Solved Model Question Paper-I with effect from 2022-23

(CBCS Scheme)

First/Second Semester B.E. Degree Examination

Applied Physics for Computer Science Stream

TIME: 03 Hours **Max. Marks: 100**

Note:

01. Answer any FIVE full questions, choosing at least ONE question from each MODULE.

02. Draw neat sketches where ever necessary.

03. Constants : Speed of Light 'c' = 3 ×108 ms^{-1}, Boltzmann Constant 'k' = 1.38 ×10-23 JK^{-1} , Planck's Constant 'h' = 6.625 × 10-34 Js, Acceleration due to gravity 'g'= 9.8 ms^{-2} , Permittivity of free space 'ε_0'=8.854 ×10−12 F m^{-1}

Module-1

Q.01a. Define LASER and Discuss the interaction of radiation with matter.

The word Laser stands for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation. It is a device which amplifies light. It has properties like Coherence, Unidirectional, Monochromatic, Focus ability, etc.

Interaction of an electromagnetic wave with matter leads to transition of an atom or a molecule from one energy state to another. If the transition is from lower state to higher state it absorbs the incident energy. If the transition is from higher state to lower state it emits a part of its energy.

Emission or Absorption takes through quantum of energy called photons. $h\nu$ is called quantum energy or photon energy.

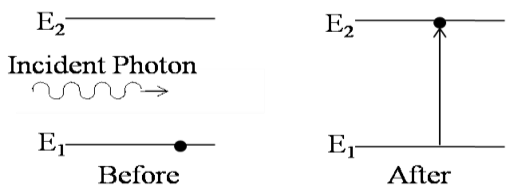
$h = 6.626\times10^{-34}$ Joules Second is Planck's constant and 'ν' is the frequency.

If ΔE is the difference between the two energy levels,

Then $\Delta E = (E_2 - E_1)$ Joule

According to Max Planck, $\Delta E = h\nu = (E_2 - E_1)$

$\nu = (E_2 - E_1)/h$ Hz.



Three types of interactions, which are possible, are as follows:

1) **<u>Induced Absorption:</u>**

Induced absorption is the absorption of an incident photon by system as a result of which the system is elevated from a lower energy state to a higher state, wherein the difference in energy of the two states is the energy of the photon.

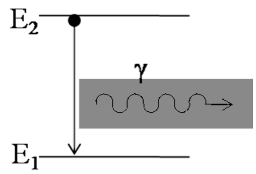
Consider the system having two energy states E_1 and E_2, $E_2 > E_1$. When a photon of energy $h\nu$ is incident on an atom at level E_1, the atom goes to a higher energy level by absorbing the energy.

When an atom is at ground level (E_1), if an electromagnetic wave of frequency ν is applied to the atom, there is possibility of getting excited to higher level (E_2). The incident photon is absorbed. It is represented as

$$\textbf{Atom + Photon} \rightarrow \textbf{Atom}^*$$

2) <u>Spontaneous Emission:</u> The emission of a photon by the transition of a system from a higher energy state to a lower energy state without the aid of an external energy is called spontaneous emission.

Let 'E_1' and 'E_2' be two energy levels in a material, such that $E_2>E_1$. E_1 is ground level and E_2 is the higher level. $h\nu=E_2-E_1$ is the difference in the energy. The atom at higher level (E_2) is more unstable as compared to that at lower level (E_1).



The life time of an atom is less in the excited state, In spontaneous emission atom emits the photon without the aid of any external energy. It is called spontaneous emission. The process is represented as

$$\text{Atom}^* \rightarrow \text{Atom} + \text{Photon}$$

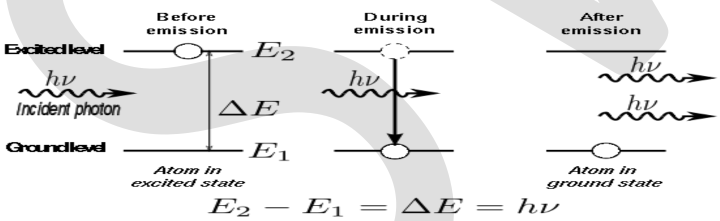
The photons emitted in spontaneous emission may not have same direction and phase similarities. It is incoherent.

Ex: Glowing electric bulbs, Candle flame etc.

3) <u>Stimulated Emission:</u>

Stimulated emission is the emission of a photon by a system under the influence of a passing photon of right energy due to which the system transits from a higher energy state to a lower energy state.

