MODULE - 01

LASERS AND OPTICAL FIBERS

LASERS

The word Laser stands for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation. It is a device which amplifies light. It has properties like Coherence, Unidirectional, Monochromatic, Focus ability, etc.

Interaction of an electromagnetic wave with matter leads to transition of an atom or a molecule from one energy state to another. If the transition is from lower state to higher state it absorbs the incident energy. If the transition is from higher state to lower state it emits a part of its energy.

Emission or Absorption takes through quantum of energy called photons. $h\nu$ is called quantum energy or photon energy.

$h = 6.626\times10^{-34}$ Joules Second is Planck's constant and 'ν' is the frequency.

If ΔE is the difference between the two energy levels,

Then $\Delta E = (E_2 - E_1)$ Joule

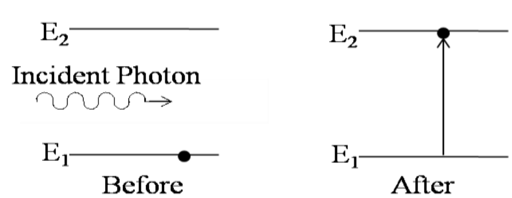
According to Max Planck, $\Delta E = h\nu = (E_2 - E_1)$

$$\nu = (E_2 - E_1)/h \quad \text{Hz.}$$

Three types of interactions, which are possible, are as follows:

1)Induced Absorption:

Induced absorption is the absorption of an incident photon by system as a result of which the system is elevated from a lower energy state to a higher state, wherein the difference in energy of the two states is the energy of the photon.



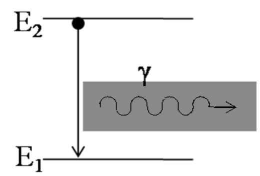
Consider the system having two energy states E_1 and E_2, $E_2 > E_1$. When a photon of energy $h\nu$ is incident on an atom at level E_1, the atom goes to a higher energy level by absorbing the energy.

When an atom is at ground level (E_1), if an electromagnetic wave of frequency ν is applied to the atom, there is possibility of getting excited to higher level (E_2). The incident photon is absorbed. It is represented as

$$\text{Atom} + \text{Photon} \rightarrow \text{Atom}^*$$

2) Spontaneous Emission: The emission of a photon by the transition of a system from a higher energy state to a lower energy state without the aid of an external energy is called spontaneous emission.

Let 'E_1' and 'E_2' be two energy levels in a material, such that $E_2>E_1$. E_1 is ground level and E_2 is the higher level. $h\nu=E_2-E_1$ is the difference in the energy. The atom at higher level (E_2) is more unstable as compared to that at lower level (E_1).



The life time of an atom is less in the excited state, In spontaneous emission atom emits the photon without the aid of any external energy. It is called spontaneous emission. The process is represented as

$$\text{Atom}^* \rightarrow \text{Atom} + \text{Photon}$$

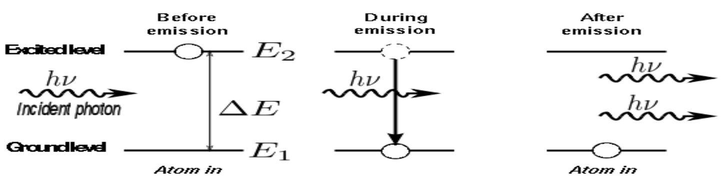
The photons emitted in spontaneous emission may not have same direction and phase similarities. It is incoherent.

Ex: Glowing electric bulbs, Candle flame etc.

3) Stimulated Emission:

Stimulated emission is the emission of a photon by a system under the influence of a passing photon of right energy due to which the system transits from a higher energy state to a lower energy state.

The photon thus emitted is called stimulated photon and will have the same phase, energy and direction of movement as that of the passing photon called the stimulation photon.



$$E_2 - E_1 = \Delta E = h\nu$$

Initially the atom is at higher level E_2. The incident photon of energy hν forces the atom to get de-excited from higher level E_2 to lower level E_1.

i.e. $h\nu = E_2 - E_1$ is the change in energy.

The incident photon stimulates the excited atom to emit a photon of exactly the same energy as that of the incident photons. The emitted two photons have same phase, frequency, direction and polarization with the incident photon and results in coherent beam of radiation. This kind of action is responsible for lasing action.

$$\text{Atom}^* + \text{Photon} \rightarrow \text{Atom} + (\text{Photon} + \text{Photon})$$

<u>Expression for energy density in terms of Einstein's Coefficients</u>

Consider two energy levels E_1 and E_2 of a system of atoms with N_1 and N_2 are population of energy levels respectively.

Let U_ν be the energy density of incident beam of radiation of frequency γ.

Let us consider the absorption and two emission process

1) Induced absorption:

Induced absorption is the absorption of an incident photon by system as a result of which the system is elevated from a lower energy state to a higher state.

The rate of absorption is proportional to $N_1 U_\nu$

$$\text{Rate of absorption} = B_{12} N_1 U_\nu \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where 'B_{12}' is the proportionality constant called Einstein Coefficient of induced absorption.

2) Spontaneous emission:

The emission of a photon by the transition of a system from a higher energy state to a lower energy state without the aid of an external energy is called spontaneous emission.

Spontaneous emission depends on N_2 and independent of energy density.

$$\text{The rate of spontaneous emission} = A_{21} N_2 \quad \ldots\ldots\ldots\ldots\ldots(2)$$

Where 'A_{21}' is called proportionality constant called Einstein coefficient of spontaneous emission.

3) Stimulated emission:

Stimulated emission is the emission of a photon by a system under the influence of a passing photon of just the right energy due to which the system transits from a higher energy state to a lower energy state

The rate of stimulated emission is directly proportional to N_2U_γ.

The rate of stimulated emission = $B_{21}N_2U_\nu$ .... (3)

Where 'B_{21}' is the proportionality constant called Einstein's Coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = (Rate of spontaneous emission + Rate of stimulated emission)

$$B_{12}N_1U_\nu = A_{21}N_2 + B_{21}N_2U_\nu$$

$$U_\nu\,(B_{12}N_1 - B_{21}N_2) = A_{21}N_2$$

$$U_\nu = \frac{A_{21}N_2}{(B_{12}N_1 - B_{21}N_2)}$$

$$\text{i.e. } U_\nu = \frac{A_{21}}{B_{21}}\left[\frac{N_2}{\left(\frac{B_{12}}{B_{21}}N_1 - N_2\right)}\right]$$

$$U_\nu = \frac{A_{21}}{B_{21}}\left[\frac{1}{\left(\frac{B_{12}N_1}{B_{21}N_2}\right) - 1}\right] \qquad \rightarrow (4)$$

By Boltzmann's law, $N_2 = N_1 e^{-\left(\frac{E_2 - E_1}{KT}\right)} = N_1\, e^{-h\nu/KT}$

i.e., $N_1/N_2 = e^{h\nu/KT}$

Eqn. (4) becomes $$U_\upsilon = \frac{A_{21}}{B_{21}}\left[\frac{1}{\left(\frac{B_{12}}{B_{21}}e^{\left(\frac{h\upsilon}{kT}\right)} - 1\right)}\right] \qquad \rightarrow (5)$$

By Planck's law, $$U_\upsilon = \frac{8\pi h\upsilon^3}{c^3}\left[\frac{1}{\left(e^{\left(\frac{h\upsilon}{kT}\right)}-1\right)}\right] \quad \rightarrow (6)$$

Comparing equation (5) & (6)

$\frac{A_{21}}{B_{21}} = 8\pi h\nu^3/c^3$ & $\frac{B_{12}}{B_{21}} = 1$ i.e. $B_{12} = B_{21}$

The probability of induced absorption is equal to the stimulated emission.

Therefore, $A_{12}$is written as A and B_{12}, B_{21} written as B.

Equation (5) becomes

$$U_\upsilon = \frac{A}{B}\left[\frac{1}{\left(e^{\left(\frac{h\upsilon}{kT}\right)}-1\right)}\right]$$

Above equation is the expression for energy density

Condition for laser action:

1) **Meta Stable State:**

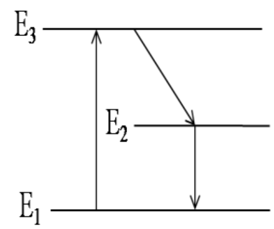
It is the special type of excited state where in the life time of atom is more than the normal excited state.

- This state plays an important role in lasing action.
- In metastable state, atoms stay for a time period of the order of 10^{-3} to 10^{-2} second.
- In normal excited state other than metastable atom stay of the order of 10^{-8} to10^{-9} seconds.
- It is possible to achieve population inversion condition in certain system which possesses a metastable state.

2) **Population Inversion:**

It is the state of the system at which the population of a higher energy level is greater than that of the lower energy level.

Let E_1, E_2, E_3 be the energy levels of the system $E_3>E_2>E_1$. E_2 is the metastable state of the system. Atoms get excited from the state E_1 to E_3 by means of external source and stay there for short time. These atoms undergo spontaneous transitions to E_2 and E_1. The atoms at the state E_2 stay for longer time. A stage is reached in which the number of atoms at state E_2 is more than the number of atoms at E_1 which is known as population inversion.

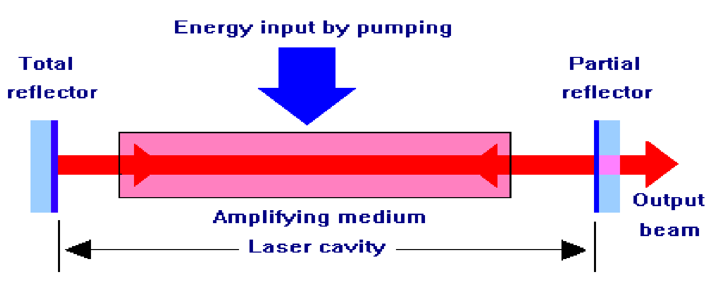


Requisites of a Laser System:

1) **The pumping process:**
It is the process of supplying energy to the medium in order to transfer it to the state of population inversion is known as pumping process
Optical Pumping: It is the process of exciting atoms from lower energy level to higher energy level by using high intensity light or by operating flash tube as an external source called optical pumping.
Electrical pumping: It is the process of exciting atoms from lower energy level to higher energy level by using dc power supply as an external source called electrical pumping.

2) **Active medium:**
It is a medium which supports population inversion and promotes stimulated emission leading to light amplification
Active centers: In a medium consisting of different species of atoms only small fraction of the atoms of a particular type are responsible for stimulated emission and consequent light amplification they are known as Active centers

3) **Laser cavity**.
An active medium bounded between two mirrors is called as a laser cavity.

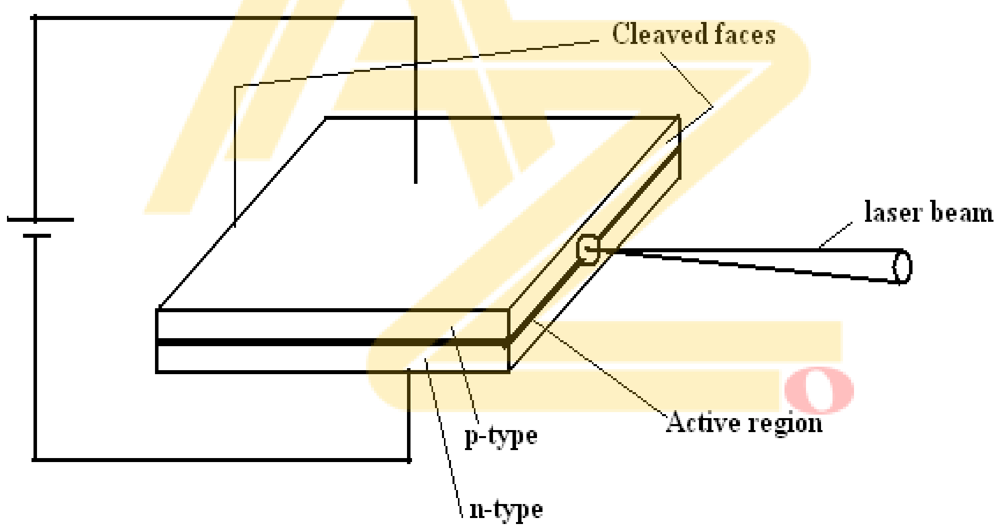


<u>Gallium-Arsenide Laser Semiconductor laser:</u>

A Semiconductor diode laser is one in which the active medium is formulated by semiconducting materials.

Construction:

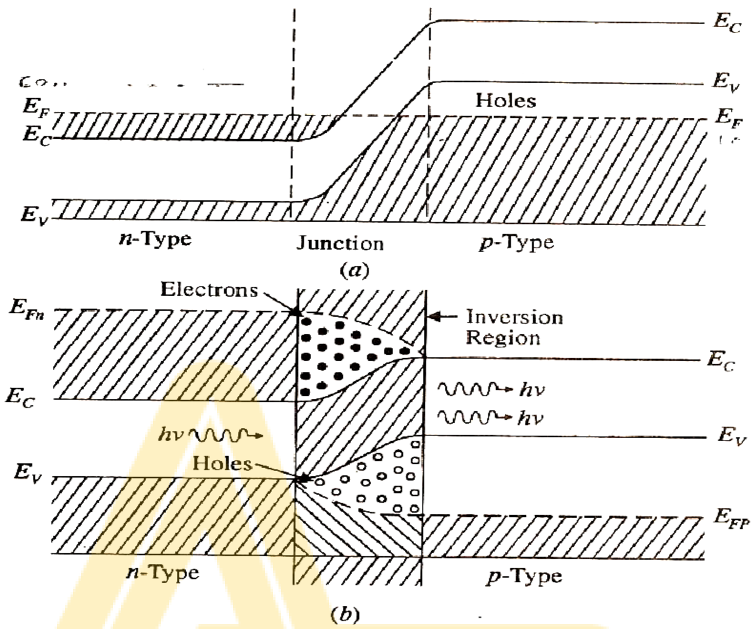
- Gallium-Arsenide Laser is a single crystal of GaAs consists of heavily doped n-type and p-type.
- The diode is very small size with sides of the order of 1mm.
- The width of the junction varies from 1-100μm.
- The top and bottom surfaces are metalized and Ohmic contacts are provided for external connection.
- The front and rear faces are polished. The polished faces functions as the resonant cavity. The other two faces are roughened to prevent lasing action in that direction.



Working:

- The energy band diagram of heavily doped p-n junction is as shown. At thermal equilibrium the Fermi level is uniform.
- Because of very high doping on **n- side**, the Fermi level is pushed in to the conduction band and electrons occupy the portions of the conduction band that lies below the Fermi level and

- on **p-side**, the Fermi level lies within the valence band and holes occupy the portions of the valence band that lies above the Fermi level.



Energy level diagram
(a) Before biasing (b) After biasing

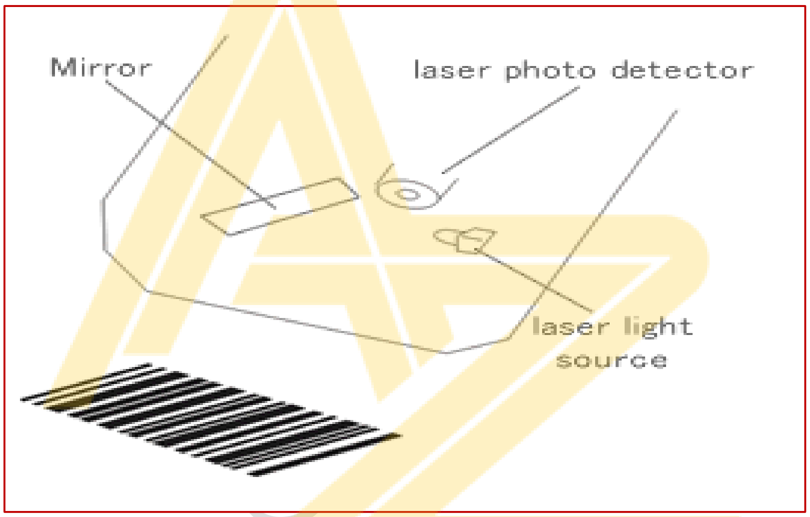
- A suitable forward bias is applied to overcome the potential barrier. As a result, electrons from n-region and holes from p-region injected into the junction.
- The current begins to flow following which there will be a region in junction in which the population inversion can be achieved.
- Initially concentration of electrons in the energy levels at the bottom of the conduction band will be less than that of energy levels at top of valence band. So that the recombination of electrons and holes result only in spontaneous emission.
- When the current exceeds the threshold value, population inversion is achieved in the active region which is formulated in the junction.
- At this stage the photons emitted by spontaneous emission triggers stimulated emission, over a large number of recombination leading to build up laser.
- Since the energy gap of GaAs is 1.4eV, the wavelength of emitted light is 8400 A^o .

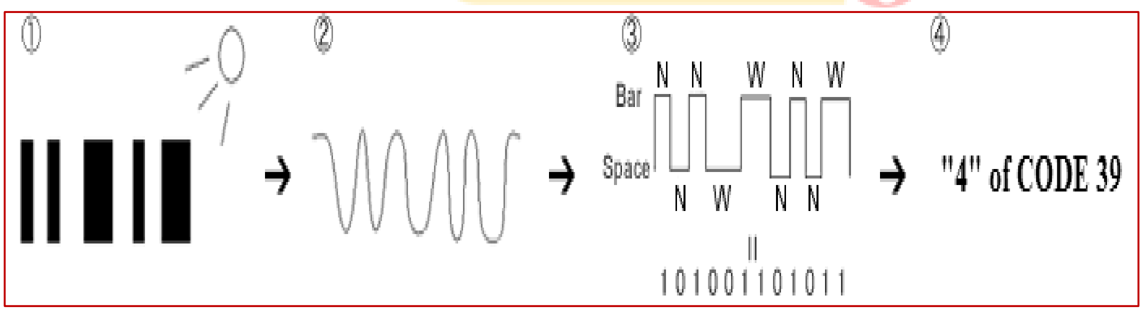
Properties of laser:

1. ***Coherence***: The emitted radiation after getting triggered is in phase with the incident radiation.
2. ***Monochromaticity***: The laser beam is highly monochromatic than any other radiations.
3. ***Unidirectionality***: Laser beam travels in only one direction. It can travel long distance without spreading.
4. ***Focusability***: A laser beam can be focused to an extremely fine spot.

Applications of Laser:

1. Laser barcode reading





- A bar code consists of a series of strips of dark and white bands. Each strip has a width of about 0.3 mm and the total width of the bar code is about 3 cm.

- Laser light is shine on the label surface and its reflection is captured by a sensor (laser photo detector) to read a bar code.
- A laser beam is reflected off a mirror and swept left and right to read a bar code, using laser allows reading of distant and wide bar code labels.
- Data retrieval is achieved when bar code scanners shine a light at a bar code, capture the reflected light and replace the black and white bars with binary digital signals.
- Reflections are strong in white areas and weak in black areas. A sensor receives reflections to obtain analog waveforms.
- The analog signal is converted into a digital signal via an A/D converter.
- Data retrieval is achieved when a code system is determined from the digital signal obtained. (Decoding process).

NOTE: Information such as the country of origin, manufacturer of the product, the direction of scan, price, reading error checking, weight of the product, and expiry date can be stored in the pattern of dark and white strips. By a simple scanning, complete information regarding the product can be obtained.

2. Laser Printing:

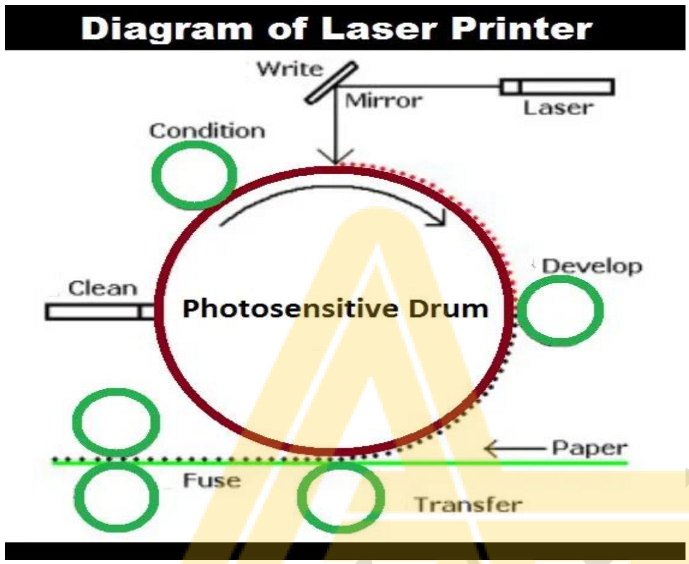
It is a digital printing device that are used to create the high quality text and graphics on a plain paper**. It reads the electronic data from your computer and beam this information onto a drum inside the printer which builds up a patron.**

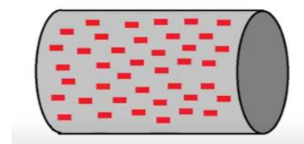
It mainly consists of six steps as follows

1. **Image Processing:** Once you click on print command on a certain document the computer immediately send the information to laser printer internal memory and prepare for image processing.
2. **Charging:** The printer starts warmer, the corona wire heats up and get ready to transfer its positive static charge to the metallic drum.
3. **Exposing:** When the drum rotates it receives a positive charge throughout its surface. A laser is then activated and reflected throughout the drum surface. The reflected laser beam creates an outline of print through the negative electrical charge.
4. **Developing:** In the areas where the laser beam hits the drum, the charge is changed from negative to positive using developer. The positive charged areas now represent where toner particles will adhere to the drum and be directly transferred onto the paper. The ink roller now begins to coat the drum with toner.
5. **Transferring:** When a paper is now passed close to the drum, charged toner particles adhere onto the page in the same pattern of the Image.

