MODULE - 2

I . AC Fundamentals:

Syllabus : Generation of sinusoidal voltage, frequency of generated voltage, definition and numerical values of average value, root mean square value, form factor and peak factor of sinusoidally varying voltage and current, phasor representation of alternating quantities.

Introduction:

- An Alternating Current is one in which the magnitude and direction of an electrical quantity changes with respect to time.

Terminologies:

1. Amplitude
   - It is the maximum value attained by an alternating quantity. Also called as maximum or peak value.

2. Time Period (T)
   - It is the Time Taken in seconds to complete one cycle of an alternating quantity.

3. Instantaneous Value
   - It is the value of the quantity at any instant.

4. Frequency (f)
   - It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec.
   - The relationship between frequency and time period can be derived as follows.
   - Time taken to complete f cycles = 1 second
   - Time taken to complete 1 cycle = 1/f second

$$T = 1/f$$

5. Angular Frequency (ω)
   - Angular frequency is defined as the number of radians covered in one second(ie the angle covered by the rotating coil).
   - The unit of angular frequency is rad/sec.

Average Value

- The arithmetic average of all the values of an alternating quantity over one cycle is called its average value

$$\text{Average value} = \frac{\text{Area under one cycle}}{\text{Base}} = 2\, I_m / \pi$$

RMS or Effective Value

- The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

  $I_{rms} = I_m / \sqrt{2}$

Form Factor

The ratio of RMS value to the average value of an alternating quantity is known as Form Factor

$$FF = \frac{RMS\ Value}{Average\ Value}$$

Peak Factor or Crest Factor

The ratio of maximum value to the RMS value of an alternating quantity is known as the peak factor

$$PF = \frac{MaximumValue}{RMSValue}$$

Phasor Representation

- An alternating quantity can be represented using

(i) Waveform

(ii) Equations

(iii) Phasor

- A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor.**
- A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity.

Phase

- Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.

Phase Difference

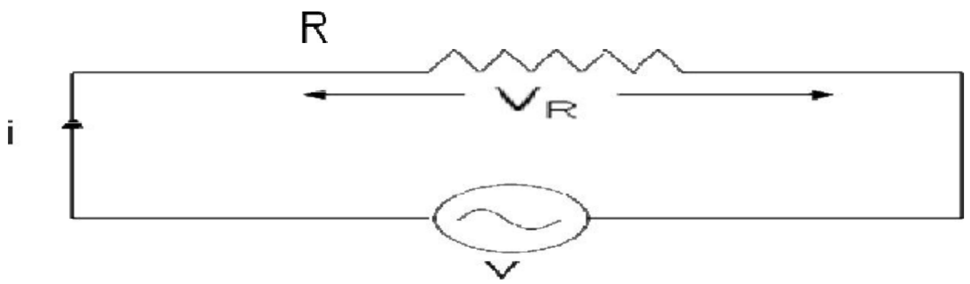
- When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

In Phase

- Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same.

- The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.

AC circuit with a pure resistance



Consider an AC circuit with a pure resistance R as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \quad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the resistor is given as V_R which is the same as v.

Using ohms law, we can write the following relations

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

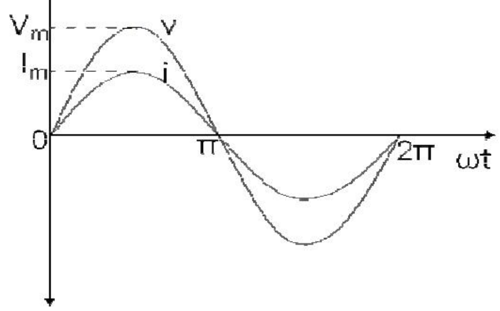
$$i = I_m \sin \omega t \qquad (2)$$

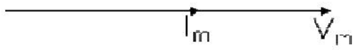
Where $I_m = \dfrac{V_m}{R}$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase. Hence the voltage and current waveforms and phasor can be drawn as below.

