

## Module - 5

### Numerical methods

Divided differences :- Suppose  $y = f(x)$  be a function in the variable  $x$ . Let the set of values of  $y$  at value of  $x$  are  $y_0, y_1, y_2, \dots, y_n$  corresponding to the value of arguments  $x_0, x_1, x_2, x_3, \dots, x_n$  the divided difference are classified into 2 types in the equal length of Interval

- 1] forward divided difference
- 2] backward divided difference

Forward divided differences :- the symbol  $\Delta$  (delta) is called the forward divided differences operator and  $\Delta^1, \Delta^2, \Delta^3, \dots$  are called the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> order divided forward differences

$x$	$y = f(x)$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$
$x_0$	$y_0$	$y_1 - y_0 = \Delta y_0$	$\Delta y_1 - \Delta y_0 = \Delta^2 y_0$	$\Delta^2 y_1 - \Delta^2 y_0 = \Delta^3 y_0$
$x_1$	$y_1$	$y_2 - y_1 = \Delta y_1$	$\Delta y_2 - \Delta y_1 = \Delta^2 y_1$	$\Delta^2 y_2 - \Delta^2 y_1 = \Delta^3 y_1$
$x_2$	$y_2$	$y_3 - y_2 = \Delta y_2$	$\Delta y_3 - \Delta y_2 = \Delta^2 y_2$	$\Delta^2 y_3 - \Delta^2 y_2 = \Delta^3 y_2$
$x_3$	$y_3$	$y_4 - y_3 = \Delta y_3$		
$x_4$	$y_4$			
			$\Delta^4 y_1 - \Delta^3 y_0 = \Delta^4 y_0$	

Here  $\Delta y_0, \Delta^2 y_0, \Delta^3 y_0, \Delta^4 y_0$  is called leading forward divided differences

Backward differences :- the symbol dell ( $\nabla$ ) is called the Backward divided differences Operator.  $\nabla, \nabla^2, \nabla^3, \dots$  are called the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> ... order backward divided differences

$x$	$y$	$\nabla y$	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
$x_0$	$y_0$				
$x_1$	$y_1$	$y_1 - y_0 = \nabla y_1$	$\Delta y_2 - \nabla y_1 = \nabla^2 y_2$	$\nabla^2 y_3 - \nabla^2 y_2 = \nabla^3 y_3$	
$x_2$	$y_2$	$y_2 - y_1 = \nabla y_2$	$\nabla y_3 - \nabla y_2 = \nabla^2 y_3$		$\nabla^3 y_4 - \nabla^3 y_3 = \nabla^4 y_4$
$x_3$	$y_3$	$y_3 - y_2 = \nabla y_3$		$\nabla^2 y_4 - \nabla^2 y_3 = \nabla^3 y_4$	
$x_4$	$y_4$	$y_4 - y_3 = \nabla y_4$	$\nabla y_4 - \nabla y_3 = \nabla^2 y_4$		

Here  $\nabla y_n, \nabla^2 y_n, \nabla^3 y_n, \dots$  are called the leading Backward divided differences

### Interpolation and Extrapolation

The Evaluation of  $y$  in the given range  $x_0$  to  $x_n$  is called in interpolation and outside of  $x_0$  to  $x_n$  is called Extrapolation

### Interpolation formula

$$i) y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \Delta^4 y_0 + \dots$$

when  $p = \frac{x-x_0}{h}$  is called the Newton's forward Interpolation formula

$$ii) y(x) = y_n + p y_{n-1} + \frac{p(p+1)}{2!} \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!} \nabla^4 y_n$$

--- where  $p = \frac{x-x_n}{h}$  is called the Newton's Backward Interpolation formula

i) The population of a town is given by the table

Year	1951	1961	1971	1981	1991
Population	19.6	39.65	58.81	72.21	94.61

Using Newton's forward and backward interpolation formula calculate the percentage in population from the year 1955 to 1985

Year	Population (y)	I DD	II DD	III DD	IV DD
$x_0 = 1951$	$y_0 = 19.6$				
$x_1 = 1961$	$y_1 = 39.65$	20.05		-0.89	
$x_2 = 1971$	$y_2 = 58.81$	19.16		-5.76	19.63
$x_3 = 1981$	$y_3 = 72.21$	13.4	9	-14.76	
$x_4 = 1991$	$y_4 = 94.61$	22.4			

To find  $y(1955)$

Since 1955 is near to  $x_0 = 1951$

$$\Rightarrow p = \frac{x - x_0}{h}$$

$$\Rightarrow p = \frac{1955 - 1951}{10}$$

$$\Rightarrow p = \frac{4}{10}$$

$$\Rightarrow p = 0.4$$

$\therefore$  By the newton's forward interpolation formula

$$\Rightarrow y(x) = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \Delta^4 y_0$$

$$\Rightarrow y(1955) = 19.6 + (0.4)(0.05) + \frac{(0.4)(-0.6)(-0.84)}{2!} + \frac{(0.4)(0.6)(-1.6)(-4.8)}{6}$$

$$+ \frac{(0.4)(-0.6)(-1.6)(-2.6)(19.63)}{24}$$

$$\Rightarrow y(1955) = 19.6 + 8.008 + 0.1008 - 0.31168 - 0.18166$$

$$\Rightarrow y(1955) = 26.60$$

To find  $y(1985)$

Since 1985 is near to  $x_0 = 1951$

$$p = \frac{x - x_0}{h} = \frac{1985 - 1991}{10} = -\frac{6}{10} = -0.6$$

$\therefore$  The newton's backward interpolation formula

$$\Rightarrow y(x) = y_4 + p \nabla y_4 + \frac{p(p+1)}{2!} \nabla^2 y_4 + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_4 +$$

$$\frac{p(p+1)(p+2)(p+3)}{4!} \nabla^4 y_4$$

$$\Rightarrow y(1985) = 94.61 + (-0.6)(2.4) + \frac{(-0.6)(0.4)(9)}{2!} +$$

$$\frac{(-0.6)(0.4)(1.4)(14.76)}{6} + \frac{(-1.6)(0.4)(1.4)(2.4)(19.6)}{24}$$

$$\Rightarrow y(1985) = 94.61 - 13.44 - 1.88 - 0.826 - 0.659$$

$$\Rightarrow y(1985) = 78.605$$

∴  $y$  from population between 1955 to 1985

$$\Rightarrow 78.605 - 26.60$$

$$\Rightarrow 52.005 //$$

Q] Use an appropriate interpolation formula to compute  $y(42)$   
Using the following data

x	40	50	60	70	80	90
y	184	204	226	250	276	304

x	y	I DD	II DD	III DD	IV DD	V
40	184					
50	204	20				
60	226	22				
70	250	24				
80	276	26				
90	304	28				

To find  $y(42)$

$y(42)$  is near to  $x_0 = 40$

$$P = \frac{x - x_0}{h} = \frac{42 - 40}{10} = \frac{2}{10} = 0.2$$

By newton forward interpolation formula

$$\Rightarrow y(x) = y_0 + P \Delta y_0 + \frac{P(P-1)}{2!} \Delta^2 y_0 \dots$$

$$\Rightarrow y(42) = 184 + (0.2)(20) + \frac{(0.2)(0.8)(2)}{(2)}$$

$$\Rightarrow y(42) = 184 + 4 - 0.16$$

$$\Rightarrow \underline{\underline{y(42) = 187.84}}$$

- 3) Use an appropriate interpolation formula to Compute  $y(8_2)$  and  $y(9_2)$  for the data

