Collaborative Transportation of Point Mass Cable-suspended Payload using Two Quadcopters

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Considering the less payload carrying capacity of the single quadcopter, one way to increase the overall payload carrying is use of more than one quadcopter which carry the payload simultaneously. Also, transporting the payload by suspended it using cables, beneficial as compared to fixed attached on the quadcopter chassis. Hence, present report focus on the trajectory planning and control of two quadcopter with cable-suspended payload. A leader-follower scheme is considered where leader quadcopter direct the motion of the payload, while follower quadcopter stabilize the oscillations occurs in the cables or payload. Simulations are carried out in in SIMSCAPE multibody dynamics tool in the MATLAB.

1 Introduction

Aerial robotics is an emerging field with various potential applications in different sectors. Quadcopter is one big example of the recent advancement in aerial robotics. The propulsive force is provided by four different rotors in a quadcopter instead of a fixed-wing as in their fixed-wing counterparts. Quadcopters are relatively easy to design, manufacture, and control because the blades have fixed pitch and the rotation for the propellers is in one direction [1]. Simple motors and controllers can be used to design a quadcopter.

Currently, quadcopter is the most popular Unmanned Aerial Vehicle (UAVs) due to being convenient in size for transportation, cheap in price, and easy to control manually [2]. UAVs are now not only used for fun times, instead they are being effectively utilized in numerous applications like payload transportation e.g. delivering amazon packages, spraying insecticides on the agricultural land, police surveillance, aerial photography etc. Payloads can be transported by two ways; either by attaching the payload rigidly to the quadcopter [3] or by connecting the payload using a cable [4]. Attaching the payload using a mechanical gripper [5] or a robotic arm [6] is not as preferred since they may increase the overall inertia of the system that reduces the quadcopter agility [7]. The promptness of the quadcopter is not affected at all when the payload is suspended with cable since the use of cable decouples the quadcopter attitude dynamics from the payload dynamics.

Quadcopters are inefficient as compared to the fixed-wing counterparts because a quadcopter has to produce the energy for flying via their rotors which is not an energy-efficient process, whereas the generated lift depends on wing-structure in a fixed-wing UAV. The battery contributes to 30% of the total mass which yields only around 30 minutes of flight time [1]. Another key challenge is related to the design of the controllers for the entire system.

Considering the quadcopter as rigid body in space, it has total four control inputs from each motor and a total of six degrees of freedom (three translational motion and three rotational motion). There are number of control schemes have been presented to control the motion the quadcopter in the literature. Even though the agility of a cable-suspended quadcopter is good, the payload may swing during travel and such swing can destabilize the quadcopter motion. Thus, it is required to control the oscillations of the quadcopter during the transportation.

The control of a payload suspended with cable from two-quadcopters is even more complicated since it requires delicate balance between the dynamics of the payload and one of the quadcopters that works as a leader of the entire unit. A multi-quadcopter system is more promising since it offers more payload capacity. Interactive controller design is necessary for such systems that can offer accurate height control, improved stability, and safe transportation. A multi-quadcopter system can then be deployed to even more areas of research after having improved controllers.

In this work, we have considered a problem of collaborative transportation of the cable-suspended payload using two quadcopters. A PID attitude controller is designed to control the attitude of each quadcopter and PD controller is designed to control the altitude of the quadcopter. To verify the developed control law, SIMSCAPE multibody dynamic toolbox of SIMULINK is used.

2 Equations of Motion (EOM)

In this section, dynamical model of the single quadcopter is described.

