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M-2 PASSING PACKAGE 2023

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Module - 01

Integral Calculus

Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

$\Rightarrow$ We know that,

$$\Gamma(m) = 2\int_0^\infty e^{-x^2} x^{2m-1} dx$$

$$\Gamma(n) = 2\int_0^\infty e^{-y^2} y^{2n-1} dy$$

$$\Gamma(m+n) = 2\int_0^\infty e^{-r^2} r^{2(m+n)-1} dr$$

$$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta$$

$$\Gamma(m)\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2m-1} dx \cdot 2\int_0^\infty e^{-y^2} y^{2n-1} dy$$

$$\Gamma(m)\Gamma(n) = 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} x^{2m-1} y^{2n-1} dx\,dy$$

$x = r\cos\theta$, $y = r\sin\theta$

$x^2+y^2 = r^2$

$dx\,dy = r\,dr\,d\theta$

$r \to 0$ to ∞, $\theta \to 0$ to $\pi/2$

$$\Gamma(m)\Gamma(n) = 4\int_{r=0}^{r=\infty}\int_{\theta=0}^{\theta=\pi/2} e^{-r^2} r^{2m-1} \cos^{2m-1}\theta \; r^{2n-1} \sin^{2n-1}\theta \; r^1 \, dr\, d\theta$$

$$\Gamma(m)\Gamma(n) = 4\int_0^\infty\int_0^{\pi/2} e^{-r^2} r^{2m-1+2n-1+1} dr \cos^{2m-1}\theta \sin^{2n+1}\theta \, d\theta$$

$$\Gamma(m)\Gamma(n) = 2\int_{r=0}^{\infty} e^{-r^2} r^{2(m+n)-1} dr \cdot 2\int_{\theta=0}^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta$$

$$\Gamma(m)\Gamma(n) = \Gamma(m+n)\beta(m,n)$$

$$\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} = \beta(m,n)$$

Hence proved

Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ by changing into polar coordinates

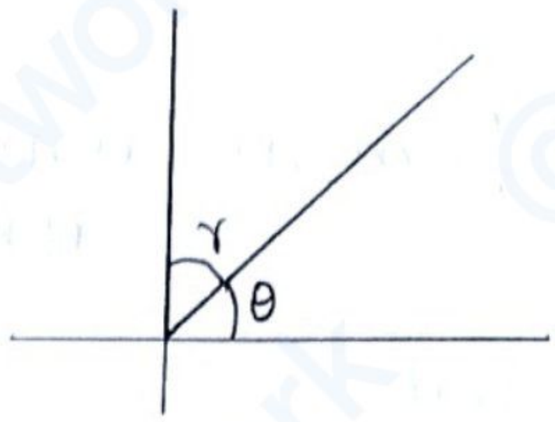
Soln:- $x = r\cos\theta$, $y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta$

$x^2+y^2 = r^2$

$r \to 0$ to ∞

$\theta \to 0$ to $\pi/2$



$$I = \int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} e^{-r^2}\cdot r\,dr\,d\theta$$

$$I = \int_0^\infty\int_0^{\pi/2} e^{-r^2}\cdot r\,d\theta\,dr$$

$$I = \int_0^\infty e^{-r^2}\cdot r\theta\Big|_{\theta=0}^{\pi/2}\,dr$$

$$I = \int_0^\infty e^{-r^2}\cdot r\left[\frac{\pi}{2}-0\right]dr$$

$v = r^2$

$dv = 2r\,dr$

$$I = \frac{\pi}{2}\int_0^\infty e^{-v}\,\frac{dv}{2}$$

$$I = \frac{\pi}{4}\left[\frac{e^{-v}}{-1}\right]_{r=0}^{\infty}$$

$$I = \frac{-\pi}{4}\left[\frac{e^{-v}}{-1}\right]_{r=0}^{\infty}$$

$$I = \frac{-\pi}{4}\left[e^{-r^2}\right]\Big|_{r=0}^{\infty}$$

$$I = \frac{-\pi}{4}\left[e^{-\infty}-e^{0}\right]$$

$$I = \frac{\pi}{4}$$

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Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$

Sol^n: $I = \int_0^a\int_0^x\int_0^{x+y} e^x e^y e^z\, dz\, dy\, dx$

$$I = \int_0^a\int_0^x e^x e^y e^z \Big|_{z=0}^{x+y} dy\, dx$$

$$I = \int_0^a\int_0^x e^x e^y (e^{x+y} - e^0)\, dy\, dx$$

$$I = \int_0^a\int_0^x e^{2x} e^{2y} - e^x e^y\, dy\, dx$$

$$I = \int_0^a e^{2x}\frac{e^{2y}}{2} - e^x e^y \Big|_{y=0}^{x} dx$$

$$I = \int_0^a \frac{e^{2x}}{2}(e^{2x} - e^0) - e^x(e^x - e^0)\, dx$$

$$I = \int_0^a \frac{e^{4x}}{2} - \frac{e^{2x}}{2} - e^{2x} + e^x\, dx$$

$$I = \int_0^a \frac{e^{4x}}{2} - \frac{3e^{2x}}{2} + e^x\, dx$$

$$I = \frac{e^{4x}}{8} - \frac{3e^{2x}}{4} + e^x \Big|_0^a$$

$$I = \frac{e^{4a}}{8} - \frac{3e^{2a}}{4} + e^a - \left(\frac{1}{8} - \frac{3}{4} + 1\right)$$

$$I = \frac{e^{4a}}{8} - \frac{3e^{2a}}{4} + e^a - \left(\frac{1-6+8}{8}\right)$$

$$I = \frac{e^{4a}}{8} - \frac{3e^{2a}}{4} + e^a - \frac{3}{8}$$

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Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dx\,dy\,dz$

Sol^n:- $I = \int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a} x^2+y^2+z^2\,dx\,dy\,dz$

$$I = \int_{-c}^{c}\int_{-b}^{b} \frac{x^3}{3} + y^2x + z^2x \Big|_{x=-a}^{x=a} dy\,dz$$

$$I = \int_{-c}^{c}\int_{-b}^{b} \frac{1}{3}[a^3+a^3] + y^2[a+a] + z^2[a+a]\,dy\,dz$$

$$I = \int_{-c}^{c}\int_{-b}^{b} \frac{2a^3}{3} + 2ay^2 + 2az^2\,dy\,dz$$

$$I = \int_{-c}^{c} \frac{2a^3}{3}y + 2a\frac{y^3}{3} + 2az^2y \Big|_{x=-b}^{x=b} dz$$

$$I = \int_{-c}^{c} \frac{2a^3}{3}[b+b] + \frac{2a}{3}[b^3+b^3] + 2az^2[b+b]\,dz$$

$$I = \int_{-c}^{c} \frac{4a^3b}{3} + \frac{4ab^3}{3} + 4abz^2\,dz$$

$$I = \frac{4a^3b}{3}z + \frac{4ab^3}{3}z + 4ab\frac{z^3}{3}\Big|_{x=-c}^{x=c}$$

$$I = \frac{4a^3b}{3}[c+c] + \frac{4ab^3}{3}[c+c] + \frac{4ab}{3}[c^3+c^3]$$

$$I = \frac{8a^3bc}{3} + \frac{8ab^3c}{3} + \frac{8abc^3}{3}$$

$$I = \frac{8abc}{3}[a^2+b^2+c^2]$$

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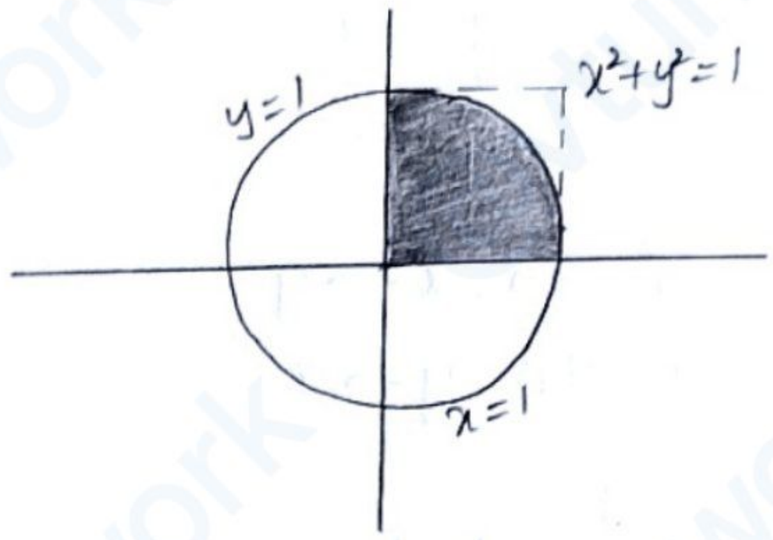
Evaluate $\int_0^1\int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$ by changing into polar coordinates

Sol^n:-



$x = r\cos\theta$, $y = r\sin\theta$

$x^2+y^2 = r^2$

$dx\,dy = r\,dr\,d\theta$

$y=0$, $y=1$

$x=0$, $x=\sqrt{1-x^2}$

$x^2 = 1-y^2$

$x^2-y^2 = 1$

$r \to 0$ to 1

$\theta \to 0$ to $\pi/2$

$$I = \int_0^1\int_0^{\pi/2} r^2\, r\,dr\,d\theta$$

$$I = \int_0^1\int_0^{\pi/2} r^3\,dr\,d\theta$$

$$I = \int_0^1 \frac{r^4}{4}\Big|_0^{\pi/2} d\theta$$

$$I = \frac{1}{4}\int_0^1 \left(\frac{\pi}{4}\right)^2 - 0\,d\theta$$

$$I = \frac{1}{4}\int_0^1 \frac{\pi^4}{16}\,d\theta$$

$$I = \frac{\pi^4}{64}[\theta]_0^1$$

$$I = \frac{\pi^4}{64}$$

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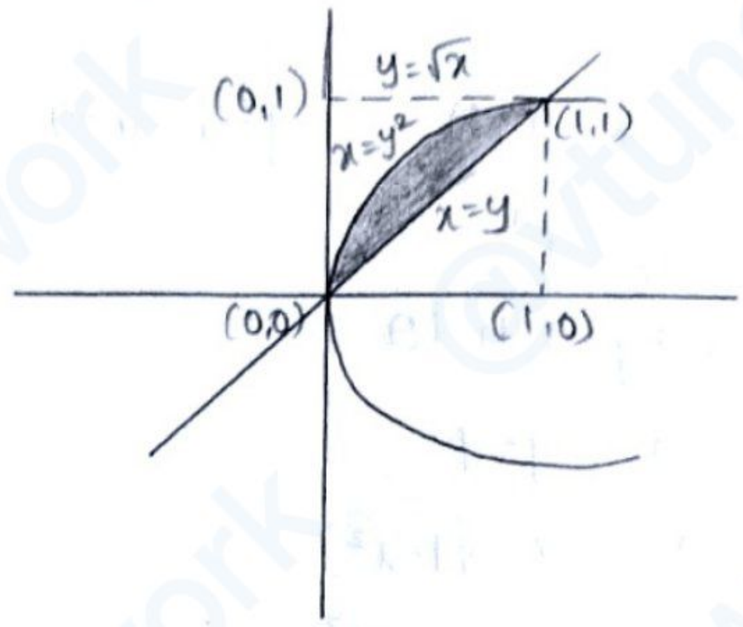
Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$ by changing the order of Integration

Soln:-

$y = x$, $y = \sqrt{x}$

$y^2 = x$



$x \to y^2$ to $x \to y$

$y \to 0$ to $y \to 1$

$$I = \int_{y=0}^{y=1} \int_{x=y^2}^{y} xy\,dx\,dy$$

$$I = \int_0^1 y \frac{x^2}{2} \Big|_{y^2}^{y} dy$$

$$I = \int_0^1 \frac{y}{2} [y^2 - y^4]\,dy$$

$$I = \frac{1}{2} \int_0^1 y^3 - y^5\,dy$$

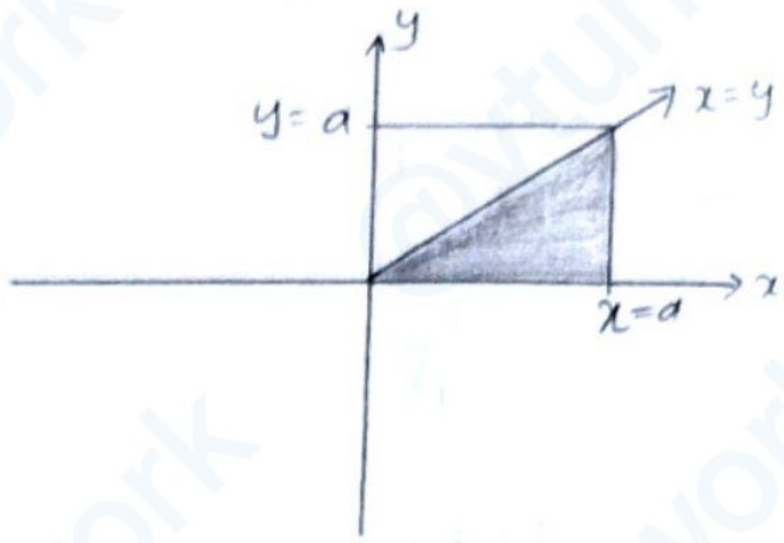
$$I = \frac{1}{2} \left(\frac{y^4}{4} - \frac{y^6}{6} \right) \Big|_0^1$$

$$I = \frac{1}{2} \left[\frac{1}{4} - \frac{1}{6} - (0) \right]$$

$$I = \frac{1}{2} \left[\frac{1}{12} \right]$$

$$I = \frac{1}{24}$$

Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$ by changing the order of Integration



Soln:- $y=0$, $y=a$

$x=y$, $x=a$

$x \to 0$ to $x \to a$

$y \to 0$ to $y \to x$

$$I = \int_{x=0}^{x=a}\int_{y=0}^{y=x} \frac{x}{x^2+y^2}\,dy\,dx$$

$$I = \int_0^a x \int_0^x \frac{1}{x^2+y^2}\,dy\,dx$$

$$I = \int_0^a x \frac{1}{x}\left[\tan^{-1}\left(\frac{y}{x}\right)\right]_0^x dx$$

$$I = \int_0^a \tan^{-1}\left(\frac{x}{x}\right) - \tan^{-1}(0)\,dx$$

$$I = \int_0^a \frac{\pi}{4}\,dx$$

$$I = \frac{\pi}{4}\int_0^a dx$$

$$I = \frac{\pi}{4}x \Big|_0^a$$

$$I = \frac{\pi}{4}a$$

Using double integration find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Soln:-

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$$

$$I=\iint_R dx\,dy$$

$$y^2=b^2\left[\frac{a^2-x^2}{a^2}\right]$$

$$y=\frac{b}{a}\sqrt{a^2-x^2}$$

$y\to 0$ to $y\to\frac{b}{a}\sqrt{a^2-x^2}$

$x\to 0$ to $x\to a$

$$I=\int_0^a\int_0^{\frac{b}{a}\sqrt{a^2-x^2}} dy\,dx$$

$$I=\int_0^a y\Big|_0^{\frac{b}{a}\sqrt{a^2-x^2}} dx$$

$$I=\int_0^a \frac{b}{a}\sqrt{a^2-x^2}-0\,dx$$

$$I=\frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$I=\frac{b}{a}\left[0+\frac{a^2}{2}\times\frac{\pi}{2}-(0)\right]$$

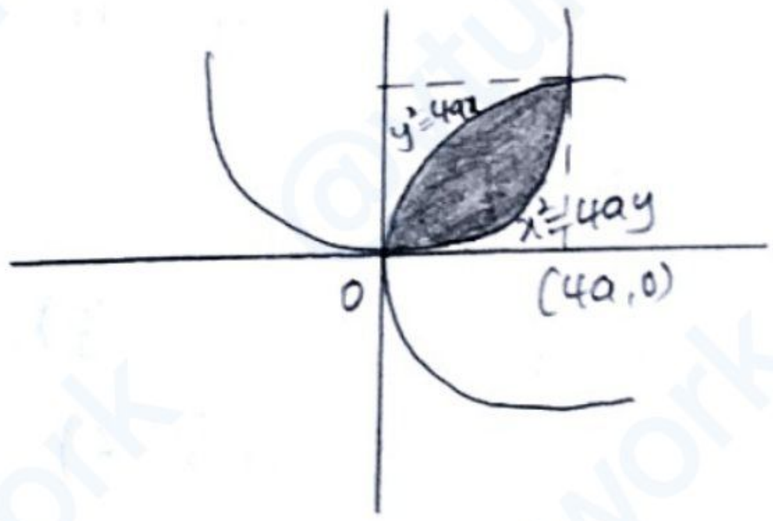
$$I=\frac{\pi ab}{4}$$

$$A=4I$$

$$A=4\,\frac{\pi ab}{4}$$

$$\underline{\underline{A=\pi ab}}$$

Find by double integration, the area between the parabolas $y^2=4ax$ and $x^2=4ay$.



