Bayesian Model of Behaviour in Economic Games

Debajyoti Ray
dray@caltech.edu
Computation and Neural Systems,
California Institute of Technology

Brooks King-Casas
bkcasas@cpu.bcm.tmc.edu
Computational Psychiatry Unit,
Baylor College of Medicine

P. Read Montague
montague@hnl.bcm.tmc.edu
Human NeuroImaging Lab,
Baylor College of Medicine

Peter Dayan
dayan@gatsby.ucl.ac.uk
Gatsby Computational Neuroscience Unit,
University College London

Classical game theoretic approaches that make strong rationality assumptions have difficulty modeling human behaviour in economic games. We investigate the role of finite levels of iterated reasoning and non-selfish utility functions in a Partially Observable Markov Decision Process model that incorporates game theoretic notions of interactivity. Our generative model captures a broad class of characteristic behaviours in a multi-round Investor-Trustee game. We invert the generative process for a recognition model that is used to classify 200 subjects playing this game against randomly matched opponents.

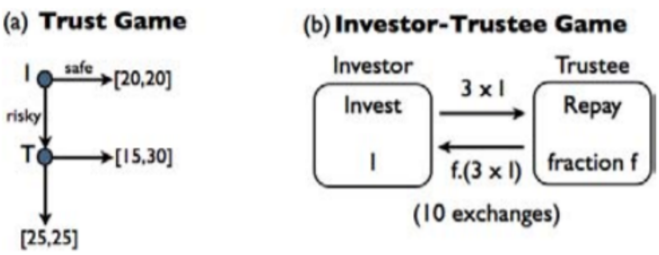

Figure 1: (a) In a simple Trust game, the Investor can take a safe option with a payoff of $[Investor=20,Trustee=20](game ends).Alternatively, he can pass the decision to the Trustee.The Trustee can now choose a fair option $[25,25] or choose to defect $[15,30]. (b) In the multi-round version, the Investor gets $20 dollars at every round. He can invest integer(I) part; this quantity becomes 3I way to the Trustee. In turn, she has the option of repaying any (integer)amount of her resulting allocation to the Investor. The game continues for 10 rounds

One strategy is to retain the notion that subjects attempt to optimize their utilities, but to include in these utilities social factors that penalize cases in which opponents win either more (crudely envy, parameterized by α) or less (guilt, parameterized by β) than themselves . One popular Inequity-Aversion utility function characterizes player i by the type Ti = (αi , βi) of her utility function:

$$U(\alpha_i,\beta_i) = x_i - \alpha_i \max\{(x_j - x_i), 0\} - \beta_i \max\{(x_i - x_j), 0\}$$

where x_i, x_j are the amounts received by players i and j respectively.

In order to capture the range of behavior exhibited by subjects in these games, we built a finite belief hierarchy model, using inequity averse utility functions in the context of a partially observable hidden Markov model of the ignorance each subject has about its opponent's type and in the light of sequential choice. We used inference strategies from machine learning to find approximate solutions to this model. In this paper, we use this generative model to investigate the qualitative classes of behaviour that can emerge in these games.

As in the framework of Bayesian games, player i's inequity aversion type Ti = (αi ,βi) is known to it, but not to the opponent. Player i does have a prior distribution over the type of the other player j, b ; and, if suitably sophisticated, can also have higher-order priors over the whole hierarchy of recursive beliefs about types. The utility U is calculated at every round for each player i for action a by marginalizing over the current beliefs b

$$U_i^{(t)}(a_i^{(t)}) = \sum_{k=1:K} b_{ik} Q_i^{(t)}(b_{ik}^{(t)}, a_i^{(t)})$$

Reputation-formation plays a particularly critical role in the Investor-Trustee game, with even the most selfish players trying to benefit from cooperation, at least in the initial rounds. In order to reduce complexity in analyzing this, we set αI = βI = 0 (i.e., a purely selfish Investor) and consider 2 values of βT (0.3 and 0.7) such that in the last round the Trustee with type βT = 0.3 will not return any amount to the Investor and will choose fair outcome if βT = 0.7. We generate a rich tapestry of behavior by varying the prior expectations as to βT and the values of strategic (k) level (0,1,2) for the players.

Since the game is Markovian we can calculate the probability of player i taking the action sequence a , t = 1, ..., 10 given his Type Ti and prior beliefs b as:

$$P(\{a_i^t\}|T_i, \vec{b}_i^{(0)}) = P(a_1^{(1)}|T_i, \vec{b}_i^{(0)}) \prod_{t=2}^{10} P(a_i^{(t)}|\mathcal{D}^{(t)}, T_i)$$

The recognition model classify subject pairs playing the 10-round Investor-Trustee game. The data included 48 student pairs playing an Impersonal task for which the opponents' identities were hidden and 54 student pairs playing a Personal task for which partners met

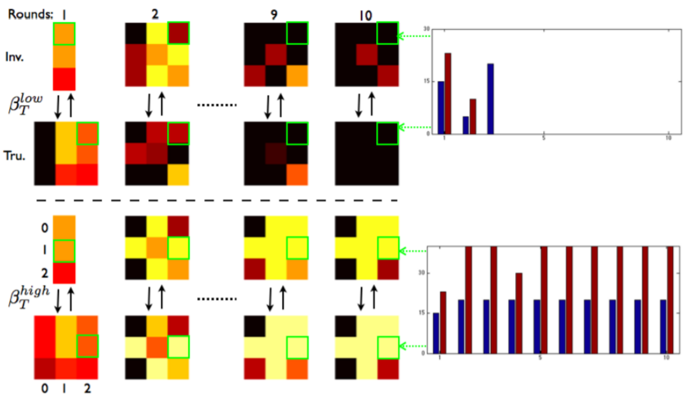

Figure 2: The dyadic interactions between the Investor and Trustee across the 10 rounds of the game. Lighter colours reveal higher amounts (with amount given by Investor in first round being 15 dollars).

The inference method have wider application, for instance identifying which of a collection of Bayes-Nash equilibria is most likely to arise, given psychological factors about human utilities.

References

[1] K.A. McCabe, M.L. Rigdon and V.L. Smith. Positive Reciprocity and Intentions in Trust Games (2003). Journal of Economic Behaviour and Organization.

[2] E. Fehr and K.M. Schmidt. A Theory of Fairness, Competition and Cooperation (1999). The Quarterly Journal of Economics.

[3] E. Fehr and S. Gachter. Fairness and Retaliation: The Economics of Reciprocity (2000). Journal of Economic Perspectives.

[4] E. Fehr and U. Fischbacher. Social norms and human cooperation (2004). *TRENDS in Cog. Sci.* 8:4.

[5] P.J. Gmytrasiewicz and P. Doshi. A Framework for Sequential Planning in Multi-Agent Settings (2005). Journal of Artificial Intelligence Research.

[6] V. Conitzer and T. Sandholm (2002). Complexity Results about Nash Equilibria. Technical Report CMU-CS-02-135, School of Computer Science, Carnegie-Mellon University.

[7] S. Thrun. Monte Carlo POMDPs (2000). Advances in Neural Information Processing Systems 12.

[8] JC Harsanyi (1967). Games with Incomplete Information Played by "Bayesian" Players, I-III. Management Science.

[9] J.F. Mertens and S. Zamir. Formulation of Bayesian analysis for games with incomplete information (1985). International Journal of Game Theory.