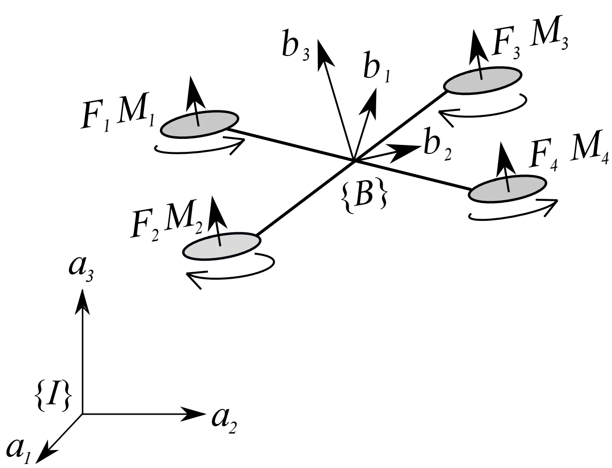

Fig. 1. Free Body Diagram of the Quadcopter showing the inertial and the body frames. Each rotor produces a thrust and a moment.

As shown in the Fig. 1, the free body diagram of the single quadcopter is shown. A single quadcopter has four motors which generates thrust, i.e., $F_i, i = \{1,2,3,4\}$ and corresponding reaction moments, i.e., $M_i, i = \{1,2,3,4\}$. The inertial frame is represented by the three perpendicular vectors $\mathbf{a_1}$, $\mathbf{a_2}$ and $\mathbf{a_3}$. Let $\mathcal{A}$ denote the basis representing the inertial frame. Vectors $\mathbf{b_1}$, $\mathbf{b_2}$ and $\mathbf{b_3}$ constitute the basis denoted by $\mathcal{B}$ which represents the body frame of the quadcopter. Following assumptions are considered to derived the EOM of single quadcopter.

1. Quadcopter is considered as rigid body.
2. It is assumed that the body frame of the quadcopter coincides with principle axes
3. Thrust and moment generated by quadcopter is proportional to the square of the rotor's speed.
4. External force such as wind forces and drag forces are not considered while modeling the quadcopter system.

The relationship between rotor's angular speed, ω_i, and thrust force and reaction moment are $F_i = k_F \omega_i{}^2$ and $M_i = k_M \omega_i{}^2$. The constants are k_F and k_M emerge from results of experiments and matching the performance of a simulation to that of a real system. $Z-X-Y$ Euler angle parameterization is used to describe the attitude of the quadcopter. This will help model the rotation of the quadcopter in the world frame. A rotation matrix ${}^{\mathcal{A}}R_{\mathcal{B}}$ is used to transform the coordinates from the body frame of the quadcopter to the world frame.

$$ {}^{\mathcal{A}}R_{\mathcal{B}} = \begin{bmatrix} c\psi c\theta - s\phi s\psi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\ c\theta s\psi + c\psi s\phi s\theta & c\phi c\psi & s\psi s\theta - c\psi c\theta s\phi \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix} \quad (1) $$

where $c \equiv cos$ and $s \equiv sin$ This rotation matrix is actually a multiplication of the three rotation matrices corresponding to the motions of roll, pitch and yaw, i.e., a rotation about each of the axis with corresponding angles of ϕ, θ and ψ respectively i.e. ${}^{\mathcal{A}}R_{\mathcal{B}} = R_3 R_2 R_1$, where

$$ R_1 = Rot(x,\phi), \quad Roll = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi & cos\phi \end{bmatrix} \quad (2) $$

$$ R_2 = Rot(y,\theta), \quad Pitch = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix} \quad (3) $$

$$ R_3 = Rot(z,\psi), \quad Yaw = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi\ 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4) $$

Now, the angular velocity of the quadcopter represented in the body frame would be:

$$ {}^{\mathcal{A}}\omega_{\mathcal{B}} = p\mathbf{b_1} + q\mathbf{b_2} + r\mathbf{b_3} $$

The coefficients of the vectors are related to the derivatives of the roll, pitch and yaw angles by:

$$ \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (5) $$

Newton- Euler Equations of Motion

We take gravity in the $-\mathbf{a_3}$ direction and the thrust forces F_i for each rotor to be in the $\mathbf{b_3}$ direction. Correspondingly, the translational dynamics are written in Eq. (6).