6. **Fussing:** The paper, now containing the inked content, is passed into the fuser unit where the rollers fuse the toner particles to the paper. The page is then passed through the other side of the copier and you now have one successful printout.

Before Drum completes its revolution it is cleaned from residual toner using cleaner.





Negatively charged Drum

Advantages of Laser Printer

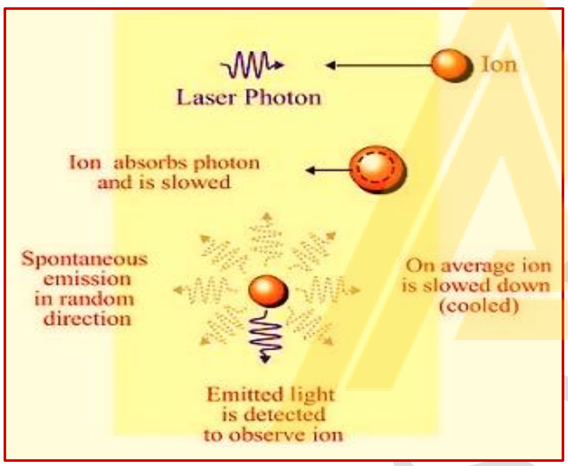
- The main advantage of Laser printer is its speed & efficiency at which prints high quality graphics & text.
- Laser printers produce high-quality output as compared to other printers.
- Laser printers are quite and does not produce disturbing sounds.
- They are also capable to produce color prints.

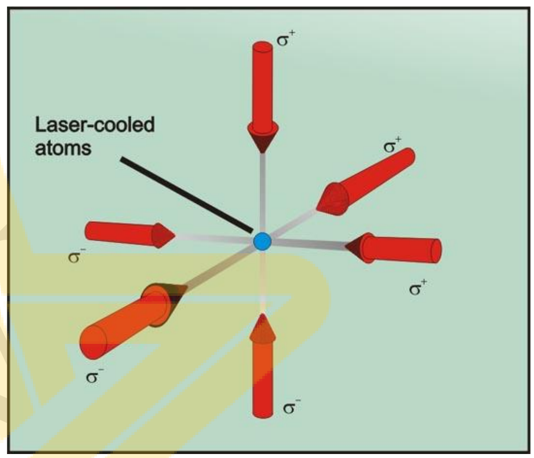
Disadvantages of Laser Printer

- The main disadvantage of Laser printer is its cost; they are relatively costly as compared to other printers.
- The maintenance, repair & servicing charges are also high of these printers.
- Laser printers emit small amount of ozone and are hazardous to health and the atmosphere.

3. Laser cooling

- In this technique, heat can be removed optically with the help of laser.
- Atoms can be cooled using lasers because light particles from the laser beam are absorbed and re-emitted by the atoms, causing them to lose some of their kinetic energy.
- Reduction in the momentum results in the reduction in temperature of atom i.e $P = \frac{E}{C} = \frac{h}{\lambda}$.
- After thousands of such impacts, the atoms will be chilled near to zero Kelvin.
- This cooling is also called Doppler cooling.





OPTICAL FIBERS

An optical fiber is a cylindrical wave guide made of transparent dielectric material (glass or plastic) which guides light waves along its length by total internal reflection.

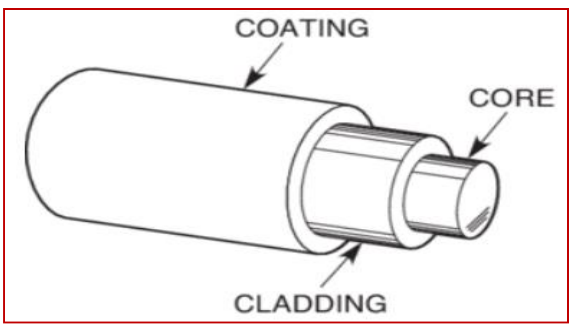
Principle

The propagation of light in an optical fiber from one end to the other end is based on the principle of Total internal reflection(TIR). They are used in optical communication.

When a light enters one end of the fiber, it undergoes successive total internal reflections from side walls and travels down the length of the fiber along zigzag path.

Construction

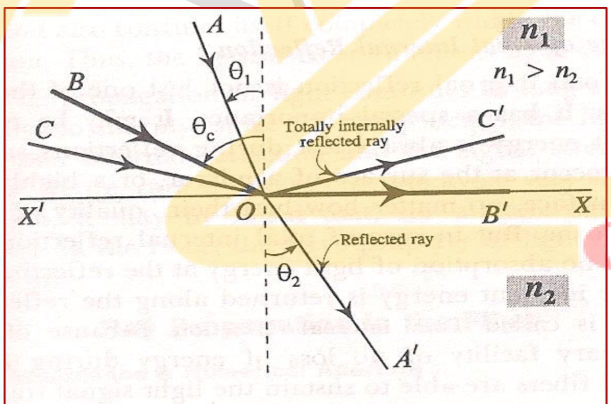
- A practical optical fiber is cylindrical in shape and has three regions.
- The innermost cylindrical region is the light guiding region called as core which is usually made up of glass or plastic.
- The outer part which is a concentric cylinder surrounding the core is called as cladding and is also made up of similar material but of lesser refractive index.
- The outermost region is called a Sheath or Protective buffer coating, nothing but the plastic coating providing a physical and environmental protection for the fiber. Number of such fibers is grouped to form a cable.



Total Internal Reflection:

- When a ray of light travels from denser to rarer medium it bends away from the normal.
- As the angle of incidence increases in the denser medium, the angle of refraction also increases. For a particular angle of incidence called the "*critical angle*" (θ_c), the refracted ray grazes the surface separating the media or the angle of refraction is equal to 90°.
- If the angle of incidence is further increased beyond the critical angle, the light ray is reflected back to the same medium. This is called *"Total Internal Reflection".*
- In total internal reflection, there is no loss of energy. The entire incident ray is reflected back.

Let XX^1 is the surface separating medium of refractive index n_1 and medium of refractive index n_2, $n_1 > n_2$. AO and OA^1 are incident and refracted rays. θ_1 and θ_2 are angle of incidence and angle of refraction, $\theta_2 > \theta_1$. For the ray BO, θ_c is the critical angle. OB^1 is the refracted ray which grazes the interface. The ray CO incident with an angle greater than θ_C is totally reflected back along OC^1.



TOTAL INTERNAL REFLECTION

From Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

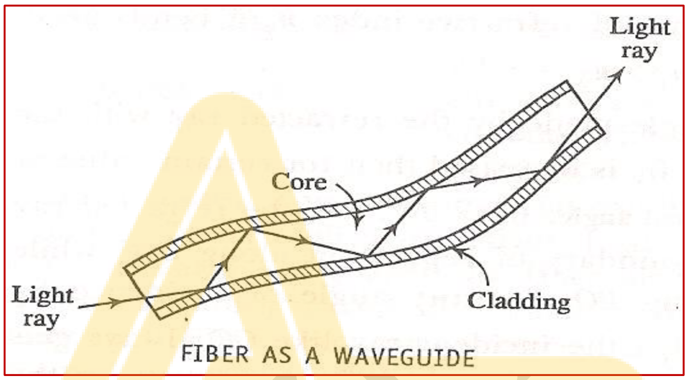
For total internal reflection, $\theta_1 = \theta_c$ and $\theta_2 = 90°$

$$n_1 \sin\theta_c = n_2 \quad \text{(because } \sin 90° = 1)$$

$$\therefore \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

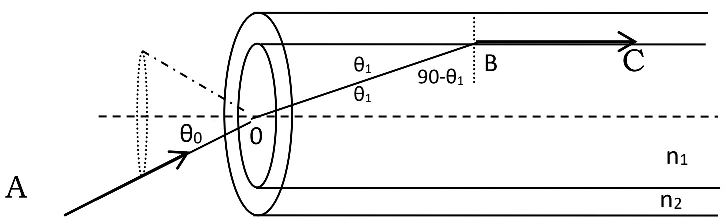
In total internal reflection there is no loss or absorption of light energy. The entire energy is returned along the reflected light. Thus is called Total internal reflection.

Propagation mechanism



- The cladding in an optical fiber always has a lower refractive index than that of the core.
- The light signal which enters into the core and strikes the interface of the core and cladding with an angle greater than the critical angle will undergo total internal reflection.
- Thus the light signal undergoes multiple reflections within the core and propagates through the fiber.
- Since each reflection is a total internal reflection, there is no absorption of light energy at the reflecting surface.
- Therefore, the signal sustains its strength and also confines itself completely within the core during the propagation.
- After series of such total internal reflection, it emerges out of the core. Thus the optical fiber works as a waveguide. Care must be taken to avoid very sharp bends in the fiber because at sharp bends, the light ray fails to undergo total internal reflection.

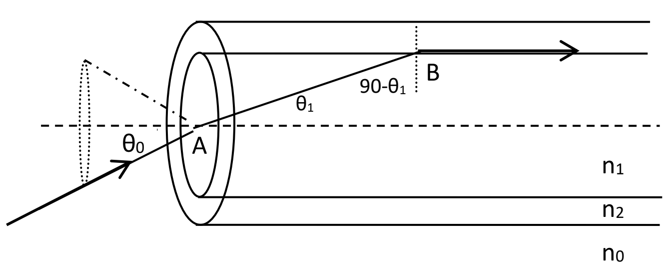
Acceptance angle and numerical aperture



Surrounding medium (n_0)

- Consider a light ray entering into the core of an optical fiber with an angle of incidence(θ_0), such that after entering, the ray incidents on the core-cladding interface with an angle of incidence equal to the critical angle.
- From figure it is clear that any ray which enters into the core with an angle more than θ_0, will have to be incident at an angle less than the critical angle at the core-cladding interface.
- Therefore, the ray does not undergo total internal reflection and the ray will be lost. Thus for any ray to propagate through the fiber it must enter with an angle less than θ_0. This maximum angle is called as '<u>Acceptance angle</u>' and the conical surface described by the ray when rotated about the axis of the fiber is called '<u>Acceptance cone</u>'.
- Thus acceptance angle is defined as **"The maximum angle that a light ray can take relative to the axis of the fiber to propagate through the fiber".**
- **Sine of the acceptance angle of an optical fiber is called as "Numerical aperture".**

Expression for Numerical aperture and condition for propagation



Consider a light ray entering into the core of an optical fiber with an angle of incidence(θ_0), such that after entering, the ray incidents on the core-cladding interface with an angle of incidence equal to the critical angle.

Let n_0, n_1 and n_2 are the refractive indices of the surrounding medium, core and cladding respectively.

Now, applying Snell's law at the point of entry of the ray i.e., at A,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

$$\sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1 \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Applying Snell's law at B,

$$n_1 \sin(90-\theta_1) = n_2 \sin 90$$

$$n_1 \cos\theta_1 = n_2 \sin 90$$

$$\Rightarrow \cos\theta_1 = \frac{n_2}{n_1}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From expression (1) $\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\cos^2\theta_1}$

Substituting for $\cos\theta_1$ from (2)

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{\frac{n_1^2-n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{\sqrt{n_1^2-n_2^2}}{n_0}$$

If n_0=1 i.e., surrounding medium if it is air

$$\sin\theta_0 = \sqrt{n_1^2-n_2^2}$$

i.e., $N.A. = \sqrt{n_1^2-n_2^2}$

Condition for propagation:

If θ_i is the angle of incidence of the incident ray, then the ray will be able to propagate,

$$\text{if } \theta_i < \theta_0$$

$$\Rightarrow if\ sin\,\theta_i < sin\,\theta_0$$

$$or\ sin\,\theta_i < \sqrt{n_1^2 - n_2^2}$$

$$i.e., sin\,\theta_i < N.A.$$

Fractional index change(Δ)

The ratio of the difference in refractive index of core and cladding to the refractive index of core of an optical fiber. $i.e., \Delta = \frac{n_1 - n_2}{n_1}$

Refractive index profile:

The curve which represents the variation of refractive index when it moves radially outwards from the fiber axis is called refractive index profile.

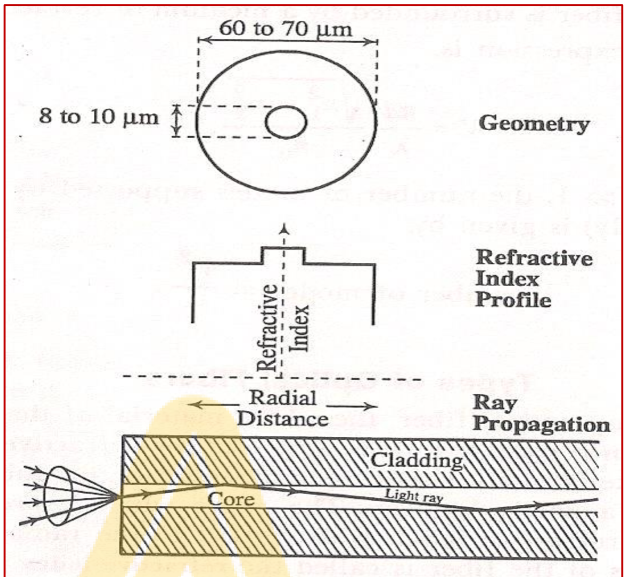
Types of optical fibers

Based on the refractive index profile and mode of propagation, there are three types of optical fibers,

1. Single mode fiber
2. Step index multimode fiber
3. Graded index multimode fiber

(i) Single mode fiber

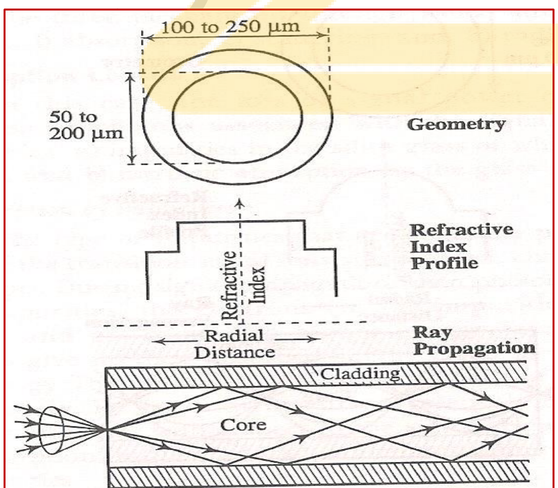
- Single mode fibers have a core material of uniform refractive index value.
- Cladding material also has a uniform refractive index but of lesser value than that of core.
- Thus its refractive index profile takes a shape of a step. The diameter of the core is about 8-10 μm and the diameter of the cladding is about 60-70 μm.



SINGLE MODE FIBER

- Because of its narrow core, it can guide just a single mode as shown in above figure.
- Single mode fibers are the extensively used ones ant they are less expensive. They need LASERs as the source of light.

(ii) <u>Step index multimode fiber</u>

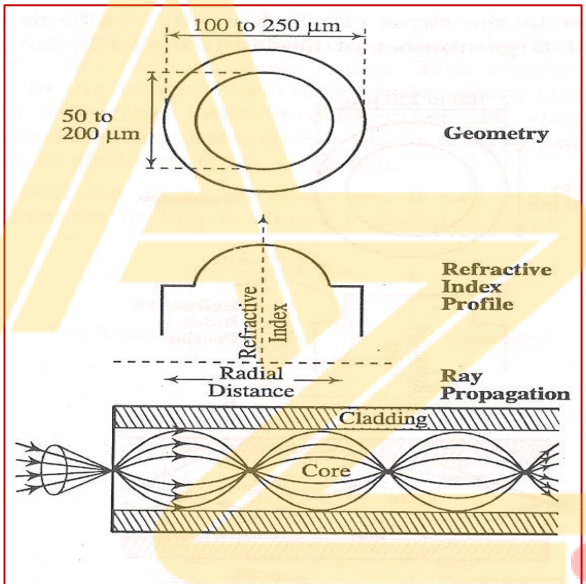


STEP - INDEX MULTIMODE FIBER

- A step index multimode fiber is very much similar to the single mode fiber except that its core is of large diameter. A typical fiber has a core diameter 50 to 200 μm and a cladding about 100 to 250μm outer diameter.

- Its refractive index profile is also similar to that of a single mode fiber but with a larger plane region for the core.
- Due to the large core diameter it can transmit a number of modes of wave propagation.
- The step index multimode fiber can accept either a LASER or an LED as source of light.
- It is the least expensive of all and its typical application is in data links which has lower bandwidth requirements.

(iii) <u>Graded index multimode fiber</u>



GRADED - INDEX MULTIMODE FIBER

- It is also called GRIN.
- The refractive index of core decreases in the radially outward direction from the axis of the fiber and becomes equal to that of cladding at the interface but the refractive index of the cladding remains uniform.
- Laser or LED is used as a source of light.
- It is the expensive of all. It is used in telephone trunk between central offices.

Signal attenuation in optical fibers

- Attenuation is the loss of optical power suffered by the optical signal as it propagates through a fiber also called as the fiber loss.
- There are three mechanisms through which attenuation takes place.

Attenuation co-efficient

- The attenuation of a fiber optic cable is expressed in decibels.

i.e., $$\alpha = -\frac{10}{L} log\left[\frac{P_{out}}{P_{in}}\right] \quad \frac{dB}{km}$$

- The main reasons for the loss in light intensity over the length of the cable is due to light absorption, scattering and due to bending losses.

Attenuation can be caused by three mechanisms.

(i) Absorption losses

- Absorption of photons by impurities like metal ions such as iron, chromium, cobalt and copper in the silica glass of which the fiber is made of.
- During signal propagation photons interact with electrons of impurity atoms and the electrons are excited to higher energy levels.
- Then the electrons give up their absorbed energy either in the form of heat or light energy.
- The re-emission of light energy will usually be in a different wavelength; hence it is referred as loss of energy.
- The other impurity such as hydroxyl (OH) ions which enters into the fiber at the time of fabrication causes significant absorption loss.
- The absorption of photons by fiber itself assuming that there are no impurities and in-homogeneities in it is called as *intrinsic absorption*.

(ii) Scattering losses

- Scattering of light waves occurs whenever a light wave travels through a medium having scattering objects whose dimensions are smaller than the wavelength of light.
- Similarly, when a light signal travels in the fiber, the photons may be scattered due to the sharp changes in refractive index values inside the core over distances and also due to the structural impurities present in the fiber material.
- This type of scattering is called as Rayleigh scattering. Scattering of

photons also takes place due to trapped gas bubbles which are not dissolved at the time of manufacturing.

- A scattered photon moves in random direction and leaves the fiber.

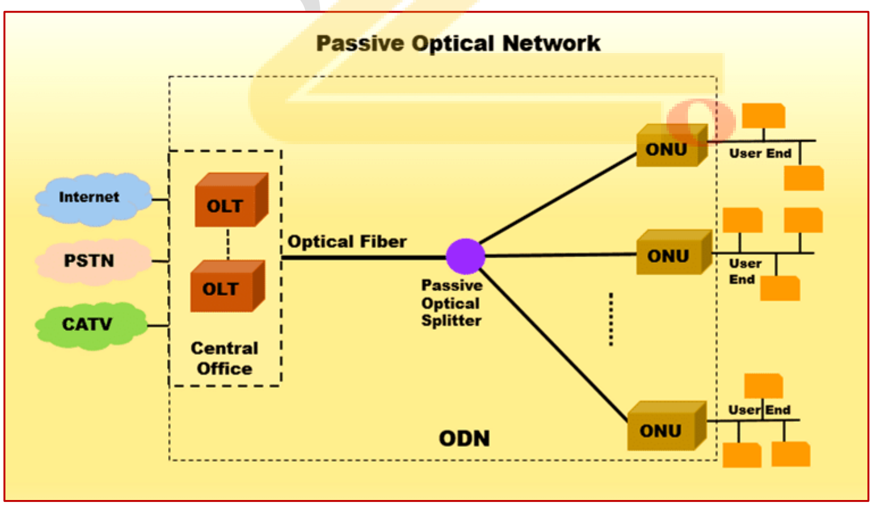
(iii) Radiation losses

Radiation losses occur due to macroscopic bends and microscopic bends.