Instantaneous power

The instantaneous power in the above circuit can be derived as follows





$$p = vi$$

$$\mathrm{p} = (V_m \sin \omega t)(I_m \sin \omega t)$$

$$\mathrm{p} = V_m I_m \sin^2 \omega t$$

$$\mathrm{p} = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average power

From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right) d\omega t$$

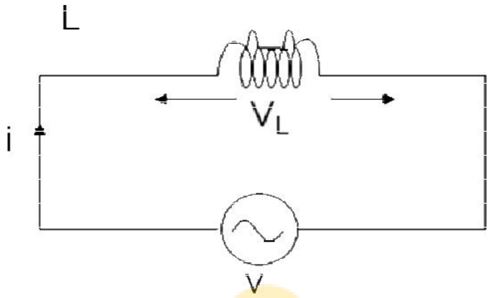
$$P = \frac{V_m I_m}{2} - \frac{1}{2\pi}\int_0^{2\pi}\frac{V_m I_m}{2}\cos 2\omega t \, d\omega t$$

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$$

$$P = V.I$$

As seen above the average power is the product of the rms voltage and the rms current.

AC circuit with a pure inductance



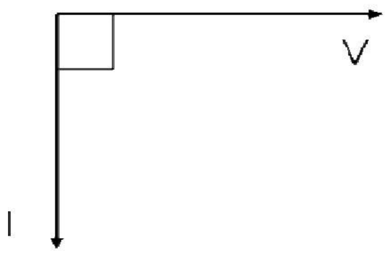
Consider an AC circuit with a pure inductance L as shown in the figure. The alternating voltage v is given by

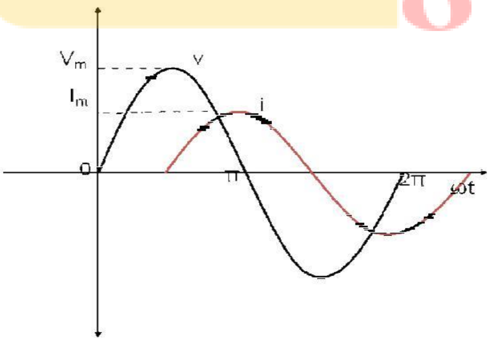
$$v = V_m \sin \omega t \quad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the inductor is given as V_L which is the same as v.

We can find the current through the inductor as follows

From equation (1) and (2) we observe that in a pure inductive circuit, the current lags behind the voltage by 90°. Hence the voltage and current waveforms and phasors can be drawn as below.





Inductive reactance

The inductive reactance X_L is given as

$$X_L = \omega L = 2\pi f L$$

$$I_m = \frac{V_m}{X_L}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms ( )

Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$P = vi$$

$$= (V_m \sin \omega t)\,(I_m \sin(\omega t - \pi/2))$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

Average power

From the instantaneous power we can find the average power over one cycle as follows

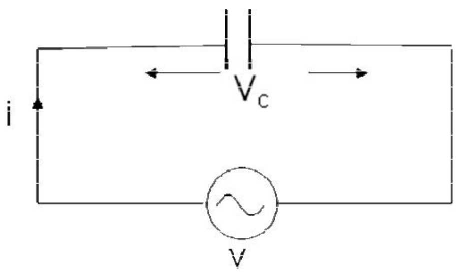
$$P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d\omega t$$

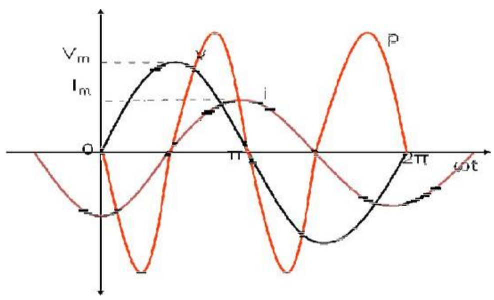
$$P = 0$$

The average power in a pure inductive circuit is zero. Or in other words, the power consumed by a pure inductance is zero.

The voltage, current and power waveforms of a purely inductive circuit is as shown in the figure.

