8E&F_LR_SVM

January 29, 2021

8E and 8F: Finding the Probability P(Y==1|X)

8E: Implementing Decision Function of SVM RBF Kernel

After we train a kernel SVM model, we will be getting support vectors and their corresponsing coefficients α_i

Check the documentation for better understanding of these attributes:

https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html

As a part of this assignment you will be implementing the `decision_function()` of kernel SVM, here decision_function() means based on the value return by `decision_function()` model will classify the data point either as positive or negative

Ex 1: In logistic regression After traning the models with the optimal weights w we get, we will find the value $\frac{1}{1+\exp(-(wx+b))}$, if this value comes out to be < 0.5 we will mark it as negative class, else its positive class

Ex 2: In Linear SVM After traning the models with the optimal weights w we get, we will find the value of $sign(wx + b)$, if this value comes out to be -ve we will mark it as negative class, else its positive class.

Similarly in Kernel SVM After traning the models with the coefficients α_i we get, we will find the value of $sign(\sum_{i=1}^{n}(y_i \alpha_i K(x_i, x_q)) + intercept)$, here $K(x_i, x_q)$ is the RBF kernel. If this value comes out to be -ve we will mark x_q as negative class, else its positive class.

RBF kernel is defined as: $K(x_i, x_q) = exp(-\gamma ||x_i - x_q||^2)$

For better understanding check this link: https://scikit-learn.org/stable/modules/svm.html#svm-mathematical-formulation

0.1 Task E

1. Split the data into X_{train}(60), X_{cv}(20), X_{test}(20)
2. Train $SVC(gamma = 0.001, C = 100.)$ on the (X_{train}, y_{train})
3. Get the decision boundry values f_{cv} on the X_{cv} data i.e. f_{cv} = `decision_function`(X_{cv}) you need to implement this decision_function()

[68]:
```
import numpy as np
import pandas as pd
from sklearn.datasets import make_classification
import numpy as np
from sklearn.svm import SVC
from sklearn.metrics.pairwise import rbf_kernel
import warnings
```

```
warnings.filterwarnings("ignore")
```

[ ]:

0.1.1 Pseudo code

clf = SVC(gamma=0.001, C=100.) clf.fit(Xtrain, ytrain)

def decision_function(Xcv, ...): #use appropriate parameters for a data point x_q in Xcv: #write code to implement $(\sum_{i=1}^{\text{all the support vectors}}(y_i\alpha_i K(x_i, x_q)) + intercept)$, here the values y_i, α_i, and *intercept* can be obtained from the trained model return # the decision_function output for all the data points in the Xcv

fcv = decision_function(Xcv, ...) # based on your requirement you can pass any other parameters

Note: Make sure the values you get as fcv, should be equal to outputs of clf.decision_function(Xcv)

[ ]:

[ ]:

[69]:
```
# you can write your code here
X, Y = make_classification(n_samples=5000, n_features=5, n_redundant=2,
                           n_classes=2, weights=[0.7], class_sep=0.7,␣
 ↪random_state=15)

from sklearn.model_selection import train_test_split
x_train, x_test, y_train, y_test = train_test_split(X, Y,stratify=Y,test_size=0.
 ↪20,random_state=42)

from sklearn.preprocessing import StandardScaler
scaler = StandardScaler().fit(x_train)
x_train = scaler.transform(x_train)
x_tr,x_cv,y_tr,y_cv = train_test_split(x_train,␣
 ↪y_train,stratify=y_train,test_size=0.20,random_state=42)

svc = SVC(C=100,gamma=0.001)
svc.fit(x_tr,y_tr)
```

[69]:
```
SVC(C=100, break_ties=False, cache_size=200, class_weight=None, coef0=0.0,
    decision_function_shape='ovr', degree=3, gamma=0.001, kernel='rbf',
    max_iter=-1, probability=False, random_state=None, shrinking=True,
    tol=0.001, verbose=False)
```