- ***Macroscopic bending:*** All optical fibers are having critical radius of curvature provided by the manufacturer. If the fiber is bent below that specification of radius of curvature, the light ray incident on the core cladding interface will not satisfy the condition of total internal reflection. This causes loss of optical power.
- ***Microscopic bending:*** Microscopic bends are repetitive small scale fluctuations in the linearity of the fiber axis. They occur due to non-uniformities in the manufacturing and also lateral pressure built up on the fiber. They cause irregular reflections and some of them leak through the fiber. The defect due to non-uniformity (micro-bending) can be overcome by introducing optical fiber inside a good strengthen polyurethane jacket.

Applications of Optical fibers:

1. Optical fiber Networking:



- The 'passive' part of the nomenclature refers to the fact that while the optical signal is traversing the network, there are no active electronic parts, and no power is needed.
- A typical PON is comprised of multiple ONUs *(optical network units)* and an OLTs *(optical line terminations)*. Generally, an OLT is located at the central office of the server provider, with as many as 32 ONUs situated close to the end users.
- A PON uses non-powered optical splitters to separate signals as they progress through the network, as a splitter can take a single input and separate the signal to transmit to multiple users, sharing strands of fiber optics for different parts of network architecture.
- PONs only require power at the transmitting and receiving ends of the network and can serve up to 32 users with a single strand of fiber, they offer an option that's both cheaper to build and to maintain.

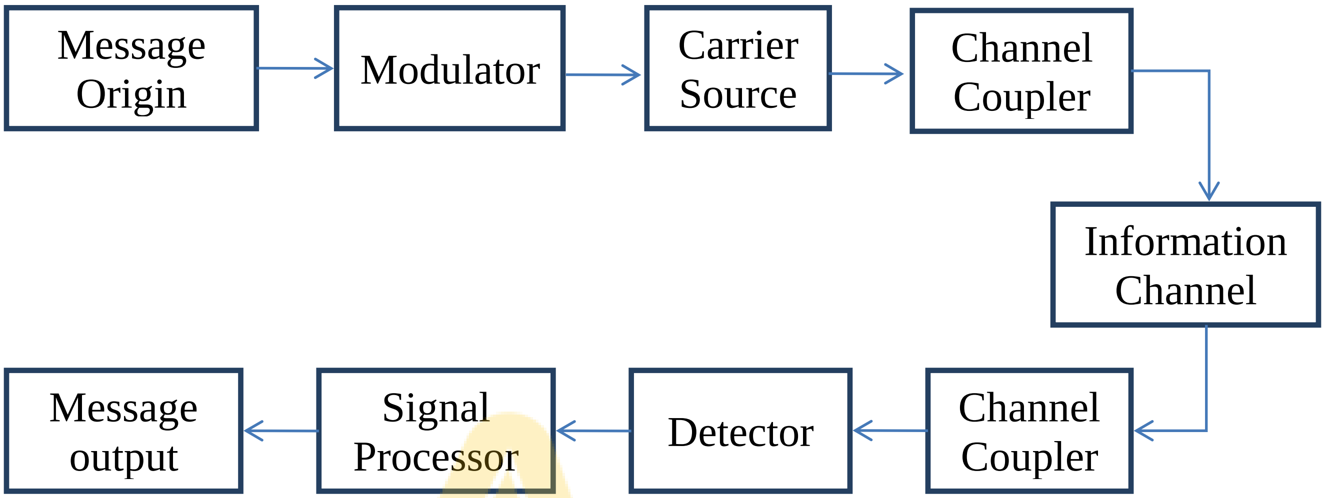
The Benefits of a PON

The reason they became so popular is that they come with several benefits:

- Reduced operational costs
- Lower installation costs
- Reduced network energy costs
- A reduction in required network infrastructure
- No requirement for network switches
- IDF real estate can be reclaimed

2. Point to point optical fiber communication System

Optical fiber communication system consists of transmitter, information channel and receiver. Transmitter converts an electrical signal into optical signal. Information channel carries the signal from transmitter to receiver. The receiver converts optical signal to electrical form. The block diagram of optical fiber communication system is shown in fig.



Message origin: It converts a non-electrical message into an electrical signal.

Modulator: It converts the electrical message into proper format and it helps to improve the signal onto the wave which is generated by the carrier source.

There are two types of format. They are Analog and digital. Analog signal is continuous and it doesn't make any change in the original format. But digital signal will be either in ON or OFF state.

Carrier source: It generates the waves on which the data is transmitted. These carrier waves are produced by the electrical oscillator. Light emitting diodes (LED) and laser diodes (LD) are the different sources.

Channel Coupler: (Input) The function of the channel coupler is to provide the information to information channel. It can be an antenna which transfers all the data.

Information channel: It is path between transmitter and receiver. There are two types of information channel. They are guided and unguided. Atmosphere is the good example for unguided information channel. Co-axial cable, two-wire line and rectangular wave guide are example for guided channel.

Channel Coupler: (Output) The output coupler guides the emerged light from the fiber on to the light detector.

Detector: The detector separates the information from the carrier wave. Here a photo-detector converts optical signal to electronic signal.

Signal processor: Signal processor amplifies the signals and filters the undesired frequencies.

Message output: The output message will be in two forms. Either person can see the information or hear the information. The electrical signal can be converted into sound wave or visual image by using CRO.

Advantages of optical fibers:

- Optical fibers are cheaper, small in size, light weight, mechanically strong and signal carrying capacity is high.
- They are immune to electromagnetic and RF interferences.
- The optical fibers have wider bandwidth so capable of carry more channels of information than electrical cables.
- It is compatible with electronic systems and tapping of signal is not possible.
- They have low loss per unit length (~2 dB/Km).
- It does not get affected by nuclear radiations, corrosion and moisture.
- No sparks are generated because the signal is optical signal.

Limitations:

- Optical fibers undergo expansion and contraction with temperature which upset little alignments that lead to loss in signal power.
- Because of some accidents or when fiber bent to circles of smaller radius, signal loss takes place or the fiber may break.
- Joining of two strands of a fiber (i.e., splicing) needs skill full work.
- High end maintenance is required.

***********ALL THE BEST***********

MODULE- 2

QUANTUM MECHANICS

Dual nature of matter (de-Broglie Hypothesis)

Dual nature of light:

The concept of photoelectric effect and Compton Effect gives the evidence for particle nature of light. Where as in physical optics the phenomenon like interference, diffraction, superposition was explained by considering wave nature of light. This is wave particle duality of light.

Dual nature of matter:

On the basis of above concept (dual nature of light), in 1923, Louis de Broglie gave a hypothesis

"Since nature loves symmetry, if the radiation behaves as particles under certain conditions and as waves under certain conditions, then one can expect that, the entities which ordinarily behaves as particles (ex. Like electrons, protons, neutrons) must also exhibit properties attributable to waves under appropriate circumstances". This is known as **deBroglie hypothesis**

Matter is made up of discrete constituent particles like atoms, molecules, protons, neutrons and electrons, hence matter has particle nature. Wave nature of matter is experimentally observed by Davisson and Germer and G.P Thomson experiments. Hence matter also exhibit wave particle duality.

The waves associated with the moving particles are called de Broglie waves or matter waves or pilot waves.

Characteristics of matter waves:

1. Waves associated with moving particles are called matter waves. The wavelength 'λ' of a de-Broglie wave associated with particle of mass 'm' moving with velocity 'v' is
   $\lambda = h/(mv)$
2. Matter waves are not electromagnetic waves because the de Broglie wavelength is independent of charge of the moving particle.

3. The amplitude of the matter wave depends on the probability of finding the particle in that position.
4. The speed of matter waves depends on the mass and velocity of the particle associated with the wave.

Debroglie's Wavelength:

A particle of mass 'm' moving with velocity 'c' possess energy given by

$E = mc^2$ → **(**Einstein's Equation) (1)

According to Planck's quantum theory the energy of quantum of frequency 'ʋ' is

$E = h\upsilon$ → (2)

From (1) & (2)

$mc^2 = h\upsilon = hc/\lambda$ since $\upsilon = c/\lambda$

$\lambda = hc/mc^2 = h/mc$

$\lambda = h/mv$ since $v \approx c$

De Broglie wavelength of a free particle in terms of its kinetic energy

Consider a particle, since the particle is free, the total energy is same as

$$E = \frac{1}{2}m\mathrm{v}^2 = \frac{p^2}{2m}$$

Where 'm' is the mass, 'v' is the velocity and 'p' is the momentum of the particle.

$$p = \sqrt{2mE}$$

The expression for de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

Debroglie Wavelength of an Accelerated Electron:

If an electron accelerated with potential difference 'V' the work done on the 'eV', which is converted to kinetic energy.

Then

$$eV = \frac{1}{2}mv^2 \rightarrow (1)$$

If 'p' is the momentum of the electron, then p=mv

Squaring on both sides, we have

$p^2 = m^2v^2$

$mv^2 = p^2/m$

Using in equation (1) we have

$eV = p^2/(2m)$

or $p = \sqrt{2meV}$

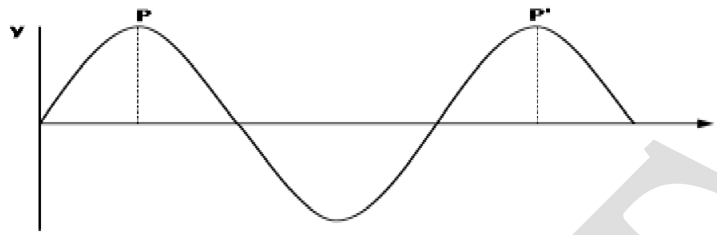
According to de-Broglie $\lambda = h/p$

Therefore $\lambda = \left[\frac{h}{\sqrt{2meV}}\right]$

$$\lambda = \frac{1}{\sqrt{V}}\left[\frac{6.626\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.602\times10^{-19}}}\right] = \frac{1.226\times10^{-9}}{\sqrt{V}}\,\text{m}, \qquad \lambda = \frac{1.226}{\sqrt{V}}\,\text{nm}$$

Phase Velocity (V_{phase}) :

A progressive wave travelling along x-direction is represented by

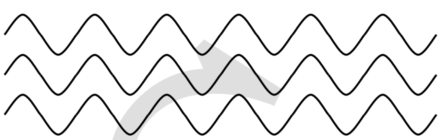


If 'p' is the point on a progressive wave, then it is the representative point for a particular phase of the wave, the velocity with which it is propagated owing to the motion of the wave is called *phase velocity.*

The phase velocity of a wave is given by $v_{phase} = (\omega / k)$.

Group Velocity (V_{group}) :

When a group of two or more waves of slightly different wavelengths superimposed on each other, the resultant pattern is in the variation in amplitude, represents the wave group called wave packet. The velocity with which the wave packet is moving called group velocity of the waves and is given by $v_{group} = \frac{d\omega}{dk}$



Individual Waves

Wave Packet

Heisenberg's Uncertainty Principle:

According to classical mechanics a particle occupies a definite place in space and possesses a definite momentum. If the position and momentum of a particle is known at any instant of time, it is possible to calculate its position and momentum at any later instant of time. The path of the particle could be traced.

This concept breaks down in quantum mechanics leading to Heisenberg's Uncertainty Principle.

Heisenberg's Uncertainty Principle states that "It is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa".

If Δx and ΔP_x are the uncertainties in the measurement of position and momentum of the particle then the uncertainty can be written as

$$\Delta x \, . \, \Delta P_x \geq (h/4\pi)$$

In any simultaneous determination of the position and momentum of the particle, the product of the corresponding uncertainties inherently present in the measurement is equal to or greater than h/4π.

Similarly, 1) $\Delta E.\Delta t \geq h/4\pi$ 2) $\Delta L.\Delta\theta \geq h/4\pi$

Significance of Heisenberg's Uncertainty Principle:

Heisenberg's Uncertainty Principle asserts that it is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa. Therefore, one should think only of the probability of finding the particle at a certain position or of the probable value for the momentum of the particle.

Application of Uncertainty Principle:

Non-existence of electrons in the atomic nucleus:

The energy of a particle is given by

$$E=\frac{1}{2}mv^2 = \frac{p^2}{2m} \qquad (1)$$

Heisenberg's uncertainty principle states that

$$\Delta x \,.\, \Delta P_x \geq \frac{h}{4\pi} \rightarrow (4)$$

The diameter of the nucleus is of the order 10^{-14}m. If an electron is to exist inside the nucleus, the uncertainty in its position Δx must not exceed 10^{-14}m.

i.e. $\Delta x \leq 10^{-14}$m

The minimum uncertainty in the momentum

$$(\Delta P_x)_{\min} \geq \frac{h}{4\pi\, (\Delta x)_{\max}} \geq \frac{6.63\times10^{-34}}{4\pi\times10^{-14}} \geq 0.527 \times 10^{-20} \text{ kg. m/s}$$

By considering minimum uncertainty in the momentum of the electron

i.e., $(\Delta P_x)_{\min} \geq 0.5 \times 10^{-20}$ kg.m/s = p → (2)

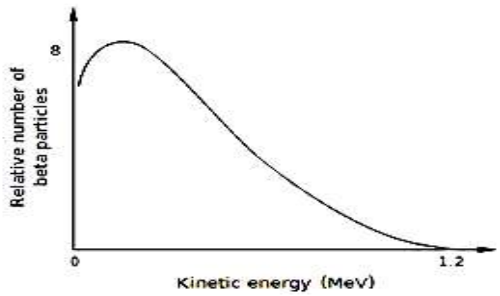
Consider eqn (1)

$$E= \frac{p^2}{2m} = \frac{(0.5\times10^{-20})^2}{2\times9.1\times10^{-31}} = 1.531\times10^{-11} = 95.68 \text{ MeV}$$

Where m_o= 9.11 × 10^{-31} kg

If an electron exists in the nucleus its energy must be greater than or equal to 95.68 MeV. It is experimentally measured that the beta particles ejected from the nucleus during beta decay have energies of about 3 to 4 MeV. This shows that electrons cannot exist in the nucleus.

[**Beta decay:** In beta decay process, from the nucleus of an atom, when neutrons are converting into protons in releasing an electron (beta particle) and an antineutrino. When proton is converted into a neutron in releasing a positron (beta particle) and a neutrino. In both the processes energy sharing is statistical in nature. When beta particles carry maximum energy neutrino's carries minimum energy and vice-versa. In all other processes energy sharing is in between maximum and minimum energies. The maximum energy carried by the beta particle is called as the end point energy (E_{max}).



Principle of complementarity:

Statement: Principle of complementarity as stated by Bohr "In a situation where the wave aspect of the system is revealed, its particle aspect is concealed (hidden) and in a situation where the particle aspect is revealed its wave aspect is concealed (hidden). Revealing both simultaneously is impossible; the wave and aspects are complementary."

Note: Meaning of complementary: things are different from each other but make a good combination.

Explanation: If an experiment is designed to measure the particle nature of matter, during this experiment errors of measurement of both position and time is zero and hence and hence momentum, energy and the wave nature of the matter are completely unknown. and vice versa.

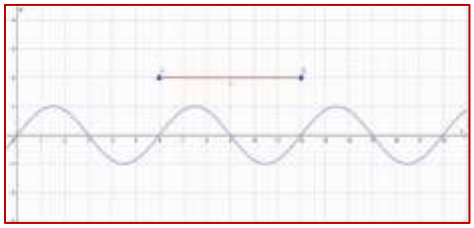
Correlation between Heisenberg's Uncertainty Principle, Debroglie wavelength and wave packet:

Although the uncertainty principle deals with many non-commute operators. if you certainly know the wavelength of the matter wave associated with the particle, you certainly know the momentum of the particle.

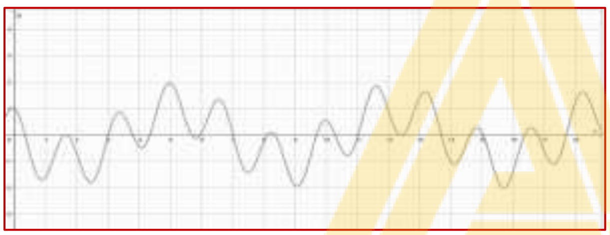
Given a wave function if you can tell its wavelength you will know the momentum but its position will be uncertain.

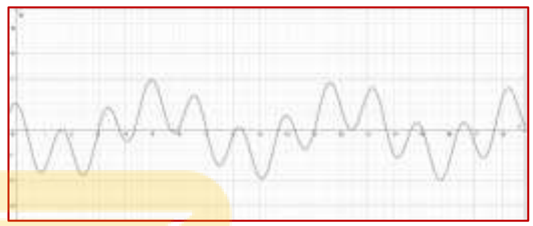
Note: In all the images only real part of the wave function is shown.

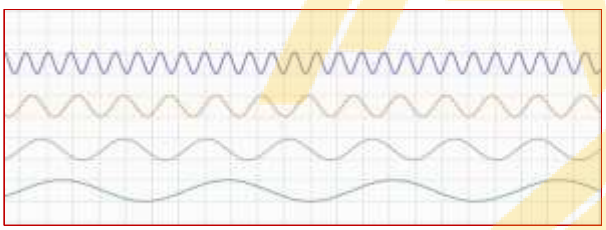
The wavelength of the following matter wave. The wavelength is the length of the red segment"

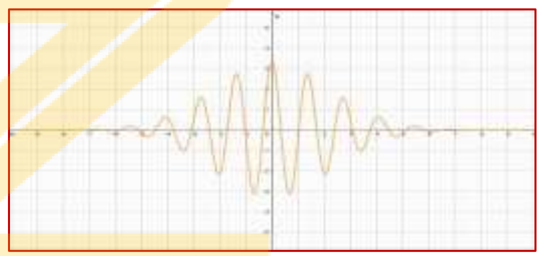


In this case you can certainly point out the wavelength; so you certainly know the momentum but you can't tell the position of the particle— it may be anywhere on the x-axis i.e the position uncertainty is very very high.









In any of the above three cases you cannot tell the wavelength of the matter wave.

For example, in the last case the wavelength uncertainty is large. This is in fact a superposition of many waves whose wavelength can be found. Such a wave packet is made up of many waves. So if you try to measure the wavelength of this wave packet you will get the wavelength of any of the waves shown above. Upon large number of observations, you will have many wavelengths and then calculate the standard deviation (the uncertainty). This will lead you to uncertainty in momentum which will of course be large (because you got a large number of wavelengths). But as you see the uncertainty in position will be comparatively smaller.

If the uncertainty in the de Broglie wavelength is large(small), the uncertainty in momentum is large(small) and consequently the uncertainty in position is small(large).

Wave Function:

A physical situation in quantum mechanics is represented by a function called wave function. It is denoted by 'ψ'. It accounts for the wave like properties of particles. Wave function is obtained by solving Schrodinger equation.

Mathematically it is given by

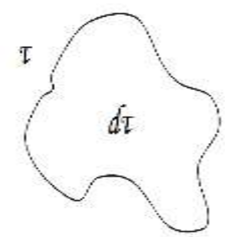
$$\psi = Ae^{i(kx-\omega t)}$$

Physical significance of wave function:

The wave function itself has no physical significance, the physical significance is given by a function called probability density or probability function.

Probability density:

If ψ is the wave function associated with a particle, then $|\psi|^2$ is the probability of finding a particle in unit volume. If 'τ' is the volume in which the particle is present but where it is exactly present is not known. Then the probability of finding a particle in certain elemental volume dτ is given by $|\psi|^2 d\tau$. Thus $|\psi|^2$ is called probability density. The probability of finding an event is real and positive quantity. In the case of complex wave functions, the probability density is $|\psi|^2 = \psi^* \psi$, where ψ* is Complex conjugate of ψ.



Max Born interpretation of wave function:

We know $\psi = Ae^{i(kx-\omega t)}$

Complex conjugate of ψ is given by $\Psi^* = Ae^{-i(kx-\omega t)}$

probability density is $|\psi|^2 = \psi\, \psi^* = A^2$, Where A = Square of amplitude.

According to max born interpretation, as square of the amplitude A^2 for electromagnetic waves represent Intensity of the wave. In quantum mechanics square of the amplitude A^2 represent the probability of finding the particle in certain position.