$x$	80	85	90	95	100
$y$	5026	5674	6362	7088	7854

$x$	$y$	I DD	II DD	III DD	IV DD
80	5026				
85	5674	648			
90	6362	688	40	-2	
95	7088	-166	38	2	4
100	7854	-66	40		

i) To find  $y(8_2)$

Since  $8_2$  is near to  $x_0 = 80$

$$p = \frac{x - x_0}{h} = \frac{8_2 - 80}{5} = \frac{2}{5} = 0.4$$

By the Newton's forward interpolation formula

$$\Rightarrow y(x) = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 + \dots$$

$$\Rightarrow y(8_2) = 5026 + (0.4)(648) + \frac{(0.4)(-0.6)(40)}{2!} +$$

$$\frac{(0.4)(-0.6)(-1.6)(-2)}{6} + \frac{4 \times 0.4(-0.6)(-1.6)(-2)(1.6)}{24}$$

$$\Rightarrow y(8_2) = 5026 + 359.2 - 4.8 - 0.228 - 0.1664$$

$$\Rightarrow y(8_2) = 5280.0056$$

ii) To find  $y(9_2)$

Since  $9_2$  is near to  $x_4 = 90$

$$P = \frac{x - x_0}{h} = \frac{92 - 100}{5} = \frac{-8}{5} = -1.6$$

By the newton's Backward Interpolation formula

$$\Rightarrow y(x) = y_4 + P \Delta y_4 + \frac{P(P+1)}{2!} \Delta^2 y_4 + \frac{P(P+1)(P+2)}{3!} \Delta^3 y_4 + \dots$$

$$\Rightarrow y(92) = 7854 + (-16)(-66) + \frac{(-1.6)(-0.6)(40)}{2} + \frac{(-1.6)(-0.6)(0.4)(2)}{6}$$

$$+ \frac{(-1.6)(-0.6)(0.4)(1.4)(4)}{24}$$

$$\Rightarrow y(92) = 7854 - 1296 + 19.2 + 0.128 + 0.0896$$

$$\Rightarrow y(92) = 6647.8176$$

4] Using find  $f(12.5)$  for the following data (NBIF)

$x$	10	11	12	13
$y$	22	24	28	34

$x$	$y$	I <sub>00</sub>	I <sub>00</sub>	III <sub>00</sub>
10	22	2		
11	24	4	2	
12	28		2	0
13	34	6		

To find  $y(12.5)$

$$P = \frac{x - x_4}{h} = \frac{12.5 - 12}{1} = -0.5$$

$$y(x) = y_4 + P \Delta y_4 + \frac{P(P+1)}{2!} \Delta^2 y_4 + \dots$$

$$y(12.5) = 28 + (-0.5)(6) + \frac{(-0.5)(0.5)}{2} 2 + \dots$$

$$\Rightarrow y(12.5) = 34 - 3 - 0.25$$

$$\Rightarrow y(12.5) = 30.75$$

=

- 5] Use the formula following table and estimate the number of students who obtained marks between 40 and 45

marks	30-40	40-50	50-60	60-70	70-80
No of Students	31	42	51	35	31

The number of student who obtained  $< 40 = 31$ ,  $< 50 = 73$ ,  $< 60 = 124$ ,  $< 70 = 154$ ,  $< 80 = 190$

X	y	I DD	II DD	III DD	IV DD
40	31				
50	73	42			
60	124	51	9		
70	159	35	-16	12	
80	190	51	-4		37

$$x \quad y = f(x)$$

I DD

$$x_0 \quad f(x_0)$$

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = f(x_0, x_1)$$

$$x_1 \quad f(x_1)$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = f(x_1, x_2)$$

$$x_2 \quad f(x_2)$$

$$\frac{f(x_3) - f(x_0, x_3)}{x_3 - x_0} = f(x_0, x_1, x_2)$$

$$x_3 \quad f(x_3)$$

$$\frac{f(x_3) - f(x_2)}{x_3 - x_2} = f(x_2, x_3)$$

$$\frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1} = f(x_1, x_2, x_3)$$

$$\frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0} = f(x_0, x_1, x_2, x_3)$$

$$f(x) = f(x_0) + (x-x_0)f(x_0-x_1) + (x-x_0)(x-x_1)f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0, x_1, x_2, x_3)$$

Using newton's interpolation formula to construct the polynomial by the following data

x	3	4	5	6	8	10
f(x)	10	96	196	350	868	1746

x	f(x)	I DD	II DD	III DD	IV DD	V DD
3	10		43			
4	96			19		
5	196		100		27	
6	350			154		25
8	868			259		35
10	1746			45		45

To find  $y(45)$

Since 45 is near to  $x_0 = 40$

$$p = \frac{x-x_0}{h} = \frac{45-40}{10} = 0.5$$

By the Newton's forward interpolation formula

$$\Rightarrow y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}$$

$$\Delta^4 y_0 + \dots$$

$$\Rightarrow y(45) = 31 + (0.5)(42) + \frac{(0.5)(-0.5)(9)}{2!} + \frac{(0.5)(-0.5)(-1.5)(-2.5)}{6}$$
$$+ \frac{(0.5)(-0.5)(-1.5)(-2.5)(37)}{34}$$

$$\Rightarrow y(45) = 31 + 21 - 1.125 - 1.5625 - 1.4453$$

$$\Rightarrow y(45) = 47.687 \approx 48$$

The number of students who obtained less than 45 marks = 48

The number of students who obtained between 40 and 45 marks =  $48 - 31 = 17$

### Divided difference for unequal intervals

Newton's divided differences formula for unequal intervals :-

Suppose  $y = f(x)$  be a function in  $x$  and  $f(x_0), f(x_1), f(x_2), f(x_3), \dots$  be the values of  $f(x)$  corresponding to the values of  $x_0, x_1, x_2, x_3, \dots$  with unequal intervals