AC circuit with a pure capacitance





$$q = Cv$$

$$q = CV_m \sin \omega t$$

$$i = \frac{dq}{dt}$$

$$\mathrm{i} = CV_m \omega \cos \omega t$$

$$\mathrm{i} = \omega CV_m \sin(\omega t + \pi / 2)$$

$$i = I_m \sin(\omega t + \pi / 2) \quad \text{------------------(2)}$$

Where $I_m = \omega CV_m$

Numericals:

1. **The equation for an AC voltage is given by V = 0.04Sin (2000t +60)̊. Determine the frequency, angular frequency, instantaneous voltage when t =160µs.**

V = 0.04Sin (2000t +60°) Comparing with general equation

$V = V_m \text{ Sin } (wt + ø)$

w = 2∏f ;

f = 2000/2∏ = 318.30Hz.

T = 1/f = 3.14ms

For V = 0.04Sin (2000t +60°) put t = 160µs

$V = 0.04\text{Sin } (2000(160*10^{-6}) +60°) = 0.034V$

2. Calculate the rms value and average value of an alternating quantity represented by

I = 141.42 Sin (314t-30°)

$I = I_m \text{ Sin } (\theta+\varphi)$

I_m = 141.42 A

$I_{rms} = I_m /\sqrt{2} = 141.42 /\sqrt{2} = 99.99A$

$I_{av} = 2 I_m / ∏ = (2*141.42) / ∏ = 90.03A$

3. A pure Inductor of Inductance 20mH is connected across an ac supply of 230V, 50Hz. Find the current drawn by an Inductor.

Solution: $L = 20 \times 10^{-3}$ H, V=230V and f=50Hz I=?

w.k.t $X_L = 2\pi f L = 2*\Pi*50*20 \times 10^{-3}$

$X_L = 6.284\ \Omega$

Therefore $I = \frac{V}{X_L} = \frac{230}{6.284} = 36.60$ **A**

4. The current drawn by pure capacitor 20μF is 1.382 A. connected across an ac supply of 220V. Find the supply frequency.

Solution: $C = 20 \times 10^{-6}$ F, V=220V and I= 1.382A f=?

w.k.t $X_c = \frac{V}{I} = \frac{220}{1.382}$

$X_c = 159.18\Omega$

Therefore $X_c = \frac{1}{2\pi f C}$

$f = \frac{1}{2\pi C X_c} = \frac{1}{2*\pi*159.18*20 \times 10^{-6}}$ **=49.99 =50Hz**

5. A 1mH inductor is connected across 200V 50Hz AC supply. Determine inductive reactance, current taken by the inductor.

Sol :

Given: L= 1mH, v = 200∟0V, f = 50Hz.

To find: X_L =? I =?

Sol:

$X_L = 2\prod fL = 2 \times \prod \times 50 \times 1 \times 10^{-3} = 0.31\ \Omega$,

$I = V / X_L = 200∟0 / 0.31 = 645.16∟-90$ A

6. A capacitance of 100μF is connected across a 300V, 50Hz supply. Calculate the capacitive reactance, current.

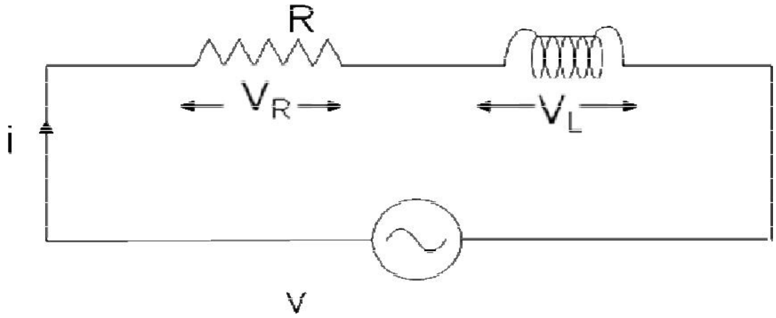
Sol: Given: C = 100μF, V = 300∟0V, f = 50Hz

To find: X_C=? I =?

$X_C = 1 / 2\prod fC = 1/ (2 * \prod *50*100*10^{-6}) = 31.83\ \Omega$

$I = V / X_C = 300 \llcorner 0 / 31.83 = 9.42 \llcorner 90$ A

R-L Series circuit



Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the inductor is V_L.

$V_R = IR$ is in phase with I

$V_L = IX_L$ leads current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.

The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_L leads the current by 90°. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ.