Soln:

$y^2=4ax \quad , x^2=4ay$

$y^2=16a^2x^2$

$y^4=16a^2.4ay$

$y^4=64a^3y$

$y^3=(4a)^3$

$y=4a$

$(4a,4a)$

$y \to \frac{x^2}{4a}$ to $y \to 2\sqrt{ax}$

$x \to 0$ to $x \to 4a$

$$I=\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$$

$$I=\int_0^{4a} y\Big|_{\frac{x^2}{4a}}^{2\sqrt{ax}} dx$$

$$I=\int_0^{4a}\left[2\sqrt{ax}-\frac{x^2}{4a}\right]dx$$

$$I=2\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]-\left[\frac{x^3}{12a}\right]\Big|_0^{4a}$$

$$I=2\sqrt{a}\,\frac{2}{3}\left[(4a)^{3/2}-0\right]-\frac{1}{12a}\left[(4a)^3-0\right]$$

$$I=2\sqrt{a}\,\frac{2}{3}\,8a^{3/2}-\frac{64a^3}{12a}$$

$$I=\frac{32}{3}\sqrt{a}-a\sqrt{a}-\frac{16}{3}a^2$$

$$I=\frac{a^2}{3}\left[32-16\right]$$

$$A=\frac{16a^2}{3}$$

Show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

Soln:- $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

$m = \frac{1}{2} = n$

$\beta(\frac{1}{2}, \frac{1}{2}) = \dfrac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}+\frac{1}{2})}$

$2\int_0^{\pi/2} \sin^0\theta \cos^0\theta \, d\theta = \dfrac{\Gamma(\frac{1}{2})^2}{\Gamma(1)}$

$2\int_0^{\pi/2} 1 \, d\theta = \dfrac{\Gamma(\frac{1}{2})^2}{1}$

$2[\theta]\Big|_0^{\pi/2} = \Gamma(\frac{1}{2})^2$

$2[\frac{\pi}{2} - 0] = \Gamma(\frac{1}{2})^2$

$\Gamma(\frac{1}{2})^2 = \pi$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$

$\Gamma(1) = \int_0^\infty e^{-x} \cdot 1 \, dx$

$\Gamma(1) = \dfrac{e^{-x}}{-1}\Big|_0^\infty$

$\Gamma(1) = -(\cancel{e^{-\infty}}^{0} - e^0)$

$\Gamma(1) = 1$

Module - 2

Vector Calculus

Show that the spherical coordinate system is orthogonal

Sol^n: r, θ, ϕ

$x = r\sin\theta\cos\phi \quad y = r\sin\theta\sin\phi \quad z = r\cos\theta$

$\vec{r} = xi + yj + zk$

$\vec{r} = r\sin\theta\cos\phi\, i + r\sin\theta\sin\phi\, j + r\cos\theta\, k$

$$\hat{e}_1 = \frac{\frac{\partial\vec{r}}{\partial r}}{\left|\frac{\partial\vec{r}}{\partial r}\right|} = \frac{\sin\theta\cos\phi\, i + \sin\theta\sin\phi\, j + \cos\theta\, k}{\sqrt{\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta}} = \sin\theta\cos\phi\, i + \sin\theta\sin\phi\, j + \cos\theta\, k$$

$$\hat{e}_2 = \frac{\frac{\partial\vec{r}}{\partial\theta}}{\left|\frac{\partial\vec{r}}{\partial\theta}\right|} = \frac{r\cos\theta\cos\phi\, i + r\cos\theta\sin\phi\, j - r\sin\theta\, k}{\sqrt{r^2\cos^2\theta\cos^2\phi + r^2\cos^2\theta\sin^2\phi + r^2\sin^2\theta}} = \cos\theta\cos\phi\, i + \cos\theta\sin\phi\, j - \sin\theta\, k$$

$$\hat{e}_3 = \frac{\frac{\partial\vec{r}}{\partial\phi}}{\left|\frac{\partial\vec{r}}{\partial\phi}\right|} = \frac{-r\sin\theta\sin\phi\, i + r\sin\theta\cos\phi\, j + 0k}{\sqrt{r^2\sin^2\theta\sin^2\phi + r^2\sin^2\theta\cos^2\phi + 0}} = -\sin\phi\, i + \cos\phi\, j + 0k$$

$\hat{e}_1 \cdot \hat{e}_2 = (\sin\theta\cos\phi\, i + \sin\theta\sin\phi\, j + \cos\theta\, k) \cdot (\cos\theta\cos\phi\, i + \cos\theta\sin\phi\, j - \sin\theta\, k)$

$\hat{e}_1 \cdot \hat{e}_2 = \sin\theta\cos^2\phi\cos\theta + \sin\theta\cos\theta\sin^2\phi - \cos\theta\sin\theta$

$\hat{e}_1 \cdot \hat{e}_2 = 0$

$\hat{e}_1 \cdot \hat{e}_3 = (\sin\theta\cos\phi\, i + \sin\theta\sin\phi\, j + \cos\theta\, k) \cdot (-\sin\phi\, i + \cos\phi\, j + 0k)$

$\hat{e}_1 \cdot \hat{e}_3 = -\sin\theta\cos\phi\sin\phi + \sin\theta\cos\phi\sin\phi$

$\hat{e}_1 \cdot \hat{e}_3 = 0$

$\hat{e}_2 \cdot \hat{e}_3 = (\cos\theta\cos\phi\, i + \cos\theta\sin\phi\, j - \sin\theta\, k) \cdot (-\sin\phi\, i + \cos\phi\, j + 0k)$

$\hat{e}_2 \cdot \hat{e}_3 = -\cos\theta\cos\phi\sin\phi + \cos\theta\sin\phi\cos\phi$

$\hat{e}_2 \cdot \hat{e}_3 = 0$

$\therefore$ Spherical polar coordinate system is orthogonal

Show that Cylindrical coordinate system is orthogonal.

ρ, ϕ, z

$x = \rho\cos\phi \quad , y = \rho\sin\phi \quad , z = z$

$\vec{r} = xi + yj + zk$

$\vec{r} = \rho\cos\phi\, i + \rho\sin\phi\, j + zk$

$$\hat{e}_1 = \frac{\frac{\partial\vec{r}}{\partial\rho}}{\left|\frac{\partial\vec{r}}{\partial\rho}\right|} = \frac{\cos\phi i + \sin\phi j + 0k}{\sqrt{\cos^2\phi + \sin^2\phi}} = \cos\phi i + \sin\phi j + 0k$$

$$\hat{e}_2 = \frac{\frac{\partial\vec{r}}{\partial\phi}}{\left|\frac{\partial\vec{r}}{\partial\phi}\right|} = \frac{-\rho\sin\phi i + \rho\cos\phi j + 0k}{\sqrt{\rho^2\sin^2\phi + \rho^2\cos^2\phi}} = -\sin\phi i + \cos\phi j + 0k$$

$$\hat{e}_3 = \frac{\frac{\partial\vec{r}}{\partial z}}{\left|\frac{\partial\vec{r}}{\partial z}\right|} = \frac{0i + 0j + 1k}{\sqrt{0 + 0 + 1}} = 0i + 0j + 1k$$

$\hat{e}_1 \cdot \hat{e}_2 = (\cos\phi i + \sin\phi j + 0k) \cdot (-\sin\phi i + \cos\phi j + 0k)$

$\hat{e}_1 \cdot \hat{e}_2 = -\cos\phi\sin\phi + \sin\phi\cos\phi$

$\hat{e}_1 \cdot \hat{e}_2 = 0$

$\hat{e}_2 \cdot \hat{e}_3 = (-\sin\phi i + \cos\phi j + 0k) \cdot (0i + 0j + 1k)$

$\hat{e}_2 \cdot \hat{e}_3 = 0$

$\hat{e}_3 \cdot \hat{e}_1 = (0i + 0j + 1k) \cdot (\cos\phi i + \sin\phi j + 0k)$

$\hat{e}_3 \cdot \hat{e}_1 = 0$

$\therefore$ Cylindrical coordinate system is orthogonal

Find $\nabla\phi$, if $\phi = x^3 + y^3 + z^3 - 3xyz$ at the point $(1,-1,2)$

Sol^n: $\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$

$$\nabla\phi = (3x^2 - 3yz) i + (3y^2 - 3xz) j + (3z^2 - 3xy) k$$

$$\nabla\phi_{(1,-1,2)} = (3(1)^2 - 3(-1)(2)) i + (3(-1)^2 - 3(1)(2)) j + (3(2)^2 - 3(1)(-1)) k$$

$$\nabla\phi_{(1,-1,2)} = (3+6) i + (3-6) j + (12+3) k$$

$$\nabla\phi_{(1,-1,2)} = 9i - 3j + 5k$$

If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find div$\vec{F}$ and curl$\vec{F}$

Sol^n: $\phi = x^3 + y^3 + z^3 - 3xyz$

$$\vec{F} = \nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$$

$$\vec{F} = (3x^2 - 3yz) i + (3y^2 - 3xz) j + (3z^2 - 3xy) k$$

$$\vec{A} = a_1 i + a_2 j + a_3 k$$

$a_1 = 3x^2 - 3yz \quad a_2 = 3y^2 - 3xz \quad a_3 = 3z^2 - 3xy$

$$\text{div}\vec{F} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}$$

$$\text{div}\vec{F} = 6x + 6y + 6z$$

$$\text{div}\vec{F} = 6(x+y+z)$$

$$\text{Curl}\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_1 & a_2 & a_3 \end{vmatrix}$$

$$\text{Curl}\vec{F} = i\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) - j\left(\frac{\partial a_3}{\partial x} - \frac{\partial a_1}{\partial z}\right) + k\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$

$$\text{Curl}\vec{F} = i(-3x + 3x) - j(-3y + 3y) + k(-3z + 3z)$$

$$\text{Curl}\vec{F} = 0i - 0j + 0k$$

$$\text{Curl}\vec{F} = 0$$

If $\vec{F} = \nabla(xy^3z^2)$, find div $\vec{F}$ and Curl $\vec{F}$ at the point $(1,-1,1)$

Sol^n:- $\phi = xy^3z^2$

$$\vec{F} = \nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$$

$$\vec{F} = (y^3z^2)i + (3xy^2z^2)j + (2xy^3z)k$$

$$\vec{A} = a_1 i + a_2 j + a_3 k$$

$$a_1 = y^3z^2, \quad a_2 = 3xy^2z^2 \quad a_3 = 2xy^3z$$

$$\text{div}\vec{F} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}$$

$$\text{div}\vec{F} = 0 + 6xyz + 2xy^3$$

$$\text{div}\vec{F}_{(1,-1,1)} = 0 + 6(1)(-1)(1) + 2(1)(-1)^3$$

$$\text{div}\vec{F}_{(1,-1,1)} = \underline{-8}$$

$$\text{Curl}\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_1 & a_2 & a_3 \end{vmatrix}$$

$$\text{Curl}\vec{F} = i\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) - j\left(\frac{\partial a_3}{\partial x} - \frac{\partial a_1}{\partial z}\right) + k\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$

$$\text{Curl}\vec{F} = i(6xy^2z - 6xy^2z) - j(2y^3z - 2y^3z) + k(3y^2z^2 - 3y^2z^2)$$

$$\text{Curl}\vec{F} = \underline{\underline{0}}$$

Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction of the vector $2i - j - 2k$.

Sol^n: $d.d = \nabla\phi \cdot \hat{n}$

$$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$$

$$\nabla\phi = (2xyz + 4z^2) i + (x^2z) j + (x^2y + 8xz) k$$

$$\nabla\phi_{(1,-2,-1)} = (2(1)(-2)(-1) + 4(-1)^2) i + ((1)^2(-1)) j + ((1)^2(-2) + 8(1)(-1)) k$$

$$\nabla\phi_{(1,-2,-1)} = 8i - j - 10k$$

$$\hat{n} = \frac{\vec{d}}{|\vec{d}|}$$

$$\hat{n} = \frac{2i - j - 2k}{\sqrt{4+1+4}}$$

$$\hat{n} = \frac{2i - j - 2k}{3}$$

$$d.d = \nabla\phi \cdot \hat{n}$$

$$= (8i - j - 10k) \cdot \left(\frac{2i - j - 2k}{3}\right)$$

$$d.d = \frac{1}{3}[16 + 1 + 20]$$

$$d.d = \frac{37}{3}$$

Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1,-2,1)$ in the direction of the vector $2i - j + k$

Sol^n: $d.d = \nabla\phi \cdot \hat{n}$

$$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$$

$$\nabla\phi = (2xyz + 4z^2) i + (x^2z) j + (x^2y + 8xz) k$$

$$\nabla\phi_{(1,-2,1)} = (2(1)(-2)(1) + 4(1)^2) i + ((1)^2(1)) j + ((1)^2(-2) + 8(1)(1)) k$$

$$\nabla\phi_{(1,-2,1)} = 0i + j + 6k$$

$$\hat{n} = \frac{\vec{a}}{|\vec{a}|}$$

$$\hat{n} = \frac{2i - j + k}{\sqrt{4+1+1}}$$

$$\hat{n} = \frac{2i - j + k}{\sqrt{6}}$$

$$d \cdot d = \nabla\phi \cdot \hat{n}$$

$$d \cdot d = (0i + j + 6k) \cdot \left(\frac{2i - j + k}{\sqrt{6}}\right)$$

$$d \cdot d = \frac{1}{\sqrt{6}}(0 - 1 + 6)$$

$$d \cdot d = \frac{5}{\sqrt{6}}$$

Find the directional derivative of $\phi = xy^3 + yz^3$ at $(2,-1,1)$ along $i + 2j + 2k$

Sol^n:- $d \cdot d = \nabla\phi \cdot \hat{n}$

$$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$$

$$\nabla\phi = (y^3 + 0) i + (3xy^2 + z^3) j + (0 + 3yz^2) k$$

$$\nabla\phi_{(2,-1,1)} = (-1)^3 i + (3(2)(-1)^2 + (1)^3) j + (3(-1)(1)^2) k$$

$$\nabla\phi_{(2,-1,1)} = -i + 7j - 3k$$

$$\hat{n} = \frac{\vec{a}}{|\vec{a}|}$$

$$\hat{n} = \frac{i + 2j + 2k}{\sqrt{1+4+4}}$$

$$\hat{n} = \frac{i + 2j + 2k}{3}$$

$$d \cdot d = \nabla\phi \cdot \hat{n}$$

$$d \cdot d = (-i + 7j - 3k) \cdot \left(\frac{i + 2j + 2k}{3}\right)$$

$$d \cdot d = \frac{1}{3}(-1 + 14 - 6)$$

$$d \cdot d = \frac{7}{3}$$

Find the angle between the surfaces $x^2+y^2-z^2=4$ and $z=x^2+y^2-13$ at $(2,1,2)$

Sol^n: $x^2+y^2-z^2=4$ and $z=x^2+y^2-13$

$\phi_1=0 \qquad \phi_2=0$

$\phi_1=x^2+y^2-z^2-4 \quad , \quad \phi_2=x^2+y^2-z-13$

$\nabla\phi_1=\frac{\partial\phi_1}{\partial x}i+\frac{\partial\phi_2}{\partial y}j+\frac{\partial\phi_1}{\partial z}k \qquad \nabla\phi_2=\frac{\partial\phi_2}{\partial x}i+\frac{\partial\phi_2}{\partial y}j+\frac{\partial\phi_2}{\partial z}k$

$\nabla\phi_1=2xi+2yj-2zk \qquad \nabla\phi_2=2xi+2yj-k$

$\nabla\phi_{1(2,1,2)}=4i+2j-4k=\vec{a} \qquad \nabla\phi_{2(2,1,2)}=4i+2j-k=\vec{b}$

$\cos\theta=\frac{\vec{a}}{|\vec{a}|}\cdot\frac{\vec{b}}{|\vec{b}|}$

$\cos\theta=\frac{4i+2j-4k}{\sqrt{16+4+16}}\cdot\frac{4i+2j-k}{\sqrt{16+4+1}}$

$\cos\theta=\frac{16+4+4}{\sqrt{36}\cdot\sqrt{21}}=\frac{24}{6\sqrt{21}}$

$\cos\theta=\frac{4}{\sqrt{21}}$

$\theta=\cos^{-1}\left(\frac{4}{\sqrt{21}}\right)$

Find the angle between the surfaces $xy=z^2$ at $(4,1,2)$ and $(3,3,-3)$

Sol^n: $xy=z^2$

$xy-z^2=0$

$\phi=0$

$\phi=xy-z^2$

$\nabla\phi=\frac{\partial\phi}{\partial x}i+\frac{\partial\phi}{\partial y}j+\frac{\partial\phi}{\partial z}k$

$\nabla\phi=yi+xj+(-2z)k$

$\nabla\phi_{(4,1,2)}=i+4j-4k=\vec{a}$

$\nabla\phi_{(3,3,-3)}=3i+3j+6k=\vec{b}$

$\cos\theta=\frac{\vec{a}}{|\vec{a}|}\cdot\frac{\vec{b}}{|\vec{b}|}$

$$\cos\theta = \frac{i+4j-4k}{\sqrt{1+16+16}} \cdot \frac{3i+3j+6k}{\sqrt{9+9+36}}$$

$$\cos\theta = \frac{3+12-24}{\sqrt{33}\ \sqrt{54}}$$

$$\cos\theta = \frac{-9}{\sqrt{33\times 54}} = \frac{-9}{9\sqrt{22}}$$

$$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{22}}\right)$$

Show that the vector $\vec{F} = \frac{xi+yj}{x^2+y^2}$ is both solenoidal and irrotational.

