particle Filter (PF), and Ensemble Kalman Filter (EnKF) have been proposed so far. EnKF yields various advantages, e.g., it can be applied to strong nonlinear system, requires much less particles than PF, and does not require any Jacobian matrix or backward state-transition function. In this paper, we propose an algorithm to improve accuracy of using EnKF with OOSMs. The algorithm concerns estimating an ensemble of the process noises. We validate the proposed algorithm by simulations of the aircraft tracking system. The results show another competitive solution for OOSM filtering.

Keywords: Out of Sequence Measurements, Ensenble Kalman filters, Signal delay, Sensor fusion.

1. INTRODUCTION

In target tracking scheme, multiple sensors are required to achieve accurate estimated position of the target. Different types of sensors enable the sensor to correct weak points of one another. For example, visual sensor which is observed with delay can help us correct long term accumulated error of inertial sensor, or we can use multiple bearing sensor to observe target in multidimension space or improve accuracy of the filter. Observation of various sensors always come at different rates, is usually delayed, and arrive out of sequence. Such measurements are known as "out-of-sequence" measurements (OOSMs) which are prior observations that are reported together with their time stamp. We can use this late measurement (OOSM) for updating the most present estimated state, to get more accurate estimation. Generally, OOSMs are categorized into two groups. OOSMs whose delay time less than one sampling interval are considered to be one-step lag OOSMs and to be multistep lag OOSMs, otherwises. Our goal is to find a data-association method which effectively update the present state with these OOSMs.

OOSM filtering is an interesting tracking problem and practically occurs. The simplest way to handle OOSMs is to go back to the observed time of OOSM, τ, and perform re-filtering again with all measurements we have since τ. Although the algorithm is easy and straightaway, but it consumes too much processing time. Alternatively, solutions to OOSMs have been broadly proposed. Bar-Shalom (2002) proposed an exact solution to one-step lag OOSMs problem using Extended Kalman filter (EKF). Also, one-step solution to l-step lag problem is available in Bar-Shalom et al. (2004). However, EKF is limited to mildly nonlinear system whose noise is assumed to be Gaussian and whose Jacobian matrix can be determined. Otherwise, solutions via particle filter shall be applied to handle non-Gaussian/strong nonlinear system, see Orton and Marrs (2005) and Mallick et al. (2002). To make the number of particles in PF algorithm efficient, a smoothing filter can be applied as in Orguner and Gustafsson (2008).

In our previous research, assuming the probability of all random variables are Gaussian, we proposed a solution to OOSMs via Ensemble Kalman filter (EnKF) as an improvement of our unpublished work, see Pornsarayouth and Yamakita (2011). One appeal of EnKF is its history of the estimated particles. An old track of particles allows us to find cross/auto covariance without any requirement of backward state-transition function. Another appeal is because it is a particle-base algorithm which we can easily handle strong nonlinear system where the Jacobian matrix cannot or can hardly be founded. The proposed technique requires much less particles than that of particle filter technique in Mallick et al. (2002). In this paper, we propose an improvement which includes an extra step which estimating an ensemble of the accumulated process noise. The algorithm greatly improve accuracy of the estimated state in high delay case.

The paper comprises six sections. Section 1 introduces a background of the problem; section 2 presents a formulation of OOSM problem and show a general solution to Gaussian dynamics; Section 3 describes our previous research which applies EnKF to OOSM problem and presents a new improvement of EnKF technique for OOSMs; section 4 describes how we conduct an experiment via simulations; section 5 compares the results of using EKF, PF, EnKF (of our previous research), and a new EnKF (proposed in this paper) techniques to OOSMs; finally, section 6 concludes advantages and disadvantages of each algorithm.

2. PROBLEM FORMULATION

Consider a time-varying discrete-time system

$$x(k) = f_{k,k-1}(x(k-1)) + v(k,k-1), \tag{1}$$

$$z(k) = h_k(x(k)) + w(k), \tag{2}$$

where $x(k)$ is the state at t_k; $f_{k,k-1}$ is a state transition function; $v(k,k-1)$ is a Gaussian zero-mean process noise accumulated between t_{k-1} and t_k with the corresponding covariance $Q(k,k-1)$; $z(k)$ is the measurement of the state $x(k)$; h_k is the measurement function; $w(k)$ is the Gaussian zero-mean observation noise with the corresponding covariance $R(k)$. We assume that $v(k,k-1)$ and $w(k)$ are sampled from the independent white Gaussian noise;