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V_R = IR$$

$$V_L = IX_L$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2}$$

$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + X_L^2}$

The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms

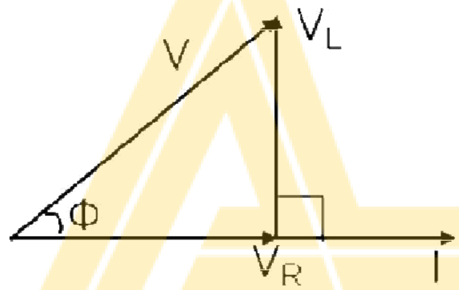
Instantaneous power

The instantaneous power in an RL series circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t - \Phi))$$

$$p = \frac{V_m I_m}{2}\cos\Phi - \frac{V_m I_m}{2}\cos(2\omega t - \Phi)$$



The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average power

From the instantaneous power we can find the average power over one cycle as follows

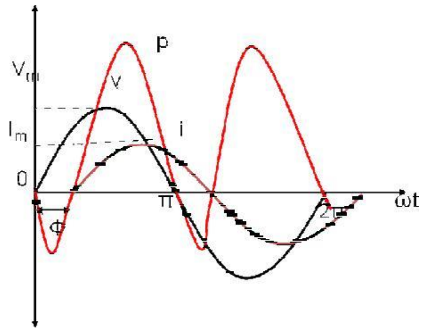
$$P = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{V_m I_m}{2}\cos\Phi - \frac{V_m I_m}{2}\cos(2\omega t - \Phi)\right)d\omega t$$

$$P = \frac{V_m I_m}{2}\cos\Phi$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\Phi$$

$$P = VI\cos\Phi$$

The voltage, current and power waveforms of a RL series circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the load and when the power in negative, the power flows from the load to the source. The positive power is not equal to the negative power and hence the average power in the circuit is not equal to zero.

From the phasor diagram,

$$P = VI\cos\Phi$$

$$P = (IZ)\cdot I\cdot\frac{R}{Z}$$

$$P = I^2R$$

Hence the power in an RL series circuit is consumed only in the resistance.
The inductance does not consume any power.

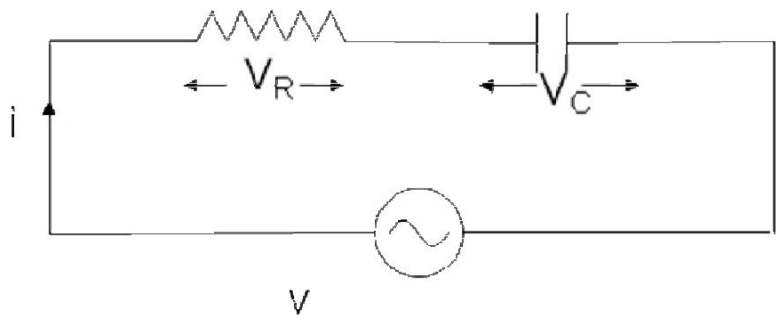
Power Factor

The power factor in an AC circuit is defined as the cosine of the angle between voltage and current i e cosΦ

$$P = VI\cos\Phi$$

The power in an AC circuit is equal to the product of voltage, current and power factor

R-C Series circuit



Consider an AC circuit with a resistance R and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