Soln:- $a_1 i + a_2 j + a_3 k$

$$a_1 = \frac{x}{x^2+y^2} \qquad a_2 = \frac{y}{x^2+y^2} \qquad a_3 = 0$$

$$\text{div}\,\vec{F} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}$$

$$\text{div}\,\vec{F} = \frac{(x^2+y^2)\cdot 1 - x\cdot 2x}{(x^2+y^2)^2} + \frac{(x^2+y^2)\cdot 1 - y(0+2y)}{(x^2+y^2)^2} + 0$$

$$\text{div}\,\vec{F} = \frac{y^2-x^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$\text{div}\,\vec{F} = \frac{\cancel{y^2}-\cancel{x^2}+\cancel{x^2}-\cancel{y^2}}{(x^2+y^2)^2}$$

$$\text{div}\,\vec{F} = 0$$

$\therefore$ The Given vector is Solenoidal

$$\text{Curl}\,\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_1 & a_2 & a_3 \end{vmatrix}$$

$$\text{Curl}\,\vec{F} = i\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) - j\left(\frac{\partial a_3}{\partial x} - \frac{\partial a_1}{\partial z}\right) + k\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$

$$\text{Curl}\,\vec{F} = i(0-0) - j(0-0) + k\left(\frac{(x^2+y^2)0 - y(2x)}{(x^2+y^2)^2} - \frac{(x^2+y^2)0 - x(2y)}{(x^2+y^2)^2}\right)$$

$$\text{Curl}\,\vec{F} = 0$$

$\therefore$ The Given vector is irrotational

Express $\vec{A} = zi - 2xj + yk$ in cylindrical coordinates

Soln:- $\vec{A} = A_1\hat{e}_1 + A_2\hat{e}_2 + A_3\hat{e}_3 \longrightarrow (1)$

$\vec{A}\cdot\hat{e}_1 = A_1$, $\vec{A}\cdot\hat{e}_2 = A_2$, $\vec{A}\cdot\hat{e}_3 = A_3$

W.K.T.

$x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$

$\vec{A} = zi - 2\rho\cos\phi\, j + \rho\sin\phi\, k$

$$\hat{e}_1 = \frac{\frac{\partial\vec{r}}{\partial\rho}}{\left|\frac{\partial\vec{r}}{\partial\rho}\right|} = \frac{\frac{\partial\vec{r}}{\partial\rho}}{h_1} = \cos\phi\, i + \sin\phi\, j + 0k$$

$A_1 = \vec{A}\cdot\hat{e}_1$

$A_1 = z\cos\phi - 2\rho\cos\phi\sin\phi$

$$\hat{e}_2 = \frac{\frac{\partial\vec{r}}{\partial\phi}}{\left|\frac{\partial\vec{r}}{\partial\phi}\right|} = \frac{\frac{\partial\vec{r}}{\partial\phi}}{h_2} = -\sin\phi\, i + \cos\phi\, j + 0k$$

$A_2 = \vec{A}\cdot\hat{e}_2$

$A_2 = -z\sin\phi - 2\cos^2\phi\,\rho$

$$\hat{e}_3 = \frac{\frac{\partial\vec{r}}{\partial z}}{\left|\frac{\partial\vec{r}}{\partial z}\right|} = \frac{\frac{\partial\vec{r}}{\partial z}}{h_3} = 0i + 0j + 1k$$

$A_3 = \vec{A}\cdot\hat{e}_3$

$A_3 = \rho\sin\phi$

$\vec{A} = (z\cos\phi - 2\rho\cos\phi\sin\phi)\,\hat{e}_1 - (z\sin\phi + 2\rho\cos^2\phi)\,\hat{e}_2 + (\rho\sin\phi)\,\hat{e}_3$

Using Newton's appropriate interpolation formula to find y at $x=8$ and $x=22$ for the given data

x	0	5	10	15	20	25
$f(x)$	7	11	14	18	24	32

x	y	I D	II D	III D	IV D	V D
0	7					
		4				
5	11		-1			
		3		2		
10	14		1		-1	
		4		1		0
15	18		2		-1	
		6		0		
20	24		2			
		8				
25	32					

$h=5$, $x=8$, $r=\frac{x-x_0}{h}=\frac{8-0}{5}=1.6$, $y=?$ N.F.I.F.

$$y(x)=y_0+r\Delta y_0+\frac{r(r-1)}{2!}\Delta^2 y_0+\frac{r(r-1)(r-2)}{3!}\Delta^3 y_0+\frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0+\frac{r(r-1)(r-2)(r-3)(r-4)}{5!}\Delta^5 y_0+\cdots$$

$$y(8)=7+1.6(4)+\frac{1.6(1.6-1)}{2}(-1)+\frac{1.6(1.6-1)(1.6-2)}{6}(2)+\frac{1.6(1.6-1)(1.6-2)(1.6-3)}{24}(-1)+0+\cdots$$

$$y(8)=7+6.4-0.48-0.128-0.0224$$

$$y(8)=\underline{\underline{12.7696}}$$

$h=5$, $x=22$, $r=\frac{x-x_n}{h}=\frac{22-25}{5}=-0.6$, $y=?$ N.B.I.F.

$$y(x)=y_n+r\nabla y_n+\frac{r(r+1)}{2!}\nabla^2 y_n+\frac{r(r+1)(r+2)}{3!}\nabla^3 y_n+\frac{r(r+1)(r+2)(r+3)}{4!}\nabla^4 y_n+\frac{r(r+1)(r+2)(r+3)(r+4)}{5!}\nabla^5 y_n+\cdots$$

$$y(22) = 32 + (-0.6)8 + \frac{(-0.6)(-0.6+1)}{2}(2) + \frac{(-0.6)(-0.6+1)(-0.6+2)}{6} \times 0 +$$

$$\frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{24}(-1) + 0 + \cdots$$

$$y(22) = 32 - 4.8 - 0.24 + 0 + 0.0336$$

$$y(22) = \underline{26.9936}$$

Given $f(40) = 184$, $f(50) = 204$, $f(60) = 226$, $f(70) = 250$, $f(80) = 276$, $f(90) = 304$. find $f(42)$ and $f(75)$ by using appropriate interpolation formula.

x	y	I D	II D	III D
40	184			
		20		
50	204		2	
		22		0
60	226		2	
		24		0
70	250		2	
		26		0
80	276		2	
		28		
90	304			

$h = 10$, $x = 42$, $r = \frac{x - x_0}{h} = \frac{42-40}{10} = \frac{2}{10} = 0.2$, $y = ?$ N.F.I.F

$$y(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \cdots$$

$$y(42) = 184 + (0.2)(20) + \frac{(0.2)(0.2-1)}{2}(2) + \frac{(0.2)(0.2-1)(0.2-2)}{6}(0) + \cdots$$

$$y(42) = 184 + 4 - 0.16 + 0$$

$$y(42) = \underline{187.84}$$

$h = 10$, $x = 75$, $r = \frac{x - x_n}{h} = \frac{75-90}{10} = -1.5$, $y = ?$ N.B.I.F

$$y(x) = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n + \cdots$$

$$y(75) = 304 + (-1.5)(28) + \frac{(-1.5)(-1.5+1)}{2}(2) + \frac{(-1.5)(-1.5+1)(-1.5+2)}{6}(0) + \cdots$$

$y(75) = 304 - 42 + 0.75$

$y(75) = \underline{262.75}$

Given $\sin 45° = 0.7072$ $\sin 50° = 0.7660$ $\sin 55° = 0.8192$ $\sin 60° = 0.8660$ find $\sin 47°$ and $\sin 48°$ using appropriate interpolation formula.

x	y	I D	II D	III D
45	0.7072			
		0.0588		
50	0.7660		−0.0056	
		0.0532		−0.0008
55	0.8192		−0.0064	
		0.0468		
60	0.8660			

$x = 47$, $h = 5$, $r = \frac{x - x_0}{h} = \frac{47-45}{5} = \frac{2}{5} = 0.4$, $y = ?$ N.F.I.F.

$$y(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \cdots$$

$$y(47) = 0.7072 + (0.4)(0.0588) + \frac{(0.4)(0.4-1)}{2}(-0.0056) + \frac{(0.4)(0.4-1)(0.4-2)}{6}(-0.0008) + \cdots$$

$$y(47) = 0.7072 + 0.02352 + 0.000672 - 0.0000512$$

$$y(47) = \underline{0.7313}$$

$x = 48$, $h = 5$, $r = \frac{x - x_n}{h} = \frac{48-45}{5} = 0.6$, $y = ?$ N.B.I.F.

$$y(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \cdots$$

$$y(48) = 0.7072 + (0.6)(0.0588) + \frac{(0.6)(0.6-1)}{2}(-0.0056) + \frac{(0.6)(0.6-1)(0.6-2)}{6}(-0.0008) + \cdots$$

$$y(48) = 0.7072 + 0.03528 + 0.000672 - 0.0000448$$

$$y(48) = \underline{0.7431}$$

From the following table estimate the number of students who have obtained (i) less than 45 marks

(ii) marks between 40 and 45

Marks	30-40	40-50	50-60	60-70	70-80
Number of Students	31	42	51	35	31

Sol^n:-

x	y	I D	II D	III D	IV D
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

$x = 45$, $h = 10$ $\quad r = \frac{x - x_0}{h} = \frac{45-40}{10} = 0.5$, $y = ?$ $\quad$ N.F.I.F.

$$y(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0 + \cdots$$

$$y(45) = 31 + (0.5)(42) + \frac{(0.5)(0.5-1)}{2}(9) + \frac{(0.5)(0.5-1)(0.5-2)}{6}(-25) + \cdots$$

$$\frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{24}(37) + \cdots$$

$y(45) = 31 + 21 - 1.125 - 1.5625 - 1.4453$

$y(45) = 47.8672 \approx 48$

$y(\text{marks b/w 40 and 45}) = y(45) - y(40)$

$= 48 - 31$

$= \underline{\underline{17}}$

The area A of a circle of diameter d

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

Calculate the area of circle A=? of diameter 105

x	y	I D	II D	III D	IV D
80	5026				
		648			
85	5674		40		
		688		-2	
90	6362		38		4
		726		2	
95	7088		40		
		766			
100	7854				

$x = 105$, $h = 5$, $y = ?$, $r = \frac{x - x_n}{h} = \frac{105-100}{5} = 1$ $\quad$ N.B.I.F

$$y(x) = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n + \frac{r(r+1)(r+2)(r+3)}{4!}\nabla^4 y_n + \cdots$$

$y(105) = 7854 + 1(766) + \frac{1(1+1)}{2}(40) + \frac{1(1+1)(1+2)}{6}(2) + \frac{1(1+1)(1+2)(1+3)}{24}(4) + \ldots$

$y(105) = 7854 + 766 + 40 + 2 + 4$

$y(105) = \underline{\underline{8666}}$

Find y_{35} using appropriate interpolation formula for the data ;

$y_{20} = 612 \quad y_{30} = 539, \quad y_{40} = 446, \quad y_{50} = 343$

x	y	I D	II D	III D
20	612			
		-73		
30	539		-20	
		-93		10
40	446		-10	
		-103		
50	343			

$x = 35, \quad h = 10, \quad r = \frac{x - x_0}{h} = \frac{35-20}{10} = 1.5, \quad y = ?$ N.F.I.F.

$y(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \ldots$

$y(35) = 612 + 1.5(-73) + \frac{(1.5)(1.5-1)}{2}(-20) + \frac{(1.5)(1.5-1)(1.5-2)}{6}(10) + \ldots$

$y(35) = 612 - 109.5 - 7.5 - 0.625$

$y(35) = \underline{\underline{494.375}}$

Find the number of men getting wages below rupees 35 from the following data

Wages in Rs	0-10	10-20	20-30	30-40
Frequency	9	30	35	42

x	y	I D	II D	III D
10	9			
		30		
20	39		5	
		35		2
30	74		7	
		42		
40	116			

$x = 35, \quad h = 10, \quad r = \frac{x - x_n}{h} = \frac{35-40}{10} = -0.5, \quad y = ?$ N.B.I.F.

$y(x) = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n + \ldots$

$$y(35) = 116 + (-0.5)(42) + \frac{(-0.5)(-0.5+1)}{2}(7) + \frac{(-0.5)(-0.5+1)(-0.5+2)}{6}(2) + \cdots$$

$$y(35) = 116 - 21 - 0.875 - 0.125$$

$$y(35) = \underline{94}$$

Find $f(9)$ for the following data

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

Using appropriate interpolation formula

x	y	I DD	II DD	III DD	IV DD
5	150				
		121			
7	392		24		
		265		1	
11	1452		32		0
		457		1	
13	2366		42		
		709			
17	5202				

$x = 9$, $y = ?$

$$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3) + \cdots$$

$$f(9) = 150 + (9-5)(121) + (9-5)(9-7)(24) + (9-5)(9-7)(9-11)(1) + 0 + \cdots$$

$$f(9) = 150 + 484 + 192 - 16$$

$$f(9) = \underline{810}$$

Using Newton's divided difference formula to find $f(8)$ and $f(15)$ for the following rate of

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

x	y	I DD	II DD	III DD	IV DD
4	48				
		52			
5	100		15		
		97		1	
7	294		21		0
		202		1	
10	900		27		0
		310		1	
11	1210		33		
		409			
13	2028				

$x=8 \quad y=?$

$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0 x_1 x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0 x_1 x_2 x_3) + \cdots$

$f(8) = 48 + (4)(52) + (8-4)(8-5)(15) + (8-4)(8-5)(8-7) \times 1 + 0$

$f(8) = 48 + 208 + 180 + 12$

$f(8) = \underline{448}$

$x=15, \quad y=?$

$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0 x_1 x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0 x_1 x_2 x_3) + \cdots$

$f(15) = 48 + (15-4)(52) + (15-4)(15-5)15 + (15-4)(15-5)(15-7)(1) + 0 + \cdots$

$f(15) = 48 + 572 + 1650 + 880$

$f(15) = \underline{3150}$

Using Newton's divided difference interpolation formula Construct the interpolating polynomial for the given data.

x	2	4	5	6	8	10
$f(x)$	10	96	196	350	868	1746

and hence find the value of $f(3)$

x	$f(x)$	I DD	II DD	III DD	IV DD	V DD
2	10					
		43				
4	96		19			
		100		2		
5	196		27		0	
		154		2		
6	350		35		0	
		259		2		
8	868		45			
		439				
10	1746					

$x=3 \quad y=?$

$f(x) = f(x_0) + (x-x_0)f(x_0 x_1) + (x-x_0)(x-x_1)f(x_0 x_1 x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0 x_1 x_2 x_3) + \cdots$

$f(3) = 10 + (3-2)(43) + (3-2)(3-4)(19) + (3-2)(3-4)(3-5)(2) + 0 + 0 + \cdots$

$f(3) = 10 + 43 - 19 + 4$

$f(3) = \underline{38}$

(or)