$$Q(k_2,k_1) \triangleq Q(|t_{k_2} - t_{k_1}|). \tag{3}$$

We define an estimation of the state and its covariance about t_k as

$$\hat{x}(k|k) \triangleq \mathbb{E}[x(k)|Z_k], \tag{4}$$

$$P(k|k) \triangleq COV[x(k)|Z_k], \tag{5}$$

where Z_k represents all measurements up to time t_k

$$Z_k \triangleq \{z(1), z(2), z(3), \dots, z(k)\}. \tag{6}$$

Suppose the earlier measurement about

$$\tau \triangleq t_l, \qquad t_l \le t_k \tag{7}$$

arrives at t_k. The subscript $\bullet_l$ denotes the time step l invoked by an OOSM. The objective of OOSM filtering is to determine a new estimation $\hat{x}(k|l^+)$ which includes an extra information about an OOSM, $z(l)$. Subsequently, we define

$$\hat{x}(k|l^+) \triangleq \mathbb{E}[x(k)|Z_l^+], \tag{8}$$

$$P(k|l^+) \triangleq COV[x(k)|Z_l^+], \tag{9}$$

$$Z_l^+ \triangleq \{Z_k, z(l)\}. \tag{10}$$

2.1 A Solution for Gaussian Dynamics

Using an optimal filter, one can update $\hat{x}(k|k)$ by $z(l)$ using the following equations,

$$\begin{aligned}\hat{x}(k|l^+) = \hat{x}(k|k) + P_{\mathrm{xz}}(k,l|k)P_{\mathrm{zz}}(l|k)^{-1} \\ \cdot (z(l) - \hat{z}(l|k)),\end{aligned} \tag{11}$$

$$\begin{aligned}P(k|l^+) = P(k|k) - P_{\mathrm{xz}}(k,l|k)P_{\mathrm{zz}}(l|k)^{-1} \\ \cdot P_{\mathrm{xz}}(k,l|k)^{\mathrm{T}},\end{aligned} \tag{12}$$

where the estimation of $z(l)$ subject to Z_k and its corresponding (cross) covariances are defined as

$$P_{\mathrm{xz}}(k,l|k) \triangleq \mathbb{E}[x(k) - \hat{x}(k|k)][z(l) - \hat{z}(l|k)]^{\mathrm{T}}, \tag{13}$$

$$P_{\mathrm{zz}}(l|k) \triangleq \mathbb{E}[z(l) - \hat{z}(l|k)][z(l) - \hat{z}(l|k)]^{\mathrm{T}}, \tag{14}$$

$$\hat{z}(l|k) \triangleq \mathbb{E}[z(l)|Z_k]. \tag{15}$$

Equation (11) and (12) are the main equations being used in EKF for OOSMs and our previous algorithm. The problem is how can we find those of (13), (14), and (15). For OOSM-EKF solution, one may use a state-transition function to predict back to the state of the prior time, t_l.

$$\hat{x}(l|k) = f_{l,k}(\hat{x}(k|k)), \tag{16a}$$

$$f_{l,k} \triangleq f_{k,l}^{-1}. \tag{16b}$$

Then use EKF technique of other prior researches, e.g., Bar-Shalom et al. (2004) (OOSM-EKF) to find $P_{\mathrm{xz}}(k,l|k)$, $P_{\mathrm{zz}}(k,l|k)$, and $\hat{z}(l|k)$ and apply these variables to (11) and (12).

If the system/measurement model is strongly nonlinear but Gaussian, EnKF can be applied as a particle-base algorithm (OOSM-EnKF); furthermore, the algorithm requires much less particle than filtering OOSMs with particle filter technique (OOSM-PF), Mallick et al. (2002).

3. A SOLUTION VIA EnKF

3.1 Previous Research

This section describes our previous work. Ensemble Kalman filter (EnKF) represents the estimated state by a cluster (ensemble) of distinct filtered states which are called particles. Each of particles are propagated and filtered independently like how we apply the conventional Kalman filter except we determine mean and (cross/auto) covariance of the estimation by calculating the sampled mean and covariance from an ensemble of the estimated particles and use it in the updating equation. We define an ensemble of the estimation $\hat{x}(k_2|k_1)$ as

$$\mathcal{X}(k_2|k_1) \triangleq \{x_1(k_2|k_1), x_2(k_2|k_1), \dots, x_p(k_2|k_1)\}, \tag{17}$$

where p is the number of particles of one ensemble. We approximate mean and covariance of an ensemble by the sampled mean and covariance as

$$\hat{x}(k_2|k_1) = \frac{1}{p}\sum_{i=1}^{p} x_i(k_2|k_1), \tag{18}$$

$$P(k_2|k_1) = \frac{1}{p-1}\sum_{i=1}^{p}[x_i(k_2|k_1)][x_i(k_2|k_1)]^{\mathrm{T}}. \tag{19}$$