$$ m\ddot{X} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + {}^{\mathcal{A}}R_{\mathcal{B}} \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} \quad (6) $$

where, $X = [x, y, y]^T$, $u_1 = \Sigma_{i=1}^{4} F_i$ is the total thrust generated by all the propellers. Each rotor also produces a moment perpendicular to the plane of rotation. Two of its rotors produce moment in clockwise ($-\mathbf{b_3}$) direction while the other two rotate in anticlockwise ($\mathbf{b_3}$) direction. Balancing the torques, we have the angular accelerations to be in the form of:

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = u_2 - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{7}$$

where, $u_2 = \begin{bmatrix} (F_1 + F_2 - F_3 - F_3)L \\ (-F_1 + F_2 - F_3 + F_4)L \\ M_1 - M_2 - M_3 + M_4 \end{bmatrix}$

The vector $\mathbf{u_2}$ contains three inputs and along with u_1 there are a total of four inputs in the quadcopter system. Since we know that the quadcopter is a rigid body in three dimensions exhibiting six degrees of freedom with four inputs, it is an underactuated system.

3 Controller

The controller is designed based on the linearized model of the quadcopter. The dynamical equations are linearized about hover equilibrium configuration. At the hover equilibrium configuration all the translational and rotational states are zeros. However, more specifically, yaw angle is kept at ψ_0. The control inputs at hover equilibrium point are $u_1 = mg$ and $u_2 = [0, 0, 0]^T$. Correspondingly, the linearized EOM of the quadcopter are written in Eqs. (8-11).

$$\ddot{x} = (\theta cos\psi_0 + \phi sin\psi_0)g \tag{8}$$
$$\ddot{y} = (\theta sin\psi_0 - \phi cos\psi_0)g \tag{9}$$
$$\ddot{z} = -g + \frac{u_1}{m} \tag{10}$$
$$I\left[\ddot{\phi}\ \ddot{\theta}\ \ddot{\psi}\right]^T = u_2 \tag{11}$$

As we have 4 independent input control, so we can control 4 degree of freedoms. As shown in Fig. 2 the nested control structure is shown. Here inner loop is for attitude control and outer loop is for position control. In attitude controller, feedback of attitude and angular velocity is taken and based on that input u_2 is being calculated. In outer position control loop, error between the desired position and current position is calculated and based on the error in position, input u_1 is being calculated.

Let define second order error control law in x, y, z and ψ as,

$$(\ddot{x} - \ddot{x}_{des}) + K_{vx}(\dot{x} - \dot{x}_{des}) + K_{p_x}(x - x_{des}) = 0 \tag{12}$$
$$(\ddot{y} - \ddot{y}_{des}) + K_{vy}(\dot{y} - \dot{y}_{des}) + K_{p_y}(y - y_{des}) = 0 \tag{13}$$
$$(\ddot{z} - \ddot{z}_{des}) + K_{v_z}(\dot{z} - \dot{z}_{des}) + K_{p_z}(z - z_{des}) = 0 \tag{14}$$
$$(\ddot{\psi} - \ddot{\psi}_{des}) + K_{v_\psi}(\psi - \psi_{des}) + K_{p_\psi}(\psi - \psi_{des}) = 0 \tag{15}$$

From Eq. (14) and Eq. (10), control input u_1 will be,

$$u_1 = m(g + \ddot{z}_{des} + K_{v_z}(\dot{z}_{des} - \dot{z}) + K_{p_z}(z_{des} - z)) \tag{16}$$