$f(x) = f(x_0) + (x-x_0)f(x_0x_1) + (x-x_0)(x-x_1)f(x_0x_1x_2) + (x-x_0)(x-x_1)f(x_0x_1x_2x_3) + \ldots$

$f(x) = 10 + (x-2)43 + (x-2)(x-4)(19) + (x-2)(x-4)(x-5)(2) + 0 + 0 + \ldots$

$f(x) = 10 + 43x - 86 + (x^2-4x-2x+8)(19) + (x^2-6x+8)(2x-10)$

$f(x) = 10 + 43x - 86 + 19x^2 - 114x + 152 + 2x^3 - 12x^2 + 16x - 10x^2 + 60x - 80$

$f(x) = 2x^3 - 3x^2 + 5x - 4$

$f(3) = 54 - 27 + 15 - 4$

$f(3) = \underline{\underline{38}}$

Using Newton's divided difference interpolation formula Construct the interpolating polynomial for the given data (4, -43) (7, 83) (9, 327) (12, 1053) and hence find f(5)

x	y	I DD	II DD	III DD
4	-43			
		42		
7	83		16	
		122		1
9	327		24	
		242		
12	1053			

$f(x) = f(x_0) + (x-x_0)f(x_0x_1) + (x-x_0)(x-x_1)f(x_0x_1x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0x_1x_2x_3) + \ldots$

$f(x) = -43 + (x-4)(42) + (x-4)(x-7)(16) + (x-4)(x-7)(x-9)(1) + \ldots$

$f(x) = -43 + 42x - 168 + (x^2-7x-4x+28)16 + (x^2-11x+28)(x-9)1 + \ldots$

$f(x) = -43 + 42x - 168 + 16x^2 - 176x + 448 + x^3 - 11x^2 + 28x - 9x^2 + 99x - 252$

$f(x) = x^3 - 4x^2 - 7x - 15$

$f(5) = (5)^3 - 4(5)^2 - 7(5) - 15$

$f(5) = 125 - 100 - 35 - 15$

$f(5) = \underline{\underline{-25}}$

Use lagranges interpolation formula to find f(10) for the following data

x	5	6	9	11
$f(x)$	12	13	14	16

$x_0=5 \quad x_1=6 \quad x_2=9 \quad x_3=11 \quad x=10$

$y_0=12 \quad y_1=13 \quad y_2=14 \quad y_3=16 \quad y=?$

$$f(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$f(10)=\frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}(12)+\frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}(13)+\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}(14)+\frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}(16)$$

$$f(10)=\frac{(4)(1)(-1)}{(-1)(-4)(-6)}(12)+\frac{(5)(1)(-1)}{(1)(-3)(-5)}(13)+\frac{(5)(4)(-1)}{(4)(3)(2)}(14)+\frac{(5)(4)(1)}{(6)(5)(2)}(16)$$

$$f(10)=2-\frac{13}{3}+\frac{35}{3}+\frac{16}{3}$$

$$f(10)=\frac{44}{3}\Rightarrow 14.\underline{\underline{666}}$$

Find y at $x=5$ using lagranges method

$y(1)=-3 \quad y(3)=9 \quad y(4)=30 \quad y(6)=132$

$x_0=1 \quad x_1=3 \quad x_2=4 \quad x_3=6 \quad x=5$

$y_0=-3 \quad y_1=9 \quad y_2=30 \quad y_3=132 \quad y=?$

$$y(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$y(5)=\frac{(5-3)(5-4)(5-6)}{(1-3)(1-4)(1-6)}(-3)+\frac{(5-1)(5-4)(5-6)}{(3-1)(3-4)(3-6)}(9)+\frac{(5-1)(5-3)(5-6)}{(4-1)(4-3)(4-6)}(30)+\frac{(5-1)(5-3)(5-4)}{(6-1)(6-3)(6-4)}(132)$$

$$y(5) = \frac{-1}{5} - 6 + 40 + \frac{176}{5}$$

$$y(5) = \underline{69}$$

Find $f(9)$ for the data using lagranges interpolation formula

x	5	7	11	13	17
$f(x)$	150	392	1492	2366	5202

$x_0 = 5 \quad x_1 = 7 \quad x_2 = 11 \quad x_3 = 13 \quad x_4 = 17 \qquad x = 9$

$y_0 = 150 \quad y_1 = 392 \quad y_2 = 1492 \quad y_3 = 2366 \quad y_4 = 5202$

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}(y_0) + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}(y_1) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}(y_2) + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}(y_3) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}(y_4) + \ldots$$

$$y(9) = \frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)}(150) + \frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)}(392) +$$

$$\frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)}(1492) + \frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)}(2366) +$$

$$\frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)}(5202)$$

$$y(9) = -\frac{50}{3} + 209.0666 + 1326.2222 - 788.6666 + 115.6$$

$$y(9) = \underline{845.5555}$$

find $f(4)$ for the data

x	0	2	3	6
y	-4	2	14	158

$x_0=0 \quad x_1=2 \quad x_2=3 \quad x_3=6 \quad x=4$

$y_0=-4 \quad y_1=2 \quad y_2=14 \quad y_3=158$

$$y(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}(y_0)+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2+$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$y(4)=\frac{(4-2)(4-3)(4-6)}{(0-2)(0-3)(0-6)}(-4)+\frac{(4-0)(4-3)(4-6)}{(2-0)(2-3)(2-6)}(2)+\frac{(4-0)(4-2)(4-6)}{(3-0)(3-2)(3-6)}(14)+$$

$$\frac{(4-0)(4-2)(4-3)}{(6-0)(6-2)(6-3)}(158)$$

$$y(4)=\frac{-4}{9}-2+\frac{224}{9}+\frac{158}{9}$$

$$y(4)=40$$

find the interpolating polynomial by using lagranges interpolation method for the data

x	0	1	2	5
$f(x)$	2	3	12	147

$x_0=0 \quad x_1=1 \quad x_2=2 \quad x_3=5$

$y_0=2 \quad y_1=3 \quad y_2=12 \quad y_3=147$

$$f(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_1-x_0)(x_2-x_1)(x_2-x_3)}(y_2)+$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$f(x)=\frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2)+\frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3)+\frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12)+$$

$$\frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$f(x)=\frac{(x^2-3x+2)(x-5)}{-5}+\frac{x(x^2-7x+10)3}{1(-1)(-4)}+\frac{x(x^2-6x+5)6}{1(-3)}+\frac{x(x^2-3x+2)}{5\cdot4\cdot3}147$$

$$f(x) = \frac{x^3-3x^2+2x-5x^2-15x-10}{-5} + \frac{3x^3-21x^2+30x}{4} - (2x^3-12x^2+10x) + \frac{49x^3-147x^2+98x}{20}$$

$$f(x) = \frac{-4x^3+32x^2-68x+40+15x^3-105x^2+150x-40x^3+240x^2-200x+49x^3-147x^2+98x}{20}$$

$$f(x) = \frac{20x^3+20x^2-20x+40}{20}$$

$$f(x) = x^3+x^2-x+2$$

Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ use Trapezoidal rule, Simpsons $(1/3)^{rd}$ rule and Simpsons $(3/8)^{th}$ rule by taking 6 divisions

Trapezoidal rule

$[0,1]$ $n=6$ $h=\frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

x $x_0=0$ $x_1=x_0+h$ $x_2=2/6$ $x_3=3/6$ $x_4=4/6$ $x_5=5/6$ $x_6=6/6=1$

$x_1 = 1/6$

$y=\frac{1}{1+x^2}$ $y_0=1$ $y_1=36/37$ $y_2=9/10$ $y_3=4/5$ $y_4=9/13$ $y_5=36/61$ $y_6=1/2$

$$I = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\dots+y_{n-1})\right]$$

$$I = \frac{1/6}{2}\left[y_0+y_6+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$I = \frac{1}{12}\left[1+\tfrac{1}{2}+2\left(\tfrac{36}{37}+\tfrac{9}{10}+\tfrac{4}{5}+\tfrac{9}{13}+\tfrac{36}{61}\right)\right]$$

$$I = 0.7842$$

Simpsons $(1/3)^{rd}$ rule

$[0,1]$ $n=6$ $h=\frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

x $x_0=0$ $x_1=x_0+h$ $x_2=2/6$ $x_3=3/6$ $x_4=4/6$ $x_5=5/6$ $x_6=6/6=1$

$x_1 = 1/6$

$y=\frac{1}{1+x^2}$ $y_0=1$ $y_1=36/37$ $y_2=9/10$ $y_3=4/5$ $y_4=9/13$ $y_5=36/61$ $y_6=1/2$

$$I = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\dots+y_{n-1})+2(y_2+y_4+\dots+y_{n-2})\right]$$

$$I = \frac{1/6}{3}\left[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$I = \frac{1}{18}\left[1+\tfrac{1}{2}+4\left(\tfrac{36}{37}+\tfrac{4}{5}+\tfrac{36}{61}\right)+2\left(\tfrac{9}{10}+\tfrac{9}{13}\right)\right]$$

$$I = 0.7853$$

Simpsons $(3/8)^{th}$ rule

$[0,1]$ $n=6$ $h=\frac{b-a}{n}=\frac{1-0}{6}=\frac{1}{6}$

x $x_0=0$ $x_1=x_0+h$ $x_2=2/6$ $x_3=3/6$ $x_4=4/6$ $x_5=5/5$ $x_6=6/6=1$

$x_1=1/6$

$y=\frac{1}{1+x^2}$ $y_0=1$ $y_1=36/37$ $y_2=9/10$ $y_3=4/5$ $y_4=9/13$ $y_5=36/61$ $y_6=1/2$

$$I=\frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+y_7+\dots+y_{n-1})+2(y_3+y_6+\dots+y_{n-3})\right]$$

$$I=\frac{3(1/6)}{8}\left[y_0+y_6+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$I=\frac{1}{16}\left[1+1/2+3(36/37+9/10+9/13+36/61)+2(4/5)\right]$$

$$I=0.7853$$

Evaluate $\int_0^{\pi/2}\sqrt{\cos x}\,dx$ by using simpsons $(1/3)^{rd}$ rule, Trapezoidal rule and simpsons $(3/8)^{th}$ rule taking 7 ordinates.

Trapezoidal rule

$[0,\pi/2]$ $n=6$ $h=\frac{b-a}{n}=\frac{\pi/2-0}{6}=\frac{\pi}{12}$

x $x_0=0$ $x_1=x_0+h$ $x_2=2\pi/12$ $x_3=3\pi/12$ $x_4=4\pi/12$ $x_5=5\pi/12$ $x_6=6\pi/12=\pi/2$

$x_1=\pi/12$

$y=\sqrt{\cos x}$ $y_0=1$ $y_1=0.9828$ $y_2=0.9306$ $y_3=0.8408$ $y_4=0.7071$ $y_5=0.5087$ $y_6=0$

$$I=\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\dots+y_{n-1})\right]$$

$$I=\frac{\pi/12}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$I=\frac{\pi}{24}\left[1+0+2(0.9828+0.9306+0.8408+0.7071+0.5087)\right]$$

$$I=1.17024$$

Simpsons $(1/3)^{rd}$ rule

$[0,\pi/2]$ $n=6$ $h=\frac{b-a}{n}=\frac{\pi/2-0}{6}=\frac{\pi}{12}$

x $x_0=0$ $x_1=\pi/12$ $x_2=2\pi/12$ $x_3=3\pi/12$ $x_4=4\pi/12$ $x_5=5\pi/12$ $x_6=6\pi/12=\pi/12$

$y=\sqrt{\cos x}$ $y_0=1$ $y_1=0.9828$ $y_2=0.9306$ $y_3=0.8408$ $y_4=0.7071$ $y_5=0.5087$ $y_6=0$

$$I=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5+\dots+y_{n-1})+2(y_2+y_4+\dots+y_{n-2})\right]$$

$$I = \frac{\pi/12}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$I = \frac{\pi}{36}\left[1 + 0 + 4(0.9828 + 0.8408 + 0.5087) + 2(0.9306 + 0.7071)\right]$$

$$I = 1.18722$$

Simpsons $(3/8)^{th}$ rule

$[0, \pi/2]$ $\quad n = 6 \quad h = \frac{\pi/2 - 0}{6} = \frac{\pi}{12}$

$x \quad x_0 = 0 \quad x_1 = x_0 + h \quad x_2 = x_0 + 2h \quad x_3 = \frac{3\pi}{12} \quad x_4 = \frac{4\pi}{12} \quad x_5 = \frac{5\pi}{12} \quad x_6 = \frac{6\pi}{12} = \frac{\pi}{2}$

$x_1 = \pi/12 \quad x_2 = 2\pi/12$

$y = \sqrt{\cos x} \quad y_0 = 1 \quad y_1 = 0.9828 \quad y_2 = 0.9306 \quad y_3 = 0.8408 \quad y_4 = 0.7071 \quad y_5 = 0.5087 \quad y_6 = 0$

$$I = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + \dots + y_{n-1}) + 2(y_3 + y_6 + \dots + y_{n-3})\right]$$

$$I = \frac{3\pi/12}{8}\left[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3\right]$$

$$I = \frac{3\pi}{96}\left[1 + 0 + 3(0.9828 + 0.9306 + 0.7071 + 0.5087) + 2(0.8408)\right]$$

$$I = 1.1848$$

Evaluate $\int_0^{0.6} e^{-x^2}$ by using Trapezoidal rule taking seven ordinates.

$[0 \quad 0.6] \quad n = 6 \quad h = \frac{b-a}{n} = \frac{0.6 - 0}{6} = 0.1$

$x \quad x_0 = 0 \quad x_1 = x_0 + h \quad x_2 = 0.2 \quad x_3 = 0.3 \quad x_4 = 0.4 \quad x_5 = 0.5 \quad x_6 = 0.6$

$x_1 = 0.1$

$y = e^{-x^2} \quad y_0 = 1 \quad y_1 = 0.9900 \quad y_2 = 0.9607 \quad y_3 = 0.9139 \quad y_4 = 0.8521 \quad y_5 = 0.7788 \quad y_6 = 0.6976$

$$I = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\right]$$

$$I = \frac{0.1}{2}\left[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$I = 0.05\left[1 + 0.6976 + 2(0.9900 + 0.9607 + 0.9139 + 0.8521 + 0.7788)\right]$$

$$I = 0.5344$$

Evaluate $\int_0^{0.6} e^{-x^2}$ by using Simpson's $(3/8)$th rule taking seven ordinates

$[0, 0.6]$ $\quad n=6 \quad h=\frac{b-a}{n}=\frac{0.6-0}{6}=0.1$

$x \quad x_0=0 \quad x_1=x_0+h \quad x_2=0.2 \quad x_3=0.3 \quad x_4=0.4 \quad x_5=0.5 \quad x_6=0.6$

$x_1=0.1$

$y=e^{-x^2} \quad y_0=1 \quad y_1=0.9900 \quad y_2=0.9607 \quad y_3=0.9139 \quad y_4=0.8521 \quad y_5=0.7788 \quad y_6=0.6976$

$$I=\frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+\cdots+y_{n-1})+2(y_3+y_6+\cdots+y_{n-3})\right]$$

$$I=\frac{3(0.1)}{8}\left[y_0+y_6+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$I=0.0375\left(1+0.6976+3(0.9900+0.9607+0.8521+0.7788+2(0.9139)\right]$$

$$I=\underline{0.5351}$$

Evaluate $\int_4^{5.2} \log_e x\,dx$ by using simpsons $(1/3)$rd rule by taking five ordinates

$[4, 5.2] \quad n=4 \quad h=\frac{b-a}{n}=\frac{5.2-4}{4}=0.3$

$x \quad x_0=4 \quad x_1=x_0+h \quad x_2=4.6 \quad x_3=4.9 \quad x_4=5.2$

$x_1=4.3$

$y=\ln x \quad y_0=1.3862 \quad y_1=1.4586 \quad y_2=1.5260 \quad y_3=1.5892 \quad y_4=1.6486$

$$I=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\cdots+y_{n-1})+2(y_2+y_4+\cdots+y_{n-2})\right]$$

$$I=\frac{0.3}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$I=0.1\left[1.3862+1.6486+4(1.4586+1.5892)+2(1.5260)\right]$$

$$I=\underline{1.8278}$$

Evaluate $\int_{0.2}^{1.4} (\sin x - \log x + e^x)\,dx$ by using simpsons $(3/8)^{th}$ taking 6 equal parts

$[0.2, 1.4]$ $n=6$ $h = \frac{b-a}{n} = \frac{1.4-0.2}{6} = 0.2$

x $x_0 = 0.2$ $x_1 = x_0 + h$ $x_2 = 0.6$ $x_3 = 0.8$ $x_4 = 1.0$ $x_5 = 1.2$ $x_6 = 1.4$

$x_1 = 0.4$

$y = (\sin x - \ln x + e^x)$ $y_0 = 3.0295$ $y_1 = 2.7975$ $y_2 = 2.8975$ $y_3 = 3.1660$

$y_4 = 3.5597$ $y_5 = 4.0678$ $y_6 = 4.7041$

$$I = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{n-1}) + 2(y_3 + y_6 + \dots + y_{n-3})\right]$$

$$I = \frac{3(0.2)}{8}\left[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)\right]$$

$$I = 0.075\left(3.0295 + 4.7041 + 3(2.7975 + 2.8975 + 3.5597 + 4.0698) + 2(2.8975)\right)$$

$I = \underline{\underline{4.050}}$

Evaluate $\int_0^1 \frac{x}{1+x^2}\,dx$ trapezoidal rule taking 10 equal parts.