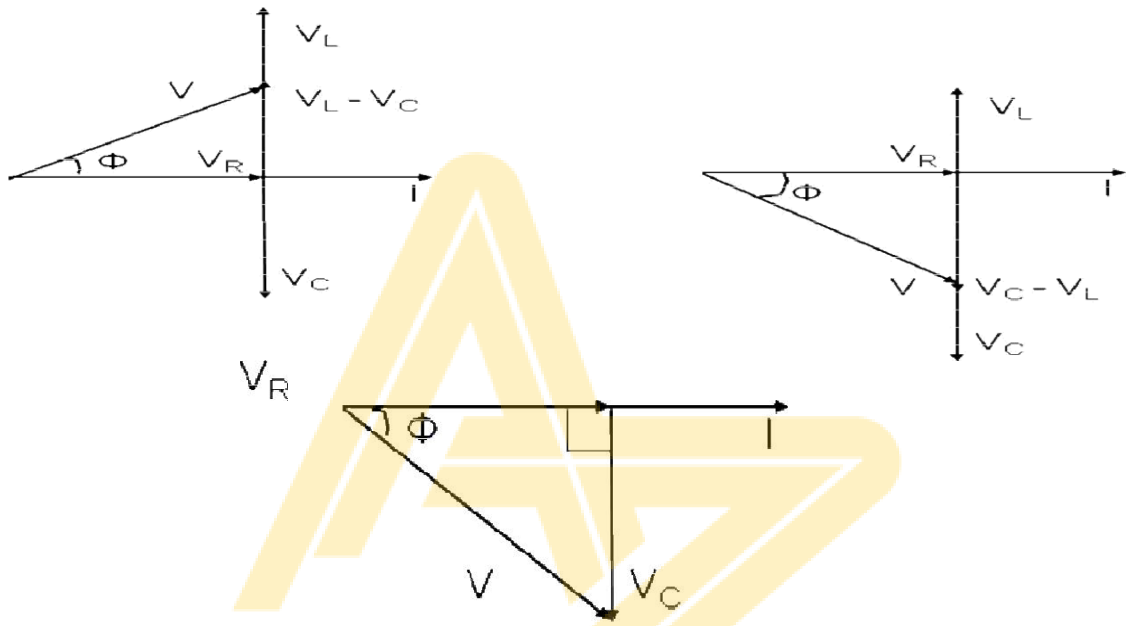
$$v = V_m \sin \omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the capacitor is V_C.

V_R=IR is in phase with I

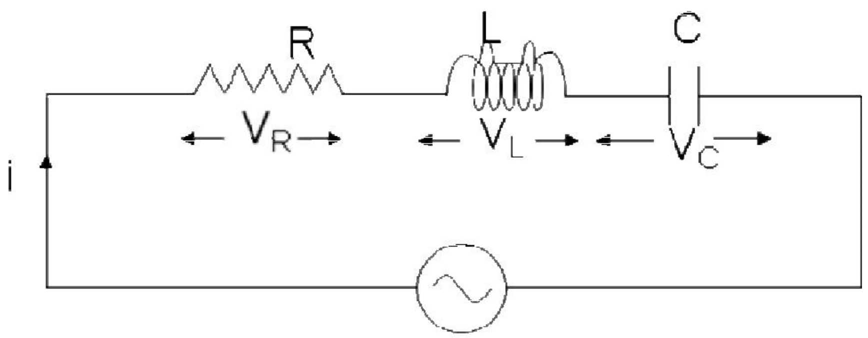
V_C=IX_C lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_C lags behind the current by 90^o. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

R-L-C Series circuit



Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R, the voltage across the inductor is V_L and that across the capacitor is V_C.

$V_R = IR$ is in phase with I

$V_L = IX_L$ leads the current by 90 degrees

$V_C = IX_C$ lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown. The current I is taken as the reference phasor. The voltage V_R is in phase with I, the voltage V_L leads the current by 90^o and the voltage V_C lags behind the current by 90^o. There are two cases that can occur $V_L > V_C$ and $V_L < V_C$ depending on the values of X_L and X_C. And hence there are two possible phasor diagrams. The phasor $V_L - V_C$ or $V_C - V_L$ is drawn and then the resultant voltage V is drawn.

Three Phase AC circuits

Syllabus: Advantages of 3-phase power, Generation of 3-phase power, Three-phase balanced circuits, voltage and current relations in star and delta connections. Measurements of three phase power using two wattmeter methods.

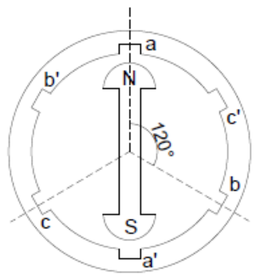
Advantages of three-phase systems:

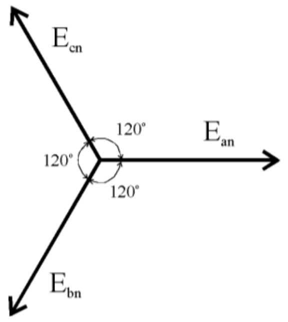
1. Three phase transmission lines require much less conductor material. The return conductor is replaced by single neutral conductor of small size.
2. Three phase machine gives higher output than a single phase machine.
3. Three phase motor develops uniform torque whereas single phase motor develops pulsating torque.
4. Three phase can generate rotating magnetic field & hence three phase induction motors are self starting.
5. Three phase system can be used to supply domestic & industrial power.
6. Voltage regulation is better in three phase system compared to single phase supply.
7. Three phase system is more efficient & less expensive compared to single phase system.