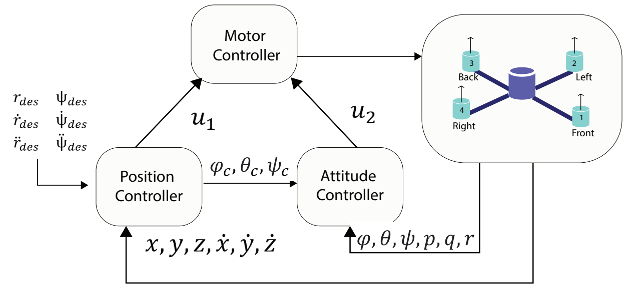

Fig. 2. Nested control structure

Let define the control input u_2 as,

$$u_2 = \begin{bmatrix} K_{p\phi}(\phi_c - \phi) + K_{v\phi}(p_c - p) \\ K_{p\theta}(\theta_c - \theta) + K_{v\theta}(q_c - q) \\ K_{p_\psi}(\psi_c - \psi) + K_{v_\psi}(r_c - r) \end{bmatrix} \tag{17}$$

In equation.(17) $\phi_c, \theta_c, , \psi_c, p_c, q_c, r_c$ are the commanded parameters that we need to find out. From Eq. (12) and Eq. (13), we can write $\ddot{x}_c$ and $\ddot{y}_c$ as

$$\ddot{x}_c = \ddot{x}_{des} + K_{vx}(\dot{x}_{des} - \dot{x}) + K_{p_x}(x_{des} - x) \tag{18}$$
$$\ddot{y}_c = \ddot{y}_{des} + K_{vy}(\dot{y}_{des} - \dot{y}) + K_{p_y}(y_{des} - y) \tag{19}$$

Also from equation.(8) and equation.(9), we can write $\ddot{x}_c$ and $\ddot{y}_c$ as

$$\ddot{x}_c = (\theta_c cos\psi_{des} + \phi_c sin\psi_{des})g \tag{20}$$
$$\ddot{y}_c = (\theta_c sin\psi_{des} - \phi_c cos\psi_{des})g \tag{21}$$

After some manipulation, from equation.(20) and (21) commanded θ_c and ϕ_c can be written as follows.

$$\theta_c = \frac{1}{g}(\ddot{y}_c sin\psi_{des} + \ddot{x}_c cos\psi_{des}) \tag{22}$$
$$\phi_c = \frac{1}{g}(\ddot{x}_c sin\psi_{des} - \ddot{y}_c cos\psi_{des}) \tag{23}$$

ψ_c will be equal to the predefined trajectory in ψ. To calculate p_c, q_c and r_c, from Eq. (5), we can write commanded p_c, q_c and r_c as follows,

$$\begin{bmatrix} p_c \\ q_c \\ r_c \end{bmatrix} = \begin{bmatrix} cos\theta_c & 0 & -sin\theta_c cos\theta_c \\ 0 & 1 & sin\phi_c \\ sin\theta_c & 0 & cos\theta_c cos\phi_c \end{bmatrix} \begin{bmatrix} \dot{\phi}_c \\ \dot{\theta}_c \\ \dot{\psi}_c \end{bmatrix} \tag{24}$$

4 Simulation Results

The simulations were carried out using SIMSCAPE multibody dynamic toolbox of SIMULINK software. The simulation was divided out into three portions, (i) Set-point