The photon thus emitted is called stimulated photon and will have the same phase, energy and direction of movement as that of the passing photon called the stimulation photon.



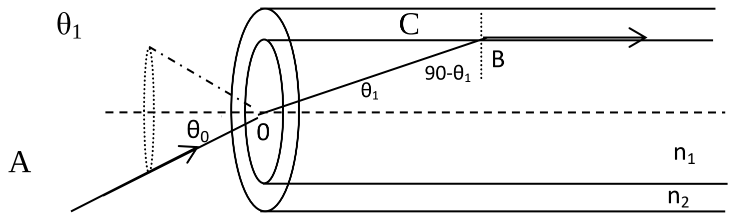
Initially the atom is at higher level E_2. The incident photon of energy hν forces the atom to get de-excited from higher level E_2 to lower level E_1.

i.e. $h\nu=E_2–E_1$ is the change in energy.

The incident photon stimulates the excited atom to emit a photon of exactly the same energy as that of the incident photons. The emitted two photons have same phase, frequency, direction and polarization with the incident photon and results in coherent beam of radiation. This kind of action is responsible for lasing action.

$$\text{Atom}^* + \text{Photon} \rightarrow \text{Atom} + (\text{Photon} + \text{Photon})$$

1b. Define Acceptance angle and Numerical Aperture and hence derive an expression for NA in terms of RIs core, cladding and surrounding.



Surrounding medium (n_0)

Acceptance angle is defined as "The maximum angle that a light ray can take relative to the axis of the fiber to propagate through the fiber".

Sine of the acceptance angle of an optical fiber is called as "Numerical aperture".

Consider a light ray entering into the core of an optical fiber with an angle of incidence(θ_0), such that after entering, the ray incidents on the core-cladding interface with an angle of incidence equal to the critical angle.

From figure it is clear that any ray which enters into the core with an angle more than θ_0, will have to be incident at an angle less than the critical angle at the core-cladding interface.Therefore, the ray does not undergo total internal reflection and the ray will be lost. Thus for any ray to propagate through the fiber it must enter with an angle less than θ_0. This maximum angle is called as 'Acceptance angle' and the conical surface described by the ray when rotated about the axis of the fiber is called 'Acceptance cone'.

Let n_0, n_1 and n_2 are the refractive indices of the surrounding medium, core and cladding respectively.

Now, applying Snell's law at the point of entry of the ray i.e., at A,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

$$\sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Applying Snell's law at B,

$$n_1 \sin(90-\theta_1) = n_2 \sin 90$$

$$n_1 \cos\theta_1 = n_2 \sin 90$$

$$\Rightarrow \cos\theta_1 = \frac{n_2}{n_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From expression (1) $\quad \sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\cos^2\theta_1}$

Substituting for $\cos\theta_1$ from (2)

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$sin\,\theta_0 = \frac{n_1}{n_0}\sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$sin\,\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

If n_0=1 i.e., surrounding medium if it is air

$$sin\,\theta_0 = \sqrt{n_1^2 - n_2^2}$$

i.e., $N.A. = \sqrt{n_1^2 - n_2^2}$

1c. A LASER source has a power output of 10^{-3} W. Calculate the number of photons emitted per second given the wavelength of LASER 692.8 nanometer.

Given : $t = 1sec$ $\quad \lambda = 692.8\times10^{-9}$ $\quad P = 1\times10^{-3}$

$$n = \frac{\lambda t P}{hc} \qquad h = 6.626\times10^{-34}$$

$$c = 3\times10^{8}\ ms^{-1}$$

$$n = \frac{692.8\times10^{-9}\times1\times1\times10^{-3}}{6.626\times10^{-34}\times3\times10^{8}}$$