Generation of three phase power :

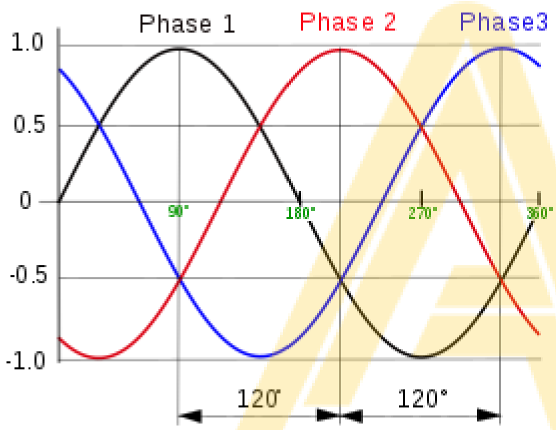
Three phase power is generated using alternator. Alternator contains stator (stationary part) and rotor (rotating part). The stator is cylindrical in shape and has slots in its inner periphery as shown in the figure below. The conductors are placed in the slots. They are connected either in star or delta. Rotor is a magnet with two poles 'N & S'.

Generation of three phase emf:





Stator conductors aa' , bb' and cc' are mutually displaced by 120^0. As the rotor rotates , the stator conductor cuts the flux and hence emf is induced in all 3 conductors.



$e_A = E_m \sin wt$

$e_B = E_m \sin(wt-120)$

$e_C = E_m \sin(wt-240)$

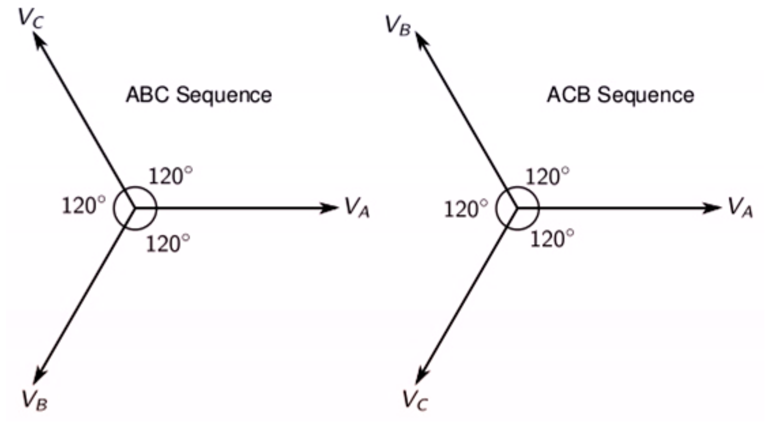
Phase sequence :

Phase sequence is the order in which the 3 phase voltages reach their maximum. It is either 'abc' or 'acb' .

'abc' sequence -- waveform 'a' reaches the peak first , followed by 'b' and 'c' .

'acb' sequence -- waveform 'a' reaches the peak first , followed by 'c' and 'b' .

In the figure , phase1 reaches the peak first , followed by 'phase 2' and 'phase 3' .