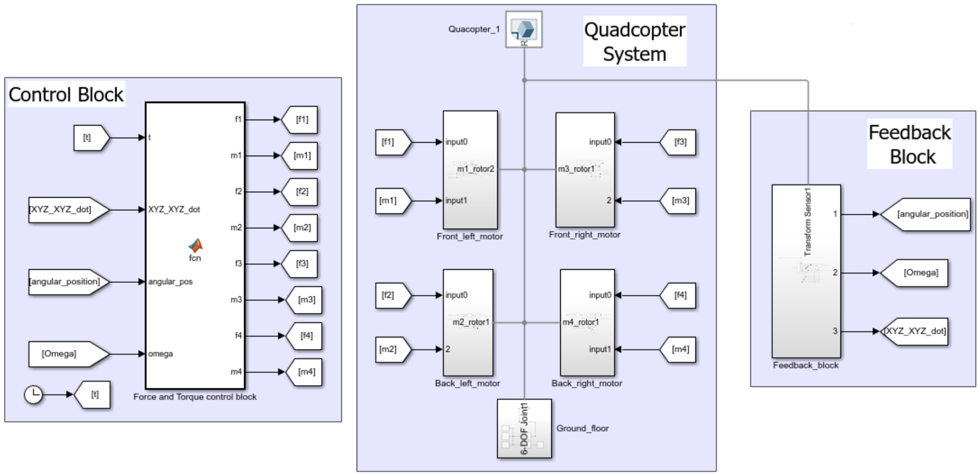

Fig. 3. Screenshot of the SIMSCAPE model. The control block calculate the required thrust and moment to the quadcopter such that it tracks the desired position. Quadcopter block consist of rigid dynamical model of the quadcopter and propeller. Feedback block take the real time feedback of the quadcopter's state and sent it to the control block.

tracking of the single quadcopter, and (ii) Set-point tracking of cable-suspended payload with two quadcopters. The physical parameters used for the simulations are listed in the Table. 1

parameter	value
m	$1.2\ kg$
J	$diag(0.0164, 0.017, 0.032)\ kg-m^2$
L	$0.164\ m$
K_F	$0.01\ Nrad^{-2}$
K_M	$0.005\ Nmrad^{-2}$

Table 1. Physical parameters of the system used while doing the simulations

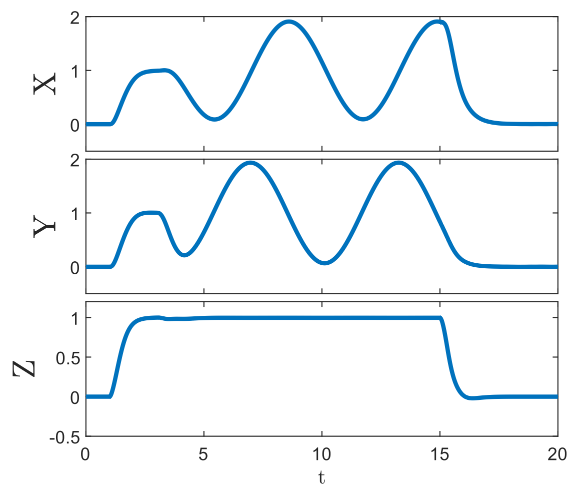

Fig. 4. Simulation results of the single quadcopter: Translational positions

4.1 Set-point tracking of the single quadcopter

In this case, we have kept the quadcopter initially at origin with zero initial conditions. After 1 second, the quadcopter is commanded to go from initial position, i.e., $(x_i, y_i, z_i) = (0,0,0)m$ to first position $(x_1, y_1, z_1) = (1,1,1)m$ for up to 3 seconds. After 4^{th} second, the quadcopter is commanded such that it follow circular trajectory of $1\,m$ keeping $(1,1,1)m$ point as the origin. After 15^{th} seconds, it is commanded to return back to the origin $(0,0,0)$. The screenshot of the SIMULINK model is shown in the Fig. 3. Corresponding simulations results are shown in the Figs. (4, 5).

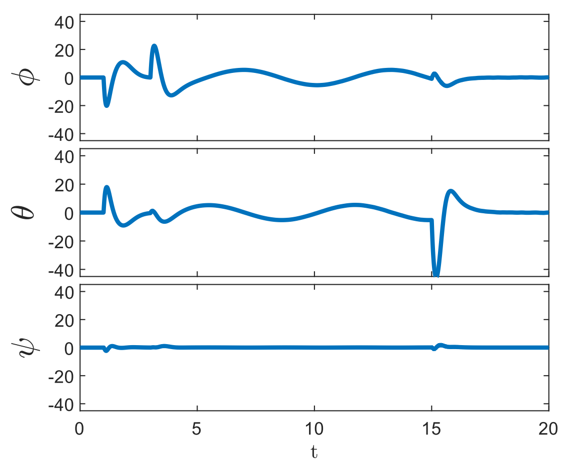

Fig. 5. Simulation results of the single quadcopter: Rotational positions

As shown in the Fig. 4, initially the quadcopter was at origin. After 1 second, quadcopter reached to set-point $(1,1,1)m$ and then it followed the sinusoidal trajectory. The simulation video is available here https://youtu.be/RneF8wiCj-8.