Normalization:

The probability of finding a particle having wave function 'ψ' in a volume 'dτ' is '|ψ|²dτ'. If it is certain that the particle is present in finite volume 'τ', then

$$\int_0^{\tau} |\psi|^2 \, d\tau = 1$$

If we are not certain that the particle is present in finite volume, then

$$\int_{-\infty}^{\infty} |\psi|^2 d\tau = 1$$

In some cases $\int |\psi|^2 d\tau \neq 1$ and involves constant.

The process of integrating the square of the wave function within a suitable limits and equating it to unity the value of the constant involved in the wave function is estimated. The constant value is substituted in the wave function. This process is called as normalization.

The wave function with constant value included is called as the normalized wave function and the value of constant is called normalization factor.

Expectation value:

In Quantum mechanics, the expectation value is the probabilistic expected value of the result (measurement) of an experiment, Can be thought of as an average of all the possible outcomes of a measurement.

Expectation value as such is not the most probable value of the measurement.

Explanation: The result of measurement of the position x isa continuous random variable. Consider a wave function ψ(x). The $|\psi|^2$ value is the probability density for the position and $|\psi|^2 dx$ is the probability of finding the particle between x and x+dx. If the measurement is repeated many times, many possible outcomes are possible and the expectation value of these out comes can be expressed as

$$< x >= \int_{-\infty}^{\infty} |\psi|^2 \, dx$$

Time independent Schrodinger wave equation

Consider a particle of mass 'm' moving with velocity 'v'. The de-Broglie wavelength 'λ' is

$$\lambda = \frac{h}{mv} = \frac{h}{P} \rightarrow (1)$$ Where 'mv' is the momentum of the particle.

The wave eqn is

$$\psi = Ae^{i(kx-\omega t)} \rightarrow (2)$$

Where 'A' is a constant and 'ω' is the angular frequency of the wave.

Differentiating equation (2) with respect to 't' twice

$$\frac{d^2\psi}{dt^2} = -A\omega^2 e^{i(kx-\omega t)} = -\omega^2\psi \quad \rightarrow (3)$$

The equation of a travelling wave is

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

Where 'y' is the displacement and 'v' is the velocity.

Similarly, for the de-Broglie wave associated with the particle

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2} \rightarrow (4)$$

where 'ψ' is the displacement at time 't'.

From eqns (3) & (4)

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2}\psi$$

But $\omega = 2\pi\upsilon$ and $v = \upsilon\lambda$ where 'υ' is the frequency and 'λ' is the wavelength.

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi \quad \text{or} \quad \frac{1}{\lambda^2} = -\frac{1}{4\pi^2\psi}\frac{d^2\psi}{dx^2} \rightarrow (5)$$

$$K.E = \frac{1}{2}m\text{v}^2 = \frac{m^2\text{v}^2}{2m} = \frac{P^2}{2m} \rightarrow (6)$$

$$= \frac{h^2}{2m\lambda^2} \rightarrow (7)$$

Using eqn (5)

$$K.E = \frac{h^2}{2m}\left(-\frac{1}{4\pi^2\psi}\right)\frac{d^2\psi}{dx^2} = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2} \rightarrow (8)$$

Total Energy E = K.E + P.E

$$E = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2} + V$$

$$E - V = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2}$$

$$\frac{d^2\psi}{dx^2} = -\frac{8\pi^2 m}{h^2}(E - V)\psi$$

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0}$$

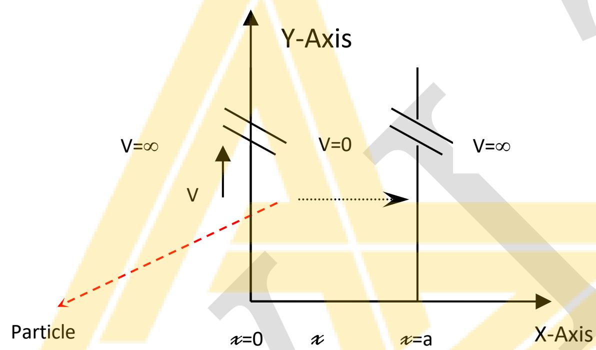
This is the time independent Schrodinger wave equation for one dimensional case.

For three dimensional case it can be eritten as follows.

$$\boxed{[\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2}] + \frac{8\pi^2 m}{h^2}(E - V)\psi(x, y, z) = 0}$$

Application of Schrodinger wave equation:

Energy Eigen values of a particle in one dimensional, infinite potential well (potential well of infinite depth) or of a particle in a box



Consider a particle of a mass 'm' free to move in one dimension along positive x-direction between x=0 to x=a. The potential energy outside this region is infinite and within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box. The Schrödinger equation outside the well is

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}(E-\infty)\psi=0 \rightarrow (1) \qquad \because V = \infty$$

For outside, the equation holds good if ψ = 0 & |ψ|² = 0. That is particle cannot be found outside the well and also at the walls

The Schrodinger's equation inside the well is:

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}E\psi=0 \rightarrow (2) \qquad \because V = 0$$

Let $\frac{8\pi^2 m}{h^2}E = k^2 \quad \rightarrow (3)$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The solution of above equation is:

$\psi = C \cos kx + D \sin kx \rightarrow (4)$

at $x = 0 \rightarrow \psi = 0$

$0 = C \cos 0 + D \sin 0$

$\therefore C = 0$

Also $x = a \rightarrow \psi = 0$

$0 = C \cos ka + D \sin ka$

But C = 0

$\therefore D \sin ka = 0 \longrightarrow (5)$

$D \neq 0$ (because the wave concept vanishes)

i.e. $ka = n\pi$ where $n = 0, 1, 2, 3, 4\ldots$ (Quantum number)

$k = \frac{n\pi}{a} \rightarrow (6)$

sub eqn (5) and (6) in (4)

$$\psi_n = D \sin \frac{n\pi}{a} x \rightarrow (7)$$

This gives permitted wave functions.

The Energy Eigen value given by

Substitute equation (6) in (3)

$$\frac{8\pi^2 m}{h^2}E = k^2 = \frac{n^2\pi^2}{a^2}$$

$$E = \frac{n^2 h^2}{8ma^2}$$

This is the expression for energy Eigen value.

For n = 0 is not acceptable inside the well because $\psi_n = 0$. It means that the electron is not present inside the well which is not true. Thus the lowest energy value for n = 1 is called zero point energy value or ground state energy.

i.e. $E_{zero\text{-}point} = \frac{h^2}{8ma^2}$

The states for which n >1 are called exited states.

To find out the value of D, normalization of the wave function is to be done.

i.e. $\int_0^a |\psi_n^2| dx = 1 \rightarrow (8)$

using the values of ψ_n from eqn (7)

$$\int_0^a D^2 \sin^2 \frac{n\pi}{a} x dx = 1$$

$$D^2 \int_0^a \left[\frac{1-\cos(2n\pi/a)x}{2}\right] dx = 1 \qquad \because \sin^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)$$

$$\frac{D^2}{2}\left[\int_0^a dx - \int_0^a \cos\frac{2n\pi}{a} x dx\right] = 1$$

$$\frac{D^2}{2}\left[x - \frac{a}{2n\pi}\sin\frac{2n\pi}{a}x\right]_0^a = 1$$

$$\frac{D^2}{2}[a-0] = 1$$

$$\frac{D^2}{2}a = 1$$

$$D = \sqrt{\frac{2}{a}}$$

Substitute D in equation (7)

the normalized wave functions of a particle in one dimensional infinite potential well is:

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x \rightarrow (9)$$

Eigen functions:

Eigen functions are those wave functions in Quantum mechanics which possesses the following properties:

1. They are single valued.

2. Finite everywhere and

3. The wave functions and their first derivatives with respect to their variables are continuous.

Eigne values:

If the wave function is operated by a quantum mechanical operator such that we get back the wavefunction back multiplied by some constant is called as Eigne value.

$\hat{H}(\psi) = \lambda(\psi)$

Where λ = Constant → is called as eigen value

If the quantum mechanical operator is Energy operator, then λ is termed as energy eigen value.

Wave functions, probability densities and energy levels for particle in an infinite potential well:

Let us consider the most probable location of the particle in the well and its energies for first three cases.

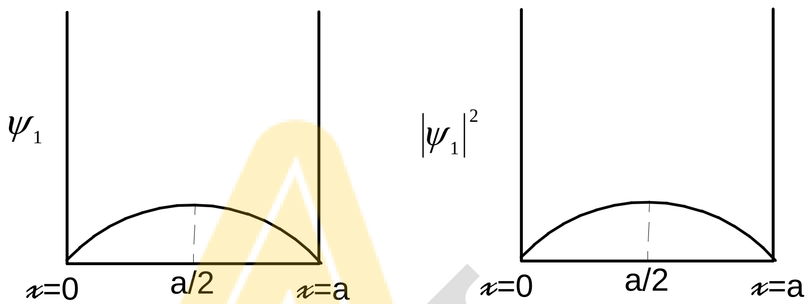
Case I → n=1 It is the ground state and the particle is normally present in this state.

The Eigen function is

$\psi_1 = \sqrt{\frac{2}{a}} Sin \frac{\pi}{a} x$ ∵from eqn (7)

$\psi_1 = 0$ for $x = 0$ and $x = a$

But ψ_1 is maximum when $x = a/2$.



The plots of ψ_1 versus x and $|\psi_1|^2$ verses x are shown in the above figure.

$|\psi_1|^2 = 0$ for $x = 0$ and $x = a$ and it is maximum for $x = a/2$. i.e. in ground state the particle cannot be found at the walls, but the probability of finding it is maximum in the middle.

The energy of the particle at the ground state is

$$E_1 = \frac{h^2}{8ma^2} = E_0$$

Case II → n=2

In the first excited state the Eigen function of this state is

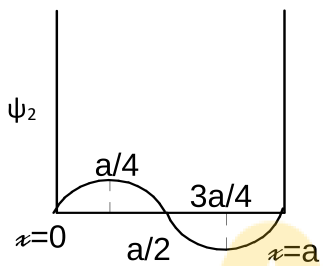
$$\psi_2 = \sqrt{\frac{2}{a}} Sin \frac{2\pi}{a} x$$

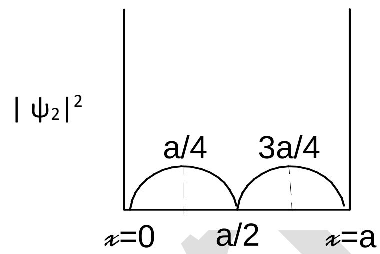
$\psi_2 = 0$ for the values $x = 0, a/2, a$.

Also ψ_2 is maximum for the values $x = a/4$ and $3a/4$.

These are represented in the graphs.

$|\psi_2|^2 = 0$ at $x = 0, a/2, a$, i.e. particle cannot be found either at the walls or at the centre. $|\psi_2|^2 = \text{maximum}\ \textit{for}\ x = \frac{a}{4}, x = \frac{3a}{4}$





The energy of the particle in the first excited state is $E_2 = 4E_0$.

Case III → n=3

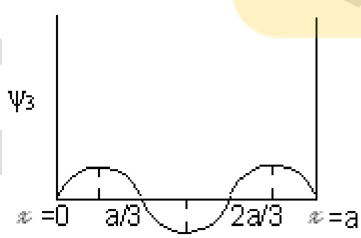
In the second excited state,

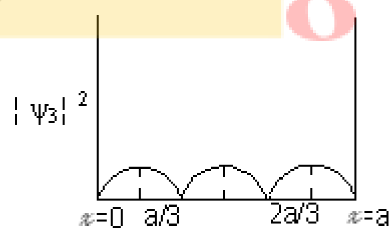
$$\psi_3 = \sqrt{\frac{2}{a}} Sin \frac{3\pi}{a} x$$

$\psi_3 = 0$, for $x = 0, a/3, 2a/3$ and a.

ψ_3 is maximum for $x = a/6, a/2, 5a/6$.

These are represented in the graphs.





$|\psi_3|^2 = 0$ for $x = 0, a/3, 2a/3$ and a. $|\psi_3|^2 = \text{maximum}\ \textit{for}\ x = \frac{a}{6}, x = \frac{a}{2}, x = \frac{5a}{6}$

The energy of the particle in the second excited state is $E_3 = 9 E_0$.

QUESTION BANK MODULE-2

Q1).State and explain Heisenberg Uncertainty principle with its physical significance. 4M (MQP-2 2018-19, July 2019, Jan 2019, Jan 2020, Sep 2020).

Q2). Show that the electron emitted during β-decay does not pre-exist inside the nucleus using uncertainty principle. 6M (MQP-2 2018-19, July 2019, Jan 2019, Jan 2020, Sep 2020, Jan /Feb 2021).

Q3). Setup 1-dimensional time independent Schrodinger wave equation also mention the equation for 3-dimensional case 8M (MQP-1 2018-19, Jan 2019, 8M (Jan /Feb 2021).

Q4). What is wave function and Probability density? Give the qualitative explanation of Max Born's interpretation of wave function. 6M (MQP-2 2018-19).

Q5). Assuming the time independent Schrodinger equation discuss the solution for a particle in one dimensional potential well of infinite height. Obtain the normalized wave function & Energy Eigen value. 10M (MQP11 2018-19, July 2019, Jan 2020, Sep 2020)

Q6). Explain about eigen functions and eigen values. 4M.

Q7). Explain about the principle of complementarity and Expectation value. 4M.

Q8). Sketch the Wave functions, probability densities and energy levels for particle in an infinite potential well for first three permitted states. 6M.

***************ALL THE BEST***************

MODULE-3

Quantum Computing

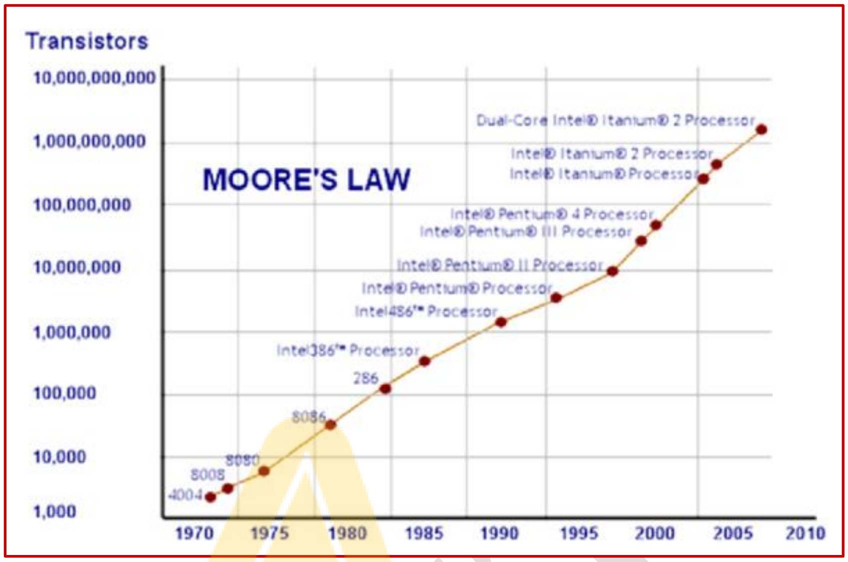
Introduction to Quantum Computing

Quantum Computing is the area of study focused on developing computing methods based on the principle of quantum theory. Quantum computing is based on the principle of quantum superposition. In Quantum computing, the information is encoded in quantum system such as atoms, ions or quantum dots. One quantum rule in particular creates enormous incentives to apply quantum mechanics to computing. The algorithms are also written based on quantum principles in which, Shor's algorithm for factorization and Grover's search algorithm are basics. (Grover is an Indian born Physicist working in Bell Labs). The process of computation is incredibly fast but it has to be done by the help of quantum computers which are yet to be realized in practice. It is expected that 140 digit log number could be factored a billion (109) times faster than classical computation. It is so powerful that a search engine can search every part of internet in half an hour.

Moore's law & its End

In the year 1965, Gordon Moore observed increasing performance in the first few generations of the integratedcircuit (IC) technology. Moore predicted that it would continue to improve at an exponential rate with the performance per unit cost increasing by a factor or two every 18 months are so. The computer industry has followed this prediction since then. But actually the doubling was occurring in every 24 months or 2 years. The following plot shows the 50 years of Moore's law. The question that arises is how long can Moore's law continues to hold and what are the ultimate limitations?. According to the semiconductor size data the size has reached 5 nanometer in 2021. The Demise of the Transistor in the quantum scale could be expected as the dimensions decrease further. Quantum effects can cascade it he micro scale realm causing problems for current microelectronics. The most typical effects are Electron tunneling among the circuit lines. Thus Quantum Computation is the option for the further generation.

Statement: "The number of transistors on a microchip doubles every year"



Differences Between Classical and Quantum Computing

Classical computing	Quantum computing
It is large scale integrated multi-purpose computer.	It is high speed parallel computer based on quantum mechanics.
Information storage is bit based on voltage or charge etc.	Information storage is Quantum bit based on direction of an electron spin.
Information processing is carried out by logic gates e.g. NOT, AND, OR etc.	Information processing is carried out by Quantum logic gates.
Classical computers use binary codes i.e. bits 0 or 1 to represent information.	Quantum computers use Qubits i.e. 0, 1 and both of them simultaneously to run machines faster.
Operations are defined by Boolean Algebra.	Operations are defined by linear algebra over Hilbert Space and can be represented by unitary matrices with complex elements.
Circuit behaviour is governed by classical physics.	Circuit behavior is governed explicitly by quantum mechanics.

Concept of Qubit and its properties

Quantum bits, called qubits are similar to bits having two measurable states called 0 and 1 states. Qubits can also be in a superposition state of these 0 and 1 states as shown in the figure. A qubit can be in a superposition of both 0 and 1. Qubits can be expressed in quantum mechanical states with mathematical formula, Dirac or "brac–ket" notation is commonly used in quantum mechanics and quantum computing. The state of a qubit is enclosed in the right half of an angled bracket, called the "ket". A qubit $|\psi\rangle$ could be in $|0\rangle$ or $|1\rangle$ state which is the superposition of both $|0\rangle$ and $|1\rangle$ state.

This is written as, $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

Where α and β called the amplitude of the states which are a complex number.

Properties of Qubits

Qubit is a basic unit in which of information in a quantum computer. Superposition, Entanglement, and Tunneling are all special properties that define a qubit.

i) A qubit can be in a superposed state of the two states 0 and 1.
Qubit is a superposition of both $|0\rangle$ and $|1\rangle$ state is given by
$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.

ii) If measurements are carried out with a qubit in superposed state then the results that we get will be probabilistic unlike how it's deterministic in a classical computer.
The total probability of all the states of the quantum system must be 100%.
i.e. $|\alpha|^2 + |\beta|^2 = 1$ is called Normalization rule.

iii) Owing to the quantum nature, the qubit changes its state at once when subjected to measurement. This means, one cannot copy information from qubits the way we do in the present computers and is known as "no cloning principle".

A Qubit can be physically implemented by the two states of an electron or horizontal and vertical polarizations of photons as $|\downarrow\rangle$ and $|\uparrow\rangle$.

Representation of qubit by Bloch Sphere

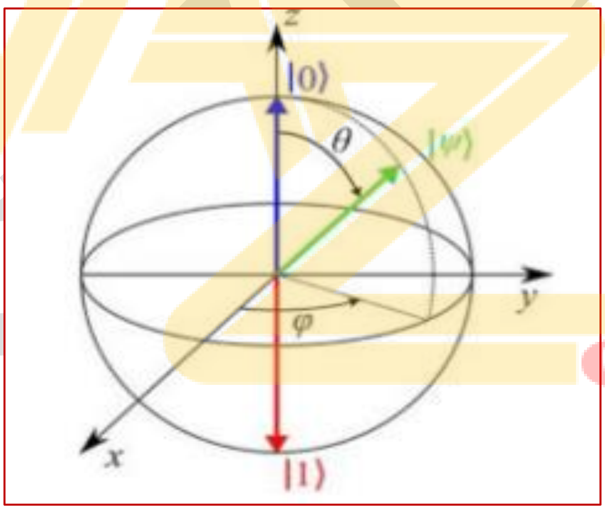
The pure state space qubits (Two Level Quantum Mechanical Systems) can be visualized using an imaginary sphere called Bloch Sphere. It has a unit radius.

The Arrow on the sphere represents the state of the Qubit. The north and south poles are used to represent the basis states $|0\rangle$ and $|1\rangle$ respectively. The other locations are the superposition of $|0\rangle$ and $|1\rangle$ states and represented by $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$.

Thus a Qubit can be any point on the Bloch Sphere. The Bloch sphere allows the state of the qubit to be represented unit spherical co-ordinates. They are the polar angle θ and the azimuth angle ϕ.