[70]:
```
from numpy.linalg import norm
from sklearn.metrics.pairwise import rbf_kernel
def decision_function(x_cv):
  sv = svc.support_vectors_
  ker_mat = rbf_kernel(X=x_cv,Y=sv,gamma=0.001)
  pred=list()
```

```
  for i,test in enumerate(x_cv):
    res = 0
    for j,sv_ in enumerate(sv):
      res += ( ker_mat[i][j] * svc.dual_coef_[0][j] )
    pred.append((res + (svc.intercept_)))
  return pd.DataFrame(pred,columns=['y_hat'])
```

[71]:
```
fcv = svc.decision_function(x_cv) # sklearn's decision function output.
```

[72]:
```
implemented_cv = decision_function(x_cv) # implemented code for deciscion␣
↪function as a part of task
```

[73]:
```
implemented_cv['fcv'] = fcv # appendeing output of sklearn's decision function␣
↪and implemented deciscion function
```

[74]:
```
implemented_cv # comparing values .
```

[74]:
```
        y_hat       fcv
0   -2.021215 -2.021215
1    1.646756  1.646756
2   -1.862529 -1.862529
3   -2.583835 -2.583835
4   -0.569751 -0.569751
..        ...       ...
795  2.312139  2.312139
796 -2.357006 -2.357006
797 -0.358116 -0.358116
798  1.426593  1.426593
799 -2.528678 -2.528678

[800 rows x 2 columns]
```

8F: Implementing Platt Scaling to find P(Y==1|X)
Check this PDF

0.2 TASK F

4. Apply SGD algorithm with (f_{cv}, y_{cv}) and find the weight W intercept b `Note: here our data is of one dimensional so we will have a one dimensional weight vector i.e W.shape (1,)`

Note1: Don't forget to change the values of y_{cv} as mentioned in the above image. you will calculate y+, y- based on data points in train data

Note2: the Sklearn's SGD algorithm doesn't support the real valued outputs, you need to use the code that was done in the `'Logistic Regression with SGD and L2'` Assignment after modifying loss function, and use same parameters that used in that assignment. if Y[i] is 1, it will be replaced with y+ value else it will replaced with y- value

5. For a given data point from X_{test}, $P(Y=1|X) = \frac{1}{1+exp(-(W*f_{test}+b))}$ where f_{test} = `decision_function`(X_{test}), W and b will be learned as metioned in the above step

Note: in the above algorithm, the steps 2, 4 might need hyper parameter tuning, To reduce the complexity of the assignment we are excluding the hyerparameter tuning part, but intrested students can try that

If any one wants to try other calibration algorithm istonic regression also please check these tutorials

1. http://fa.bianp.net/blog/tag/scikit-learn.html#fn:1
2. https://drive.google.com/open?id=1MzmA7QaP58RDzocB0RBmRiWfl7Co_VJ7
3. https://drive.google.com/open?id=133odBinMOIVb_rh_GQxxsyMRyW-Zts7a
4. https://stat.fandom.com/wiki/Isotonic_regression#Pool_Adjacent_Violators_Algorithm