3] find the Interpolating polynomial

$x$	0	1	2	3	4	5
$f(x)$	3	8	7	24	59	118

$x$	$f(x)$	I <sup>DD</sup>	II <sup>DD</sup>	III <sup>DD</sup>	IV <sup>DD</sup>	V <sup>DD</sup>
0	3	-1				
1	8	5	3			
2	7	17	6	1	0	
3	24	35	9		0	
4	59	12				
5	118	59				

$$f(x) = f(x_0) + (x - x_0) f(x_0 - x_1) + (x - x_0)(x - x_1) + f(x_0, x_1, x_2) + \dots$$

$$f(x) = 3 + (-1)(x-0)f_3(x-0)(x-1) + 1(x-0)(x-1)(x-2)$$

$$f(x) = 3 - x + 3x^2 - 3x + x^3 - 3x^2 + 8x$$

$$f(x) = x^3 - 8x + 3$$

$$\text{when } x=6$$

$$f(6) = (6)^3 - (8 \times 6) + 3$$

$$f(6) = 207$$

8 Find the number of men getting wages less than Rs 3.5 from the following data

Wages in Rs	0-10	10-20	20-30	30-40
frequency	9	30	35	42

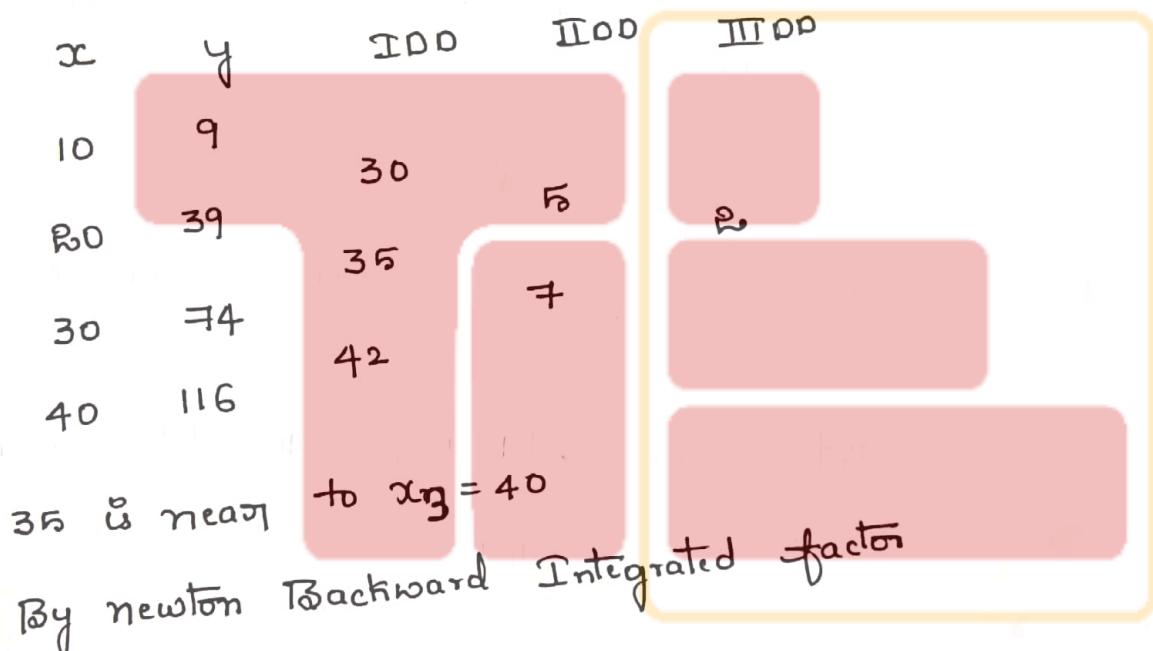
24

The number of men who getting less than 10 = 9

$$< 20 = 30$$

$$< 30 = 35$$

$$< 40 = 42$$



$$P = \frac{x - x_3}{h} = \frac{35 - 40}{10} = -0.5$$

$$y(x) = y_3 + P \nabla y_3 + \frac{P(P+1)}{2!} \nabla^2 y_3 + \frac{P(P+1)(P+2)}{3!} \nabla^3 y_3$$

$$y(35) = 116 + (-0.5)4^2 + \frac{(-0.5)(0.5)7}{2} + \frac{2(-0.5)(0.5)(1.5)}{6}$$

$$y(35) = 116 - 21 - 0.875 - 0.185$$

$$\underline{\underline{y(35) = 94}}$$

9] Determine the Interpolating formula construct the polynomial for the following data hence find

$x$	3	7	9	10
$f(x)$	168	120	72	63

Soln	$x$	$f(x)$	$f(x_0, x_1)$	$f(x_0, x_1, x_2)$	$f(x_0, x_1, x_2, x_3)$
	3	168			
	7	120	-12		
	9	72	-24	-8	
	10	63	-9	5	1

$$\Rightarrow f(x) = 168 - 12(x-3) - 8(x-3)(x-7) + 1(x-3)(x-7)(x-9)$$

$$\Rightarrow f(x) = 168 - 12x + 36 - \frac{8}{2}x^2 + 20x - 48 + x^3 - 16x + 63x$$

$$-3x^2 + 48x - 189$$

$$\Rightarrow f(x) = x^3 - \frac{8}{2}x^2 + 119x - 189$$

$$\Rightarrow f(8) = 8^3 - \frac{8}{2}(8^2) + 119(8) - 189$$

$$\Rightarrow f(8) = 93$$

10] From the table of half year premium for policy measuring different states estimate the premium for policy measuring on the age of (46)

age	45	50	55	60	65
premium	114.84	96.16	83.32	74.48	68.48

age (x)	potassium	I <sub>00</sub>	II <sub>00</sub>	III <sub>00</sub>	IV <sub>00</sub>
45	114.84		-18.68		
50	96.16			5.84	
55	83.32	-18.84			-1.84
60	74.48		-8.84		-1.16
65	68.48	-6		2.64	

To find  $f(46)$  : it is near to  $x_0 = 45$

$$p = \frac{x - x_0}{h} = \frac{46 - 45}{5} = 0.2$$

By newton forward integrated factor

$$y(x) = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \Delta^4 y_0$$

$$y(46) = 114.84 + (0.2)(-18.68) + \frac{(0.2)(-0.8)}{2} \times 5.84 \\ + \frac{(0.2)(-0.8)(-1.8)(-1.84)}{6} + \frac{(0.2)(-0.8)(-1.8)(-2.8)}{24} \times 0.68$$

$$y(46) = 114.84 - 3.76 - 0.4672 - 0.08832 - 0.022846$$

$$\underline{\underline{y(46) = 110.5256}}$$

## Lagrange's Interpolation formula for Unequal Intervals

Suppose  $y_0, y_1, y_2, y_3, \dots, y_n$  be the set of values of  $y = f(x)$  corresponding to  $x_0, x_1, x_2, \dots, x_n$  then the Lagrange's Interpolation formula is follows as