$[0, 1]$ $n=10$ $h = \frac{1-0}{10} = 0.7$

x $x_0 = 0$ $x_1 = x_0 + h$ $x_2 = 0.2$ $x_3 = 0.3$ $x_4 = 0.4$ $x_5 = 0.5$ $x_6 = 0.6$ $x_7 = 0.7$

$x_1 = 0.1$

$x_8 = 0.8$ $x_9 = 0.9$ $x_{10} = 1.0$

$y = \frac{x}{1+x^2}$ $y_0 = 0$ $y_1 = 0.0990$ $y_2 = 0.1923$ $y_3 = 0.2752$ $y_4 = 0.3448$ $y_5 = 0.4$

$y_6 = 0.4411$ $y_7 = 0.4697$ $y_8 = 0.4878$ $y_9 = 0.4972$ $y_{10} = 0.5$

$$I = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots + y_{n-1})\right]$$

$$I = \frac{0.1}{2}\left[y_0 + y_{10} + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9)\right]$$

$$I = 0.05\left[0 + 0.5 + 2(0.0990 + 0.1923 + 0.2752 + 0.3448 + 0.4 + 0.4411 + 0.4697 + 0.4878 + 0.4972)\right]$$

$I = \underline{\underline{0.3457}}$

Evaluate $\int_0^5 \frac{1}{4x+5}\,dx$ using trapezoidal rule taking elven ordinates

$[0,5]$ $n=10$ $h=\frac{5-0}{10}=0.5$

x $x_0=0$ $x_1=0.5$ $x_2=1$ $x_3=1.5$ $x_4=2$ $x_5=2.5$ $x_6=3$ $x_7=3.5$ $x_8=4$

$x_9=4.5$ $x_{10}=5$

$y=\frac{1}{4x+5}$ $y_0=\frac{1}{5}$ $y_1=\frac{1}{7}$ $y_2=\frac{1}{9}$ $y_3=\frac{1}{11}$ $y_4=\frac{1}{13}$ $y_5=\frac{1}{15}$ $y_6=\frac{1}{17}$ $y_7=\frac{1}{19}$ $y_8=\frac{1}{21}$

$y_9=\frac{1}{23}$ $y_{10}=\frac{1}{25}$

$$I=\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\dots+y_{n-1})\right]$$

$$I=\frac{0.5}{2}\left[y_0+y_{10}+2(y_1+y_2+y_3+y_4+y_5+y_6+y_7+y_8+y_9)\right]$$

$$I=\frac{0.5}{2}\left[\frac{1}{5}+\frac{1}{25}+2\left(\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\frac{1}{13}+\frac{1}{15}+\frac{1}{17}+\frac{1}{19}+\frac{1}{21}+\frac{1}{23}\right)\right]$$

$$I=0.4055$$

Evaluate $\int_0^3 \frac{1}{4x+5}\,dx$ using simpsons $(1/3)^{rd}$ rule by taking 7 ordinates

$[0,3]$ $n=6$ $h=\frac{b-a}{n}=\frac{3-0}{6}=\frac{3}{6}=0.5$

x $x_0=0$ $x_1=x_0+h$ $x_1=0.5$ $x_2=1$ $x_3=1.5$ $x_4=2$ $x_5=2.5$ $x_6=3$

$y=\frac{1}{4x+5}$ $y_0=\frac{1}{5}$ $y_1=\frac{1}{7}$ $y_2=\frac{1}{9}$ $y_3=\frac{1}{11}$ $y_4=\frac{1}{13}$ $y_5=\frac{1}{15}$ $y_6=\frac{1}{17}$

$$I=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5+\dots+y_{n-1})+2(y_2+y_4+\dots+y_{n-2})\right]$$

$$I=\frac{0.5}{3}\left[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$I=\frac{0.5}{3}\left(\frac{1}{5}+\frac{1}{17}+4\left(\frac{1}{7}+\frac{1}{11}+\frac{1}{15}\right)+2\left(\frac{1}{9}+\frac{1}{13}\right)\right]$$

$$I=0.3061$$

Find the real root of an equation $x^3-2x-5=0$ by applying regula false method correct to four places of decimals.

$f(x)=x^3-2x-5$

$f(0)=-5<0 \quad f(1)=-6<0 \quad f(2)=-1<0 \quad f(3)=16>0$

$(2,3)$

$f(2)=-1<0 \quad f(3)=16>0$

$f(2)=-1<0 \quad f(2.1)=0.061>0$

I iteration

$$x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}=\frac{2f(2.1)-2.1f(2)}{f(2.1)-f(2)}=\frac{2(0.061)-2.1(-1)}{0.061-(-1)}=2.09425$$

II iteration

$f(x_1)=f(2.09425)=-3.364\times10^{-3}=-0.003364<0$

$f(2.1)=0.061>0$

$(2.09425, 2.1)$

$$x_2=\frac{af(b)-bf(a)}{f(b)-f(a)}=\frac{2.09425(0.061)-2.1(-0.003364)}{0.061-(-0.003364)}=2.09455$$

III iteration

$f(x_2)=f(2.09455)=-0.0000165<0$

$f(2.1)=0.061>0$

$(2.09455, 2.1)$

$$x_3=\frac{af(b)-bf(a)}{f(b)-f(a)}=\frac{2.09455(0.061)-2.1(-0.0000165)}{0.061-(-0.0000165)}=2.09455$$

Find the real root of an equation $x^3-3x+4=0$ use method of false position correct to four decimal place which lies between -3 and -2.

$f(x)=x^3-3x+4$

$(-3, -2)$

I iteration

$f(-2.2)=-0.048<0 \quad f(-2.1)=1.039>0$

$(-2.2, -2.1)$

$$x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}=\frac{-2.2f(-2.1)-(2.1)f(-2.2)}{f(-2.1)-f(-2.2)}=\frac{-2.2(1.039)-(-2.1)(-0.048)}{1.039-(-0.048)}$$

$x_1 = -2.1955$

II iteration

$f(x_1) = f(-2.1955) = 0.003706$

$f(-2.2) = -0.048$

$(-2.2, -2.1955)$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{-2.2(0.003706) - (-2.1955)(-0.048)}{0.003706 + 0.048} = -2.1958$$

III iteration

$f(x_2) = f(-2.1958) = 0.0002676$

$f(-2.2) = -0.048$

$(-2.2, -2.1958)$

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{-2.2(0.0002676) - (-2.1958)(-0.048)}{(0.0002676) + 0.048} = -2.1958$$

Find the real root of the equation $xe^x = 3$ using method of false position correct to four decimal place

$xe^x = 3$

$xe^x - 3 = 0$

$f(x) = xe^x - 3$

I iteration

$(1, 1.1)$

$f(1) = -0.2817 \qquad f(1.1) = 0.3045$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(0.3045) - 1.1(-0.2817)}{(0.3045) - (-0.2817)} = 1.0480$$

II iteration

$f(x_1) = f(1.0480) = -0.0111$

$f(1.1) = 0.3045$

$(1.0480, 1.1)$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1.0480(0.3045) - 1.1(-0.0111)}{0.3045 + 0.0111} = 1.0498$$

III iteration

$f(x_2) = f(1.0498) = -0.0006377$

$f(1.1) = 0.3045$

$(1.0498, 1.1)$

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1.0498(0.3045) - 1.1(-0.0006377)}{0.3045 + 0.0006377} = 1.0499$$

IV iteration

$f(x_3) = f(1.0499) = -0.00005210$

$f(1.1) = 0.3045$

$(1.0499, 1.1)$

$$x_4 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1.0499(0.3045) - 1.1(-0.00005210)}{0.3045 + 0.00005210} = 1.0499$$

Find the Real root of the equation $\cos x = 3x - 1$ using method fake position correct to 3 decimal place

$\cos x = 3x - 1$

$\cos x - 3x + 1 = 0$

$f(x) = \cos x - 3x + 1$

I iteration

$f(0.6) = 0.0253 \qquad f(0.7) = -0.3351$

$(0.6, 0.7)$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0.6(-0.3351) - 0.7(0.0253)}{(-0.3351) - 0.0253} = 0.607$$

II iteration

$f(x_1) = f(0.607) = 3.6292 \times 10^{-4} = 0.0003629$

$f(0.7) = -0.3351$

$(0.607, 0.7)$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0.607(-0.3351) - 0.7(0.0003629)}{-0.3351 - 0.0003629} = 0.607$$

Find the Real root of the equation $x\log_{10}x = 1.2$ using method fake position

$x\log_{10}x = 1.2$

$x\log_{10}x - 1.2 = 0$

$f(x) = x\log_{10}x - 1.2$

I iteration

$f(2.7) = -0.0353 \qquad f(2.8) = 0.0520$

$(2.7, 2.8)$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.7(0.0520) - 2.8(-0.0353)}{0.0520 + 0.0353} = 2.7404$$

II iteration

$f(x_1) = f(2.7404) = -0.0002146$

$f(2.8) = 0.0520$

$(2.7404,\ 2.8)$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.7404(0.0520) - 2.8(-0.0002146)}{0.0520 + 0.0002146} = 2.7406$$

III iteration

$f(x_2) = f(2.7406) = -0.00004020$

$f(2.8) = 0.0520$

$(2.7406,\ 2.8)$

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.7406(0.0520) - 2.8(-0.00004020)}{0.0520 + 0.00004020} = 2.7406$$

Find the Real root of the equation $2x - \log_{10}x - 7 = 0$ using method false position correct to 4 decimal place

$2x - \log_{10}x - 7 = 0$

$f(x) = 2x - \log_{10}x - 7$

I iteration

$f(3.7) = -0.1682 \quad f(3.8) = 0.0202$

$(3.7,\ 3.8)$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{3.7(0.0202) - 3.8(-0.1682)}{0.0202 + 0.1682} = 3.7892$$

II iteration

$f(x_1) = f(3.7892) = -0.0001475$

$f(3.8) = 0.0202$

$(3.7892,\ 3.8)$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{3.7892(0.0202) - 3.8(-0.0001475)}{0.0202 + 0.0001475} = 3.7892$$

Find the real root of $3x = \cos x + 1$ using Newton Raphson method

$f(x) = 3x - \cos x - 1$

$f'(x) = 3 + \sin x$

$f(0) = -2 < 0 \quad f(1) = 1.4596 > 0$

$\therefore$ root lies in $(0,1)$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

I iteration

$$n=0 \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{f(0)}{f'(0)} = 0.6666$$

II iteration

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.6666 - \frac{f(0.6666)}{f'(0.6666)} = 0.6074$$

III iteration

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.6074 - \frac{f(0.6074)}{f'(0.6074)} = 0.6071$$

IV iteration

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.6071 - \frac{f(0.6071)}{f'(0.6071)} = 0.6071$$

Find the real root of $\cos x = xe^x$ using Newton Raphson method near $x = 0.5$

$f(x) = \cos x - xe^x$

$f'(x) = -\sin x - [xe^x + e^x \cdot 1]$

$f'(x) = -\sin x - xe^x - e^x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

I iteration

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.5180$$

II iteration

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.5180 - \frac{f(0.5180)}{f'(0.5180)} = 0.5177$$

III iteration

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.5177 - \frac{f(0.5177)}{f'(0.5177)} = 0.5177$$

Find the real root of $x\sin x + \cos x = 0$ using Newton Raphson method near $x = \pi$

$f(x) = x\sin x + \cos x$

$f'(x) = x\cos x + \sin x - \sin x$

$f'(x) = x\cos x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

I iteration

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \pi - \frac{f(\pi)}{f'(\pi)} = 2.8232$$

II iteration

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.8232 - \frac{f(2.8232)}{f'(2.8232)} = 2.7985$$

III iteration

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7985 - \frac{f(2.7985)}{f'(2.7985)} = 2.7983$$

IV iteration

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.7983 - \frac{f(2.7983)}{f'(2.7983)} = 2.7983$$

Find the real root of the equation $x^3 + x^2 + 3x + 4 = 0$ using newton Raphson method

$f(x) = x^3 + x^2 + 3x + 4$

$f'(x) = 3x^2 + 2x + 3$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(0) = 4 \quad f(-1) = 1 \quad f(-2) = -6$

the root lies between $(-2, -1)$

I iteration

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1 - \frac{f(-1)}{f'(-1)} = -1.25$$

II iteration

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -1.25 - \frac{f(-1.25)}{f'(-1.25)} = -1.2228$$

III iteration

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -1.2228 - \frac{f(-1.2228)}{f'(-1.2228)} = -1.2224$$

IV iteration

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -1.2224 - \frac{f(-1.2224)}{f'(-1.2224)} = -1.2224$$

Find the real root of the equation $x\log_{10}x = 1.2$ using netwon raphson method.

$x\log_{10}x - 1.2 = 0$

$f(x) = x\log_{10}x - 1.2$

$f(1) = -1.2 \quad f(2) = -0.5979 \quad f(3) = 0.2313$

root lies between $(2,3)$

$f(x) = x\log_{10}x - 1.2$

$f(x) = x \cdot \frac{\ln x}{\ln 10} - 1.2$

$f(x) = \frac{1}{\ln 10}[x\ln x] - 1.2$

$f'(x) = 0.4342\left[x \cdot \frac{1}{x} + \ln x \cdot 1\right] - 0$

$f'(x) = 0.4342[1 + \ln x]$

I iteration

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 2.8133$$

II iteration

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.8133 - \frac{f(2.8133)}{f'(2.8133)} = 2.7410$$

III iteration

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7410 - \frac{f(2.7410)}{f'(2.7410)} = 2.7406$$

IV iteration

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.7406 - \frac{f(2.7406)}{f'(2.7406)} = 2.7406$$

Module - 5

Numerical Method - 2

Employ Taylor's series method to obtain appropriate value of y at $x=0.2$ for D.E. $y' = 2y+3e^x$, $y(0)=0$

Sol^n:- $x_0=0$, $y_0=0$ $y=?$ at $x=0.2$

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y^{iv}(x_0) + \cdots$$

$$y(x) = y(0) + xy'(x_0) + \frac{x^2}{2}y''(x_0) + \frac{x^3}{6}y'''(x_0) + \frac{x^4}{24}y^{iv}(x_0) + \cdots \rightarrow (1)$$

$y'(x) = y' = 2y+3e^x \Rightarrow y'(0) = 2y_0 + 3e^{x_0} = 0 + 3\times 1 = 3$ $\boxed{y'(0)=3}$

$y'' = 2y'+3e^x \Rightarrow y''(0) = 2(3)+3e^{x_0}$ $\boxed{y''(0)=9}$

$y''' = 2y''+3e^x \Rightarrow y'''(0) = 2(9)+3e^{x_0}$ $\boxed{y'''(0)=21}$

$y^{iv} = 2y'''+3e^x \Rightarrow y^{iv}(0) = 2(21)+3e^{x_0}$ $\boxed{y^{iv}(0)=45}$

$(1) \Rightarrow y(x) = 0 + xy'(0) + \frac{x^2}{2}y''(0) + \frac{x^3}{6}y'''(0) + \frac{x^4}{24}y^{iv}(0) + \cdots$

$y(x) = 0 + x\times 3 + \frac{x^2}{2}(9) + \frac{x^3}{6}(21) + \frac{x^4}{24}(45) + \cdots$

$x=0.2$ $y(0.2) = 3(0.2) + \frac{(0.2)^2}{2}9 + \frac{(0.2)^3}{6}21 + \frac{(0.2)^4}{24}45 + \cdots$