Subsequently, the distribution of $\hat{z}(k_1)$, $\hat{v}(k_1,k_2)$, and $\hat{w}(k_1)$ are represented by $\mathcal{Z}(k_1)$, $\mathcal{V}(k_1,k_2)$, and $\mathcal{W}(k_1)$, respectively. We can also approximate $P_{\mathrm{xz}}(k,l|k)$, $P_{\mathrm{zz}}(k,l|k)$, and $\hat{z}(l|k)$ by the same way,

$$\hat{z}(l|k) \approx \frac{1}{p}\sum_{i=1}^{p} z_i(l|k), \quad \hat{x}(k|k) \approx \frac{1}{p}\sum_{i=1}^{p} z_i(k|k), \tag{20}$$

$$\begin{aligned}P_{\mathrm{zz}}(l|k) \approx \frac{1}{p-1}\sum_{i=1}^{p}[z_i(l|k) - \hat{z}(l|k)] \\ \cdot [z_i(l|k) - \hat{z}(l|k)]^{\mathrm{T}},\end{aligned} \tag{21}$$

$$\begin{aligned}P_{\mathrm{xz}}(k,l|k) \approx \frac{1}{p-1}\sum_{i=1}^{p}[x_i(k|k) - \hat{x}(k|k)] \\ \cdot [z_i(l|k) - \hat{z}(l|k)]^{\mathrm{T}}.\end{aligned} \tag{22}$$

We apply (11) and (12) in particle case,

$$\begin{aligned}x_i(k|l^+) = x_i(k|k) + P_{xz}(k,l|k)P_{zz}(l|k)^{-1}(z(l) - z_i(l|k)), \\ \{i = 1, \dots, p\}.\end{aligned} \tag{23}$$

$P(k|l^+)$ is calculated by the same way as (19). While $\mathcal{X}(k|k)$ is available, we only have to find the ensemble

$\mathcal{Z}(l|k)$. First, we approximate $\mathcal{X}(l|l)$ from the nearby ensembles on the track by an appropriate interpolation technique. For example, if the system dynamics is linear, bilinear is a suitable technique.

$$x_i(l|l) \approx x_i(a|a) + \frac{t_l - t_a}{t_b - t_a}(x_i(b|b) - x_i(a|a)),$$
$$\{i = 1, \dots, p\}, \quad \text{and} \quad t_a \le t_l \le t_b. \quad (24)$$

We get $\mathcal{X}(l|l)$, randomly generate $\mathcal{V}(k,l)$ from $Q(k,l)$ and predict $\mathcal{X}(l|l)$ to $\mathcal{X}(k|l)$ by

$$x_i(k|l) = f_{k,l}\left(x_i(l|l)\right) + v_i(k,l), \quad \{i = 1, \dots, p\}. \quad (25)$$

Then use a one-step smoothing filter to update $\mathcal{X}(l|l)$ by $\mathcal{X}(k|k)$, and obtain $\mathcal{X}(l|k)$ as

$$x_i(l|k) = x_i(l|l) + P_{\mathrm{xd}}(l,k)P_{\mathrm{dd}}(k)^{-1}(x_i(k|l) - x_i(k|k)),$$
$$\{i = 1, \dots, p\}. \quad (26)$$

where

$$e_i(l|l) \triangleq x_i(l|l) - \hat{x}(l|l), \quad (27)$$
$$d_i(k|l) \triangleq x_i(k|l) - \hat{x}(k|l), \quad (28)$$
$$P_{\mathrm{xd}}(l,k) = \frac{1}{p-1}\sum_{i=1}^{p}[e_i(l|l)][d_i(k|l)]^{\mathrm{T}}, \quad (29)$$
$$P_{\mathrm{dd}}(k) = \frac{1}{p-1}\sum_{i=1}^{p}[d_i(k|l)][d_i(k|l)]^{\mathrm{T}}. \quad (30)$$