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\dots(x_0-x_n)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\dots(x_1-x_n)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\dots(x_2-x_n)} y_2 \\ + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_n-x_0)(x_n-x_1)(x_n-x_2)} \dots \frac{(x-x_{n-1})}{(x_n-x_{n-1})} y_n$$

By the Inverse Lagrange's Interpolation formula can be

$$x = g(y) = \frac{(y-y_1)(y-y_2)(y-y_3)\dots(y-y_n)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)\dots(y_0-y_n)} x_0 +$$

$$\frac{(y-y_0)(y-y_2)(y-y_3)\dots(y-y_n)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)\dots(y_1-y_n)} x_1 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)\dots(y-y_n)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)\dots(y_2-y_n)} x_2 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)\dots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)(y_n-y_2)\dots(y_n-y_{n-1})} x_n$$

II Use the Lagrange's Interpolation formula to find  $y$  at  $x=0$

$x$	5	6	9	11
$y$	12	13	14	16

Given :-

x	y
$x_0 = 5$	$y_0 = 12$
$x_1 = 6$	$y_1 = 13$
$x_2 = 9$	$y_2 = 14$
$x_3 = 11$	$y_3 = 16$

To find  $y(x) = y(10)$

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y(10) = \frac{(10-6)(10-9)(10-11)(12)}{(5-6)(5-9)(5-11)} + \frac{(10-5)(10-6)(10-11)}{(6-5)(6-9)(6-11)} (13)$$

$$+ \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} (14) + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} (16)$$

$$y(10) = \frac{4(1)(-1)(12)}{(-1)(-4)(-6)} + \frac{5(1)(-1)(13)}{(1)(-3)(-5)} + \frac{(5)(4)(-1)(14)}{(-4)(3)(-1)} + \frac{(5)(4)(1)(16)}{(6)(5)(2)}$$

$$y(10) = -4.33 + 11.66 + 5.33$$

$$y(10) = \underline{\underline{14.66}}$$

[2] By the polynomial  $f(x)$  using Lagrange's formula from the following data

x	0	1	2	5
y	2	3	12	147

<u>Soln</u> :-	x	y
	$x_0 = 0$	$y_0 = 2$
	$x_1 = 1$	$y_1 = 3$
	$x_2 = 2$	$y_2 = 12$
	$x_3 = 5$	$y_3 = 147$

$$f(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \\ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$f(x) = \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)} (2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)} (3) +$$

$$\frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)} (12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)} (147)$$

$$f(x) = \frac{2(x-1)(x-2)(x-5)}{(-1)(-2)(-5)} + \frac{3(x-0)(x-2)(x-5)}{(1)(-1)(-4)} + \frac{12(x)(x-1)(x-5)}{2(1)(-4)} \\ + \frac{147(x-0)(x-1)(x-2)}{5(4)(3)}$$

$$f(x) = \frac{1}{5} (x-1)(x-2)(x-5) + \frac{3}{4} (x-2)(x-5)(x-0) - 24x(x-1)(x-5) \\ + \frac{49}{20} x(x-1)(x-2)$$

$$f(x) = \frac{1}{20} \left| -4(x-1)(x^2-7x+10) + 15x(x^2-7x+10) - 40x(x^2-5x) \right| \\ + 49x(x^2-3x+2)$$

$$f(x) = \frac{1}{20} \left| -4x^3 + 32x^2 - 68x + 40 + 15x^3 - 15x^2 + 150x - 40x^3 + 240x^2 \right. \\ \left. - 800x + 44x^3 - 147x^2 - 98x \right|$$

$$f(x) = \frac{1}{20} \left| 80x^3 + 20x^2 - 80x + 40 \right|$$

$$f(x) = x^3 + x^2 - x + 2 //$$

## Numerical Integration

Suppose  $y = f(x)$  be a function and let  $y_0, y_1, y_2, \dots, y_n$  are the set of values corresponding to the partition of

$$P = [x_0 = a, x_0 + h = x_1, x_1 + h = x_2, x_2 + h = x_3 + \dots + x_{n-1} + h = x_n = b]$$

with the equal length of the partition  $h = \frac{b-a}{n}$  where

$n$  is the number of equal strips [or  $n+1$  ordinates] we can evaluate  $\int_a^b f(x) dx$  by the following method

i) Simpson's  $\frac{1}{3}$ rd rule

$$\int_a^b f(x) dx = \frac{h}{3} [(y_0 + y_n) + 2(y_2 + y_4 + y_6 + \dots + y_{n-2}) + 4(y_1 + y_3 + y_5 + \dots + y_{n-1})]$$

ii) Simpson's  $\frac{3}{8}$ th rule

$$\int_a^b f(x) dx = \frac{3h}{8} [(y_0 + y_n) + 2(y_3 + y_6 + y_9 + \dots + y_{n-3}) + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{n-1})]$$

iii) Weddle's rule for  $n=6$

$$\int_a^b f(x) dx = \frac{3h}{10} [y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6]$$

[3] Evaluate  $\int_0^1 \frac{1}{(1+x^2)} dx$  by taking equal strips and hence deduce an approximate value of  $\pi$  by following

method i) Simpson's  $\frac{1}{3}$ rd rule

ii) Simpson's  $\frac{3}{8}$ th rule

iii) Weddle's rule

$$\underline{\text{Soln}} : \text{Let } I = \int_0^1 \frac{1}{1+x^2} dx$$

$$a=0, b=1, n=6, y = \frac{1}{1+x^2}$$

$$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$$

$$P = \left\{ a = x_0 = 0, x_1 = \frac{1}{6}, x_2 = \frac{2}{6}, x_3 = \frac{3}{6}, x_4 = \frac{4}{6}, x_5 = \frac{5}{6}, x_6 = 1 = b \right\}$$

$x$	$y = \frac{1}{1+x^2}$
$x_0 = 0$	$y_0 = \frac{1}{1+0} = 1$
$x_1 = \frac{1}{6}$	$y_1 = \frac{1}{1+\frac{1}{36}} = 0.9730$
$x_2 = \frac{1}{3}$	$y_2 = \frac{1}{1+\frac{1}{9}} = 0.9$
$x_3 = \frac{1}{2}$	$y_3 = \frac{1}{1+\frac{1}{4}} = 0.8$
$x_4 = \frac{2}{3}$	$y_4 = \frac{1}{1+\frac{4}{9}} = 0.6923$
$x_5 = \frac{5}{6}$	$y_5 = \frac{1}{1+\frac{25}{36}} = 0.5902$
$x_6 = 1$	$y_6 = \frac{1}{1+1^2} = 0.5$

i) Simpson's  $\frac{1}{3}$  rd rule

$$\int_a^b f(x) dx = \frac{h}{3} \left[ (y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5) \right]$$

$$\Rightarrow \int_0^1 \frac{1}{1+x^2} dx = \frac{1}{3} \left[ (1+0.5) + 2(0.9+0.6923) + 4(0.97304+0.87) \right] \\ \Rightarrow \int_0^1 \frac{1}{1+x^2} dx = \frac{1}{18} \left[ 1.5 + 3.1846 + 9.45028 \right]$$

