INTEGRAL CALCULUS

M2 SIMP 2023 - TIE

QUESTIONS:

MODULE I

(1) Evaluate the following double and triple integrals –

(i) $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$

(ii) $\int_{-1}^{1} \int_0^z \int_{x-z}^{x+z} (x+y+z)\, dy\, dx\, dz$.

(iii) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$

(iv) $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2+y^2+z^2)\, dx\, dy\, dz$.

(2) Evaluate the following integrals by changing the order of integration.

(i) $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$.

(ii) $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dx\, dy$

(iii) $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\, dy\, dx$

(iv) $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dy\, dx$.

(v) $\int_0^1 \int_x^1 \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx$.

(vi) $\int_0^1 \int_{\sqrt{y}}^{2-y} xy\, dx\, dy$

(3) Evaluate the following integrals by changing into polar coordinates:

(i) $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \sqrt{x^2+y^2}\, dy\, dx$

(ii) $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)}\, dy\, dx$.

(4) (a) Find the area between the parabolas $y^2 = 4ax$ & $x^2 = 4ay$.

(b) Find the area of ellipse by double integration.

(c) Find the volume of the solid bounded by the plane $x=0$, $y=0$, $z=0$, $x+y+z=1$ using double integrals.

(d) By double integration find the area of $r = a(1+\cos\theta)$ between $\theta = 0$ and $\theta = \pi$.

(e) Find the area lying inside the circle $r = a\sin\theta$ outside the cardoid $r = a(1-\cos\theta)$.

(5) Solve the following Bela Gamma Functions:

(i) Prove that $\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$

(ii) Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}\, d\theta = \pi$

(iii) Show that $\int_0^1 \frac{x^2\,dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$

(iv) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$

(v) Show that $\int_0^{\pi/2} \sqrt{\cot\theta}\,d\theta = \frac{\pi}{\sqrt{2}}$

(6) Find the center of gravity of the curve $r = a(1+\cos\theta)$