#SGD ALGO

[105]:
```
def sigmoid(z):
    ''' In this function, we will return sigmoid of z'''
    # compute sigmoid(z) and return

    return 1/(1+np.exp(-z))

def initialize_weights(dim):
    ''' In this function, we will initialize our weights and bias'''
    #initialize the weights to zeros array of (1,dim) dimensions
    #you use zeros_like function to initialize zero, check this link https://
↪docs.scipy.org/doc/numpy/reference/generated/numpy.zeros_like.html
    #initialize bias to zero
    b = 0
    w = np.zeros_like(dim)

    return w,b


def gradient_dw(x,y,w,b,alpha,N):
    '''In this function, we will compute the gardient w.r.to w '''
    #print(x.shape,w.shape)
    z = np.dot(x,w) + b
    #print(z)
    #print(((y-sigmoid(z))),x)
    a = ((y-sigmoid(z))*x)
    #print(a)
    b = ((alpha/N)*w)
    dw = a-b
    return dw
```

```
def gradient_db(x,y,w,b):
     '''In this function, we will compute gradient w.r.to b '''
     z = ( (np.dot(x,w)) + b )
     db = (y-(sigmoid(z)))
     return (db)


def pred(w,b, X):
    N = len(X)
    predict = []
    #w = list(w)
    for i in range(N):
        z = np.dot(X[i],w) + b
        predict.append(sigmoid(z))
    return np.array(predict)
#print(1-np.sum(y_train - pred(w,b,x_train))/len(x_train))
#print(1-np.sum(y_test  - pred(w,b,x_test))/len(x_test))


import math
def logloss(y_true,y_pred,N_pos,N_neg):
    '''In this function, we will compute log loss '''
    l=0
    #print(y_pred)
    for i in range(len(y_true)):
      if y_true[i]==1:
        y_true[i] = N_pos
      else:
        y_true[i] = N_neg
      l+= ( (y_true[i]*(math.log10(y_pred[i]))) + ((1-y_true[i])*(math.
↪log10(1-y_pred[i]))) )
    loss = (-1/len(y_true))*l
    return loss


from tqdm import tqdm
from collections import OrderedDict
def train(x_train,y_train,x_test,y_test,epochs,alpha,eta0,N_pos,N_neg):
    w,b = initialize_weights(x_train[0])
    N = len(x_train)
    tr_loss,ts_loss = [],[]
    #loss = OrderedDict()
    initial_loss = 0
    for epoch in tqdm(range(epochs)):
      #print(epoch)
      for i in range(x_train.shape[0]):
```

```
            dw = gradient_dw(x_train[i],y_train[i],w,b,alpha,N)
            db = gradient_db(x_train[i],y_train[i],w,b)
            w += (eta0*dw)
            b += (eta0*db)
        #x = decision_function(x_train)
        y_pred_tr = pred(w,b,x_train)
        y_pred_ts = pred(w,b,x_test)
        tr_loss.append(logloss(y_train,y_pred_tr,N_pos,N_neg))
        ts_loss.append(logloss(y_test,y_pred_ts,N_pos,N_neg))
        #loss[logloss(y_test,y_pred_ts)] = w
        #print(y_train,y_pred_tr)
        #print(y_test,y_pred_ts)
        #ts_los = logloss(y_test,y_pred_ts)
        #if abs(initial_loss - ts_los  ) <= (10**(-3)):
        #  print('(ts_los - initial_loss):',(ts_los, initial_loss),abs(ts_los -␣
 ↪initial_loss),'\n')
        #  break
        #print('(ts_los - initial_loss):',(ts_los, initial_loss),abs(ts_los -␣
 ↪initial_loss),'\n')
        #initial_loss = ts_los
    return w,b,tr_loss,ts_loss
```

[115]:
```
alpha=0.0001
eta0=0.0001
N=len(x_train)
epochs=14
```

[116]:
```
'''
creating the dataset for calibration
'''
#D_cv = decision_function(x_cv)
fcv = pd.DataFrame(svc.decision_function(x_cv),columns=['f_cv'])
D_cv = fcv
D_cv['y_true'] = y_cv
#D_cv.sort_values(by='y_hat',inplace=True)
#c = dict(pd.DataFrame(y_tr).value_counts()) # getting counts
#N_pos = (c[1,]+1)/(c[1,]+2) # computing N_pos
#N_neg = 1/(c[0,]+2) # computing N_neg

y_train = pd.DataFrame(y_train)
num_pos = y_train[y_train==1].sum()
num_neg = y_train[y_train==0].sum()
N_pos = (num_pos+1)/(num_pos+2) # computing N_pos
N_neg = 1/(num_neg+2) # computing N_neg
```