0.5902

$$\Rightarrow \int_0^1 \frac{1}{1+x^2} dx = 0.7854$$

$$\Rightarrow \tan'(x) \Big|_0^1 = 0.7854$$

$$\Rightarrow \tan'(1) - \tan'(0) = 0.7854$$

$$\Rightarrow \frac{\pi}{4} = 0.7854$$

$$\Rightarrow \pi = 0.7854 \times 4$$

$$\Rightarrow \pi \approx 3.1416$$

ii) Simpson's  $\frac{3h}{8}$  rule

$$\int_a^b f(x) dx = \frac{3h}{8} [c(y_0 + y_6) + 2(y_3) + 3(y_1 + y_2 + y_3 + y_4 + y_5)]$$

$$\Rightarrow \int_0^1 \frac{1}{1+x^2} dx = \frac{3(1/6)}{8} \left[ (1+0.5) + 2(0.8) + 3(0.973 + 0.9 + 0.6923) + 0.5402 \right]$$

$$\Rightarrow \frac{1}{16} [1.5 + 1.6 + 9.4665]$$

$$\Rightarrow \tan'(x) \Big|_0^1 = 0.7854$$

$$\Rightarrow \tan'(1) - \tan'(0) = 0.7854$$

$$\Rightarrow \frac{\pi}{4} = 0.7854$$

$$\Rightarrow \pi = 0.7854 \times 4$$

$$\Rightarrow \pi \approx 3.1416$$

iii) Weddle rule

$$\int_a^b f(x) dx = \frac{3L}{10} [y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6]$$

$$\Rightarrow \int_0^1 \frac{1}{1+x^2} dx = \frac{3(1/6)}{10} [1 + 4.665 + 0.9 + 4.8 + 0.6923 + 0.195]$$

$$\Rightarrow \int_0^1 \frac{1}{1+x^2} dx = \frac{1}{60} [15.7063]$$

$$\Rightarrow \tan^{-1}(1) - \tan^{-1}(0) = 0.7854$$

$$\Rightarrow \frac{\pi}{4} = 0.7854$$

$$\Rightarrow \pi \approx 3.1416$$

14] Evaluate  $\int_0^1 \frac{1}{1+x} dx$  taking 7 ordinates and hence deduce the value of  $\log_e$  by using i) Simpson's

Soln :- Let  $a=0$ ,  $b=1$ ,  $n=6$

$$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$$

$$P = (a=x_0=0, x_1=1/6, x_2=2/6, x_3=3/6, x_4=4/6, x_5=5/6, x_6=6/6)$$

$x$	$y = \frac{1}{1+x}$
$x_0=0$	$y_0=1$
$x_1=1/6$	$y_1=0.8571$
$x_2=2/6$	$y_2=0.73$
$x_3=3/6$	$y_3=0.667$
$x_4=4/6$	$y_4=0.6$
$x_5=5/6$	$y_5=0.545$
$x_6=1$	$y_6=0.5$

i) Simpson's  $\frac{1}{3}$  rd rule

$$\int_a^b f(x) dx = \frac{n}{3} [(y_0+y_6) + 2(y_1+y_4) + 4(y_2+y_3+y_5)]$$

$$= \left(\frac{1/6}{3}\right) [(1+0.5) + 2(0.75+0.6) + 4(0.857 + 0.667 + 0.545)]$$

$$= \frac{1}{18} [1.5 + 2.7 + 8.8784]$$

$$\log_e^{(x+1)} \Big|_0^1 = 0.6932$$

$$\log_e = 0.6932$$

15] Find the approximate value of  $\int_0^{\frac{\pi}{2}} \sqrt{\cos \theta} d\theta$  by Simpson's  $\frac{1}{3}$ -rd rule by dividing  $[0, \frac{\pi}{2}]$  into 6 equal parts

Given :-  $y(\theta) = \sqrt{\cos \theta}$

$$a = 0, b = \frac{\pi}{2}, n = 6$$

$$h = \frac{b-a}{n} = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12} = 15^\circ (0.2619)$$

$$\begin{aligned} P = \{ & a = \theta_0 = 0, \theta_1 = 15^\circ, \theta_2 = 30^\circ, \theta_3 = 45^\circ, \theta_4 = 60^\circ, \theta_5 = 75^\circ \\ & \theta_6 = 90^\circ = b \} \end{aligned}$$

$\theta$	$y = \sqrt{\cos \theta}$
$\theta = 0$	$y = \sqrt{\cos 0} = 1 = y_0$
$\theta = 15$	$y = \sqrt{\cos 15} = 0.982 = y_1$
$\theta = 30$	$y = \sqrt{\cos 30} = 0.9306 = y_2$
$\theta = 45$	$y = \sqrt{\cos 45} = 0.8109 = y_3$
$\theta = 60$	$y = \sqrt{\cos 60} = 0.7071 = y_4$
$\theta = 75$	$y = \sqrt{\cos 75} = 0.508 = y_5$
$\theta = 90$	$y = \sqrt{\cos 90} = 0 = y_6$

By the Simpson's  $\frac{1}{3}$ -rd rule  
 $n=6$

$$\Rightarrow \int_a^b f(x) dx = \frac{h}{3} [(y_0 + y_6) + 2(y_2 + y_5) + 4(y_1 + y_3 + y_4)]$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \sqrt{\cos \theta} d\theta = \frac{0.2619}{3} [(1+0) + 2(0.9306 + 0.7071) + 4(0.9828 + 0.8109 + 0.508)]$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \sqrt{\cos \theta} d\theta = 1.1777$$

10) Evaluate  $\int_0^1 \frac{x}{1+x^2} dx$  taking 7 ordinates using wedge rule  
Hence find  $\log_e$