$y(0.2) = \underline{0.811}$

By Taylor's series method find the value of y at $x=0.1$ & $x=0.2$ to 5 place of decimals for D.E. $\frac{dy}{dx} = x^2y-1$ with $y(0)=1$

Sol^n:- $x_0=0$, $y_0=1$, $y=?$ at $x=0.1$ and $x=0.2$

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y^{iv}(x_0) + \cdots$$

$$y(x) = y(0) + xy'(0) + \frac{x^2}{2}y''(0) + \frac{x^3}{6}y'''(0) + \frac{x^4}{24}y^{iv}(0) + \cdots \rightarrow (1)$$

$y' = x^2y-1 \Rightarrow y'(0) = 0(1)-1 = -1$

$y'' = x^2y'+y\cdot 2x - 0 \Rightarrow y''(0) = 0+0 = 0$

$y''' = x^2y''+y'\cdot 2x + y\cdot 2 + 2xy' \Rightarrow x^2y''+4xy'+2y \Rightarrow y'''(0) = 0+0+2 = 2$

$y^{iv} = x^2y'''+y''2x + 4xy''+y'\cdot 4 + 2y' \Rightarrow y^{iv}(0) = -4-2 = -6$

① $\Rightarrow y(x) = 1 + x(-1) + \frac{x^2}{2}(0) + \frac{x^2}{6}(2) + \frac{x^4}{24}(-6) + \cdots$

$y(x) = 1 - x + \frac{x^2}{3} - \frac{x^4}{4} + \cdots$

$y(0.1) = 0.90030$

$y(0.2) = 0.80226$

Solve the D.E. $y' = x+y$ with $y(1)=0$ by using taylor's series method considering upto 5th degree term for $x=1.1$ and $x=1.2$

Soln:- $x_0 = 1$ $y_0 = 0$ $y = ?$ at $x = 1.1$ and $x = 1.2$

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y^{iv}(x_0) + \cdots$$

$$\frac{(x-x_0)^5}{5!}y^{v}(x_0) + \cdots$$

$$y(x) = y(1) + (x-1)y'(1) + \frac{(x-1)^2}{2}y''(1) + \frac{(x-1)^3}{6}y'''(1) + \frac{(x-1)^4}{24}y^{iv}(1) + \frac{(x-1)^5}{120}y^{v}(1) + \cdots$$

$y' = x+y = y'(1) = 1+0 = 1$

$y'' = 1+y' = y''(1) = 1+1 = 2$

$y''' = 0+y'' = y'''(1) = 0+2 = 2$

$y^{iv} = 0+y''' = y^{iv}(1) = 0+2 = 2$

$y^{v} = 0+y^{iv} = y^{v}(1) = 0+2 = 2$

① $\Rightarrow y(x) = 0 + (x-1) + (x-1)^2 + \frac{(x-1)^3}{3} + \frac{(x-1)^4}{12} + \frac{(x-1)^5}{60} + \cdots$

$y(1.1) = 0.11034$

$y(1.2) = 0.24280$

Solve the D.E. $y' = 3x+y^2$ with $y(0)=1$ for $x=0.1$

Soln:- $x_0 = 0$, $y_0 = 1$ $y = ?$ at $x = 0.1$

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y^{iv}(x_0) + \cdots$$

$$y(x) = y(0) + (x-0)y'(0) + \frac{(x-0)^2}{2}y''(0) + \frac{(x-0)^3}{6}y'''(0) + \frac{(x-0)^4}{24}y^{iv}(0) + \cdots \rightarrow ①$$

$y' = 3x + 1y^2 \Rightarrow y'(0) = 0+1 = 1$

$y'' = 3 + 2y \cdot y' \Rightarrow y''(0) = 3 + 2(1)(1) = 5$

$y''' = 0 + 2y'y' + 2yy'' \Rightarrow y'''(0) = 0 + 2 + 10 = 12$

$y^{iv} = 0 + 2y'y'' + 2y'y'' + 2yy''' + 2y''y'$

$y^{iv}(0) = 0 + 10 + 10 + 24 + 10 = 54$

$① \Rightarrow 1 + (x-0) + \frac{(x-0)^2}{2} 5 + \frac{(x-0)^3}{6} 12 + \frac{(x-0)^4}{24} 54 + \ldots$

$y(0.1) = 1.12722$

Solve the D.E $y' = x - y^2$ with $y(0) = 1$ for $x = 0.1$

Soln:- $x_0 = 0 \quad y_0 = 1 \quad y = ?$ at $x = 0.1$

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y^{iv}(x_0) + \ldots$$

$$y(x) = y(0) + (x-0)y'(0) + \frac{(x-0)^2}{2}y''(0) + \frac{(x-0)^3}{6}y'''(0) + \frac{(x-0)^4}{24}y^{iv}(0) + \ldots \rightarrow ①$$

$y' = x - y^2 \Rightarrow y'(0) = 0 - 1 = -1$

$y'' = 1 - 2y \cdot y' \rightarrow y''(0) = 1 - 2(1)(-1) = 1 + 2 = 3$

$y''' = 0 - 2yy'' - 2y'y' \Rightarrow y'''(0) = 0 - 2(1)(3) - 2(-1)(-1) = -8$

$y^{iv} = -2yy''' - 2y'y'' - 2y'y'' - 2y'y'' \Rightarrow y^{iv}(0) = -2(1)(-8) - 2(-1)(3) - 2(-1)(3) - 2(-1)(3) = 34$

$$① \Rightarrow y(x) = 1 + (x-0) - 1 + \frac{(x-0)^2}{2} \times 3 + \frac{(x-0)^3}{6}(-8) + \frac{(x-0)^4}{24}(34) + \ldots$$

$$y(x) = 1 - x + \frac{x^2}{2}(3) + \frac{x^3}{6}(-8) + \frac{x^4}{24}(34) + \ldots$$

$y(0.1) = 0.91380$

Find $y(1.1)$ and $y(1.2)$ given $y'(x) = \log xy$ with $y(1) = 2$ using Taylor's Series method consider upto 3rd degree term

Soln:- $x_0 = 1 \quad y_0 = 2 \quad y = ? \quad x = 1.1$ and $x = 1.2$

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \ldots$$

$$y(x) = y(1) + (x-1)y'(1) + \frac{(x-1)^2}{2}y''(1) + \frac{(x-1)^3}{6}y'''(1) + \ldots \rightarrow ①$$

$y' = \log xy \Rightarrow y'(1) = \log(1)(2) = 0.6931$

$y' = \log xy \Rightarrow \log x + \log y$

$y'' = \frac{1}{x} + \frac{1}{y} \times y' \rightarrow y''(1) = \frac{1}{1} + \frac{1}{2} \times 0.6931 = 1.3465$

$y''' = \frac{-1}{x^2} + \left[\frac{1}{y} \cdot y'' + y'\left(\frac{-1(y')}{y^2}\right)\right] \Rightarrow y'''(1) = -\frac{1}{1} + \left[\frac{1}{2} \cdot 1.3465 + 0.6931\left(\frac{-0.6931}{4}\right)\right] + \ldots$

$y'''(1) = -0.4468$

$$① \Rightarrow y(x) = 2 + (x-1)0.6931 + \frac{(x-1)^2}{2}1.3465 + \frac{(x-1)^3}{6}(-0.4468) + \ldots$$

$y(1.1) = 2.0759$

$y(1.2) = 2.16495$

Solve $y'(x) = 3x + \frac{y}{2}$, $y(0)=1$, find $y(0.2)$ with $h=0.2$ using modified eulers method

Solⁿ:- $\frac{dy}{dx} = 3x + \frac{y}{2}$ $\quad \frac{dy}{dx} = f(x,y)$

$f(x,y) = 3x + \frac{y}{2}$

$y(x_0) = y_0$

$x_0 = 0, \; y_0 = 1 \quad h = 0.2$

$y_1 = ? \quad x_1 = x_0 + h$

$x_1 = 0.2$

Intial Approximation

$y_1^{(0)} = y_0 + hf(x_0, y_0)$

$y_1^{(0)} = 1 + 0.2(0.5)$

$y_1^{(0)} = 1.1$

$f(x,y) = 3x + \frac{y}{2}$

$f(x_0, y_0) = 3x_0 + \frac{y_0}{2}$

$= 3(0) + \frac{1}{2}$

$f(x_0, y_0) = 0.5$

I Approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1 + \frac{0.2}{2}[f(x_0, y_0) + (3x_0 + \frac{y_1^{(0)}}{2})]$

$= 1 + \frac{0.2}{2}[0.5 + 3(0.2) + \frac{1.1}{2}]$

$y_1^{(1)} = 1.165$

II Approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 1 + \frac{0.2}{2}[0.5 + 3(0.2) + \frac{1.165}{2}]$

$y_1^{(2)} = 1.16825$

III Approximation

$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$

$= 1 + \frac{0.2}{2}[0.5 + 3(0.2) + \frac{1.16825}{2}]$

$y_1^{(3)} = 1.1684$

IV Approximation

$y_1^{(4)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(3)})]$

$= 1 + \frac{0.2}{2}[0.5 + 3(0.2) + \frac{1.1684}{2}]$

$y_1^{(4)} = 1.1684$

Solve $\frac{dy}{dx} = 3x + \frac{y}{2}$ with $y(0)=1$ for $x=0.2$ taking $h=0.1$ by applying modified Euler's method

Solⁿ:- $\frac{dy}{dx} = 3x + \frac{y}{2}$ $\quad \frac{dy}{dx} = f(x,y)$

$f(x,y) = 3x + \frac{y}{2}$

$y(x_0) = y_0$

$x_0 = 0, \; y_1 = ? \quad x_1 = 0.1$

$y_0 = 1, \; h = 0.1$

Stage 01

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0, y_0)$

$= 1 + 0.1(0.5)$

$y_1^{(0)} = 1.05$

$f(x_0, y_0) = 3x_0 + \frac{y_0}{2}$

$f(x_0, y_0) = 0 + \frac{1}{2}$

$f(x_0, y_0) = 0.5$

I Approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1 + \frac{0.1}{2}[0.5 + 3(0.1) + \frac{1.05}{2}]$

$y_1^{(1)} = 1.06625$

II Approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 1 + \frac{0.1}{2}[0.5 + 3(0.1) + \frac{1.06625}{2}]$

$y_1^{(2)} = 1.06665$

III Approximation

$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$

$= 1 + \frac{0.1}{2}[0.5 + 3(0.1) + \frac{1.06665}{2}]$

$y_1^{(3)} = 1.0666$

$y(x_1) = y(0.1) = 1.0666$

Stage 02

$x_0 = 0.1 \quad y_0 = 1.0666, \; h = 0.1, \; y_1 = ? \quad x_1 = x_0 + h \Rightarrow x_1 = 0.2$

$f(x, y) = 3x + \frac{y}{2}$

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0, y_0)$

$= 1.0666 + 0.1(0.8333)$

$y_1^{(0)} = 1.14993$

$f(x_0, y_0) = 3x_0 + \frac{y_0}{2}$

$= 3(0.1) + \frac{1.0666}{2}$

$f(x_0, y_0) = 0.8333$

I approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1.0666 + \frac{0.1}{2}[0.8333 + 3(0.2) + \frac{1.14993}{2}]$

$y_1^{(1)} = 1.16701$

II Approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 1.0666 + \frac{0.1}{2}[0.8333 + 3(0.2) + \frac{1.16701}{2}]$

$y_1^{(2)} = 1.16744$

III approximation

$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$

$= 1.0686 + \frac{0.1}{2}[0.8333 + 3(0.2) + \frac{1.16744}{2}]$

$y_1^{(3)} = 1.1674$

$y(x_1) = y(0.2) = 1.1674$

Find given $y'(x) = \log_{10}\left(\frac{x}{y}\right)$ with $y=5$ at $x=20$ by applying modified euler's method $y(20.2)$ and $y(20.4)$

Soln:- $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$, $\frac{dy}{dx} = f(x,y)$, $f(x,y) = \log_{10}\left(\frac{x}{y}\right)$

Stage :-01

$x_0 = 20$, $y_0 = 5$, $x_1 = x_0 + h$ $h = 0.2$ $y = ?$

$x_1 = 20.2$

Intial Approximation

$y_1^{(0)} = y_0 + hf(x_0, y_0)$

$= 5 + 0.2(0.6020)$

$y_1^{(0)} = 5.1204$

$f(x_0, y_0) = \log_{10}\left(\frac{x_0}{y_0}\right)$

$= \log_{10}\left(\frac{20}{5}\right)$

$f(x_0, y_0) = 0.6020$

I Approximation

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$

$= 5 + \frac{0.2}{2}\left[0.6020 + \log_{10}\left(\frac{20.2}{5.1204}\right)\right]$

$y_1^{(1)} = 5.1198$

II Approximation

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$

$= 5 + \frac{0.2}{2}\left[0.6020 + \log_{10}\left(\frac{20.2}{5.1198}\right)\right]$

$y_1^{(2)} = 5.1198$

$y(x_1) = y(20.2) = 5.1198$

Stage :-02

$x_0 = 20.2$ $y_0 = 5.1198$ $h = 0.2$ $y_1 = ?$ $x_1 = 20.2 + 0.2 = 20.4$

$f(x,y) = \log_{10}\left(\frac{x}{y}\right)$

Intial

$y_1^{(0)} = y_0 + hf(x_0, y_0)$

$= 5.1198 + 0.2(0.5960)$

$y_1^{(0)} = 5.239$

$f(x_0, y_0) = \log_{10}\left(\frac{x}{y}\right)$

$= \log_{10}\left(\frac{20.2}{5.1198}\right)$

$f(x_0, y_0) = 0.5960$

I Approximation

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$

$= 5.1198 + \frac{0.2}{2}\left[0.5960 + \log_{10}\left(\frac{20.4}{5.239}\right)\right]$

$y_1^{(1)} = 5.2384$

II Approximation

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$

$= 5.1198 + \frac{0.2}{2}\left[0.5960 + \log_{10}\left(\frac{20.4}{5.2384}\right)\right]$

$y_1^{(2)} = 5.2384$

$y(x_1) = y(20.4) = 5.2384$

Solve $\frac{dy}{dx}+xy^2=0$ with $y(0)=2$ for $x=0.1$ by applying modified Euler's method

Sol^n:- $\frac{dy}{dx}=-xy^2$

$f(x,y)=-xy^2$

$x_1=0.1 \quad y_0=2 \quad x_0=0 \quad h=0.1 \quad y=?$

Intial approximation

$y_1^{(0)}=y_0+hf(x_0,y_0)$

$=2+0.1(0)$

$y_1^{(0)}=2$

$f(x_0,y_0)=-xy^2$

$=-(0)(4)$

$f(x_0,y_0)=0$

I approximation

$y_1^{(1)}=y_0+\frac{h}{2}[f(x_0,y_0)+f(x_1,y_1^{(0)})]$

$=2+\frac{0.1}{2}[0+(-0.1)4)]$

$y_1^{(1)}=1.98$

II Approximation

$y_1^{(2)}=y_0+\frac{h}{2}[f(x_0,y_0)+f(x_1,y_1^{(1)})]$

$=2+\frac{0.1}{2}[0+(-0.1)(1.98)^2]$

$y_1^{(2)}=1.9803$

III approximation

$y_1^{(3)}=y_0+\frac{h}{2}[f(x_0,y_0)+f(x_1,y_1^{(2)})]$

$=2+\frac{0.1}{2}[0+(-0.1)(1.9803)^2]$

$y_1^{(3)}=1.9803$

$y(x)=y(0.1)=1.9803$

Solve $\frac{dy}{dx}+xy^2=0$ with $y(0)=2$ for $x=0.1$, $h=0.05$ by applying modified euler's method.