Subsequently, generate $\mathcal{W}(l)$ and estimate $\mathcal{Z}(l|k)$ by applying the measurement model,

$$z_i(l|k) = h_l(x_i(l|k)) + w_i(l), \quad \{i = 1, \dots, p\}. \quad (31)$$

Now we can use $\mathcal{Z}(l|k)$ to complete (23). If multiple sensors report their observation at the same time, multiple filter can be processed simultaneously and independently. Sensor fusion technique can be applied for multitasking purpose. For EnKF, a fusion $\mathcal{X}^*(k|k)$ between states $\mathcal{X}^1(k|k)$ and $\mathcal{X}^2(k|k)$ which were estimated by different two sensors is

$$x_i^*(k|k) = x_i^1(k|k) + P_{\mathrm{x}^1\Delta}(k|k)P_{\Delta\Delta}(k|k)^{-1}$$
$$\cdot\, (x_i^2(k|k) - x_i^1(k|k)), \quad \{i = 1, \dots, p\}. \quad (32)$$

where,

$$e_i^1(k|k) \triangleq x_i^1(k|k) - \hat{x}^1(k|k), \quad (33)$$
$$e_i^2(k|k) \triangleq x_i^2(k|k) - \hat{x}^2(k|k), \quad (34)$$
$$P_{\mathrm{x}^1\Delta}(k|k) = \frac{1}{p-1}\sum_{i=1}^{p}[e_i^1(k|k)][e_i^1(k|k) - e_i^2(k|k)]^{\mathrm{T}}, \quad (35)$$
$$P_{\Delta\Delta}(k|k) = \frac{1}{p-1}\sum_{i=1}^{p}[e_i^1(k|k) - e_i^2(k|k)] \cdot [e_i^1(k|k) - e_i^2(k|k)]^{\mathrm{T}}. \quad (36)$$

A fusion of m observations can be done in $\lceil \log_2 m \rceil + 1$ steps of (32) including the filtered step of each sensor where $\lceil \bullet \rceil$ is a ceiling function.

3.2 An Improvement of EnKF

Inspiring by an estimation of the process noise in Bar-Shalom et al. (2004), we propose a technique for estimating an ensemble of the accumulated process noise between t_k and t_l, $\mathcal{V}(k,l|k)$. Then use it for estimating more accurate $\mathcal{X}(l|k)$. Suppose that we have $\mathcal{X}(k|k)$ and $\mathcal{X}(l|l)$, we can estimate $\mathcal{V}(k,l|k)$ by the same fashion of the EnKF fusion technique. First, we use (25) to predict an ensemble of the state from t_l to t_k.

$$v_i(k,l|k) = v_i(k,l) + P_{\mathrm{v}\delta}(k,l)P_{\delta\delta}(k)^{-1}(x_i(k|k) - x_i(k|l)),$$
$$\{i = 1, \dots, p\}. \quad (37)$$

where

$$e_i(k|k) \triangleq x_i(k|k) - \hat{x}(k|k), \quad (38)$$
$$e_i(k|l) \triangleq x_i(k|l) - \hat{x}(k|l), \quad (39)$$
$$P_{\mathrm{v}\delta}(k,l) = \frac{1}{p-1}\sum_{i=1}^{p}[v_i(k,l)][e_i(k|l) - e_i(k|k)]^{\mathrm{T}}, \quad (40)$$
$$P_{\delta\delta}(k) = \frac{1}{p-1}\sum_{i=1}^{p}[e_i(k|l) - e_i(k|k)] \cdot [e_i(k|l) - e_i(k|k)]^{\mathrm{T}}, \quad (41)$$

Since $v_i(k,l)$ is randomly and independently generated, the cross covariance between $\mathcal{V}(k,l)$ and $\mathcal{X}(k|k)$ is equal to zero and the cross covariance between $\mathcal{V}(k,l)$ and $\mathcal{X}(k|l)$ is equal to $Q(k,l)$. Therefore,

$$P_{\mathrm{v}\delta}(k,l) = Q(k,l). \quad (42)$$

An estimating equation for $\mathcal{V}(k,l|k)$ becomes

$$v_i(k,l|k) = v_i(k,l) + Q(k,l)P_{\delta\delta}(k)^{-1}(x_i(k|k) - x_i(k|l)).$$
$$\{i = 1, 2, \dots, p\}. \quad (43)$$

We subtract $\mathcal{V}(k,l|k)$ and $\mathcal{V}(k,l)$ from $\mathcal{X}(k|k)$ and $\mathcal{X}(k|l)$, respectively, and use their difference to update $\mathcal{X}(l|l)$ to be $\mathcal{X}(l|k)$.