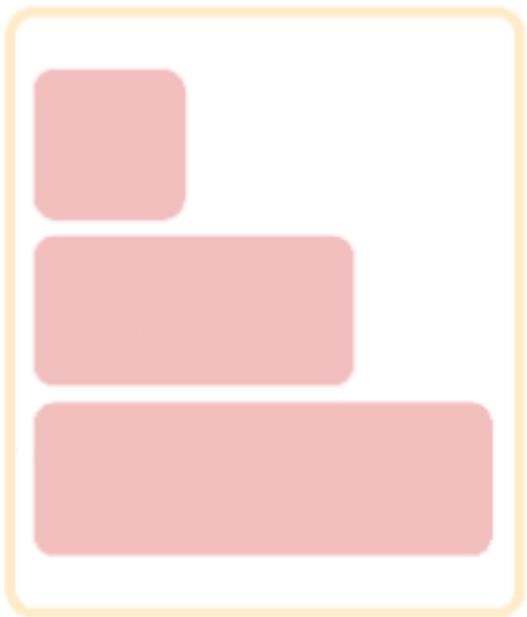
Given :-  $y = \frac{x}{1+x^2}$

$$a=0, b=1, n=6$$

$$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$$

$$P = \left\{ a-x_0=0, x_1=\frac{1}{6}, x_2=\frac{1}{3}, x_3=\frac{1}{2}, x_4=\frac{2}{3}, x_5=\frac{5}{6}, x_6=1=b \right\}$$

$x$	$\frac{x}{1+x^2}$
0	$0=y_0$
$\frac{1}{6}$	$0.1621 = y_1$
$\frac{1}{3}$	$0.30 = y_2$
$\frac{1}{2}$	$0.4 = y_3$
$\frac{2}{3}$	$0.4615 = y_4$
$\frac{5}{6}$	$0.4915 = y_5$
1	$0.5 = y_6$



By the wedge rule

$$\int_0^1 f(x) dx = \frac{3h}{10} (y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6)$$

$$\Rightarrow \int_0^1 \frac{x}{1+x^2} dx = \frac{3(1/6)}{10} \left[ 0 + 0.1621 + 0.3 + 0.4 + 0.4615 + 0.4915 + 0.5 \right]$$

$$\Rightarrow \int_0^1 \frac{x}{1+x^2} dx = 0.3466$$

$$\Rightarrow \frac{1}{2} \int_0^1 \frac{2x}{1+x^2} dx = 0.3466$$

$$\log(1+x^2) \Big|_0^1 = 9 \times 0.3466$$

$$\log(2) - \log(1) = 0.6932$$

$$\log_e^2 = 0.6932$$

F7 Use Simpson's  $\frac{1}{3}$ rd rule of ordinates to evaluate  $\int_2^8 \frac{1}{\log_{10} x} dx$

Given :-  $y = \frac{1}{\log_{10} x}$

$$a=2, b=8, n=6$$

$$h = \frac{b-a}{n} = \frac{8-2}{6} = 1$$

$$P = \left\{ a = x_0 = 2, x_1 = 3, x_2 = 4, x_3 = 5, x_4 = 6, x_5 = 7, x_6 = 8 = b \right\}$$

$x$	$y = \frac{1}{\log_{10} x}$
2	$y_0 = 3.3219$
3	$y_1 = 2.0460$
4	$y_2 = 1.6609$
5	$y_3 = 1.4306$
6	$y_4 = 1.2851$
7	$y_5 = 1.1833$
8	$y_6 = 1.1073$

By Simpson's  $\frac{1}{3}$ rd rule

$$n=6$$

$$\Rightarrow \int_a^b f(x) dx = \frac{h}{3} [(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$

$$\Rightarrow \int_2^8 \frac{1}{\log_{10} x} dx = \frac{1}{3} \left[ (3.3219 + 1.1073) + 2(1.6609 + 1.2851) + 4(2.0460 + 1.4306 + 1.1833) \right]$$

$$\Rightarrow \frac{1}{3} (4.4292 + 5.8920 + 6.9596)$$

$$\int_2^8 \frac{1}{\log x} dx = 9.6936$$

Q13] Use Simpson's 1/3rd rule to find  $\int_0^{0.6} e^{-x^2} dx$  by taking 6 Subintervals

Given :-  $y = e^{-x^2}$

$$a=0, b=0.6, n=6$$

$$h = \frac{b-a}{n} = \frac{0.6-0}{6} = 0.1$$

$$P = \{x_0 = 0, x_1 = 0, x_2 = 0.1, x_3 = 0.2, x_4 = 0.3, x_5 = 0.4, x_6 = 0.5, x_7 = 0.6\}$$

$x$	$e^{-x^2}$
0	$e^{0.0} = 1 \Rightarrow y_0$
0.1	$e^{-0.01} = 0.99 \Rightarrow y_1$
0.2	$e^{-0.04} = 0.9607 \Rightarrow y_2$
0.3	$e^{-0.09} = 0.9137 \Rightarrow y_3$
0.4	$e^{-0.16} = 0.8521 \Rightarrow y_4$
0.5	$e^{-0.25} = 0.7788 \Rightarrow y_5$
0.6	$e^{-0.36} = 0.697 \Rightarrow y_6$

## Numerical Soln for transcendental Equation

An Eqn which involves algebraic logarithmic Exponential Trigonometry function is called the transcendental eqn

Regula-falsi method (or) falsi position method

Step 1:- Rewrite the given transcendental Eqn in the form  $\text{if } f(x) = 0$

Step 2:- Choose  $x_0$  and  $x_1$  nearest to the real root for which  $f(x_0) < 0$ ,  $f(x_1) > 0$

Step 3:-

$$\text{let the real root } x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)}$$

Suppose  $f(x_2) < 0$ . We say that the root lies b/w  $x_2$  and  $x_1$  and as follows

Step 4:-

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

If  $f(x_3) > 0$  we say that the root lies b/w  $x_2$  and  $x_3$  and follows the same

Step 5:-

Continue the same procedure until  $f(x_n)$  is 0 (or) approximately zero

[Q] find the real root of the equation  $x e^x - 3 = 0$  by Regula-falsi method correct to 3 decimal places

Soln

$$x e^x - 3 = 0$$

$$\Rightarrow f(x) = 0$$

$$f(x) = x e^x - 3$$

$$\text{Let } x_0 = 0, \quad x_1 = 1.1$$

$$f(x_0) = f(0) = 1 \cdot e^0 - 3 = -0.2817 < 0$$

$$f(x_1) = f(1.1) = 1.1e^{1.1} - 3 = 0.30467$$

The root lies between  $x_0$  and  $x_1$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(1)(0.3046) - (1.1)(-0.2817)}{0.3046 + 0.2817}$$

$$\Rightarrow x_2 = 1.0480$$

$$\therefore f(x_2) = (1.048)e^{1.048} - 3 = -0.011 < 0$$

The root lies b/w  $x_2$  and  $x_1$

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$x_3 = \frac{(1.048)(0.3046) - (1.1)(-0.011)}{0.3046 + 0.011}$$