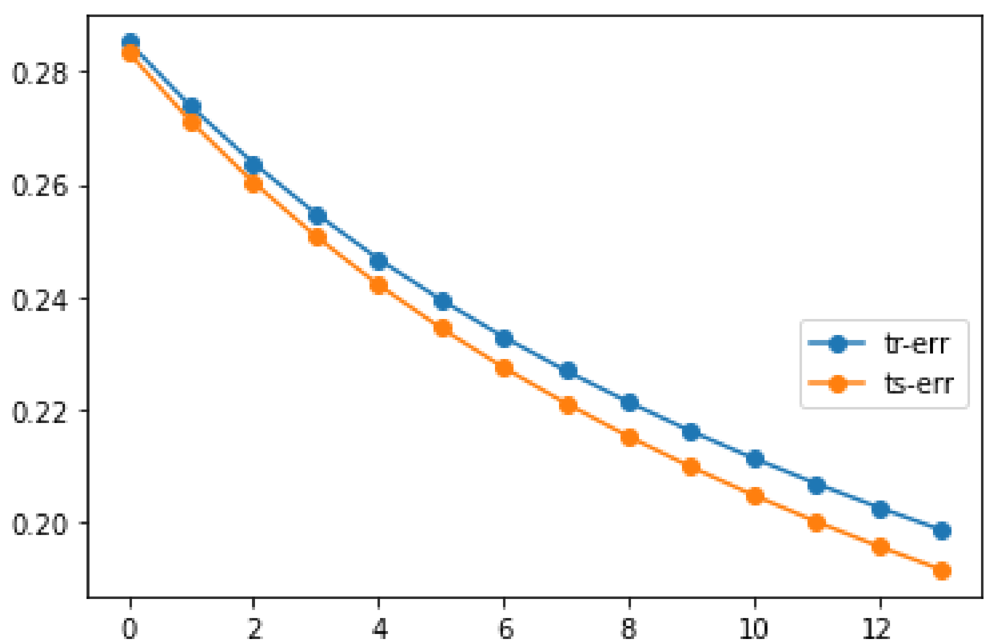
[117]:
```
# splitting cv data to find optimal w and b
calib_x_tr,calib_x_cv,calib_y_tr,calib_y_cv = train_test_split(D_cv['f_cv'],␣
 ↪D_cv['y_true'],stratify=D_cv['y_true'],test_size=0.20,random_state=42)
```

```
# passing splitted cv data to train function for optimal w and b values
w,b,tr_loss,ts_loss = train(x_train=np.array(calib_x_tr),
                            y_train=np.array(calib_y_tr),
                            x_test=np.array(calib_x_cv),
                            y_test=np.array(calib_y_cv),
                            epochs=epochs,
                            alpha=alpha,
                            eta0=eta0,
                            N_pos=N_pos,N_neg=N_neg)
```

```
100%|| 14/14 [00:00<00:00, 24.49it/s]
```

[118]:
```
import matplotlib.pyplot as plt
plt.plot(tr_loss,'-o',label='tr-err')
plt.plot(ts_loss,'-o',label='ts-err')
plt.legend(bbox_to_anchor=(1,0.5))
```

[118]:
```
<matplotlib.legend.Legend at 0x7fcaa122ff28>
```



[119]:
```
def pred_prob(w,b,x_test):
  return pd.DataFrame(1/(1+(np.exp(-((np.
↪dot(decision_function(x_test),w))+b)))),columns=['prob_values'])
```

[122]:
```
pred_prob(w,b,x_test)
```

[122]:

```
     prob_values
0       0.211260
1       0.366319
2       0.209828
3       0.305193
4       0.240818
..           ...
995     0.214169
996     0.196957
997     0.421671
998     0.295574
999     0.259279

[1000 rows x 1 columns]
```

[121]:

```
```

[121]:

```
```

[ ]:

```
```