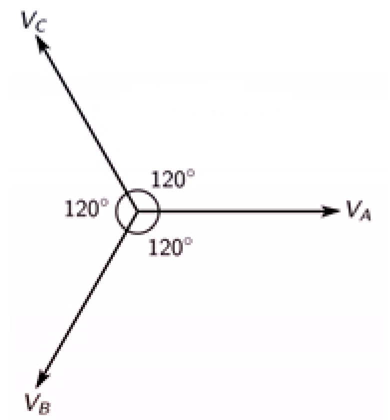
<u>Importance of phase sequence</u> **:**

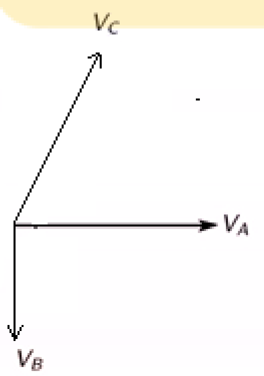
- 3 phase supply of a particular sequence is given to <u>3 phase load (static load)</u>. If the **phase sequence** is changed then the direction of current flow will also change.
- If the 3 phase supply is given to 3 phase induction motor , and if phase sequence is changed then the **direction of current flow will reverse and also the direction of rotation changes**.

<u>Balanced supply and balanced load</u> **:**

Balanced supply :

If the magnitude of 3 phases are same and are displaced by 120^0 it is said to be balanced supply.





$V_A = V_B = V_C$

All 3 vectors are displaced by 120^0

$V_A \neq V_B \neq V_C$

All 3 vectors are displaced by different angles

Balanced load: If the impedances in all the three phases are equal in magnitude, then the load is said to be balanced.

Relation between line & phase values of balanced star connections:

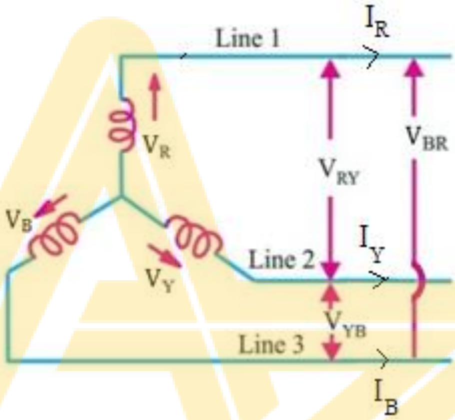
Let I_R, I_Y, I_B → Line Currents

V_{RY}, V_{YB}, V_{BR} → Line Voltages

V_R, V_Y, V_B → Phase Voltages

Phase voltage is the voltage between line & neutral

Line voltage is the voltage between any two lines

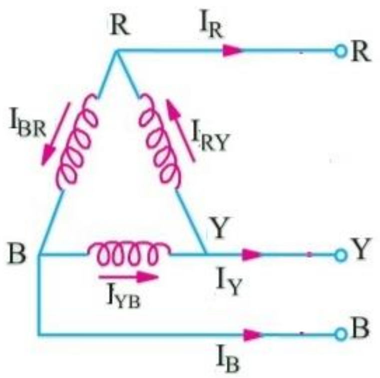


$V_L = \sqrt{3} V_p$ $I_L = I_{PH}$

Relation between line & phase values of balanced delta connections:

Let I_R, I_Y, I_B → Line currents

I_{RY}, I_{YB}, I_{BR} → Phase Currents



$$I_L = \sqrt{3}\, I_{ph} \quad V_L = V_{ph}$$

Numericals on Single & Three Phase AC Circuits

1. A circuit having a resistance of 12Ω inductance of 0.15H in series is connected across a 100V, 50Hz supply. Calculate the inductive reactance, impedance, current.

Given: R = 12Ω, L = 0.15H, v = 100∟0V, f = 50Hz,

To calculate: X_L = ?,Z = ?, I =?,

$X_L = 2\pi fL = 2 \times \pi \times 50 \times 0.15 = 47.1\ \Omega$

$Z = R + j X_L = 12 + j\,47.1 = 48.6\angle 75.70\ \Omega$

V = I Z

$I = V / Z = 100\angle 0 / 48.6\angle 75.70\ \Omega$

$= 2.05\angle -75.70A$

2. A circuit having a resistance of 12Ω inductance of 0.15H and a capacitance of 100μF in series is connected across a 100V, 50Hz supply. Calculate the impedance, current, power factor.