$$\Rightarrow x_3 = 1.0497$$

$$\therefore f(x_3) = (1.0497)e^{1.0497} - 3 = -0.0012 < 0$$

$$x_4 = \frac{x_3 f(x_1) - x_1 f(x_3)}{f(x_1) - f(x_3)}$$

$$x_4 = \frac{(1.0497)(0.3046) - (1.1)(-0.0012)}{0.3046 + 0.0012}$$

$$\Rightarrow x_4 = 1.0498$$

$$f(x_4) = (1.0498)e^{1.0498} - 3 = 0.0006 \approx 0$$

The real root is  $x = 1.0498$

=

20) Find the real root of  $Eqn xe^x - 2 = 0$  by Regula-falsi method

Soln :-

$$\text{Given: } xe^x - 2 = 0$$

$$f(x) = 0$$

$$\therefore f(x) = xe^x - 2$$

$$\text{Let } x_0 = 0.8, x_1 = 0.9$$

$$\Rightarrow f(x_0) = f(0.8) = -0.2195 < 0$$

$$f(x_1) = f(0.9) = 0.2136 > 0$$

$$\Rightarrow x_2 = \frac{(0.8)(0.2136) - (0.9)(-0.2195)}{0.2136 + 0.2195} = 0.8506$$

$$\therefore f(x_2) = f(0.8506) = -0.0087 < 0$$

$$\Rightarrow x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$x_3 = \frac{(0.8506)(0.2136) - (0.9)(-0.0087)}{0.2136 + 0.0087}$$

$$x_3 = 0.8525$$

$$\therefore f(x_3) = f(0.8525) = -0.0004 < 0$$

$$\Rightarrow x_4 = \frac{x_3 f(x_1) - x_1 f(x_3)}{f(x_1) - f(x_3)}$$

$$x_4 = \frac{(0.8525)(0.2136) - (0.9)(-0.0004)}{0.2136 + 0.0004}$$

$$x_4 = 0.8526$$

$$\therefore f(x_4) = f(0.8526) = -0.00002 < 0$$

$$\therefore f(x_4) = f(0.8526) = -0.0000 \neq 0$$

The root of the eqn is  $x = 0.8526$

- Q1 Use the Regula Falsi method to obtain a root of the eqn  $2x - \log_{10} x = 7$  which lies b/w 3.5 and 4 carry out 2 iterations

Soln:-  $2x - \log_{10} x = 7$

$$\Rightarrow 2x - \log_{10} x - 7 = 0$$

$$f(x) = 0$$

$$f(x) = 2x - \log_{10} x - 7$$

and  $x_0 = 3.5$  and  $x_1 = 4$

$$f(x_0) = f(3.5) = 2(3.5) - \log_{10}(3.5) - 7 = -0.5441 < 0$$

$$f(x_1) = f(4) = 2(4) - \log_{10}(4) - 7 = 0.397 > 0$$

The root lies b/w  $x_0$  and  $x_1$

$$x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)}$$

$$x_2 = \frac{4(-0.5441) - (3.5)(0.397)}{-0.5441 - 0.397}$$

$$x_2 = 3.788$$

$$f(x_2) = 2(3.788) - \log_{10}(3.788) - 7 = 0.0009 < 0$$

The root lies b/w  $x_2$  and  $x_1$

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$x_3 = \frac{(3.788)(0.397) - (4)(0.0009)}{0.3978 + 0.0009}$$

$$x_3 = 3.7892$$

$$f(x_3) = \ln(3.7892) - \log_{10}(3.7892) - 7$$

$$f(x_3) = -0.0001 \approx 0$$

The root lies b/w  $x = 3.7892$

- 22) find the real root of  $x \log_{10} x - 1.2 = 0$  correct to 3 decimal places lying in the interval (2, 3)  
 Using regular false method

Soln :-

$$\text{Given: } x \log_{10} x - 1.2 = 0$$

$$f(x) = 0$$

$$f(x) = x \log_{10} x - 1.2$$

$$\text{let } x_0 = 2.7, x_1 = 2.8$$

$$\Rightarrow f(x_0) = f(2.7) = -0.0353 < 0$$

$$f(x_1) = f(2.8) = 0.0520 > 0$$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\Rightarrow x_2 = \frac{(2.8)(0.0520) - (2.7)(-0.0353)}{0.0520 + 0.0353} = 2.7404$$

$$f(x_2) = f(2.7404) = -0.0002 < 0$$

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$\Rightarrow x_3 = \frac{(2.7404)(0.0520) - (2.7)(-0.0002)}{0.0520 + 0.0002}$$

$$x_3 = 2.7406$$

$$f(x_3) = f(2.7406) = -0.00002 < 0$$

The real root of the Eqn  $x = 2.7406$

Q3] Find the 4<sup>th</sup> root of 12 by using regular-falsi method

Soln :- Let  $x = 4\sqrt{12}$

$$\Rightarrow x^4 = 12$$

$$\Rightarrow x^4 - 12 = 0$$

$$\Rightarrow f(x) = 0$$

$$f(x) = x^4 - 12$$

$$\text{Let } x_0 = 1.8, \quad x_1 = 1.9$$

$$\therefore f(x_0) = -1.5024 < 0$$

$$f(x_1) = 1.0321 > 0$$

$\therefore$  The root lies between  $x_0$  and  $x_1$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\Rightarrow x_2 = \frac{(1.8)(1.0321) - (1.9)(-1.5024)}{1.0321 + 1.5024}$$

$$\Rightarrow x_2 = 1.8592$$

$$\therefore f(x_2) = -0.0517 < 0$$

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$\Rightarrow x_3 = \frac{(1.8592)(1.0321) - (1.9)(-0.0517)}{1.0321 + 0.0517}$$

$$\Rightarrow x_3 = 1.8611$$

$$\therefore f(x_3) = 0.0028 < 0$$

$$x_4 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(1.8611)(1.0321) - (1.9)(-0.0028)}{1.0321 + 0.0028}$$

$$x_4 = 1.8612$$

$$f(x_4) = -0.000100$$

$x = 1.8612$  is real root

$$\therefore \sqrt[4]{12} = 1.8612$$

Q4 Using Stegula falsi method to find the real root for the equation  $x^3 - 2x - 5 = 0$

Soln :-

$$x^3 - 2x - 5 = 0$$

$$\Rightarrow f(x) = 0$$

$$f(x) = x^3 - 2x - 5 = 0$$

$$\text{Let } x_0 = 2 \Rightarrow f(x_0) = -1 < 0$$

$$x_1 = 2.1 \Rightarrow f(x_1) = 0.061 > 0$$

$$\therefore x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\Rightarrow x_2 = \frac{(2)(0.06) - (2.1)(-1)}{0.061 + 1}$$

$$\Rightarrow x_2 = 2.0947$$

$$f(x_2) = -0.0039 < 0$$

$$\therefore x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$\Rightarrow x_3 = \frac{(2.0942)(0.061) - (2.1)(-0.0039)}{0.061 + 0.0039}$$

$$\Rightarrow x_3 = 2.0945$$

$$f(x_3) = -0.0005 < 0$$

$$\therefore x_4 = \frac{(2.0945)(0.061) - (2.1)(-0.0005)}{0.061 + 0.0005}$$

$$\Rightarrow x_4 = 2.0945$$

$$f(x_4) = -0.000500$$

$$\therefore x = 2.0945 \text{ is real root}$$

## Newton Dampen method

Step 1 :- Rewrite the given transcendental Eqn in the form  
 $f(x)=0$  and find  $f'(x)$

