MODULE - II : - Vector Calculus. M2 SIMP 2023 - TIE

1. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^3+y^3-3$ at point $P(2,-1,2)$

2. Find the directional derivative of $4xz^3-3x^2y^2z$ at $(2,-1,2)$ along $2\hat{i}-3\hat{j}+6\hat{k}$.

3. Find $\nabla\phi$ or gradϕ in the following : $3x^2y-y^3z^2$ at point $(1,-2,-1)$

4. (a) Find div $\vec{f}$ and curl $\vec{f}$ if $\vec{f}$ = grad(xy^3z^2)
   (b) Find div $\vec{f}$ and curl $\vec{f}$ if $\vec{f}$ = grad $(x^3+y^3+z^3-3xyz)$
   (c) If $\vec{f}=(x+y+1)\hat{i}+\hat{j}-(x+y)\hat{k}$, prove that $\vec{f}\cdot\text{curl}\vec{f}=0$.

5. (a) Show that $\vec{f}=\dfrac{x\hat{i}+y\hat{j}}{x^2+y^2}$ is both solenoidal & irrotational.
   (b) If $\vec{f}=x^2\hat{i}+y^2\hat{j}+z^2\vec{k}$ and $\vec{g}=yz\hat{i}+zx\hat{j}+xy\vec{k}$ then verify whether $\vec{f}\times\vec{g}$ is solenoidal.

6. (a) Show that $\vec{f}=(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$ is irrotational. Also find the scalar function ϕ such that $\vec{f}=\nabla\phi$.
   (b) Find the constants a, b, c if the vector $\vec{f}=(x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(x+cy+2z)\hat{k}$ is irrotational. Hence find scalar potential ϕ such that $\vec{f}$ = gradϕ.
   (c) Find the constant a such that the vector field $\vec{f}=(axy-z^3)\hat{i}+(a-2)\hat{j}+(1-a)xz^2\hat{k}$ is irrotational. Hence find the scalar potential ϕ such that $\vec{f}$ = grad.ϕ

7 (a) Prove that cylindrical and spherical coordinate systems are orthogonal.

(b) Represent $\vec{F} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates. Also determine F_ρ, F_θ, F_z.

(c) Express the following vectors in:

(i) cylindrical coordinates → $\vec{F} = 3y\hat{i} + x^2\hat{j} - z^2\hat{k}$

(ii) Spherical coordinates → $\vec{F} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$.

8 If $\vec{F} = (3x^2+6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate $\int_C \vec{F}\cdot d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the curve C given by $x=t,\ y=t^2,\ z=t^3$