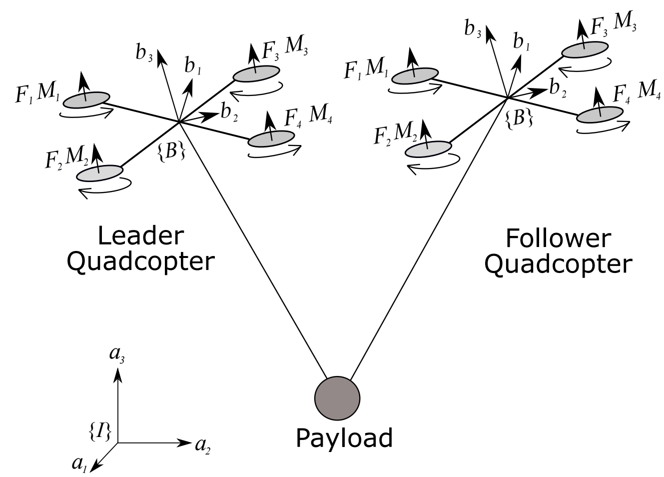

Fig. 6. Line diagram of the two quadcopter with cable-suspended payload system

4.2 Set-point tracking of cable-suspended payload with two quadcopters

As shown in the Fig. 6, the line diagram of the two quadcopters with cable-suspended payload system is shown. As done in the previous section, one simple approach to control the motion of the payload is by implementing set-point tracking for both the quadcopters. However, the oscillations of the payload might be destabilize the overall system flight. Thus, we would have to implement certain controller which controls the payload oscillations. We will introduce this problem in the upcoming future.

5 Conclusion

In this report, we have simulated single quadcopter in SIMSCAPE. Further, steps would be to simulate two quadcopter with cable-suspended payload system. In that we will develop required controller that would minimize the payload oscillations for stable transportation.

References

[1] Valavanis, K. P., and Vachtsevanos, G. J., 2015. *Handbook of unmanned aerial vehicles*, Vol. 1. Springer.

[2] Prajapati, P., Parekh, S., and Vashista, V., 2020. "On the human control of a multiple quadcopters with a cable-suspended payload system". In 2020 IEEE International Conference on Robotics and Automation (ICRA), IEEE, pp. 2253–2258.

[3] Loianno, G., and Kumar, V., 2017. "Cooperative transportation using small quadrotors using monocular vision and inertial sensing". *IEEE Robotics and Automation Letters,* **3**(2), pp. 680–687.

[4] Goodarzi, F. A., and Lee, T., 2016. "Stabilization of a rigid body payload with multiple cooperative quadrotors". *Journal of Dynamic Systems, Measurement, and Control,* **138**(12).

[5] Mellinger, D., Lindsey, Q., Shomin, M., and Kumar, V., 2011. "Design, modeling, estimation and control for aerial grasping and manipulation". In 2011 IEEE/RSJ International Conference on Intelligent Robots and Systems, IEEE, pp. 2668–2673.

[6] Kim, S., Choi, S., and Kim, H. J., 2013. "Aerial manipulation using a quadrotor with a two dof robotic arm". In 2013 IEEE/RSJ International Conference on Intelligent Robots and Systems, IEEE, pp. 4990–4995.

[7] Prajapati, P., Parekh, S., and Vashista, V., 2019. "Collaborative transportation of cable-suspended payload using two quadcopters with human in the loop". In 2019 28th IEEE International Conference on Robot and Human Interactive Communication (RO-MAN), IEEE, pp. 1–6.