Sol^n:- $\frac{dy}{dx}=-xy^2 \quad f(x,y)=-xy^2$

Stage 1 :-

$x_0=0 \quad y_0=2 \quad h=0.05 \quad x_1=x_0+h \quad y=?$

$x_1=0.05$

Intial approximation

$y_1^{(0)}=y_0+hf(x_0,y_0)$

$=2+0.05(0)$

$y_1^{(0)}=2$

$f(x_0,y_0)=-xy^2$

$=-(0)(2)^2$

$f(x_0,y_0)=0$

I Approximation

$y_1^{(1)}=y_0+\frac{h}{2}[f(x_0,y_0)+f(x_1,y_1^{(0)})]$

$=2+\frac{0.05}{2}[0+(-(0.05)(2))]$

$y_1^{(1)}=1.9975$

II Approximation

$y_1^{(2)}=y_0+\frac{h}{2}[f(x_0,y_0)+f(x_1,y_1^{(1)})]$

$=2+\frac{0.05}{2}[0+(-(0.05)(1.9975)]$

$y_1^{(2)}=1.9975$

$y(x_1) = y(0.05) = 1.9975$

Stage 2

$x_0 = 0.05 \quad y_0 = 1.9975 \quad h = 0.05 \quad x_1 = 0.05 + 0.05 = 0.1$

$f(x,y) = -xy^2$

Intial approximation

$y_1^{(0)} = y_0 + h[f(x_0, y_0)]$

$y_1^{(0)} = 1.9975 + 0.05(-0.1995)$

$y_1^{(0)} = 1.9875$

$f(x_0, y_0) = -x_0 y_0^2$

$= -0.05(1.9975)^2$

$= -0.1995$

I approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1.9975 + \frac{0.05}{2}[-0.1995 + (-x_1(y_1^{(0)})^2)]$

$= 1.9975 + \frac{0.05}{2}[-0.1995 + (-(0.1)(1.9875)^2)]$

$y_1^{(1)} = 1.9826$

II approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 1.9975 + \frac{0.05}{2}[-0.1995 + (-x_1, (y_1^{(1)})^2]$

$= 1.9975 + \frac{0.05}{2}[-0.1995 + (-(0.1)(1.9826)^2]$

$y_1^{(2)} = 1.9826$

By using modified euler's method $y(1.2)$ Given $y'(x) = 1 + \frac{y}{x}$ with $y(1) = 2$

Sol^n:- $x_0 = 1 \quad x_1 = 1.2 \quad h = 0.2 \quad y_0 = 2$

$y_1 = y(x_1) = ?$

$f(x,y) = 1 + \frac{y}{x}$

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0, y_0)$

$= 2 + 0.2(3)$

$y_1^{(0)} = 2.6$

$f(x_0, y_0) = 1 + \frac{y_0}{x_0}$

$= 1 + \frac{2}{1}$

$= 1 + 2$

$f(x_0, y_0) = 3$

I Approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 2 + \frac{0.2}{2}[3 + (1 + \frac{y_1^{(0)}}{x_1})]$

$= 2 + \frac{0.2}{2}[3 + (1 + \frac{2.6}{1.2})]$

$y_1^{(1)} = 2.6166$

II Approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 2 + \frac{0.2}{2}[3 + (1 + \frac{2.6166}{1.2})]$

$y_1^{(2)} = 2.6180$

III approximation

$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(2)})]$

$= 1.2309 + \frac{0.2}{2}[1.3094 + 0.4 + \sqrt{1.5252}]$

$y_1^{(3)} = 1.5253$

IV approximation

$y_1^{(4)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(3)})]$

$= 1.2309 + \frac{0.2}{2}[1.3094 + 0.4 + \sqrt{1.5253}]$

$y_1^{(4)} = 1.5253$

By using modified euler's method Given $y' = \frac{y-x}{y+x}$, $y=1$ at $x=0$ find $y(0.2)$ taking $h=0.1$

Sol^n:- $x_0 = 0$ $x_1 = x_0 + h$ $h = 0.1$ $y_0 = 1$

$= 0 + 0.1$

$x_1 = 0.1$

$f(x,y) = \frac{y-x}{y+x}$

Stage :- 01

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0,y_0)$

$= 1 + 0.1(1)$

$= 1 + 0.1$

$y_1^{(0)} = 1.1$

$f(x_0,y_0) = \frac{y_0 - x_0}{y_0 + x_0}$

$= \frac{1-0}{1+1}$

$f(x_0,y_0) = 1$

I approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(0)})]$

$= 1 + \frac{0.1}{2}\left[1 + \left(\frac{1.1-0.1}{1.1+0.1}\right)\right]$

$y_1^{(1)} = 1.0916$

$y(x) = y(0.1) = 1.0916$

II approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(1)})]$

$= 1 + \frac{0.1}{2}\left[1 + \left(\frac{1.0916-0.1}{1.0916-0.1}\right)\right]$

$y_1^{(2)} = 1.0916$

Stage:- 02

$x_0 = 0.1$ $y_0 = 1.0916$ $h = 0.1$ $x_1 = 0.1 + 0.1 = 0.2$

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0,y_0)$

$= 1.0916 + 0.1(0.8321)$

$y_1^{(0)} = 1.17481$

$f(x_0,y_0) = \frac{y_0 - x_0}{y_0 + x_0}$

$= \frac{1.0916 - 0.1}{1.0916 + 0.1}$

$f(x_0,y_0) = 0.8321$

I approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(0)})]$

$= 1.0916 + \frac{0.1}{2}\left[0.8321 + \left(\frac{1.17481-0.2}{1.17481+0.2}\right)\right]$

$y_1^{(1)} = 1.1686$

II approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(1)})]$

$= 1.0916 + \frac{0.1}{2}\left[0.8321 + \left(\frac{1.1686-0.2}{1.1686+0.2}\right)\right]$

$y_1^{(2)} = 1.1685$

III Approximation

$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(2)})]$

$= 2 + \frac{0.2}{2}[3 + (1 + \frac{2.6180}{1.2})]$

$y_1^{(3)} = 2.6181$

IV Approximation

$y_1^{(4)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(3)})]$

$= 2 + \frac{0.2}{2}[3 + (1 + \frac{2.6181}{1.2})]$

$y_1^{(4)} = 2.6181$

By using modified euler's method find $y' = x + |\sqrt{y}|$, $y=1$ at $x=0$ for $0 \le x \le 0.4$ taking $h=0.2$

Soln:- $x_0 = 0$ $y_0 = 1$ $h = 0.2$ $x_1 = x_0 + h$ $y_1 = y(x_1) = ?$

$x_1 = 0.2$

$f(x,y) = x + |\sqrt{y}|$

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0,y_0)$

$= 1 + 0.2(1)$

$y_1^{(0)} = 1.2$

$f(x_0,y_0) = x_0 + |\sqrt{y_0}|$

$= 0 + 1$

$f(x_0,y_0) = 1$

I approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(0)})]$

$= 1 + \frac{0.2}{2}[1 + x_0 + |\sqrt{y_1^{(0)}}|]$

$= 1 + \frac{0.2}{2}[1 + 0.2 + \sqrt{1.2}]$

$y_1^{(1)} = 1.2295$

II Approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(1)})]$

$= 1 + \frac{0.2}{2}[1 + x_1 + |\sqrt{y_1^{(1)}}|]$

$= 1 + \frac{0.2}{2}[1 + 0.2 + \sqrt{1.2295}]$

$y_1^{(2)} = 1.2308$

III approximation

$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(2)})]$

$= 1 + \frac{0.2}{2}[1 + 0.2 + |\sqrt{1.2308}|]$

$y_1^{(3)} = 1.2309$

IV approximation

$y_1^{(4)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(3)})]$

$= 1 + \frac{0.2}{2}[1 + 0.2 + \sqrt{1.2309}]$

$y_1^{(4)} = 1.2309$

Stage 02

$x_0 = 0.2$ $y_0 = 1.2309$ $h = 0.2$ $x_1 = 0.2 + 0.2 + 0.4$

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0,y_0)$

$= 1.2309 + 0.2(1.3094)$

$y_1^{(0)} = 1.4927$

$f(x_0,y_0) = x_0 + |\sqrt{y_0}|$

$= 0.2 + \sqrt{1.2309}$

$f(x_0,y_0) = 1.3094$

I approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(0)})]$

$= 1.2309 + \frac{0.2}{2}[1.3094 + 0.4 + \sqrt{1.4927}]$

$y_1^{(1)} = 1.5240$

II Approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(1)})]$

$= 1.2309 + \frac{0.2}{2}[1.3094 + 0.4 + \sqrt{1.5240}]$

$y_1^{(2)} = 1.5252$

III Approximation

$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(2)})]$

$= 1.0916 + \frac{0.1}{2}[0.8321 + (\frac{1.1685-0.2}{1.1685+0.2})]$

$y_1^{(3)} = 1.1685$

Using the modified Euler's method, find $y(0.1)$ given that $\frac{dy}{dx} = x^2+y$ and $y(0)=1$ take step $h=0.05$ and perform two modifications in each stage.

Sol^n:- Stage 1

$x_0=0$, $y_0=1$, $x_1=0.05$, $h=0.05$

$f(x,y) = x^2+y$

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0,y_0)$

$= 1 + 0.05(1)$

$y_1^{(0)} = 1.05$

$f(x_0,y_0) = x_0^2 + y_0$

$= 0+1$

$f(x_0,y_0) = 1$

I approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(0)})]$

$= 1 + \frac{0.05}{2}[1 + f(0.05, 1.05)]$

$y_1^{(1)} = 1 + \frac{0.05}{2}[1 + 1.0525]$

$y_1^{(1)} = 1.0513$

II approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(1)})]$

$= 1 + \frac{0.05}{2}[1 + f(0.05, 1.0513)]$

$= 1 + \frac{0.05}{2}[1 + 1.0538]$

$y_1^{(2)} = 1.0513$

$y_1(x) = y_1(0.05) = 1.0513$

Stage 02

$x_0=0.05$, $x_1=0.1$ $y_0=1.0513$, $h=0.05$

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0,y_0)$

$= 1.0513 + 0.05(1.0538)$

$y_1^{(0)} = 1.1039$

$f(x_0,y_0) = x_0^2 + y_0$

$= (0.05)^2 + 1.0513$

$f(x_0,y_0) = 1.0538$

I approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(0)})]$

$= 1.0513 + \frac{0.05}{2}[1.0538 + f(0.1, 1.1039)]$

$= 1.0513 + \frac{0.05}{2}[1.0538 + 1.1139]$

$y_1^{(1)} = 1.1054$

II approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(1)})]$

$= 1.0513 + \frac{0.05}{2}[1.0538 + f(0.1, 1.1054)]$

$= 1.0513 + \frac{0.05}{2}[1.0538 + 1.1154]$

$y_1^{(2)} = 1.1055$

III approximation

$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$

$= 1.0513 + \frac{0.05}{2}[1.0538 + f(0.1), 1.1055)]$

$= 1.0513 + \frac{0.05}{2}[1.0538 + 1.1155]$

$y_1^{(3)} = 1.1055$

Solve $y'(x) = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ for $x = 0.2$

Sol^n:- $y' = f(x, y)$ - $f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$

$x_0 = 0$, $y_0 = 1$, $x_1 = 0.2$, $h = 0.2$

$y(x_1) = y_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4] \longrightarrow (1)$

$k_1 = h f(x_0, y_0)$

$= 0.2 f(0, 1)$

$k_1 = 0.2$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right)$

$= 0.2 f(0.1, 1.1)$

$k_2 = 0.1967$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1967}{2}\right)$

$= 0.2 f(0.1, 1.0983)$

$k_3 = 0.1967$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.2 f(0 + 0.2, 1 + 0.1967)$

$= 0.2 f(0.2, 1.1967)$

$k_4 = 0.1891$

$(1) \Rightarrow y(0.2) = 1 + \frac{1}{6}[0.2 + 2(0.1967 + 0.1967) + 0.1891]$

$y(0.2) = 1.1959$

Solve $y'(x) = x - y^2$, $y(0) = 0$ for $x = 0.4$

Sol^n: $x_0 = 0$ $y_0 = 0$ $x_1 = 0.4$ $h = 0.4$

$y'(x) = f(x, y)$ $f(x, y) = x - y^2$

$y(x_1) = y_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4] \rightarrow (1)$

$k_1 = hf(x_0, y_0)$
$= 0.4f(0, 0)$
$k_1 = 0$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$
$= 0.4f(0 + \frac{0.4}{2}, 0 + \frac{0}{2})$
$k_2 = 0.4f(0.2, 0)$
$k_2 = 0.08$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$
$= 0.4f(0 + \frac{0.4}{2}, 0 + \frac{0.08}{2})$
$= 0.4f(0.2, 0.04)$
$k_2 = 0.0793$

$k_4 = hf(x_0 + h, y_0 + k_3)$
$= 0.4f(0 + 0.4 \quad 0 + 0.0793)$
$= 0.4f(0.4, 0.0793)$
$k_4 = 0.1574$

$(1) \Rightarrow y(0.4) = 0 + \frac{1}{6}[0 + 2(0.08 + 0.0793) + 0.1574]$
$y(0.4) = \underline{\underline{0.0793}}$

Solve $y'(x) = 3e^x + 2y$, $y(0) = 0$ for $x = 0.1$

$x_0 = 0$, $y_0 = 0$ $x_1 = 0.1$ $h = 0.1$

$y'(x) = f(x, y)$ $f(x, y) = 3e^x + 2y$

$y(x_1) = y_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4] \rightarrow (1)$

$k_1 = hf(x_0, y_0)$
$= 0.1f(0, 0)$
$k_1 = 0.3$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$
$= 0.1f(0 + \frac{0.1}{2}, 0 + \frac{0.3}{2})$
$= 0.1f(0.05, 0.15)$
$k_2 = 0.3453$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$
$= 0.1f(0 + \frac{0.1}{2}, 0 + \frac{0.3453}{2})$
$= 0.1f(0.05, 0.1726)$
$k_3 = 0.3499$

$k_4 = hf(x_0 + h, y_0 + k_3)$
$= 0.1f(0.1, 0 + 0.3499)$
$k_4 = 0.4015$

$(1) \Rightarrow y(0.1) = 0 + \frac{1}{6}[0.3 + 2(0.3453 + 0.3499) + 0.4015]$
$y(0.1) = \underline{\underline{0.3486}}$

Solve $y' = 3x + y^2$ $y(1) = 1.2$ for $x = 1.1$

$x_0 = 1$ $y_0 = 1.2$ $x_1 = 1.1$ $h = 0.1$

$y'(x) = f(x, y)$ $f(x, y) = 3x + y^2$

$y(x_1) = y_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4] \longrightarrow (1)$

$k_1 = hf(x_0, y_0)$
$= 0.1f(1, 1.2)$
$k_1 = 0.444$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$
$= 0.1f\left(1 + \frac{0.1}{2}, 1.2 + \frac{0.444}{2}\right)$
$= 0.1f(1.05, 1.422)$
$k_2 = 0.5172$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$
$= 0.1f\left(1 + \frac{0.1}{2}, 1.2 + \frac{0.5172}{2}\right)$
$= 0.1f(1.05, 1.4586)$
$k_3 = 0.5277$

$k_4 = hf(x_0 + h, y_0 + k_3)$
$= 0.1f(1 + 0.1, 1.2 + 0.5277)$
$= 0.1f(1.1, 1.7277)$
$k_4 = 0.6284$

$(1) \Rightarrow y(1.1) = 1.2 + \frac{1}{6}[0.444 + 2(0.5172 + 0.5277) + 0.6284]$

$y(1.1) = 1.7270$

USE RK Method to find $y(0.2)$ Given $\frac{dy}{dx} = y'(x) = \tan(x + y)$, $y(0) = 1$

Sol^n: $x_0 = 0$ $y_0 = 1$ $x_1 = 0.2$ $h = 0.2$

$y'(x) = f(x, y)$ $f(x, y) = \tan(x + y)$

$y(x_1) = y_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4] \longrightarrow (1)$

$k_1 = hf(x_0, y_0)$
$= 0.2f(0, 1)$
$k_1 = 0.3114$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$
$= 0.2f\left(0 + \frac{0.2}{2}, 1 + \frac{0.3114}{2}\right)$
$= 0.2f(0.1, 1.1557)$
$k_2 = 0.6135$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$
$= 0.2f\left(0 + \frac{0.2}{2}, 1 + \frac{0.6135}{2}\right)$
$= 0.2f(0.1, 1.3067)$
$k_3 = 1.2078$

$k_4 = hf(x_0 + h, y_0 + k_3)$
$= 0.2f(0.2, 1 + 1.2078)$
$= 0.2f(0.2, 2.2078)$
$k_4 = -0.1803$

$(1) \Rightarrow y(0.2) = 1 + \frac{1}{6}[0.3114 + 2(0.6135 + 1.2078) + (-0.1803)]$

$y(0.2) = 1.6289$

Using modified eulers method to find $y(0.4)$ Given $y' = x + \sin y$
$y(0) = 1$ with $h = 0.2$