$$x_i(l|k) = x_i(l|l) + P_{\mathrm{x}\rho}(l,k)P_{\rho\rho}(k)^{-1}\xi_i(k), \quad \{i = 1, \dots, p\}, \quad (44)$$

where

$$\xi_i(k) \triangleq (x_i(k|k) - v_i(k,l|k)) - (x_i(k|l) - v_i(k,l)), \quad (45)$$
$$e_i(l|l) \triangleq x_i(l|l) - \hat{x}(l|l), \quad (46)$$
$$\rho_i(k) \triangleq (x_i(k|l) - v_i(k,l)) - (\hat{x}(k|l) - \hat{v}(k,l)), \quad (47)$$
$$P_{\mathrm{x}\rho}(l,k) = \frac{1}{p-1}\sum_{i=1}^{p}[e_i(l,l)][\rho_i(k)]^{\mathrm{T}}, \quad (48)$$
$$P_{\rho\rho}(k) = \frac{1}{p-1}\sum_{i=1}^{p}[\rho(k)][\rho(k)]^{\mathrm{T}}, \quad (49)$$

Effectiveness of the new $\mathcal{X}(l|k)$ will be shown in the simulation compared to other filtering technique.

4. SIMULATION

We simulated OOSMs filtering using the following algorithms 1) an EKF technique as presented in Bar-Shalom et al. (2004) denoted as 'OOSM-EKF'; 2) a particle filter technique as presented in Mallick et al. (2002) simulated with 10000 particles denoted as 'OOSM-PF'; 3) a technique using EnKF together with RTS (Rauch-Tung-Striebel) smoothing filter denoted as 'OOSM-RTS'; 4) an EnKF technique as presented in Pornsarayouth and Yamakita (2011) denoted as 'OOSM-EnKF1'; 5) an improved EnKF technique which is proposed in this paper denoted as 'OOSM-EnKF2'. We simulate 'OOSM-EnKF1' and 'OOSM-EnKF2' algorithms with 100 particles. The following show a target system and its observation model.

4.1 Target System

We choose a two-dimension constant velocity system. So we can investigate the efficiency of the proposed algorithm compared to others including the EKF technique (OOSM-EKF). The time-varying step-size mathematical model of the system is

$$x(k) = F(\Delta t)x(k-1) + v(k,k-1), \tag{50}$$
$$v(k,k-1) \sim \mathcal{N}(0, Q(\Delta t)), \tag{51}$$

where $x = [x^{\mathrm{x}}\ \dot{x}^{\mathrm{x}}\ x^{\mathrm{y}}\ \dot{x}^{\mathrm{y}}]^{\mathrm{T}}$ is a vector of position in **x** axis, velocity in **x** axis, position in **y** axis, and velocity in **y** axis, respectively.

Note that the definition of $Q(\Delta t)$ is given by

$$Q(\Delta t) = q \times \begin{bmatrix} \Delta t^3/3 & \Delta t^2/2 & 0 & 0 \\ \Delta t^2/2 & \Delta t & 0 & 0 \\ 0 & 0 & \Delta t^3/3 & \Delta t^2/2 \\ 0 & 0 & \Delta t^2/2 & \Delta t \end{bmatrix}, \tag{52}$$

where $\Delta t \triangleq |t_k - t_{k-1}|$ and q is a constant parameter represents noisiness of the process noise.

4.2 Observation

The target is observed by two bearing sensors which measure a target from different positions. The measurement model of each sensor is described by

$$z(k) = \mathrm{atan2}(\Delta x^{\mathrm{y}}, \Delta x^{\mathrm{x}}) + w(k), \tag{53}$$
$$\Delta x^{\mathrm{y}} = x^{\mathrm{y}} - x_s^{\mathrm{y}}, \quad \Delta x^{\mathrm{x}} = x^{\mathrm{x}} - x_s^{\mathrm{x}} \tag{54}$$

where $(x_s^{\mathrm{x}}, x_s^{\mathrm{y}})$ is the coordinate of a sensor, $w(k)$ is a Gaussian zero-mean measurement noise with the covariance $R(k)$, and $\mathrm{atan2}(x_2, x_1)$ is a $\tan(x_2/x_1)$ function over 360°. Each sensor reports an intercepted angle with at a distinct scanning period. When scanning line of a bearing sensor intercepts with the target, the sensor waits for the end of scanning round and report an intercepted angle alongside with the intercepting time. Therefore, the measurement from a bearing sensor itself is practically a one-step lag OOSM.

Although this model is undefined at $x = y = 0$, discontinuous on $x < 0, y = 0$, it is differentiable elsewhere. Jacobian matrix of the measurement model is represented by

$$\nabla_{x_k} h_k(x(k)) = \frac{[\,-\Delta x^{\mathrm{y}}\ \ 0\ \ \Delta x^{\mathrm{x}}\ \ 0\,]^{\mathrm{T}}}{\left((\Delta x^{\mathrm{x}})^2 + (\Delta x^{\mathrm{y}})^2\right)}. \tag{55}$$

On real systems, $x = y = 0$ never occurs and we can easily avoid discontinuity of the function by accumulating the observation over π and $-\pi$. Thus, we can apply EKF technique.

