Statistical Physics for Computing

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Overview

1. Descriptive statistics and inferential statistics
2. Poisson distribution and modeling the probability of proton decay
3. Normal Distributions (Bell Curves)
4. Monte Carlo Method: Determination of Value of π

Descriptive statistics and inferential statistics

Descriptive Statistics	Inferential Statistics
Descriptive statistics use summary statistics, graphs, and tables to describe a data set.	Inferential statistics use samples to draw inferences.
Descriptive statistics are limited to a sample or population having a small size.	Inferential statistics are applicable to larger and unknown data sets.
Descriptive statistics include measures of central tendency (such as mean, median, and mode)	Inferential statistics include hypothesis testing, regression analysis, ANOVA, and confidence intervals
Descriptive statistics can be achieved with the help of charts, graphs, tables, etc.	Inferential statistics can be achieved by probability that helps us quantify the uncertainty and the likelihood of an event.

Poisson distribution and modeling the probability of proton decay

Poisson distribution is used to model the number of events that occur in a fixed interval of time or space, given the average rate of occurrence, assuming that the events happen independently and at a constant rate. Let X be a discrete random variable that can assume values 0, 1, 2 . . . then, the probability function Poisson distribution of X is given as:

$$P(X) = \frac{\lambda^x e^{-\lambda}}{x!} \tag{1}$$

An important feature of Poisson distribution is that mean and variance of the distribution are the same.

Poisson distribution and modeling the probability of proton decay (contd.)

The probability for observing a proton decay can be estimated from the nature of particle decay. The number of protons N can be modeled by the decay equation

$$N = N_o e^{-\lambda t} \tag{2}$$

Here,
N is the number of protons after decay,
N_o is the initial number of protons,
λ is the decay constant and
t is the elapsed time.

Poisson distribution and modeling the probability of proton decay (contd.)

The lower bound for the lifetime is projected to be of the order of $\tau = 10^{33}$ years.
Therefore, $\lambda = 10^{-33}/$ year.
Since the decay constant λ is so small,

$$e^{-\lambda t} = 1 - \lambda t \tag{3}$$

$$\therefore N = N_o(1 - \lambda t) \tag{4}$$

Poisson distribution and modeling the probability of proton decay (contd.)

For a small sample, the observation of a proton decay is infinitesmal, but suppose we consider the volume of protons represented by the Super Kameokande neutrino detector in Japan. The number of protons in the detector volume is reported by Ed Kearns of Boston University to be 7.5×10^{33} protons. For one year of observation, the number of expected proton decays is then

$$N_o - N = N_o \lambda t = 7.5 \times 10^{33} 10^{-33} 1 = 7.5. \tag{5}$$

About 40 % of the area around the detector tank is covered by photodetector tubes, and if we take that to be the nominal efficiency of detection, we expect about three observations of proton decay events per year based on a 10^{33} year lifetime.

Poisson distribution and modeling the probability of proton decay (contd.)

If we presume that $\lambda = 3$ observed decays per year is the mean, then the Poisson distribution function tells us that the probability for zero observations of a decay is

$$P(X) = \frac{\lambda^x e^{-\lambda}}{X!} \tag{6}$$

$$= \frac{3^0 e^{-3}}{0!} = 0.05 \tag{7}$$

This low probability for a null result suggests that the proposed lifetime of 10^{33} years is too short.

Normal distribution

The Normal Distribution, also known as the Gaussian distribution, is a continuous probability distribution for any dataset. The 5 common properties of Normal Distribution are:

- Normal Distribution Curve is symmetric about the mean.
- Normal Distribution has single peak value.
- Normal Distribution Curve is always bell-shaped.
- Mean, mode, and median for Normal Distribution is always same.
- Normal Distribution follows the empirical rule.

Monte Carlo Method: Determination of Value of π

Consider a function,

$$\mathbf{y} = \mathbf{f}(\mathbf{x})$$

Large number of normally distributed random numbers are generated for K input vectors x. For each vector of x, a y vector is generated.

Monte Carlo Method: Determination of Value of π (contd.)

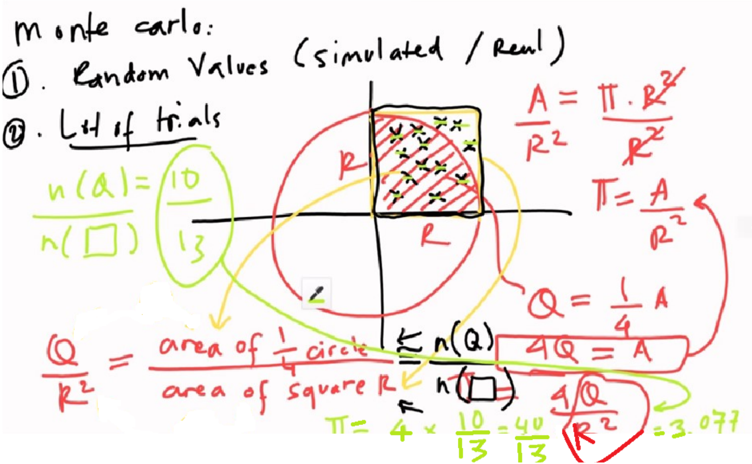
Sample mean values for the K vectors is generated using,

$$\bar{\mathbf{y}}_K = \frac{1}{K}\sum_{k=1}^{K} \mathbf{y}^{(k)} \ .$$

Sample covariance error matrix is given by

$$\mathbf{Cov} = \frac{1}{K}\sum_{k=1}^{K} (\mathbf{y}^{(k)} - \bar{\mathbf{y}})(\mathbf{y}^{(k)} - \bar{\mathbf{y}})^T$$

Monte Carlo Method: Determination of Value of π (contd.)



Numerical Problem

1. In a poisson distribution, P(x) for x=0 is 10%. Find the mean.

Ans:

$$P(X) = \frac{\lambda^X e^{-\lambda}}{X!} \tag{8}$$

$$\tag{9}$$

where,
$P(X)$ = probability of $X = 0.1$,
X = number of observations $= 0$,
λ = mean.

$$\therefore 0.1 = \frac{\lambda^0 e^{-\lambda}}{0!} \tag{10}$$

$$i.e.\ e^{-\lambda} = 0.1 \tag{11}$$

$$i.e.\ e^{\lambda} = 10 \tag{12}$$

$$\therefore\ \lambda = \ln 10 \tag{13}$$

* Thank You *