$X_L = 2\pi FL = 2\pi \times 50 \times 0.15$

$= 100 \times \pi \times 0.15$

$\therefore X_L = 47.12\Omega$

$X_C = \frac{1}{2\pi fL} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$

$\therefore X_C = 31.83\Omega$

$Z = R + jX_L - jX_C = 12 + j47.12 - j31.83$

$= 12 + j15.29\Omega$

$V = 100\text{volts} = 100\angle 0^o$

$Z = 12 + j15.29 = 19.43\angle 51.87^o\Omega$

$I = \frac{V}{Z} = \frac{100\angle 0^o}{19.43\angle 51.87^o} = 5.146\angle 51.87^o\Omega A$

$\therefore \quad I = 5.146A$

$p.f. = \cos\phi = \cos 51.87^o = 0.6174\ \text{lag}, \quad \phi = 51.87^o$

3. A series circuit with resistance of 10ohms, inductance of 0.2 H and capacitance 40micro F is supplied with a 100 V supply at 50 Hz. Find current, power and power factor

Circuit resistance $R = \mathbf{10\Omega}$

Inductive reactance of the circuit, $X_L = 2\pi fL$

$= 2\pi \times 50 \times 0.2 = \mathbf{62.83\Omega}$

Capacitive reactance of the circuit, $X_c = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 40 \times 10{-6}} = \mathbf{79.57\ \Omega}$

$Z = R - j\,(Xc - X_L) = 10 - j\,16.74 = 19.49\angle -59.14$

- $I = V / Z = 100\angle 0 / 19.49\angle -59.14 = 5.13\angle 59.14A$
- Power factor = Cos (59.14) = 0.512
- $P = VI\cos\Phi = 100*5.13*0.512 = 262.65W$

4. A resistance of 100 ohms is connected in series with an inductance of 1mH in each phase of a 400V star connected 50Hz three phase supply. Calculate inductive reactance (ii) impedance per phase (iii) Total current (iv) power factor (v) active power (vi) reactive power (vii) apparent power.

Given: R = 100, L = 1mH, VL= 400V, f= 50Hz

To Find: XL =? Zph =? Vph = ? , IL = Iph =? , Cos Φ =? , P = ?, Q=?,S =?

$XL = 2*\pi*f*L = 2*\Pi*50*1*10^{-3} = 0.314$

$Zph = R + j\,X_L = 100 + j0.314 = 100\angle 0.179$

$V\,ph = V_L / \sqrt{3} = 400 / \sqrt{3} = 230.94\ V$

$I_L = I\,ph = V\,ph / Z\,ph = 230.94\angle 0 / 100\angle 0.179 = 2.3\angle -0.179\ A$

$\cos\Phi = \cos(-0.179) = 0.99,$

$P = \sqrt{3}\,V_L\,I_L\cos\Phi = \sqrt{3} * 400 * 2.3 * 0.99 = 1577.55W$

$Q = \sqrt{3}\,V_L\,I_L\sin\Phi = \sqrt{3} * 400 * 2.3 * \sin(-0.179) = -4.97VAR,$

$S = \sqrt{3}\,V_L\,I_L = \sqrt{3} * 400 * 2.3 = 1593.48VA.$

5. A resistance of 50 ohms is connected in series with a capacitance of 150micro farad in each phase of a 450V delta connected 50Hz three phase supply. Calculate capacitive reactance (ii) impedance per phase (iii) line current and phase current (iv) power factor (v) active power (vi) reactive power (vii) apparent power.

$XC = 1 / 2\prod fC = 1/(2*\prod*50*150*10^{-6}) = 21.22\ \Omega$

$Z\,ph = R - j\,Xc = 50 - j21.22 = 54.31\angle -22.99$

$I_{ph} = V_{ph} / Z_{ph} = 450\angle 0 / 54.31\angle -22.99 = 8.28 \angle 22.99$ A

$I_L = \sqrt{3} I_{ph} = \sqrt{3} * 8.28 = 14.34 \angle 22.99$ A.

$\cos \Phi = \cos (22.99) = 0.92$

$P = \sqrt{3}\, V_L I_L \cos \Phi = \sqrt{3} * 450 * 14.34 * 0.92 = 10282.76$W,

$Q = \sqrt{3}\, V_L I_L \sin \Phi = \sqrt{3} * 450 * 14.34 * 0.39 = 4359$VAR

$S = \sqrt{3}\, V_L I_L = \sqrt{3} * 450 * 14.34 = 11176.92$VA