4.3 Environment

A target is considered to be an aircraft or an unmanned aerial vehicle (UAV) with an initial state $[-190, 6, 150, 1]^{\mathrm{T}}$. The target evolve according to the model in 4.1 and is affected by the process noise with $q = 0.1$. The first bearing sensor is located at $(-200, -100)$ scanning with 2.8s period. The second is located at $(200, 0)$ scanning with 2.6s period. Both sensors are disturbed by the noise with covariance $R = 0.01$ and the resolution of the measurement is 0.1° We did simulations in three cases 1) no additional delay on both sensors – one-step lag OOSMs; 2) 5s delay is introduced on the second sensor – multistep lag OOSM; 3) 10s delay is introduced on the second sensor – multistep lag OOSM (extremely delayed). Each of the investigated filters start with a fault initial value at $[-200, 6, 180, 1]^{\mathrm{T}}$. Note that the performance of the filter is getting worse as the target moves away from both sensors thus we stop the simulation at 80s.

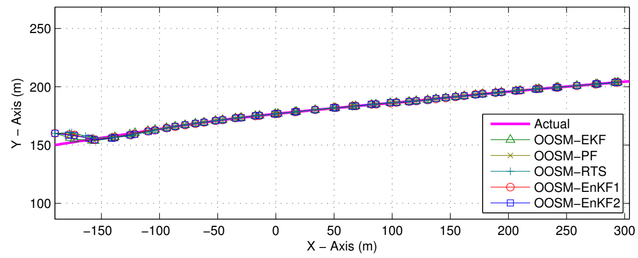

Fig. 1. Trajectory of the target and the estimated trajectory, one-step lag OOSM.

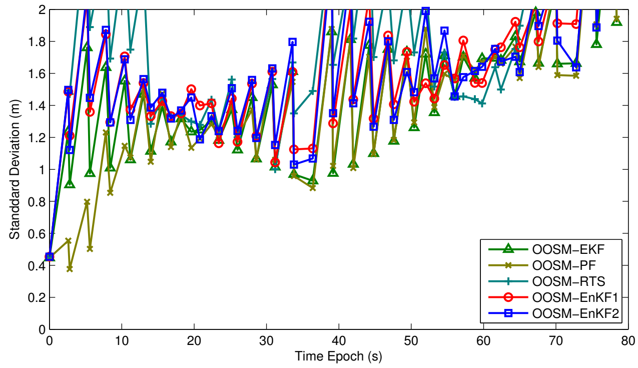

Fig. 2. Standard deviation of the estimated position, one-step lag OOSM.

5. SIMULATION RESULTS

We represent real trajectories by magenta lines; 'OOSM-EKF' algorithm is represented by dark green lines with triangle marks; 'OOSM-PF' algorithm is represented by olive lines with crossed marks; 'OOSM-RTS' is represented by cyan lines with '+' marks; 'OOSM-EnKF1' is represented by red lines with circle marks; 'OOSM-EnKF2' is represented by blue lines with square marks.

5.1 One-step Lag OOSM

The measurement from each sensor is fed directly to the filters. Fig. 1 shows the tracking trajectory of each algorithm. Fig. 2 and Fig. 3 show the standard deviation and the absolute error of the tracking trajectory, respectively, over time. We repeated the simulation four times and summarize the absolute error of each simulation in Table 1. We can see that the trajectory tracked by each algorithm can well track the real trajectory and is close to one another. The standard deviation and the absolute error of those algorithms are also at the same level.

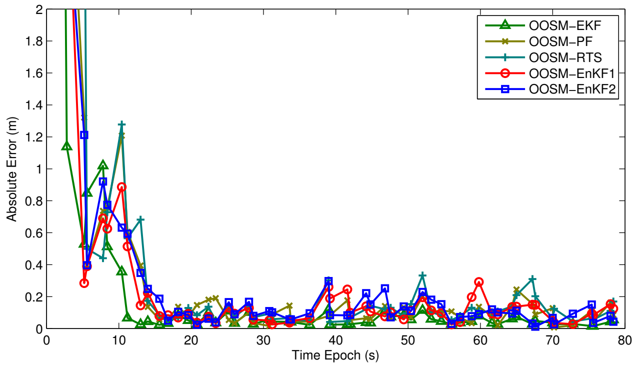

Fig. 3. Absolute error, one-step lag OOSM.