Step 2 :- Choose  $x_0$  for which  $f(x_0) < 0$

Step 3 :- Use of formula  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$  for  $n=0, 1, 2, \dots$

for the next root and continue the same  
 until to get the value of  $x$  has to be equal

**Ex** Find  $\sqrt[3]{37}$  by newton dampen method

Soln :-

$$\text{Let } x = \sqrt[3]{37}$$

$$\Rightarrow x^3 = 37$$

$$\Rightarrow x^3 - 37 = 0$$

$$\Rightarrow f(x) = 0$$

$$\therefore f(x) = x^3 - 37 \Rightarrow f'(x) = 3x^2$$

$$\text{Let } x_0 = 3.3$$

$$\Rightarrow f(x_0) = -1.063, f'(x_0) = 32.67 \neq 0$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow 3.3 - \frac{(-1.063)}{32.67}$$

$$\Rightarrow 3.3325$$

$$\Rightarrow f(x_1) = -0.0092, f'(x_1) = 33.3166$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\Rightarrow 3.3325 - \frac{(-0.0092)}{33.3166}$$

$$\Rightarrow 3.3327$$

$$f(x_2) = 0.0159 \quad f'(x_2) = 33.3206$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\Rightarrow 3.3327 - \frac{(0.0159)}{33.3206}$$

$$\Rightarrow 3.3322$$

$$x = 3\sqrt{27}$$

$$x = 3.3322$$

Q6 Find a real root of eqn  $x \sin x + \cos x = 0$  near to  $x = \pi$  radians. Correct to 4 decimal places by using  
Iteration method

Soln :-

$$x \sin x + \cos x = 0$$

$$\Rightarrow f(x) = 0$$

$$\therefore f(x) = x \sin x + \cos x$$

$$\Rightarrow f'(x) = x \cos x + \sin x - \sin x$$

$$f'(x) = x \cos x$$

$$\text{and } x_0 = \pi$$

$$\Rightarrow f(x_0) = \pi \sin \pi + \cos \pi = -1$$

$$f'(x_0) = \pi \cos \pi = -\pi$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow \pi - \frac{(-1)}{(-\pi)}$$

$$\Rightarrow \pi - \frac{1}{\pi}$$

$$\Rightarrow \frac{\pi^2}{\pi} - \frac{1}{\pi^2}$$

$$\Rightarrow x_1 \Rightarrow 3.8246$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 2.7986 - \frac{(-0.0697)}{(-2.6638)} = 2.7986$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 2.7986 - \frac{(-0.00056)}{(-2.6355)} = 2.7983$$

$$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \Rightarrow 2.7983 - \frac{(0.00022)}{(-2.6350)} = 2.7983$$

$x = 2.7983$  is a real root

[Using Newton Raphson method find the real root of equation  $3x = \cos x + 1$  [Use radians mode]

Soln :-

Given :-  $3x = \cos x + 1$

$$\Rightarrow 3x - \cos x - 1 = 0$$

$$\Rightarrow f(x) = 0$$

$$f(x) = 3x - \cos x - 1$$

$$\Rightarrow f'(x) = 3 + \sin x$$

$$\text{let } x_0 = 0.5$$

$$\Rightarrow f(x_0) = 0.3715 < 0$$

$$f'(x_0) = 3.4794$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{(-0.3715)}{3.4794} = 0.6084$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.6084 - \frac{0.00463}{3.5715} = 0.6071$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.6071 - \frac{(-0.0000056)}{3.5704} = 0.6071$$

$x = 0.6071$  is a real root

Ex] Find the real root of  $x \log_{10} x = 1.2$  using newton dropson method near to 2.5

Soln :- Given :-  $x \log_{10} x = 1.2$

$$\Rightarrow x \log_{10} x - 1.2 = 0$$

$$\Rightarrow \frac{x \log x}{\log_{10}} - 1.2 = 0$$

$$\Rightarrow (0.4343) x \log x - 1.2 = 0$$

$$\Rightarrow f(x) = 0$$

$$\therefore f(x) = (0.4343) x \log x - 1.2$$

$$f'(x) = (0.4343) [x \cdot \frac{1}{x} + (1) \log x] - 0$$

$$\Rightarrow f'(x) = (0.4343) (1 + \log x)$$

$$\text{and } x_0 = 2.5$$

$$f(x_0) = -0.2051$$

$$f'(x_0) = 0.8322$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow 2.5 - \frac{(-0.2051)}{(0.8322)} = 2.7465$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 2.7465 - \frac{(0.0051)}{(0.8322)} = 2.7406$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 2.7406 - \frac{(0.000051)}{(0.8322)} = 2.7406$$

The real root is 2.7406

Ques] Using newton dropson method find the real root that lies near  $x=4.5$  of the equation  $\tan x = x$  correct to 4 decimal places [take  $x$  as a radian]

Soln

Given,  $\tan x = x$

$$f(x) = x - \tan x$$

$$f'(x) = 1 - \sec^2 x$$

$$f'(x) = -(\sec^2 x - 1)$$

$$f'(x) = -\tan^2 x$$

and  $x_0 = 4.5$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow 4.5 - \frac{(-0.1373)}{(-20.5046)} = 4.4936$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 4.4936 - \frac{(-0.0039)}{(-20.227)} = 4.4934$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 4.493 - \frac{(0.0002)}{(-20.1869)} = 4.4934$$

The real root is  $x = 4.4934$

30] find the real root of the Eqn  $xe^x - \cos x = 0$  correct to 3 decimal places by using newton raphson method near To the root  $x = 0.5$

Soln :-  $xe^x - \cos x = 0$

$$\Rightarrow f(x) = 0$$

$$\therefore f(x) = xe^x - \cos x$$

$$f'(x) = 1 \cdot e^x + xe^x + \sin x$$

$$\Rightarrow f'(x) = e^x(1+x) + \sin x$$

and  $x_0 = 0.5$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow 0.5 - \frac{(-0.0532)}{2.4595} = 0.5180$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 0.5180 - \frac{(0.0007)}{(3.0434)} = 0.5177$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 0.5177 - \frac{(-0.0001)}{(3.0418)} = 0.5177$$

The real root is  $x = 0.5177$