Sol^n:- $x_0 = 0$ $y_0 = 1$ $x_1 = 0.2$ $h = 0.2$

Stage :-01

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0, y_0)$

$= 1 + 0.2(0.8414)$

$y_1^{(0)} = 1.16828$

$f(x_0, y_0) = x_0 + \sin y_0$

$= 0 + \sin(1)$

$= 0 + 0.8414$

$f(x_0, y_0) = 0.8414$

I approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1 + \frac{0.2}{2}[0.8414 + 0.2 + 0.9200]$

$y_1^{(1)} = 1.1961$

II approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 1 + \frac{0.2}{2}[0.8414 + 0.2 + 0.9306]$

$y_1^{(2)} = 1.1972$

III approximation

$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$

$= 1 + \frac{0.2}{2}[0.8414 + 0.2 + 0.9310]$

$y_1^{(3)} = 1.1972$

$y(x) = y(0.2) = 1.1972$

Stage :-02

$x_0 = 0.2$ $y_0 = 1.1972$ $h = 0.2$ $x_1 = 0.4$

Intial approximation

$y_1^{(0)} = y_0 + hf(x_0, y_0)$

$= 1.1972 + 0.2(1.131)$

$y_1^{(0)} = 1.4234$

$f(x_0, y_0) = x_0 + \sin y_0$

$= 0.2 + \sin(1.1972)$

$f(x_0, y_0) = 1.131$

I approximation

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1.1972 + \frac{0.2}{2}[1.131 + 0.4 + 0.9891]$

$y_1^{(1)} = 1.4492$

II approximation

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1.1972 + \frac{0.2}{2}[1.131 + 0.4 + 0.9926]$

$= 1.4495$

III approximation

$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$

$= 1.1972 + \frac{0.2}{2}[1.131 + 0.4 + 0.9926]$

$y_1^{(3)} = 1.4495$

Using the Runge-kutta method of fourth order, find $y(0.2)$ given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0.1) = 1.0912$ taking $h = 0.1$

Sol^n: $x_0 = 0.1$, $y_0 = 1.0912$ $h = 0.1$

$$y'(x) = f(x, y) = \frac{y-x}{y+x}$$

$$y(x_1) = y_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4] \longrightarrow (1)$$

$k_1 = hf(x_0, y_0)$

$= 0.1\, f(0.1, 1.0912)$

$= 0.1 \times 0.8321$

$k_1 = 0.08321$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$= 0.1 f(0.1 + \frac{0.1}{2}, 1.0912 + \frac{0.08321}{2})$

$= 0.1 f(0.15, 1.1328)$

$= 0.1(0.7661)$

$k_2 = 0.07661$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$= 0.1 f(0.1 + \frac{0.1}{2}, 1.0912 + \frac{0.07661}{2})$

$= 0.1 f(0.15, 1.1295)$

$= 0.1 \times 0.7655$

$k_3 = 0.07655$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1 f(0.1 + 0.1, 1.0912 + 0.07655)$

$= 0.1 f(0.2, 1.1677)$

$k_4 = 0.7075 \times 0.1$

$k_4 = 0.07075$

$(1) \Longrightarrow y(0.2) = 1.0912 + \frac{1}{6}[0.08321 + 2(0.07661 + 0.07655) + 0.07075)$

$y(0.2) = \underline{\underline{1.16791}}$

Use RK method to find $y(0.1)$ Given $\frac{dy}{dx} = y'(x) = x^2+y^2$, $y(0)=1$

$x_0=0 \quad y_0=1 \quad x_1=0.1 \quad h=0.1$

$y'(x)=f(x,y) \quad f(x,y)=x^2+y^2$

$y(x_1)=y_0+\frac{1}{6}[k_1+2(k_2+k_3)+k_4] \rightarrow ①$

$k_1=hf(x_0,y_0)$
$=0.1f(0,1)$
$k_1=0.1$

$k_2=hf(x_0+\frac{h}{2}, y_0+\frac{k_1}{2})$
$=0.1f(0+\frac{0.1}{2}, 1+\frac{0.1}{2})$
$=0.1f(0.05, 1.05)$
$k_2=0.1105$

$k_3=hf(x_0+\frac{h}{2}, y_0+\frac{k_2}{2})$
$=0.1f(0+\frac{0.1}{2}, 1+\frac{0.1105}{2})$
$=0.1f(0.05, 1.0552)$
$k_3=0.1115$

$k_4=hf(x_0+h, y_0+k_3)$
$=0.1f(0.1, 1+0.1115)$
$=0.1f(0.1, 1.1115)$
$k_4=0.12454$

$① \Rightarrow y(0.1)=1+\frac{1}{6}[0.1+2(0.1105+0.1115)+(0.12454)]$
$y(0.1)=\underline{\underline{1.1114}}$

Using method of RK of fourth order to find an appropriate value of y when $x=0.2$ given that $\frac{dy}{dx}=x+y$ and $y=1$ when $x=0$

Solⁿ:- $x_0=0 \quad y_0=1 \quad x_1=0.2 \quad h=0.2$

$y'(x)=f(x,y)=x+y$

$y(x_1)=y_0+\frac{1}{6}[k_1+2(k_2+k_3)+k_4] \rightarrow ①$

$k_1=hf(x_0,y_0)$
$=0.2f(0.1)$
$=0.2(1)$
$k_1=0.2$

$k_2=hf(x_0+\frac{h}{2}, y_0+\frac{k_1}{2})$
$=0.2f(0+\frac{0.2}{2}, 1+\frac{0.2}{2})$
$=0.2f(0.1, 1.1)$
$k_2=0.2\times1.2$
$k_2=0.24$

$k_3=hf(x_0+\frac{h}{2}, y_0+\frac{k_2}{2})$
$=0.2f(0+\frac{0.2}{2}, 1+\frac{0.24}{2})$
$=0.2f(0.1, 1.12)$
$=0.2\times1.22$
$k_3=0.244$

$k_4=hf(x_0+h, y_0+k_3)$
$=0.2f(0+0.2, 1+0.244)$
$=0.2f(0.2, 1.244)$
$=0.2\times1.444$
$k_4=0.2888$

$① \Rightarrow y(0.2)=1+\frac{1}{6}(0.2+2(0.24+0.244)+0.2888)$
$y(0.2)=\underline{\underline{1.2428}}$

Given $y' = 2e^x - y$ with $y(0) = 2$, $y(0.1) = 2.01$, $y(0.2) = 2.04$, $y(0.3) = 2.09$ find $y(0.4)$ by milners method

Sol^n:- $\frac{dy}{dx} = f(x, y)$

x	$x_0 = 0$	$x_1 = 0.1$	$x_2 = 0.2$	$x_3 = 0.3$
y	$y_0 = 2$	$y_1 = 2.01$	$y_2 = 2.04$	$x_4 = 2.09$
$f = 2e^x - y$	$f_0 = 0$	$f_1 = 0.2003$	$f_2 = 0.4028$	$f_3 = 0.6097$

$x_4 = 0.4$ $\quad h = 0.1$ $\quad y_4 = ?$

$$y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$$

$$= 2 + \frac{4 \times 0.1}{3}[2(0.2003) - 0.4028 + 2(0.60971)]$$

$$y_4^{(P)} = 2.1622$$

I

$$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$$

$$= 2.04 + \frac{0.1}{3}[0.4028 + 4(0.6097) + 0.82144]$$

$$y_4^{(c)} = \underline{2.1621}$$

$f = 2e^x - y$

$f_4 = 2e^{x_4} - y_4$

$= 2e^{0.4} - 2.1622$

$f_4 = 0.82144$

II

$$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$$

$$= 2.04 + \frac{0.1}{3}[0.4028 + 4(0.6097) + 0.82152]$$

$$y_4^{(c)} = 2.1621$$

$f = 2e^x - y$

$f_4 = 2e^{x_4} - y_4$

$= 2e^{0.4} - 2.1621$

$f_4 = 0.82152$

Given $y' = x - y^2$ with $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$ find $y(0.8)$ by milnes method

$$\frac{dy}{dx} = f(x, y)$$

x	$x_0 = 0$	$x_1 = 0.2$	$x_2 = 0.4$	$x_3 = 0.6$
y	$y_0 = 0$	$y_1 = 0.02$	$y_2 = 0.0795$	$y_3 = 0.1762$
$f = x - y^2$	$f_0 = 0$	$f_1 = 0.1996$	$f_2 = 0.3936$	$f_3 = 0.5689$

$x_4 = 0.8$ $\quad h = 0.2$ $\quad y_4 = ?$

$$y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$$

$$= 0 + \frac{4 \times 0.2}{3}[2(0.1996) - 0.3936 + 2(0.5689)]$$

$$y_4^{(P)} = 0.3049$$

I

$$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$$

$$= 0.0795 + \frac{0.2}{3}[0.3936 + 4(0.5689) + 0.70703]$$

$$y_4^{(c)} = 0.3045$$

$f_4 = x - y^2$
$= x_4 - (y_4)^2$
$= 0.8 - (0.3049)^2$
$f_4 = 0.70703$

II

$$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$$

$$= 0.0795 + \frac{0.2}{3}[0.3936 + 4(0.5689) + 0.7072]$$

$$y_4^{(c)} = \underline{\underline{0.3045}}$$

$f_4 = x - y^2$
$= x_4 - (y_4)^2$
$= 0.8 - (0.3045)^2$
$f_4 = 0.7072$

Given $y' = x^2 + \frac{y}{2}$ with

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514

find $y(1.4)$ by milnes method

$$\frac{dy}{dx} = f(x, y) = x^2 + \frac{y}{2}$$

x	$x_0 = 1$	$x_1 = 1.1$	$x_2 = 1.2$	$x_3 = 1.3$
y	$y_0 = 2$	$y_1 = 2.456$	$y_2 = 2.4649$	$y_3 = 2.7514$
$f = x^2 + \frac{y}{2}$	$f_0 = 2$	$f_1 = 2.3178$	$f_2 = 2.6724$	$f_3 = 3.0657$

$x_4 = 1.4$ $\quad h = 0.1$ $\quad y_4 = ?$

$$y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$$

$$= 2 + \frac{4\times 0.1}{3}[2(2.3178) - 2.6724 + 2(3.0657)]$$

$$y_4^{(P)} = 3.07928$$

I

$$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$$

$$= 2.4649 + \frac{0.1}{3}[2.6724 + 4(3.0657) + 3.4996]$$

$$y_4^{(c)} = 3.0793$$

$$f_4 = x^2 + \frac{y}{2}$$

$$= x_4^2 + \frac{y_4}{2}$$

$$= (1.4)^2 + \frac{3.07928}{2}$$

$$f_4 = 3.4996$$

II

$$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$$

$$= 2.4649 + \frac{0.1}{3}[2.6724 + 4(3.0657) + 3.4996]$$

$$y_4^{(c)} = 3.0793$$

$$f_4 = x^2 + \frac{y}{2}$$

$$= x_4^2 + \frac{y_4}{2}$$

$$= (1.4)^2 + \frac{3.0793}{2}$$

$$f_4 = 3.4996$$

Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$

x	1	1.1	1.2	1.3
y	1	0.996	0.986	0.972

find $y(1.4)$ by milnes method.

Soln:- $\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x}$

x	$x_0 = 1$	$x_1 = 1.1$	$x_2 = 1.2$	$x_3 = 1.3$
y	$y_0 = 1$	$y_1 = 0.996$	$y_2 = 0.986$	$y_3 = 0.972$
$f = \frac{1}{x^2} - \frac{y}{x}$	$f_0 = 0$	$f_1 = -0.0790$	$f_2 = -0.1272$	$f_3 = -0.1559$

$x_4 = 1.4 \quad h = 0.1 \quad y_4 = ?$

$$y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$$

$$= 1 + \frac{4\times 0.1}{3}[2(-0.0790) - (-0.1272) + 2(-0.1559)]$$

$$y_4^{(P)} = 0.95432$$

I

$$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$$

$$= 0.986 + \frac{0.1}{3}[-0.1272 + 4(-0.1559) + (-0.1714)]$$

$$y_4^{(c)} = 0.9552$$

$$f_4 = \frac{1}{x^2} - \frac{y}{x}$$

$$= \frac{1}{(1.4)^2} - \frac{0.95432}{1.4}$$

$$f_4 = -0.1714$$

II

$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$

$= 0.986 + \frac{0.1}{3}[-0.1272 + 4(-0.1559) + (-0.1720)]$

$y_4^{(c)} = 0.9552$

$f_4 = \frac{1}{x^2} - \frac{y}{x}$

$= \frac{1}{x^4} - \frac{y_4}{x_4}$

$= \frac{1}{1.4} - \frac{0.9552}{1.4}$

$f_4 = -0.1720$

Given $2\frac{dy}{dx} = (1+x^2)y^2$ $y(0)=1$, $y(0.1)=1.06$, $y(0.2)=1.12$, $y(0.3)=1.21$

find $y(0.4)$ by milnes method

Sol^n:- $\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}$

x	$x_0=0$	$x_1=0.1$	$x_2=0.2$	$x_3=0.3$
y	$y_0=1$	$y_1=1.06$	$y_2=1.12$	$y_3=1.21$
$f=\frac{(1+x^2)y^2}{2}$	$f_0=0.5$	$f_1=0.56741$	$f_2=0.65228$	$f_3=0.79793$

$x_4=0.4$, $h=0.1$, $y_4=?$

$y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$

$= 1 + \frac{4(0.1)}{3}[2(0.56741 - (0.65228) + 2(0.79793)$

$y_4^{(P)} = 1.27712$

I

$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$

$= 1.12 + \frac{0.1}{3}[0.65228 + 4(0.79793) + 0.946000]$

$y_4^{(c)} = 1.27966$

$f_4 = \frac{(1+x^2)y^2}{2}$

$= \frac{(1+(0.4)^2)(1.27712)^2}{2}$

$f_4 = 0.946000$

II

$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$

$= 1.12 + \frac{0.1}{3}[0.65228 + 4(0.79793) + 0.94976]$

$y_4^{(c)} = 1.2797$

$f_4 = \frac{(1+x^2)y^2}{2}$

$= \frac{(1+(0.4)^2)(1.27966)^2}{2}$

$= 0.94976$

III

$y_4^{(1)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$

$= 1.12 + \frac{0.1}{3}[0.65228 + 4(0.79793) + 0.94996]$

$y_4^{(1)} = 1.27979$

$f_4 = \frac{(1+x^2)y^2}{2}$

$f_4 = \frac{(1+(0.4)^2)(1.27979)^2}{2}$

$f_4 = 0.94996$

Given $5xy' + y^2 - 2 = 0$, $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0142$, $y(4.4) = 1.0186$. find $y(4.5)$ by milnes method

Sol^n: $5xy' + y^2 - 2 = 0$

$5xy' = 2 - y^2$

$y' = \frac{2-y^2}{x}$

x	$x_0 = 4$	$x_1 = 4.1$	$x_2 = 4.2$	$x_3 = 4.3$	$x_4 = 4.4$
y	$y_0 = 1$	$y_1 = 1.0049$	$y_2 = 1.0097$	$y_3 = 1.0142$	$y_4 = 1.0186$
$f = \frac{2-y^2}{x}$	$f_0 = 0.05$	$f_1 = 0.0483$	$f_2 = 0.0466$	$f_3 = 0.0451$	$f_4 = 0.0437$

$x_5 = 4.5$, $h = 0.1$ $y_5 = ?$

$y_5^{(P)} = y_1 + \frac{4h}{3}[2f_2 - f_3 + 2f_4]$

$= 1.0049 + \frac{4(0.1)}{3}[2(0.0466) - (0.0451) + 2(0.0437)]$

$y_5^{(P)} = 1.0229$

I

$y_5^{(c)} = y_3 + \frac{h}{3}[f_3 + 4f_4 + f_5]$

$= 1.0142 + \frac{0.1}{3}[0.0451 + 4(0.0437) + (0.0423)]$

$y_5^{(c)} = 1.0229$

$f_5 = \frac{2-y^2}{5x}$

$= \frac{2-(1.0229)^2}{5(4.5)}$

$f_5 = 0.0423$

II

$y_5^{(c)} = y_3 + \frac{h}{3}[f_3 + 4f_4 + f_5]$

$= 1.0142 + \frac{0.1}{3}[0.0451 + 4(0.0437) + (0.0423)]$

$y_5^{(c)} = 1.0229$

$f_5 = \frac{2-y^2}{5x}$

$= \frac{2-(1.0229)^2}{5(4.5)}$

$f_5 = 0.0423$