OOSM-/Run	1	2	3	4	Average
EKF	0.2578	0.2570	0.2566	0.2580	0.2573
PF	0.2881	0.3221	0.3044	0.3034	0.3045
RTS	0.4128	0.3781	0.4337	0.4274	0.4130
EnKF1	0.3651	0.3884	0.3554	0.3544	0.3659
EnKF2	0.3696	0.3279	0.3362	0.3331	0.3417

Table 1. Average absolute error : one-step lag OOSM.

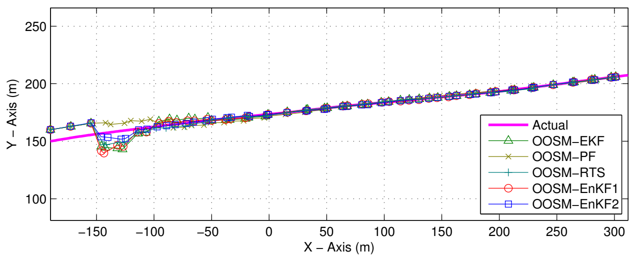

Fig. 4. Trajectory of the target and the estimated trajectory, 10s additional delayed on the second sensor.

OOSM-/Run	1	2	3	4	Average
EKF	1.4820	1.4341	1.3524	1.4285	1.4242
PF	1.2869	1.1675	1.3600	1.5238	1.3345
RTS	1.1861	1.2320	0.9689	1.0003	1.0968
EnKF1	1.1056	1.2213	1.1984	1.2350	1.1901
EnKF2	0.7541	0.6852	0.8889	0.8309	0.7898

Table 2. Absolute error of the estimated position, 5s additional delayed on the second sensor.

5.2 Multistep Lag OOSM

The delay of the second sensor is introduced by 5s. Fig. 4 shows the tracking trajectory of each algorithm. Fig. 5 and Fig. 6 show the standard deviation and the absolute error of the tracking trajectory, respectively. The absolute error of four simulations are summarized in Table 2. From Fig. 4, an additional delay on the second sensor effects the ripple of the tracking trajectory of all algorithms. While other algorithm exhibit very good results, 'OOSM-EnKF2' algorithm shows less ripple on the tracking trajectory. As in Pornsarayouth and Yamakita (2011) and Orguner and Gustafsson (2008), we inspected that 'OOSM-PF' algorithm require as many as 10000 particles to keep the algorithm stable (unstable at 5000 particles) because the dimension of the state is doubled by the requirement of the estimation of the process noise.

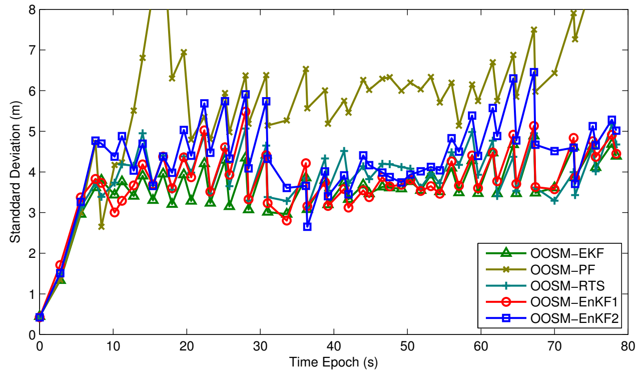

Fig. 5. Standard deviation of the estimated position, 10s additional delayed on the second sensor.

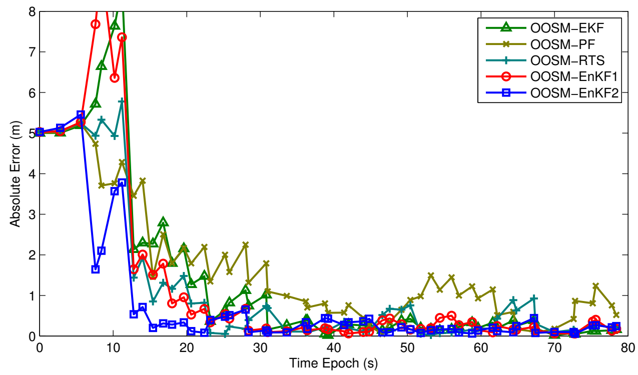

Fig. 6. Absolute error of the estimated position, 5s additional delayed on the second sensor.

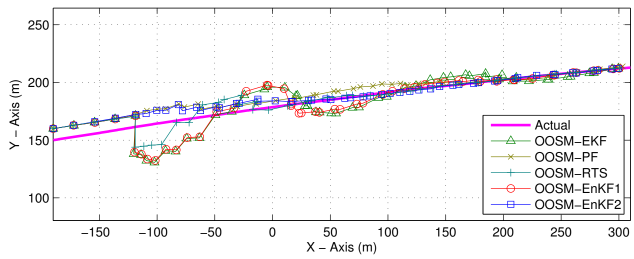

Fig. 7. Trajectory of the target and the estimated trajectory, 10s additional delayed on the second sensor.