The Bloch sphere is represented by the equation

$$|\psi\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\,\sin\frac{\theta}{2}\,|1\rangle$$



Case i) For φ =0 and θ=0 then $|\psi\rangle = |0\rangle$ which is along +z axis.

Case ii) For φ=0 and θ = 180 then $|\psi\rangle = |1\rangle$ which is along -z axis.

Case iii) For φ =0 and θ= $\frac{\pi}{2}$ *then* $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ which is along +X axis.

Case iv) For φ =0 and θ= $-\frac{\pi}{2}$ *then* $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ which is along -X axis.

Single and Two qubits and Extension to N qubits

i) Single qubit

A Single qubit has two computational basis states $|0\rangle$ and $|1\rangle$. It is in general written as by $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Such that $|\alpha|^2 + |\beta|^2 = 1$

The matrix representation of $|0\rangle$ and $|1\rangle$ is given by

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad and \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

ii) Two qubit

A two qubit system has four computational basis states denoted has $|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$. The two qubit state is given by $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle + \cdots$

iii) N qubit

A multi-qubit system of N qubits has 2^N computational basis states. For example a state of 3 qubits has 2^3 computational basis states. Thus for N-qubit the computational basis states are denoted has $|000\ldots\ldots00\rangle$ $|000\ldots..01\rangle$ $|10\ldots\ldots.00\rangle$ $|10\ldots\ldots.01\rangle$.

Dirac Representation and Matrix Operations

Matrix representation of $|0\rangle$ and $|1\rangle$

The wave function could be expressed in ket notation as $|\psi\rangle$ (ket Vector), ψ is the wave function. The quantum state is given by $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and in matrix form $|\psi\rangle = \begin{pmatrix}\alpha\\\beta\end{pmatrix}$. The matrix form of the states $|0\rangle$ and $|1\rangle$ is given by

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad and \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

Identity Operator

The operator of type $I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ is called identity operator. When an identity operator acts on a state vector its keeps the state intact. By analogy we study identity operator as an identity matrix.

Let us consider the operation of Identity operator on $|0\rangle$ and $|1\rangle$states. As per the principle of identity operation $I\,|0\rangle = |0\rangle$ and $I\,|1\rangle = |1\rangle$.

$$I\,|0\rangle = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |0\rangle$$

$$I\,|1\rangle = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} = |1\rangle$$

Thus the operation of identity matrix (operator) on $|0\rangle$ and $|1\rangle$states leaves the states unchanged.

Pauli Matrices

Pauli Matrices are set of 2×2 matrices. Which are very much useful in the study of quantum computation and quantum information. The pauli matrices are given by

$$\sigma_x = X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \quad , \sigma_y = Y = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix} \quad \text{and} \quad \sigma_z = Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

Pauli Matrices operating on $|0\rangle$ and $|1\rangle$ States

1. $\sigma_x|0\rangle = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} = |1\rangle$

$\sigma_x|1\rangle = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |0\rangle$

2. $\sigma_y|0\rangle = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\i\end{bmatrix} = i|1\rangle$

$\sigma_x|1\rangle = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-i\\0\end{bmatrix} = -i|0\rangle$

3. $\sigma_z|0\rangle = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |0\rangle$

$\sigma_z|1\rangle = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\-1\end{bmatrix} = -|1\rangle$

Conjugate of a Matrix

- It is possible to find the conjugate for a given matrix by replacing each element of the matrix with its complex conjugate.
- The conjugate of a complex number is found by switching the sign of the imaginary part.
- The complex conjugate of 1 is just 1 and the complex conjugate of +i is −i.

$$A = \begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix}$$

The conjugate of matrix A is

$$A^* = \begin{bmatrix} 1 & -i \\ 1 & -i \end{bmatrix}$$

Transpose of a matrix

Transpose of a matrix, switches rows with columns.

- The first row turns into the first column, second row turns into the second column.

$$A = \begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix}$$

- The conjugate of matrix A is

$$A^* = \begin{bmatrix} 1 & -i \\ 1 & -i \end{bmatrix}$$

$$A^+ = \begin{bmatrix} 1 & 1 \\ -i & -i \end{bmatrix}$$

Unitary Matrix (U)

- A matrix U is unitary, if the matrix product of U and its conjugate transpose U † (called U-dagger) produces the identity matrix.

$UU^\dagger = U^\dagger U = I = 1$

Let $U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Conjugate of U is $U^* = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Transpose of U is $U^\dagger = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$UU^\dagger = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = I$

$U^\dagger U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = I$

$UU^\dagger = I$

Row and Column matrix (Inner product)

- A Row matrix is a vector represented by Brac vector . < |
- A Column matrix is a vector represented by ket vector | >

then $|\psi\rangle = \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}$; Row vector $|\psi\rangle = [\alpha_1, \beta_1]$,Where Bra vector is a complex conjugate of ket vector.

$|\psi *\rangle = \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}$ and $|\psi\rangle^\dagger = [\alpha_1{}^*, \beta_1{}^*]$

Thus Brac is the complex conjugate of ket and conversely ket is the complex conjugate of Brac.

Orthogonality and Orthonormal

Two states $|\psi\rangle$ and $|\phi\rangle$ are said to be orthogonal if their inner product is Zero.

Mathematically $\langle\psi|\phi\rangle = 0$

The two states are orthogonal means they are mutually exclusive. Like spin up and Spin down of an electron.

Consider the inner product of and $\langle 0|1\rangle = [1, 0]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = [0 + 0] = 0$

Two states $|\psi\rangle$ and $|\phi\rangle$ are said to be orthonormal if their inner product is one.

Mathematically $\langle\psi|\phi\rangle = 1$

Quantum Gates

In quantum computing a quantum logic gate is a basic quantum circuit operating on a small number of qubits. A qubit is useless unless it is used to carry out a quantum calculation. The quantum calculations are achieved by performing a series of fundamental operations, known as quantum logic gates. They are the building blocks of quantum circuits similar to the classical logic gates in conventional digital circuits.

1) Quantum Not Gate:

In Quantum Computing the quantum NOT gate for qubits takes the state $|0\rangle$ to $|1\rangle$ and vice versa. It is analogous to the classical not gate. The Matrix representation of Quantum Not Gate is given by $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

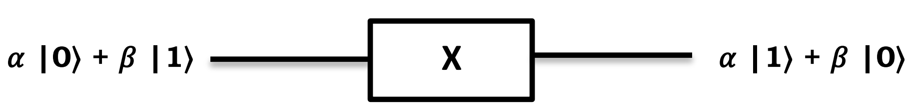
$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

A Quantum State is given by $\alpha\ |0\rangle + \beta\ |1\rangle$ and its matrix representation is given by $\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

Hence the operation of Quantum Not Gate on quantum state is given by

$$X\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

Thus the quantum state becomes $\alpha\ |1\rangle + \beta\ |0\rangle$. Similarly, The input $\alpha\ |1\rangle + \beta\ |0\rangle$ to the quantum not gates changes the state to $\alpha\ |0\rangle + \beta\ |1\rangle$. The quantum not gate circuit and the truth table are as shown below



Quantum Not Gate

Truth table of Quantum Not Gate

Input	**Output**
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$
$\alpha\ \|0\rangle + \beta\ \|1\rangle$	$\alpha\ \|1\rangle + \beta\ \|0\rangle$

2) Pauli-X,Y and Z Gates

i) Pauli X Gate

The Pauli-X Gate is nothing but Quantum Not Gate.

ii) Pauli Y Gate

Pauli Y Gate is represented by Pauli matrix σ_y or Y. This gate Maps $|0\rangle$ state to $i\ |1\rangle$ state and $|1\rangle$ state to $-i\ |0\rangle$ state. The Y Gate and its operation is as given below

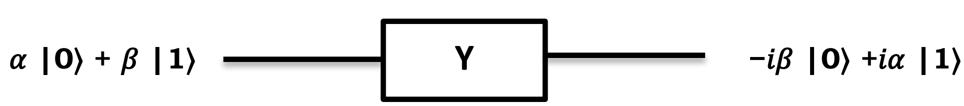
$$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$$

$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle$$

Thus the Y-Gate defines the transformation

$$Y\ (\alpha\ |0\rangle + \beta\ |1\rangle) = \alpha Y\ |0\rangle + \beta Y\ |1\rangle = -i\beta\ |0\rangle + i\alpha\ |1\rangle$$

Quantum Y-Gate is represented by



Quantum Y Gate

Truth table of Quantum Y Gate

Input	Output
$\|0\rangle$	$i\|1\rangle$
$\|1\rangle$	$-i\|0\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$-i\beta\|0\rangle + i\alpha\|1\rangle$

iii) Pauli Z Gate

The Z-gate is represented by Pauli Matrix $_z$ or Z. Z Gate maps input state $|k\rangle$ to $(-1)|k\rangle$.

1. For input $|0\rangle$ the output remains unchanged.

2. For input $|1\rangle$ the output is $-|1\rangle$.

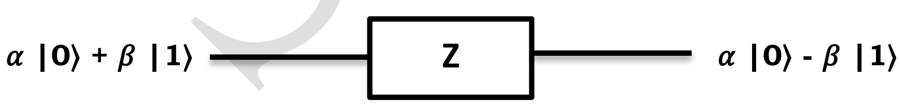
The Matrix representation and the operation of Z-Gate on $|0\rangle$ and $|1\rangle$ are as follows

$$Z|0\rangle = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |0\rangle$$

$$Z|1\rangle = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\-1\end{bmatrix} = -|1\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle) = \alpha Z|0\rangle + \beta Z|1\rangle = \alpha|0\rangle - \beta|1\rangle$$

The circuit symbol and the truth table of Z-Gate are as follows



Truth table of Quantum Z Gate

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$-\|1\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|0\rangle - \beta\|1\rangle$

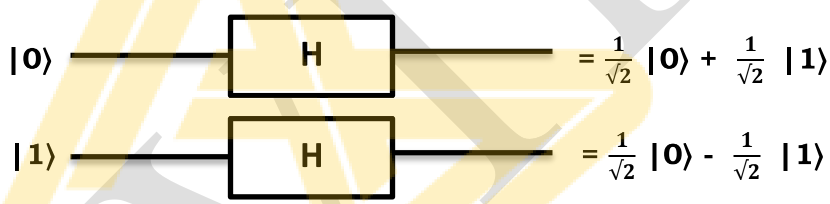
3) Hadamard Gate

The Hadamard Gate is a truly quantum gate and is one of the most important in Quantum Computing. It is has similar characteristics of $\sqrt{NOT}$ Gate. It is a self-inverse gate. It is used to create the superposition of $|0\rangle$ and $|1\rangle$ states.

The Matrix representation of Hadamard Gate is as follows $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

The Hadamard Gate and the output states for the $|0\rangle$ and $|1\rangle$ input states are represented as follows.

The Hadamard Gate satisfies Unitary Condition. $H^{\dagger}H = I$



The truth-table for the Hadamard Gate is as follows.

Input	Output
$\lvert 0\rangle$	$\frac{\lvert 0\rangle + \lvert 1\rangle}{\sqrt{2}}$
$\lvert 1\rangle$	$\frac{\lvert 0\rangle - \lvert 1\rangle}{\sqrt{2}}$
$\alpha\lvert 0\rangle + \beta\lvert 1\rangle$	$\alpha\frac{\lvert 0\rangle + \lvert 1\rangle}{\sqrt{2}} + \beta\frac{\lvert 0\rangle - \lvert 1\rangle}{\sqrt{2}}$

4) Phase Gate or S Gate

The phase gate turns a $|0\rangle$ into $|0\rangle$ and a $|1\rangle$ into $i\ |1\rangle$.

The Matrix representation of the S gate is given by

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

The effect of S gate on input $|0\rangle$ is given by

$$S\,|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

Similarly the effect of S gate on input $|1\rangle$ is given by

$$S|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$$

The transformation of state $|\psi\rangle$ is given by

$S\ |\psi\rangle = S\ (\alpha\ |0\rangle + \beta\ |1\rangle) = \alpha S\ |0\rangle + \beta S\ |1\rangle = \alpha\ |0\rangle + i\beta\ |1\rangle$

The symbol of S gate is given by

The Truth table for S gate is as follows

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$i\|1\rangle$
$\alpha\ \|0\rangle + \beta\ \|1\rangle$	$\alpha\ \|0\rangle + i\ \beta\|1\rangle$

5) T -Gate / $\frac{\pi}{8}$ Gate

The T Gate is represented by the matrix as follows

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1+i}{\sqrt{2}} \end{bmatrix}$$

The Operation of T- gate on $|0\rangle$ and $|1\rangle$ are given by

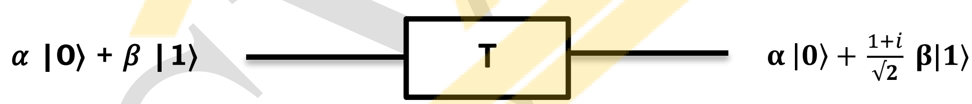
$$T|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1+i}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$T|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1+i}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1+i}{\sqrt{2}} \end{bmatrix} = \frac{1+i}{\sqrt{2}} |1\rangle$$

T -Gate is also called $\frac{\pi}{8}$ Gate is shown below

$$T = e^{\frac{i\pi}{8}} \begin{bmatrix} e^{-\frac{i\pi}{8}} & 0 \\ 0 & e^{\frac{i\pi}{8}} \end{bmatrix}$$

The symbolic representation of T-gate is given by



The Truth table for T- gate is as follows

Input	Output
$|0\rangle$	$|0\rangle$
$|1\rangle$	$\frac{1+i}{\sqrt{2}}\ |1\rangle$
$\alpha\ |0\rangle + \beta\ |1\rangle$	$\alpha\,|0\rangle + \frac{1+i}{\sqrt{2}}\,\beta|1\rangle$

Note: Important feature of T- gate is it could be related to S gate as $T^2 = S$

Multiple Qubit Gates

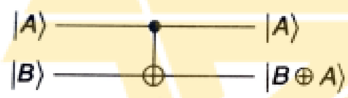
Multiple Qubit Gates operate on Two or More input Qubits. Usually on of them is a control qubit and other is target qubit.

1) Controlled Gates

The Gate with operation of kind "If 'A' is True then do 'B'" is called Controlled Gate. The $|A\rangle$ Qubit is called control qubit and $|B\rangle$ is the Target qubit. The target qubit is altered only when the control qubit state is $|1\rangle$. The control qubit remains unaltered during the transformations.

2) Controlled Not Gate or CNOT Gate

The CNOT gate is a typical multi-qubit logic gate and the circuit is as follows.



CNOT gate

The matrix representation of CNOT gate is given by

$$U_{CN} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The Transformation could be expressed as $|A, B\rangle \rightarrow |A, B \oplus A\rangle$

Consider the operations of CNOT gate on the four inputs $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$.

Operation of CNOT Gate for input $|00\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|0\rangle$. Hence no change in the state of Target qubit $|0\rangle$. $|00\rangle \rightarrow |00\rangle$

Operation of CNOT Gate for input $|01\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|0\rangle$. Hence no change in the state of Target qubit $|1\rangle$. $|01\rangle \rightarrow |01\rangle$

Operation of CNOT Gate for input $|10\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|1\rangle$. Hence the state of Target qubit flips from $|0\rangle$ to $|1\rangle$. $|10\rangle \rightarrow |11\rangle$

Operation of CNOT Gate for input $|11\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|1\rangle$. Hence the state of Target qubit flips from $|1\rangle$ to $|0\rangle$. $|11\rangle \rightarrow |10\rangle$

The Truth Table of operation of CNOT gate is as follows.

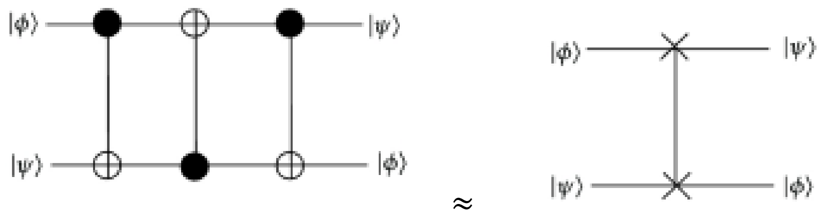
Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$

3) Swap Gate

The SWAP gate is two-qubit operation. Expressed in basis states, the SWAP gate swaps the state of the two qubits involved in the operation. The Matrix representation of the Swap Gate is as follows

$$U_{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The schematic symbol of swap gate circuit is as follows



The swap gate is a combined circuit of 3 CNOT gates and the overall effect is that two input qubits are swapped at the output. The Action and truth table of the swap gate is as follows.

Truth table of SWAP gate

Input	*Output*
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|10\rangle$
$\|10\rangle$	$\|01\rangle$
$\|11\rangle$	$\|11\rangle$

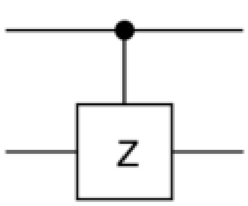
4) Controlled Z Gate

In Controlled Z Gate, The operation of Z Gate is controlled by a Control Qubit. If the control Qubit is $|A\rangle$ is equal to $|1\rangle$ then only the Z gate transforms the Target Qubit $|B\rangle$ as per the Pauli-Z operation.

The action of Controlled Z-Gate could is specified by a matrix as follows.

$$U_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The schematic circuit of controlled Z gate and the truth table are as follows



Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|10\rangle$
$\|11\rangle$	$-\|11\rangle$

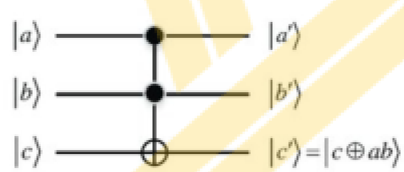
5) Toffoli Gate

The Toffoli Gate is also known as CCNOT Gate (Controlled-Controlled-Not). It has three inputs out of which two are Control Qubits and one is the Target Qubit. The Target Qubit flips only when both the Control Qubits are $|1\rangle$. The two Control Qubits are not altered during the operation.

The matrix representation of Toffoli Gate is given by

$$U_T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The schematic circuit of Toffoli Gate is as follows



The truth table for Toffoli gate is as follows

Inputs			Ouputs		
a	b	c	a'	b'	c'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

The Toffolli matrix is unitary. The Toffolli Gate is its own inverse. It could be used for NAND Gate Simulation.

MODULE 4

Electrical Properties of Materials and Applications

Assumptions of classical free electron theory:

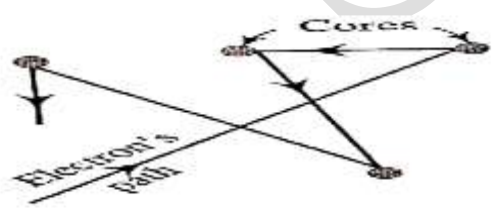
- A metal is imagined as the structure of 3-dimensional array of ions in between which, there are free moving valence electrons confined to the body of the material. Such freely moving electrons cause electrical conduction under an applied field and hence referred to as conduction electrons
- The free electrons are treated as equivalent to gas molecules and they are assumed to obey the laws of kinetic theory of gases. In the absence of the field, the energy associated with each electron at a temperature T is given by 3/2 kT, where k is a Boltzmann constant.
  It is related to the kinetic energy.
  $3/2\ kT = ½\ mv_{th}^2$
  Where v_{th} is the thermal velocity same as root mean square velocity.
- The electric potential due to the ionic cores is taken to be essentially constant throughout the body of the metal and the effect of repulsion between the electrons is considered insignificant.
- The electric current in a metal due to an applied field is a consequence of the drift velocity in a direction opposite to the direction of the field.

Drift velocity (v_d):

The average velocity of occupied by the electrons in the steady state in an applied electric field is called drift velocity.

The drift velocity $v_d = \frac{eE\tau}{m}$

Thermal velocity(V_{th}):



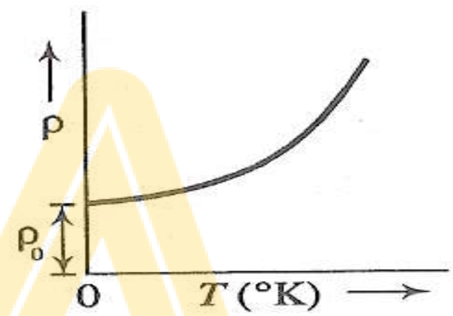
The velocity of electrons in random motion due to thermal agitation called thermal velocity.

Mean free path (λ):

The average distance travelled by the conduction electrons between any two successive collisions with lattice ions.

Temperature dependence of resistivity of a metal:

All metals are good conductors of electricity. The electrical conductivity of metal varies with the temperature. The electrical resistance of a metal, to the flow of current, is due to scattering of conduction electrons by lattice vibrations. When the temperature increases the amplitude of lattice vibrations also increases, thereby increasing the resistance. The dependence of resistance of metal (non-superconducting state) is shown in figure. The resistance decreases with temperature and reaches a minimum value at T = 0K. The residual resistance at T = 0K is due to impurities in the metal.



DEPENDENCE OF ρ ON T

By Matthiessen's rule

$\boldsymbol{\rho = \rho_0 + \rho(T)}$

Where 'ρ' is the resistivity of the given material, 'ρ_0'is the residual resistivity and 'ρ (T)' is the temperature dependent part of resistivity.

"The total resistivity of a metal is the sum of the resistivity due to phonon scattering which is temperature dependent and the resistivity due to scattering by impurities which is temperature dependent"

Expression for electrical conductivity of conductor according to classical free electron theory

According to classical free electron theory the expression for electrical conductivity is given by

$$\sigma_{CFET} = \frac{ne^2\tau}{m}$$

Where σ - Electrical conductivity
n- Electron density
τ – mean collision time
m- mass of electron

Failures of classical free electron theory:

Electrical and thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

i) Specific heat: The molar specific heat of a gas at constant volume is

$$C_v = \frac{3}{2} R$$

As per the classical free electron theory, free electrons in a metal are expected to behave just as gas molecules. Thus the above equation holds good equally well for the free electrons also.

But experimentally it was found that, the contribution to the specific heat of a metal by its conduction electrons was

$$C_V = 10^{-4} RT$$

which is for lower than the expected value. Also according to the theory the specific heat is independent of temperature whereas experimentally specific heat is proportional to temperature.

ii)Temperature dependence of electrical conductivity:

Experimentally, electrical conductivity σ is inversely proportional to the temperature T.

i.e. $\sigma_{exp} \;\alpha\; \frac{1}{T} \rightarrow (1)$

According to the assumptions of classical free electron theory

Since $V_{th} \;\alpha\; \sqrt{T}$

But $\tau \alpha \frac{1}{V_{th}}$, $\tau \alpha \frac{1}{\sqrt{T}}$,

substituting in conductivity equation we get

$$\sigma_{CFET} = \frac{ne^2 \tau}{m} = \frac{ne^2}{m\sqrt{T}}$$

Or $\sigma_{CFET} \alpha \frac{1}{\sqrt{T}}$ $\rightarrow (2)$

From equations (1) & (2) it is clear that the experimental value is not agreeing with the theory.

iii) Dependence of electrical conductivity on electron concentration:

According to classical free electron the theory

$\sigma = \dfrac{ne^2\tau}{m}$ i.e., $\sigma \propto n$, where n is the electron concentration,

Consider copper and aluminum. Their electrical conductivities are $5.88\times10^7/\Omega m$ and $3.65x10^7/\Omega m$. The electron concentrations for copper and aluminum are $8.45\times10^{28}/m^3$ and $18.06x10^{28}/m^3$. Hence the classical free electron theory fails to explain the dependence of σ on electron concentration.

Experimental results:

Metals	Electron concentration(n)	conductivity (σ)
Copper	$8.45\times10^{28}/m^3$	$5.88x10^7/\Omega m$
Aluminium	$18.06x10^{28}/m^3$	$3.65x10^7/ \Omega m$

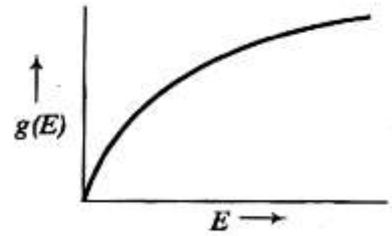
Quantum free electron theory:

<u>Assumptions of quantum free electron theory:</u>

- The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
- The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.
- The electrons travel with a constant potential inside the metal but confined within its boundaries.
- The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

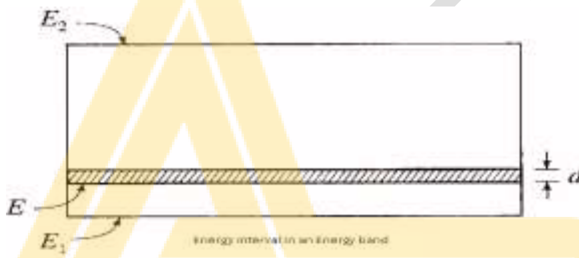
<u>Density of states g(E):</u>

Density of states is defined as the number of allowed energy states per unit energy range per unit volume in the valance band of a material. It is denoted as g(E).
A graph of g(E) verses E is shown below.



DENSITY OF STATES

Consider an energy band spread in an energy interval between E_1 and E_2. Below E_1 and above E_2 there are energy gaps. g(E) represents the density of states at E. As dE is small, it is assumed that g(E) is constant between E and E+dE. The density of states in range E and (E+dE) is denoted by g(E)dE.

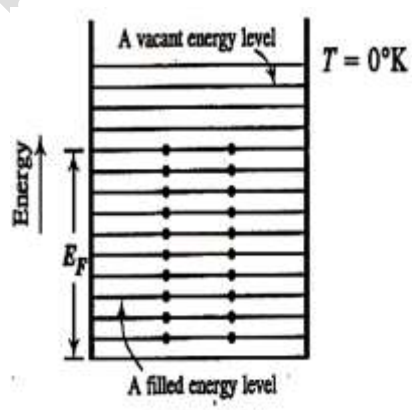


i.e. $$g(E)dE = \left[\frac{8\sqrt{2}\,\pi\, m^{\frac{3}{2}}}{h^3}\right] E^{\frac{1}{2}}\, dE$$

It is clear g(E) is proportional to $\sqrt{E}$ in the interval dE

Fermi energy and Fermi level:

The energy of electrons corresponding to the highest occupied energy level at absolute 0°K is called Fermi energy and the energy level is called Fermi level.



FERMI ENERGY

Fermi factor:

Fermi factor is the probability of occupation of a given energy state by the electrons in a material at thermal equilibrium.

The probability f(E) that a given energy state with energy E is occupied by the electrons at a steady temperature T is given by

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

f(E) is called the Fermi factor.

Dependence of Fermi factor with temperature and energy:

The dependence of Fermi factor on temperature and energy is as shown in the figure.

i) Probability of occupation for $E<E_F$ at T=0K:

When T=0K and $E<E_F$

$$f(E) = \frac{1}{e^{-\infty}+1} = \frac{1}{0+1} = 1$$

The probability of occupation of energy state is 100%

f(E)=1 for $E<E_F$.

f(E)=1 means the energy level is certainly occupied and $E<E_F$ applies to all energy levels below E_F. Therefore at T=0K all the energy levels below the Fermi level are occupied.

ii) Probability of occupation for $E>E_F$ at T=0K:

When T=0K and $E>E_F$

$$f(E) = \frac{1}{e^{\infty}+1} = \frac{1}{\infty} = 0$$

The probability of occupation of energy state is 0%

f(E)=0 for $E>E_F$

∴ At T=0K, all the energy levels above Fermi levels are unoccupied. Hence at T=0K the variation of f(E) for different energy values, becomes a step function as shown in the above figure.

iii) The probability of occupation at ordinary temperature(for E≈E_F at T>0K)

At ordinary temperatures though the value of probability remains 1, for E<E_F it starts reducing from 1 for values of E close to but lesser than E_F as in the figure.

The values of f(E)becomes ½ at E=E_F

This is because for E=E_F

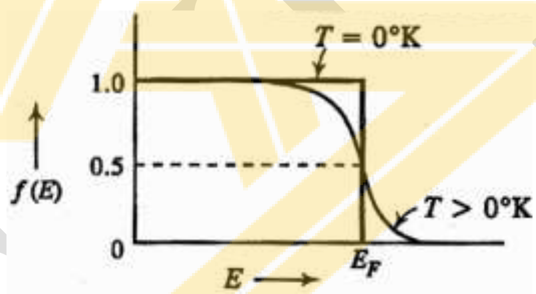
$$e^{(E-E_f)/kT} = e^0 = 1$$

$$\therefore f(E) = \frac{1}{e^{(E-E_f)/kT}+1} = \frac{1}{1+1} = \frac{1}{2}$$

The probability of occupation of energy state is 50%

Further for E>E_F the probability value falls off to zero rapidly.

Hence, the Fermi energy is the most probable or the average energy of the electrons across which the energy transitions occur at temperature above zero degree absolute.



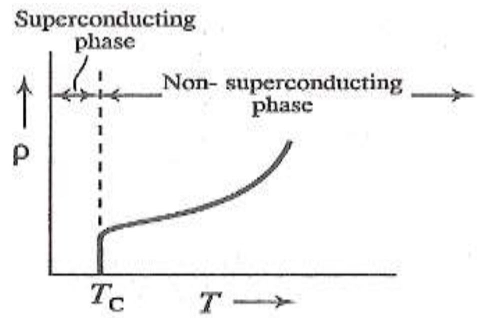
VARIATION OF f(E) WITH E

Super Conductivity:

Super conductivity is the phenomenon observed in some metals and materials. Kammerlingh Onnes in 1911 observed that the electrical resistivity of pure mercury drops abruptly to zero at about 4.2K .This state is called super conducting state. The material is called superconductor .The temperature at which they attain superconductivity is called critical temperature T_c.

Temperature dependence of resistivity of a superconductor:

One of the most interesting properties of solid at low temperature is that electrical resistivity of metals and alloys vanish entirely below a certain temperature. This zero resistivity or infinite conductivity is known as superconductivity. Temperature at which transition takes place is known as transition temperature or critical temperature (T_c). Above the transition temperature, the substance is in the normal state and below it will be in superconducting state. T_c value is different for different materials.



DEPENDENCE OF ρ ON T

"The resistance offered by certain materials to the flow of electric current abruptly drop to zero below a threshold temperature. This phenomenon is called superconductivity and threshold temperature is called "critical temperature".

<u>Meissner effect:</u>

A superconducting material kept in a magnetic field expels the magnetic flux out of its body when it is cooled below the critical temperature and thus becomes perfect diamagnet. This effect is called Meissner effect.

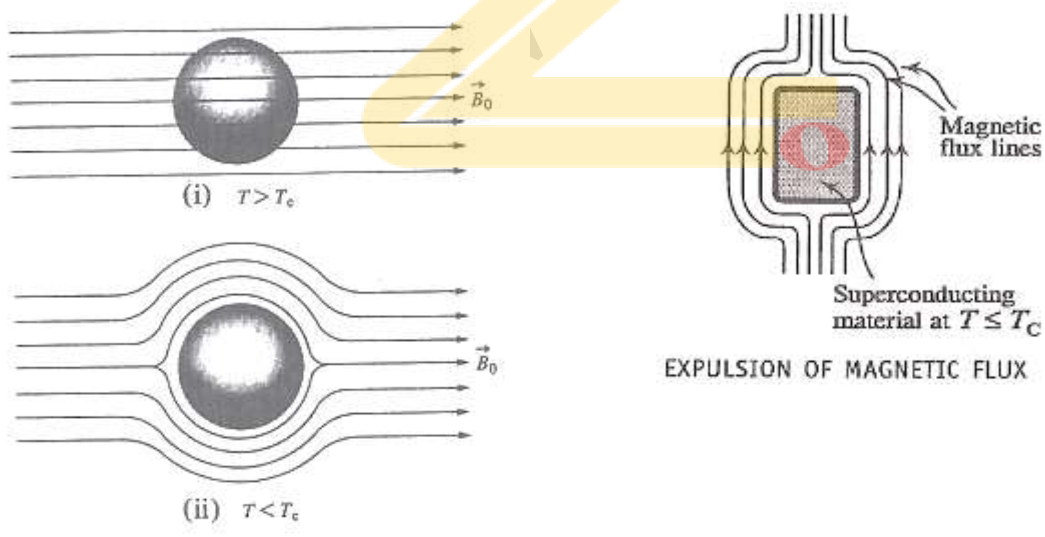


Fig: Superconductor sample subjected to an applied magnetic field with temperature (i) above and (ii) below T_C. The flux expulsion below Tc is called Meissner effect

When the temperature is lowered to T_c, the flux is suddenly and completely expelled, as the specimen becomes superconducting. The Meissner effect is reversible. When the temperature is raised the flux penetrates the material, after it reaches T_c. Then the substance will be in the normal state.

The magnetic induction inside the specimen

$$B = \mu_o (H + M)$$

Where 'H' is the intensity of the magnetizing field and 'M' is the magnetization produced within the material.

For $T < T_c$, $B = 0$

$$\mu_0 (H + M) = 0$$

$$M = -H$$

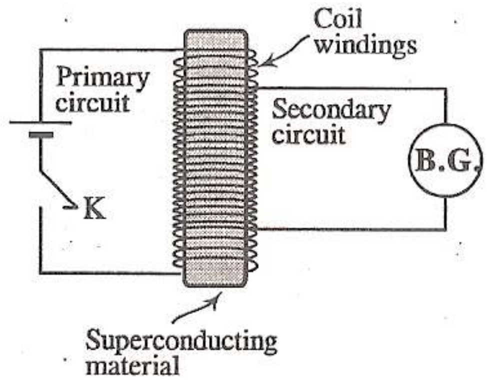
$$M/H = -1 = \chi$$

Susceptibility is -1 i.e. it is perfect diamagnetism.

Hence superconducting material do not allow the magnetic flux to exist inside the material.

Experimental demonstration of Meissner effect:

Consider a primary coil and a secondary coil, wound on a superconducting material. The primary coil is connected to a battery and a key. The secondary coil is connected to ballistic galvanometer (BG). When the key is closed the current flows through the primary coil and the magnetic field is produced. This flux is linked with the secondary coil and the current flows through the secondary coil which makes a deflection in the galvanometer. If the primary current is steady the magnetic flux and the flux linked with the coil will become steady. As the temperature of the specimen is decreased below the critical temperature, BG suddenly shows a deflection indicating that the flux linked with the secondary coil is changed. This is due to the expulsion of the magnetic flux from the specimen.



MEISSNER EFFECT

Effect of magnetic field:

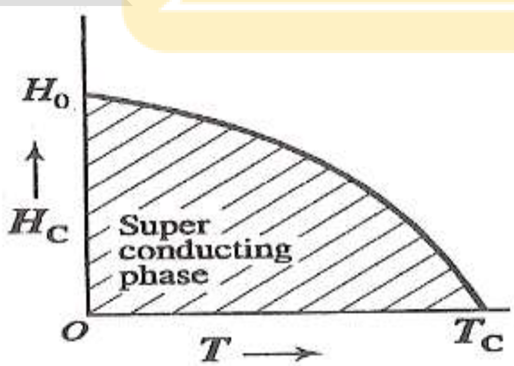
Superconductivity can be destroyed by applying magnetic field. The strength of the magnetic field required to destroy the superconductivity below the T_c is called critical field. It is denoted by $H_c(T)$.

If 'T' is the temperature of the superconducting material, 'T_c' is the critical temperature, 'H_c' is the critical field and 'H_o' is the critical field at 0^oK.

They are related by

$$H_c = H_o[1-(T/T_c)^2]$$

By applying magnetic field greater than H_o, the material can never become superconductor whatever may be the low temperature. The critical field need not be external but large current flowing in superconducting ring produce critical field and destroys superconductivity.



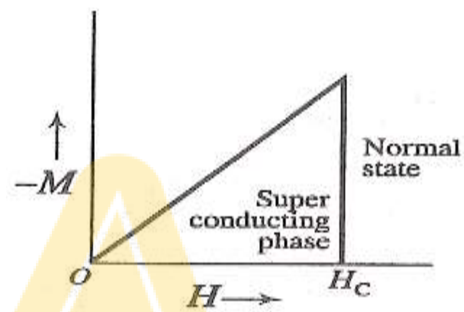
DEPENDENCE OF H_c ON T

Types of superconductors:

There are two types of superconductors. They are type-I superconductors and type-II superconductors.

i)Type-I superconductors:

Type-I superconductors exhibit complete Meissner effect. Below the critical field it behaves as perfect diamagnetic. If the external magnetic field increases beyond H_c the superconducting specimen gets converted to normal state. The magnetic flux penetrates and resistance increase from zero to some value. As the critical field is very low for type-I superconductors, they are not used in construction of solenoids and superconducting magnets.



DEPENDENCE OF MAGNETIC MOMENT ON H

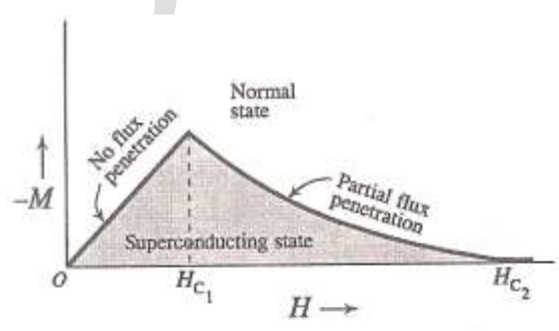
ii) Type-II superconductors

Type-II superconductors are hard superconductors. They exist in three states

i) Superconducting state

ii) Mixed state

iii) Normal state



DEPENDENCE OF MAGNETIC MOMENT ON H

They are having two critical fields H_{c1} and H_{c2}. For the field less then Hc_1, it expels the magnetic field completely and becomes a perfect diamagnetic. Between Hc_1 and Hc_2 the flux starts penetrating throughout the specimen. This state is called vortex state. Hc_2 is 100 times higher than

Hc_1. At Hc_2 the flux penetrates completely and becomes normal conductor. Type-II superconductors are used in the manufacturing of the superconducting magnets of high magnetic fields above 10 Tesla.

BCS theory superconductivity:

- Bardeen, Cooper and Schrieffer (BCS) in 1957 explained the phenomenon of superconductivity based on the formation of cooper pairs. It is called BCS theory. It is a quantum mechanical concept.
- When a current flow in a superconductor, electrons come near a positive ion core of lattice, due to attractive force. The ion core also gets displaced from its position, which is called lattice distortion. The lattice vibrations are quantized in a term called Phonons.
- Now an electron which comes near that place will interact with the distorted lattice. This tends to reduce the energy of the electron. It is equivalent to interaction between the two electrons through the lattice. This leads to the formation of cooper pairs.
- "Cooper pairs are a bound pair of electrons formed by the interaction between the electrons with opposite spin and momentum in a phonon field".
- When the electrons flow in the form of cooper pairs in materials, they do not encounter any scattering and the resistance factor vanishes or in other words conductivity becomes infinity which is called as superconductivity.
- In superconducting state electron-phonon interaction is stronger than the coulomb force of attraction of electrons. Cooper pairs are not scattered by the lattice points. They travel freely without slow down as their energy is not transferred. Due to this they do not possess any electrical resistivity.

High temperature superconductors:

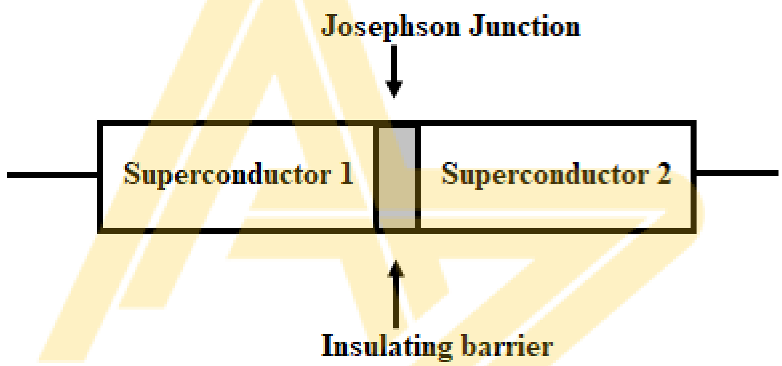
The term high-temperature superconductor was first used to designate the new family of cuprate-perovskite ceramic materials discovered by Bednorz and Müller in 1986. The first high-temperature superconductor, LaBaCuO, with a transition temperature of 30 K and in the same year LSCO ($La_{2-x}Sr_xCuO_4$) discovered with T_C of 40K. In 1987 it was shown that superconductors with T_c greater than 77K could be prepared, this temperature is greater than the liquid helium temperature. $YBa_2Cu_3O_7$ was discovered to have a Tc of 92 K. Bismuth/lead strontium Calcium Copper $(Bi\,Pb)_2Sr_2\,Ca_2Cu_3O_x$ (x<0.1) with T_c=105K. Thallium barium Calcium copper oxide ($Tl\,Ba_2Ca_2Cu_3O_4$) of T_c=115K. Mercury barium calcium copper oxide ($Hg\,Ba_2Ca_2Cu_3O_4$) with T_c=135K.