5.3 Multistep Extremely Delay OOSM

The delay of the second sensor is introduced by 10s. Fig. 7 shows the tracking trajectory of each algorithm. Fig. 8 and Fig. 9 show the standard deviation and the absolute error of the tracking trajectory, respectively. The absolute error of four simulations are summarized in Table 3. The effect of the delay to the ripple of the tracking trajectory is amplified. A fault estimation of each algorithm occurs when an OOSM about $t = 2.8$s arrives at 10.8s and causes a ripple. Despite of that, the proposed 'OOSM-EnKF2' algorithm give us the best result with a little ripple. As the result from the standard deviation, we found that 'OOSM-EnKF2' algorithm estimate the result with higher covariance than other algorithms which underestimate covariance of the state. In this case, 'OOSM-PF' algorithm is sometimes unstable with just 10000 particles.

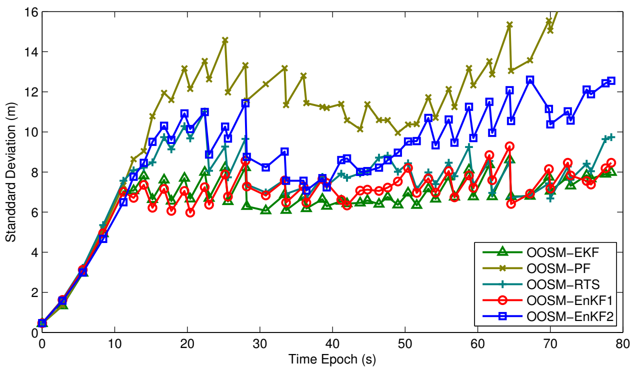

Fig. 8. Standard deviation of the estimated position, 10s additional delayed on the second sensor.

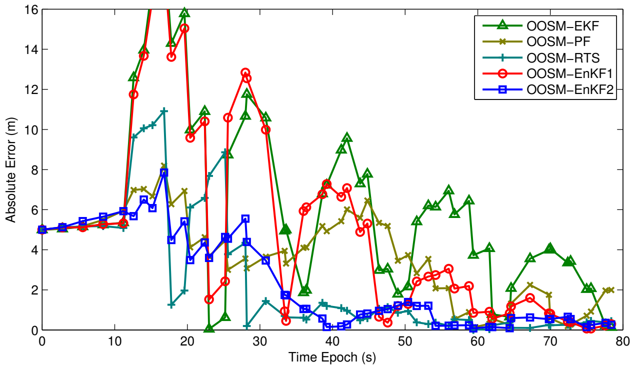

Fig. 9. Absolute error of the estimated position, 10s additional delayed on the second sensor.

OOSM-/Run	1	2	3	4	Average
EKF	6.0754	5.9324	5.8670	5.9354	5.9526
PF	3.9052	5.4714	5.9619	4.2676	4.9015
RTS	2.9940	2.6203	1.9810	2.7933	2.5972
EnKF1	4.5671	5.0608	4.0717	4.9888	4.6721
EnKF2	1.8573	2.5493	2.3162	2.6468	2.3424

Table 3. Average absolute error : 10s additional delayed on the second sensor.

6. CONCLUSIONS

In this paper, we propose a new OOSM filtering technique which applies a one-step smoothing and a process noise estimation using EnKF. Since EnKF is a particle-base filter, it is applicable to a strong nonlinear system where the Jacobian matrix cannot be determined. Comparing to other particle-base algorithm such as 'OOSM-PF', the 'OOSM-EnKF2' algorithm because it indeed reduces complexity of the calculation, consumes less computational cost, and is more stable. A smoothing technique makes the proposed algorithm requires only the state transition function which makes the algorithm also applicable to the system whose the inverse function of $f_{k,k-1}$ can hardly or cannot be found.

If the system is mildly nonlinear and an OOSM is only one-step lag, the 'OOSM-EKF' algorithm is more suitable for consuming less computational cost and has the same level of the tracking performance as other algorithms. For multistep lag to extremely delayed system with a Gaussian assumption, we highly recommend the 'OOSM-EnKF2' algorithm whose performance is superior to other techniques. The key of the 'OOSM-EnKF2' algorithm is the step where every particle in an ensemble of the process noise is estimated which enable us to determine $\hat{x}(l|k)$ with lower covariance.

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