All high temperature superconductors are different types of oxides of copper, and bear a particular type of crystal structure called Perovskite crystal structure. The number of copper layers increases the T_c value increases. The current in the high T_C materials is direction dependent. It is strong in parallel to copper-oxygen planes and weak in perpendicular to copper-oxygen planes.

High T_c materials are Type-II superconductors and they are brittle and don't carry enough current. The formation of electron pairs is not due to interaction of electron lattice as in the BCS theory. Still it is not clear what does cause the formation of pairs. Research is being conducting in this direction. The high temperature superconductors are useful in high field applications. It can carry high currents of 10^5 to 10^6 amps in moderate magnetic fields. They are used in military applications, Josephson junction in SQUIDS, under sea communication, submarines.

Quantum Tunneling:

Consider two superconductors separated by insulating barrier of thickness less than 10-20 Å, then the cooper pairs tunneling through the insulating barrier is known as **Josephson superconducting quantum tunneling**. The junction between the two superconductors with insulating barrier is known as **Josephson junction**.



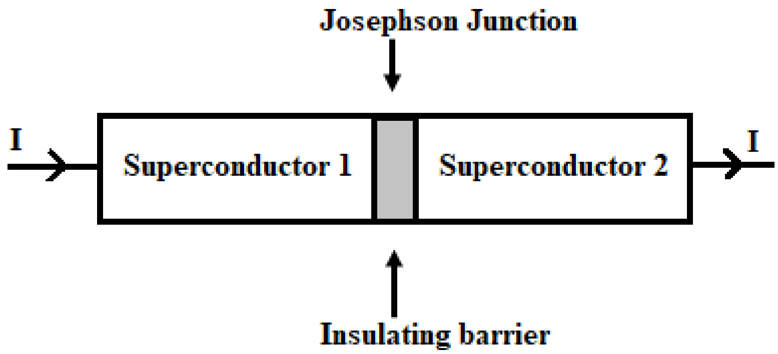
Josephson junction is an arrangement of two superconductors separated by an insulating barrier. When the barrier is thin enough, cooper pairs from one superconductor can tunnel through the barrier and reach the other superconductor.

Josephson proposed that this kind of tunneling leads to three kinds of effect, namely

1. dc Josephson effect
2. ac Josephson effect
3. quantum interference

1. dc Josephson effect

As per dc Josephson Effect, the tunneling of cooper pairs through the junction occurs without any resistance, which results in a steady dc current without any application of voltage between the two superconductors.



The super current through the junction is given by,

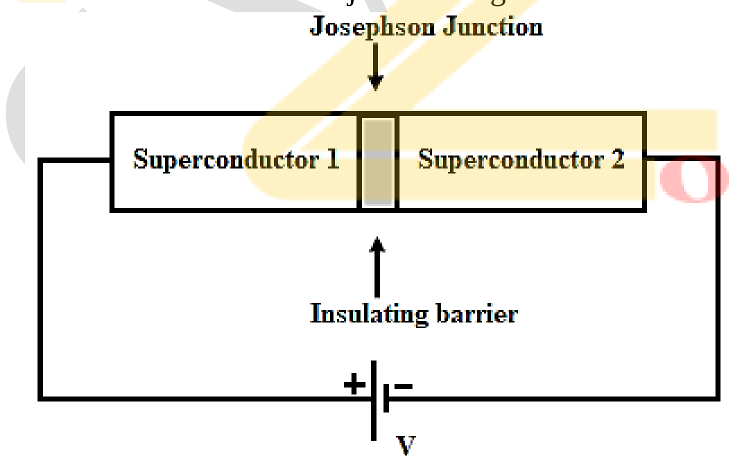
$I_S = I_C \sin \Phi_0$

Where, Φ_0 = phase difference between the wave functions describing the cooper pairs on both the sides of the junction.

I_C = critical current at zero voltage which depends on the thickness and width of the insulating layer.

2. ac Josephson effect

When a dc voltage is applied across the junction, the tunneling of cooper pairs occur in such a way than an ac current would develop in the junction and this effect is called as ac Josephson Effect. When a potential difference of 'V' is applied between the two sides of the junction then a radio frequency (RF) current oscillations across the junction is generated.



$I = I_C \sin(\Phi_0 + \Delta\Phi)$

The energies of the cooper pairs on both sides of the barrier difference is E = hν = 2eV (Calculated using quantum mechanical concept).

Therefore it can be shown that, $\Delta\Phi = \omega t = 2\pi t \left(\frac{2\,eV}{h}\right)$

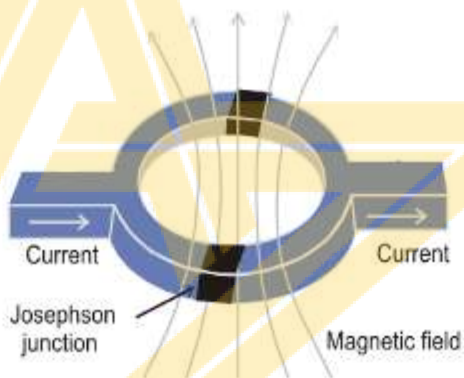
Hence, $I = I_C \sin\left(\Phi_0 + 2\pi t \left(\frac{2\,eV}{h}\right)\right)$

I = Alternating current of frequency $\nu = \frac{2\,eV}{h}$

It shows that a photon of frequency ν is emitted or absorbed when a cooper pair crosses the junction. Thus when a voltage is applied across the junction, an ac current gets generated. This is known as ac Josephson Effect.

SQUID

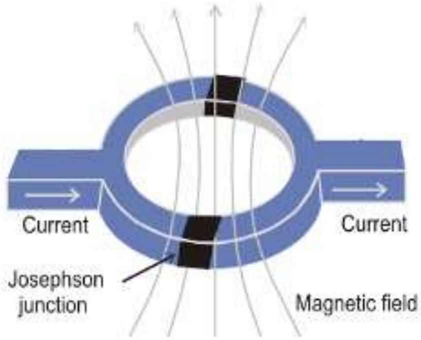
- SQUID is an acronym for Superconducting Quantum Interference Device.
- It is an Ultra-sensitive measuring instrument used for the detection of very weak magnetic fields of the order of 10^{-14} T.
- A SQUID is formed by incorporating two Josephson's junction in the loop of a superconducting material.
- When a magnetic field is applied to this superconducting circuit, it induces a circulating current which produces just that much opposing magnetic field so as to exclude the flux from the loop.

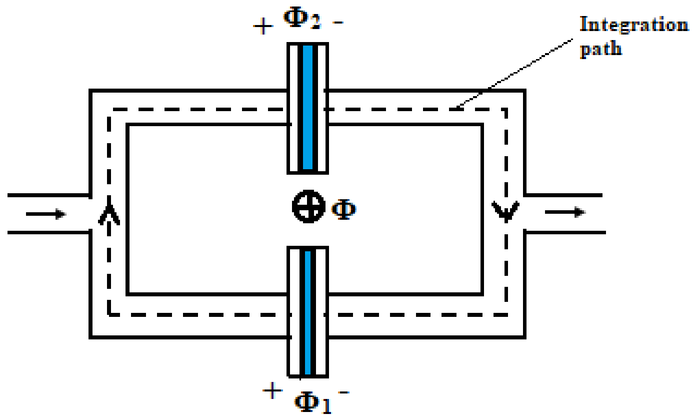


[Note: The flux remains excluded as long as the junction current do not exceed a critical value. But the circuit switches to resistive phase and thereby the flux passes into the loop once the current in either of the junction or in both the junction exceed the critical value. Thus the loop acts like a gate to allow or exclude the flux]

DC Squids

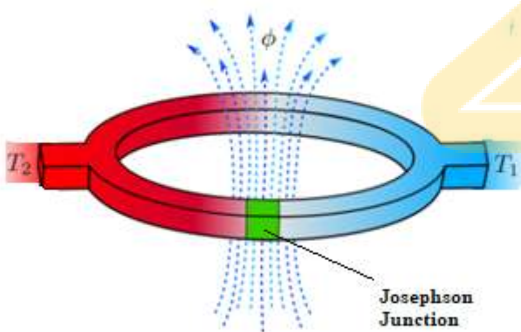
- DC Squids are nothing but a double junction SQUIDS.
- A DC SQUID consists of two Josephson junctions connected in parallel on a closed superconducting loop as shown in the Fig.
- In this device the two weak links are not shorted by superconductor path; therefore the dc current-voltage characteristics can be observed.
- This device applies current slightly greater than the critical current and one can monitor the voltage drop across the device.

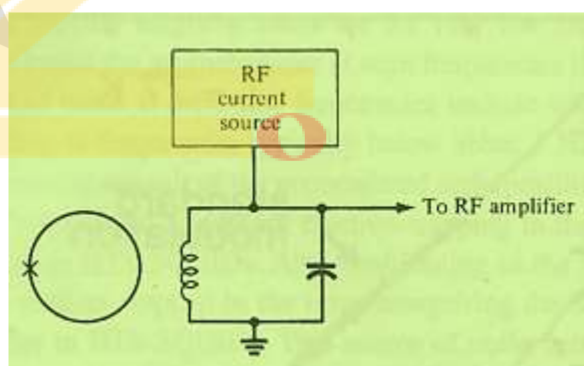




RF SQUIDS

- The single junction SQUIDS are also known as RF SQUIDS.
- The junction is shorted by superconductor path; therefore the voltage response is obtained by coupling the loop to a RF bias tank circuit.
- The RF (Radio Frequency) SQUID is **a one-junction SQUID loop that can be used as a magnetic field detector.**
- In this configuration, **the RF SQUID is inductively coupled to the inductance L_T of an L_C tank circuit**. The tank circuit is driven by an rf current, and the resultant rf voltage is periodic in the flux applied to the SQUID loop with period Φ_0.





MODULE-5- Physics of Animation

Animation

Animation is the process of displaying still images (drawings, models, or even puppets) in a rapid sequence to create the illusion of movement. Because our eyes can only retain an image for approx. $\frac{1}{10}$ th of a second, when multiple images appear in fast succession, the brain blends them into a single moving image.

Frames and frames per second.

A frame is a single image in a sequence of pictures. A frame contains the image to be displayed at a unique time in the animation. In general, one second of a video is comprised of 24 or 30 frames per second also known as FPS. The frame is a combination of the image and the time of the image when exposed to the view. An extract of frames in a row makes the animation.

The Taxonomy of Physics-Based Animation Methods

At the highest level, the field of physics-based animation and simulation can roughly be subdivided into two large groups:

1. Kinematics is the study of motion without consideration of mass or forces.

2. Dynamics is the study of motion taking mass and forces into consideration.

kinematics and dynamics come in two flavors or subgroups:

1. Inverse is the study of motion knowing the starting and ending points.

2. Forward is the study of motion solely given the starting point.

Elucidate the Importance of Size & Scale, Weight and strength in animations (8M)

Size and Scale

The size and scale of characters often play a central role in a story's plot.

We cannot imagine a Superman be without his height and bulging biceps? Some characters, like the Incredible Hulk, are even named after their body types.

We can equate large characters with weight and strength, and smaller characters with agility and speed. As it is noticeable in real life scenarios that, larger people and animals do have a larger capacity for strength, while smaller critters can move and maneuver faster than their large counterparts.

When designing characters, we can run into different situations having to do with size and scale, such as:

1. Human or animal-based characters that are much larger than we see in our everyday experience. Superheroes, Greek gods, monsters,

2. Human or animal-based characters that are much smaller than we are accustomed to, such as fairies and elves.

3. Characters that need to be noticeably larger, smaller, older, heavier, lighter, or more energetic than other characters.

4. Characters that are child versions of older characters. An example would be an animation featuring a mother cat and her kittens. If the kittens are created and animated with the same proportions and timing as the mother cat, they won't look like kittens; they'll just look like very small adult cats.

Proportion and Scale

Creating a larger or smaller character is not just a matter of scaling everything about the character uniformly.

Example: When we scale a cube, its volume changes much more dramatically than its surface area. Let us say each edge of the cube is 1 unit length. The area of one side of the cube is 1 square unit, and the volume of the cube is 1 cubed unit.

If we double the size of the cube along each dimension, its height increases by 2 times, the surface area increases by 4 times, and its volume increases by 8 times. While the area increases by squares as we scale the object, the volume changes by cubes.

Wight and strength

Body weight is proportional to volume. The abilities of our muscles and bones, however increase by area because their abilities depend more on cross-sectional area than volume.

To increase a muscle or bone's strength, we need to increase its cross- sectional area.

To double a muscle's strength, for example, you would multiply its width by $\sqrt{2}$.

To triple the strength, multiply the width by $\sqrt{3}$.

Since strength increases by squares and weight increases by cubes, the proportion of a character's weight that it can lift does not scale proportionally to its size.

Let us take an example of a somewhat average human man. At 6 feet tall, he weighs 180 pounds and can lift 90 pounds. He can lift half his body weight.

If we scale up the body size by a factor of 2, the weight increases by a factor of 8. Such a character could then lift more weight. But since he weighs more than 8 times more than he did before, he cannot lift his arms and legs as easily as a normal man. Such a giant gains strength, but loses agility.

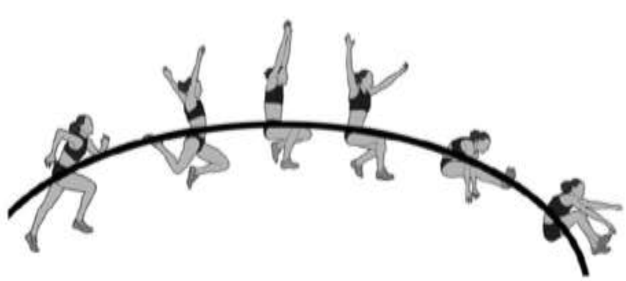
Discuss the timing in Linear motion, Uniform Motion, ease in (Slow in) and ease out (slow out) (8M)

Timing animation refers to the duration of an action.

In animation, timing of action consists of placing objects or characters in particular locations at specific frames to give the illusion of motion.

Line of action: Individual drawings or poses have a **line of action**, which indicates the visual flow of action at that single image.

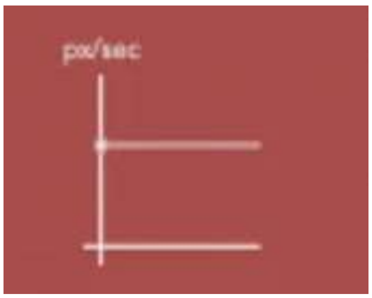
Path of action Motion has a **path of action**, which indicates the path along which the object or character moves. The path of action refers to the object's motion in space.

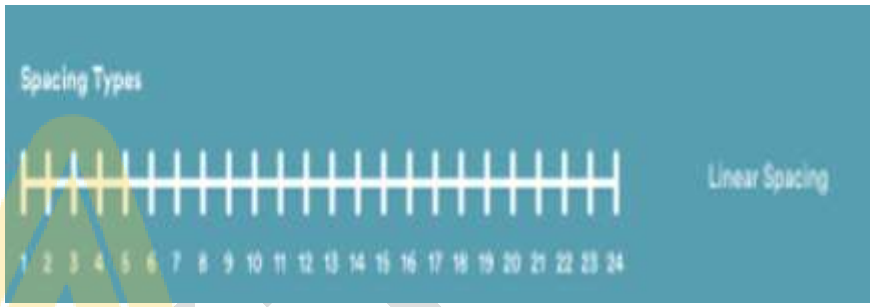


An object moving with linear motion might speed up, slow down or move with a constant speed and it follows a linear path.

1) **Uniform motion**: It is the easiest to animate because the distance the object travels between frames is always the same.

   The object moves the same distance between consecutive frames. The longer the distance between frames, the higher the speed.

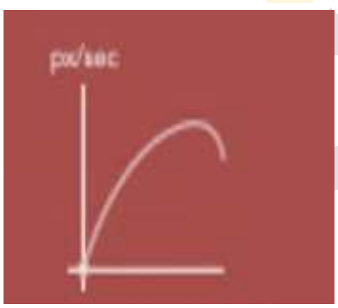


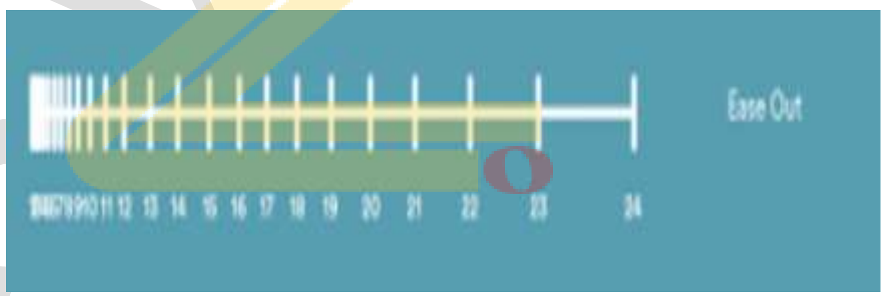


2) Ease out / Speed up

The object is speeding up i.e it's speed increases gradually, often from a still position.

The frames are located such that, initially the frames are closely spaced with gradual increase in the spacings.

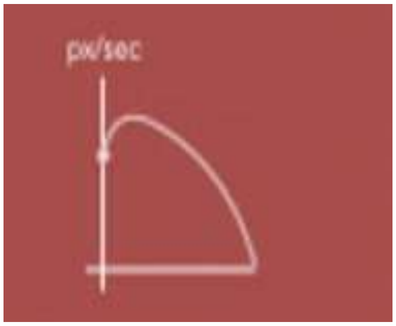


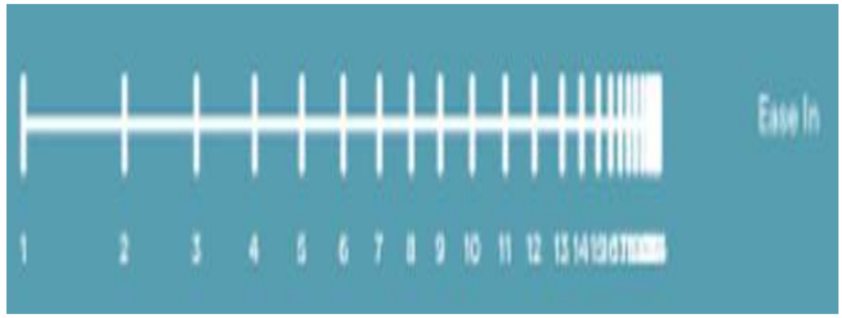


3) Ease in/ Slowed down.

The object is slowing down, it's speed decreases gradually often in preparation for stopping.

The frames are located such that, initially the frames are widely spaced with gradual decrease in the spacings of the frames.





4) Ease out- Ease in or Ease-Ease.

It is the combination of speed up and slowed down. That is the object initially gets speed up initially and finally comes to still position with slowing down.

In the beginning the frames are located such that, initially the frames are closely spaced with gradual increase in the spacings up to middle position.

From the middle position onwards, the frames are widely spaced with gradual decrease in the spacings of the frames towards the still position.

Illustrate the odd rule and odd rule multipliers with a suitable example (8M)

- When acceleration is constant, The Odd Rule is used (Simple Pattern of Odd Numbers) to time the frames.
- Between consecutive frames, the distance moved by the object is a multiple of an odd number.
- For acceleration, the distance between frames increases by multiples of 1, 3, 5, 7, etc.

- For deceleration, the multiples start at a higher odd number and decrease, for example 7, 5, 3, 1.

- The Odd Rule is a multiplying system based on the smallest distance (base distance) travelled between two frames in the sequence

- **Base distance** :For a slow-out is the distance between the first two frames and for a slow-in: the distance between the last two frames is called as the **base distance**.

Rocket speeding up using the Odd Rule.

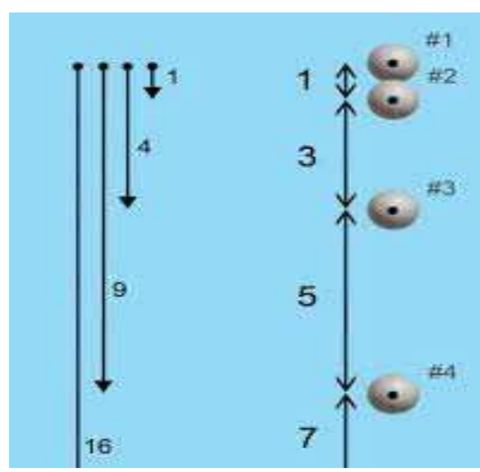
Sled coming to a stop using the Odd Rule.

Odd Rule Multipliers can be used to calculate the distance from the first frame to the current frame and use these distances to place the object on specific frames

Odd multipliers for Consecutive Frames = $((Frame\# - 1) \times 2 - 1)$

Multiplier for distance from first frame to current frame = $(Current\ Frame\# - 1)^2$

	Multiply by base distance to get distance between:	
Frame #	Consecutive frames	First frame and this frame
1	n/a	0
2	1	1
3	3	4
4	5	9
5	7	16
6	9	25
7	11	36



Odd rule scenarios

- Base Distance Known Speeding up : Base Distance * odd rule multi- pliers from the first frame
- Base Distance Known Slowing Down : The base distance * Odd rule multipliers backwards.
- Total Distance and Number of Frames Known, Speeding Up :

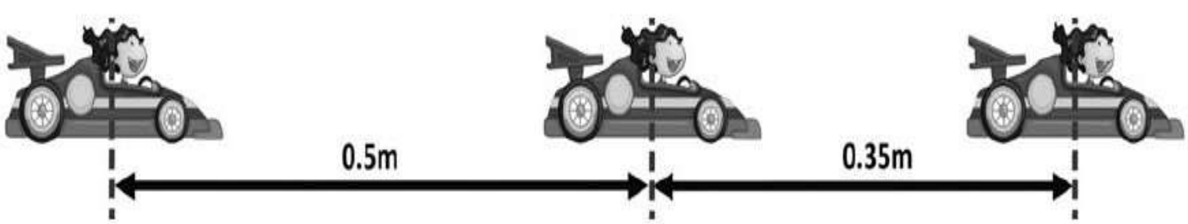
$$\textbf{Base distance} = \frac{\textit{Total distance}}{(\textit{Last frame number} - 1)^2}$$

Example: Suppose there is a jump push (take-off) with constant acceleration over 5 frames, and the total distance travelled is 0.4m. Using the formula above, we find the base distance.

$$Base\ distance = \frac{0.4m}{(5-1)^2} = \frac{0.4m}{16} = 0.025m$$

A slowing in object in an animation has a first frame distance of 0.5 m and the slow in frame 0.35m. Calculate the base distance and the number of frames in sequence (5M)

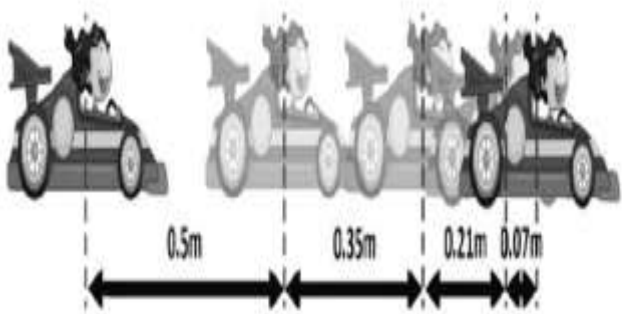
For the given example the illustration can be written as



One of the features of the Odd Rule is that the base distance is always half the difference between any two adjacent distances.

$$\text{Consecutive Frame Multiplier} = \frac{First\ Distance}{Base\ Distance} = \frac{0.5}{0.07} = 7$$

Thus, Consecutive Frame Multiplier '7' Corresponds to '4' Frames



Frame #	Consecutive frame multiplier	Distance from previous frame
1	7	7 * 0.07m = 0.5m
2	5	5 * 0.07m = 0.35m
3	3	3 * 0.07m = 0.21m
4	1	1 * 0.07m = 0.07m

Describe Jumping, parts of Jump and Jump Magnification (8M)

A jump is an action where the character's entire body is in the air, and both the character's feet leave the ground at roughly the same time.

A jump action includes a take-off, free movement through the air, and a landing.

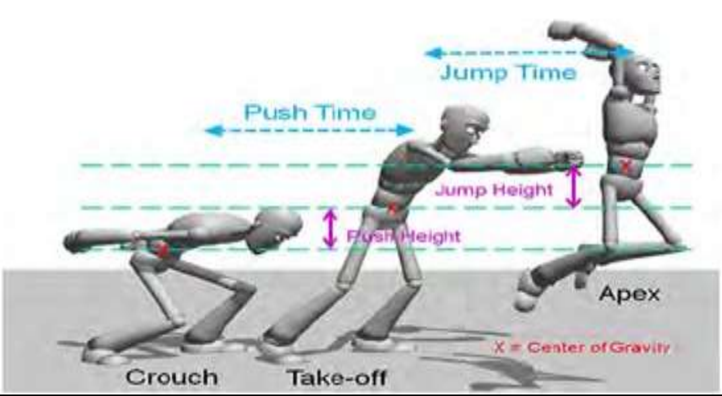
Parts of Jump :

Crouch—A squatting pose taken as preparation for jumping.

Take off—Character pushes up fast and straightens legs with feet still on the ground. The amount of time (or number of frames) needed for the push is called the push time.

In the air— Both the character's feet are off the ground, and the character's CG moves in a parabolic arc as any free-falling body would.

Landing—Character touches the ground and bends knees to return to a crouch. The distance from the character's CG when her feet hit to the ground to the point where the character stops crouching is called the stop height. The stop height is not always exactly the same as the push height.



Push height: The distance between Center of gravity (CG) in crouch position to CG of Take off position

Jump Height: The distance between CG in takeoff position to CG of position at air.

Stop Height: The distance between CG in Landing position to CG of Crouch position during landing.

$$\text{Jump Height} = 1.2\text{m}$$

$$\text{Jump Time} = t = \sqrt{\frac{2h}{g}} = 0.5s$$

$$\text{Jump Time at 30 fps} = 0.5 \times 30 = 15 \text{ frames.}$$

Jump Magnification

Jump Magnification is in fact an exact ratio that tells one how much the character has to accelerate against gravity to get in to the air.

Push time: The number of frames required to move from 'crouch position' to 'Take off position'.

Jump time: The number of frames required to move from 'Take off position' to 'In air position'.

Stop time: The number of frames required to move from 'In air position' to 'Landing position'.

$$JM = \frac{Jump\,Time}{Push\,Time} = \frac{Jump\,Height}{Push\,Height} = \frac{Push\,Acceleration}{Jump\,Acceleration} = \frac{Push\,Accelaration}{Gravitational\,Accelaration}$$

Example:

Push Time: 5 frames

Push Height: 0.4m

Stop Height: 0.5m

Stop Time = (5 * 0.5) / 0.4 = 6 frames

Define Strides and Gait.

Walking

Walks feature all the basics of mechanics while including personality. The ability to animate walk cycles is one of the

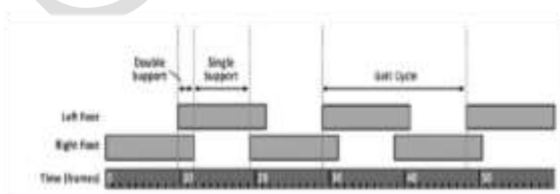
most important skills a character animator needs to master.

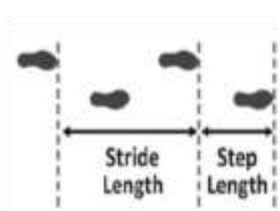
Strides and Steps

A step is one step with one foot. A stride is two steps, one with each foot. Stride length is the distance the character travels in a stride, measured from the same part of the foot. Step and stride length indicate lengthwise spacing for the feet during a walk.

Gait is the timing of the motion for each foot, including how long each foot is on the ground or in the air.

During a walk, the number of feet the character has on the ground changes from one foot (single support) to two feet (double support) and then back to one foot. You can plot the time each foot is on the ground to see the single and double support times over time. A normal walking gait ranges from 1/ 3 to 2/3 of a second per step, with 1/2 second being average.





Statistical Physics for Computing

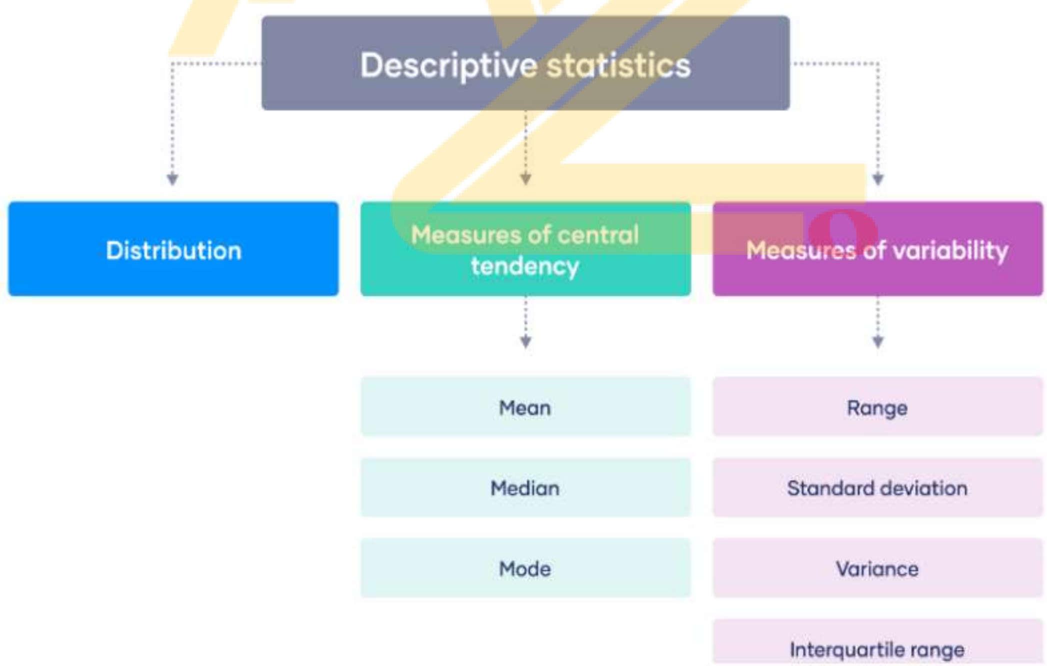
Distinguish between Descriptive Statistics and Inferential statistics

Statistical physics is a branch of physics that evolved from a foundation of statistical mechanics, which uses methods of probability theory and statistics, particularly the mathematical tools for dealing with large populations and approximations, in solving physical problems.

Descriptive statistics: The term "descriptive statistics" refers to summarizing and organizing the characteristics of a data set. A data set is a collection of responses or observations from a sample or entire population.

In quantitative research, after collecting data, the first step of statistical analysis is to describe characteristics of the responses, such as the average of one variable (e.g., age), or the relation between two variables (e.g., age and creativity).

Descriptive statistics comprises three main categories – Frequency Distribution, Measures of Central Tendency, and Measures of Variability.



Inferential Statistics:

Inferential Statistics is a method that allows us to use information collected from a sample to make decisions, predictions, or inferences from a population. The major inferential statistics are based on statistical models such as Analysis of Variance, chi-square test, student's t distribution, regression analysis, etc.

Methods of inferential statistics:

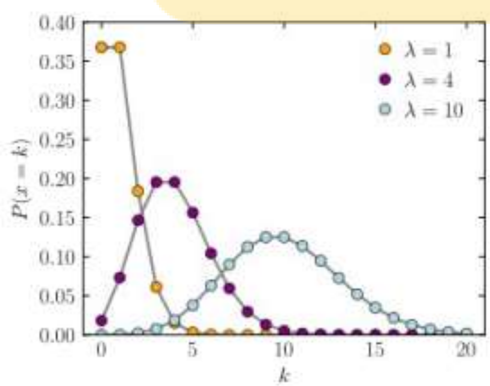
- Estimation of parameters
- Testing of hypothesis

Explain the Poisson's distribution with an example

Poisson Distribution If the probability p is so small that the function has significant value only for very small k, then the distribution of events can be approximated by the Poisson Distribution. Probability mass function A discrete Radom variable X is said to have a Poisson distribution, with parameter , if it has a probability Mass Function given by

$$f(k;\lambda)=P(X=k)=\frac{\lambda^k e^{-\lambda}}{k!}$$

Here k is the number of occurrences, e is Euler's Number,! is the factorial function. The positive real number λ is equal to the expected value of X and also to its Variance. The Poisson distribution may be used in the design of experiments such as scattering experiments where a small number of events are seen.



Example of probability for Poisson distributions On a particular river, overflow floods occur once every 100 years on average. Calculate the probability of k = 0, 1, 2, 3, 4, 5, or 6 overflow floods in a 100 year interval, assuming the Poisson model is appropriate. Because the average event rate is one overflow flood per 100 years, λ = 1

$$f(K,\lambda) = P(X = K) = \frac{\lambda^K e^{-\lambda}}{K!}$$

$$\text{P (K overflow floods in 100 years)} = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{1^K e^{-1}}{K!}$$

$$\text{P (K=0 overflow floods in 100 years)} = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{1^0 e^{-1}}{0!} = \frac{e^{-1}}{1} = 0.368$$

$$\text{P (K=1 overflow floods in 100 years)} = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{1^1 e^{-1}}{1!} = \frac{e^{-1}}{1} = 0.368$$

$$\text{P (K=2 overflow floods in 100 years)} = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{1^2 e^{-1}}{2!} = \frac{e^{-1}}{2} = 0.184$$

Discuss the modelling probability for proton decay

Proton decay

Proton decay is a rare type of radioactive decay of nuclei containing excess protons, in which a proton is simply ejected from the nucleus. The mechanism of the decay process is very similar to alpha decay. Proton decay is also a quantum tunneling process.

Modeling the Probability for Proton Decay

The probability of observing a proton decay can be estimated from the nature of particle decay and the application of Poisson Statistics. The number of protons N can be modeled by the decay equation

$$N = N_0\, e^{-\lambda t}$$

Where:

N0: is the initial quantity of the element

λ: is the radioactive decay constant

t: is time

N(t): is the quantity of the element remaining after time t.

Here $\lambda = 1/t = 10^{-33}/\ year$ is the probability that any given proton will decay in a year.

Since the decay constant λ is so small, the exponential can be represented by the first two terms of the Exponential Series.

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots \infty$$

$$e^{-\lambda t} = 1 - \lambda t$$

$$N = N_0(1 - \lambda t)$$

Most recently the experiment on proton decay has been done by Super Kamiokande, Japan which started observation in 1996. It is a large water Cherenkov detector which is the most sensitive detector in the world used to examine proton decay with the huge source with 7.5×10^{33} protons

For one year of observation, the number of expected proton decays is then

No-N = No λ t

$$= (7.5 \times 10^{33}\ protons)(10^{-33} / \ year)(1\ year)$$

$$N_0 - N = 7.5$$

$$N_0 - N = N_0(1 - \lambda t)$$

$$= (7.5 \times 10^{33}\ protons)(10^{-33} / \ year)(1\ year)$$

$$N_0 - N = 7.5$$

Proton decay has not been detected experimentally till now probably because of fact that the event is extremely rare. Assuming that λ = 3 observed decays per year is mean, then the Poisson distribution function tells us that the probability for zero observations of decay is

$$P(K) = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{3^0 e^{-3}}{0!} = 0.05$$

This low probability for a null result suggests that the proposed lifetime of 10^{33} years is too short.

Discuss the salient features of normal distribution using bell curves

Normal Distribution:

The bell curve is a normal probability distribution of variables plotted on the graph and is like a bell shape where the highest or top point of the curve represents the most probable event out of all the series data.

CHARACTERISTICS

1. The Normal Curve is Symmetrical: The normal probability curve is symmetrical around its vertical axis called ordinate which represents the mean of distribution. The symmetry about the ordinate at the central point of the curve implies that the size, shape, and slope of the curve on one side of the curve is identical to that of the other. In other words, the left and right halves of the middle central point are mirror images, as shown in the figure given here.

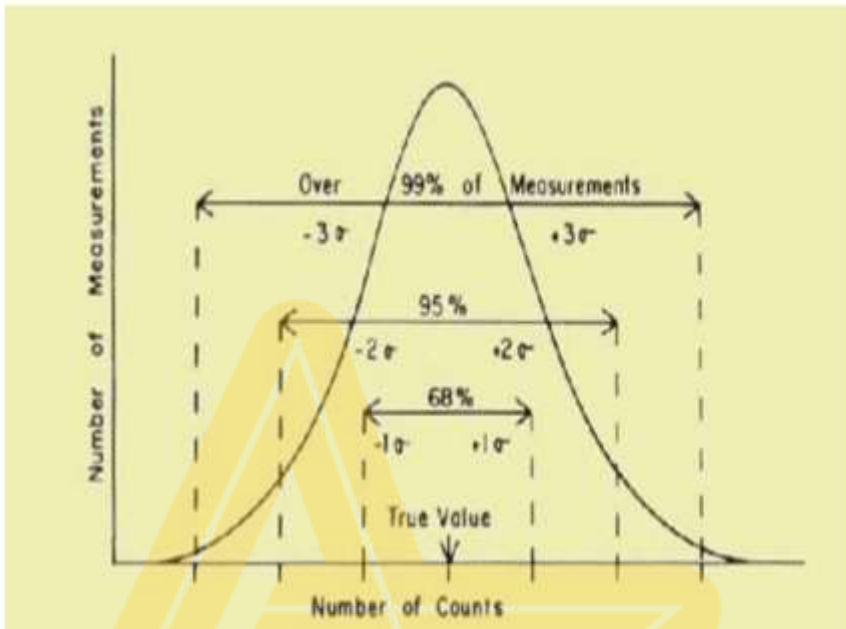
2. The Normal Curve is Unimodel: Since there is only one maximum point in the curve, thus the normal probability curve is unimodal, i.e. it has only one mode.

3. The Normal Curve is Bilateral: The total area under the curve is 1, the 50% area of the curve lies to the left side of the maximum central ordinate and 50% of the area lies to the right side. Hence the curve is bilateral.

4. The Normal Curve is a mathematical model in behavioral Sciences: This curve is used as a measurement scale. The measurement un it of this scale is ± 1σ (the unit standard deviation).

Standard Deviations: The standard normal distribution is a normal probability distribution that has a mean of 0 and a standard deviation of 1. The Standard Deviation is a measure of how spread-out numbers are. As per 3 sigma rule of normal distribution,

I. 68% of values are within 1 standard deviation of the mean.
II. 95% of values are within 2 standard deviations of the mean.
III. 99.7%of values are within 3 standard deviations of the mean



Mention the general pattern of Monte-Carlo Method and hence determine the value of pi.

Monte-Carlo Method:

Monte Carlo Simulation, also known as the Monte Carlo Method or a multiple probability simulation, is a mathematical technique, which is used to estimate the possible outcomes of an uncertain event. The Monte Carlo Method was invented by John von Neumann and Stanislaw Ulam during World War II to improve decision-making under uncertain conditions. It was named after a well-known casino town, called Monaco.

The statistical method of understanding complex physical or mathematical systems by using randomly generated numbers as input into those systems to generate a range of solutions.

How to use Monte Carlo methods

1. Define a domain of possible inputs

2. Generate inputs randomly from a probability distribution over the domain

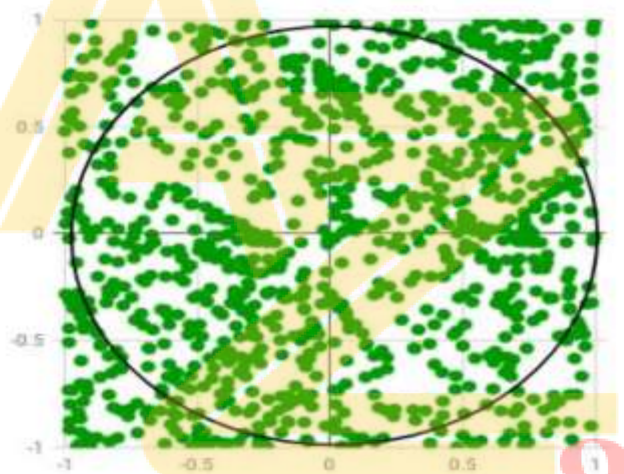
3. Perform a deterministic computation on the inputs

4. Aggregate the results

Estimation of Pi

- The idea is to simulate random (x, y) points in a 2-D plane with the domain as a square of side 2r units centered on (0,0).
- Imagine a circle inside the same domain with the same radius r and inscribed into the square.
- We then calculate the ratio of the number of points that lay inside the circle and the total number of generated points.

Refer to the image below:



We know that the area of the circle πr 2 , while that of square 4r2 . The ratio of these two areas is as follows:

$$\frac{area\ of\ the\ circle}{area of\ the\ square} = \frac{\pi r^2}{4r^2} = \frac{\pi}{4}$$

Now for a very large number of generated points $\frac{no\ of\ points\ generated\ inside\ \ the\ circle}{no\ of\ points\ generated\ inside\ the\ square} = \frac{\pi}{4}$

$$4 * \frac{no\ of\ points\ generated\ inside\ \ the\ circle}{no\ of\ points\ generated\ inside\ the\ square} = \pi$$