$$\boxed{n = 3.4852\times10^{15}}$$

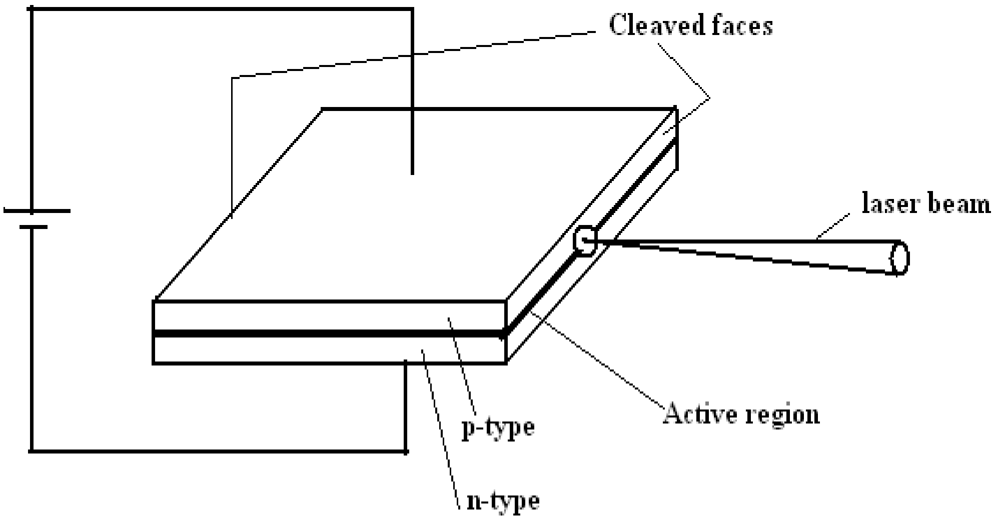
OR

Q2 a. llustrate the construction and working of Semiconductor LASER with a neat sketch and energy level diagram also mention its applications.

Semiconductor diode laser is one in which the active medium is formulated by semiconducting materials.

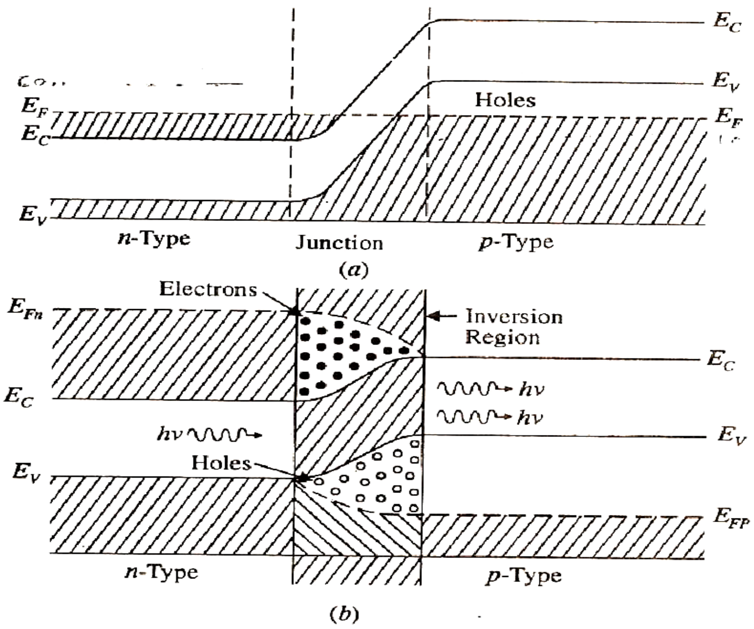
Construction:

- Gallium-Arsenide Laser is a single crystal of GaAs consists of heavily doped n-type and p-type.
- The diode is very small size with sides of the order of 1mm.
- The width of the junction varies from 1-100μm.
- The top and bottom surfaces are metalized and Ohmic contacts are provided for external connection.
- The front and rear faces are polished. The polished faces functions as the resonant cavity. The other two faces are roughened to prevent lasing action in that direction.



Working:

- The energy band diagram of heavily doped p-n junction is as shown. At thermal equilibrium the Fermi level is uniform.
- Because of very high doping on **n- side**, the Fermi level is pushed in to the conduction band and electrons occupy the portions of the conduction band that lies below the Fermi level and
- on **p-side**, the Fermi level lies within the valence band and holes occupy the portions of the valence band that lies above the Fermi level.



Energy level diagram

(a) Before biasing (b) After biasing

- A suitable forward bias is applied to overcome the potential barrier. As a result, electrons from n-region and holes from p-region injected into the junction.
- The current begins to flow following which there will be a region in junction in which the population inversion can be achieved.
- Initially concentration of electrons in the energy levels at the bottom of the conduction band will be less than that of energy levels at top of valence band. So that the recombination of electrons and holes result only in spontaneous emission.
- When the current exceeds the threshold value, population inversion is achieved in the active region which is formulated in the junction.
- At this stage the photons emitted by spontaneous emission triggers stimulated emission, over a large number of recombination leading to build up laser.
- Since the energy gap of GaAs is 1.4eV, the wavelength of emitted light is 8400 A^o.

2b. Discuss the types of optical fibers based on Modes of Propagation and RI profile.

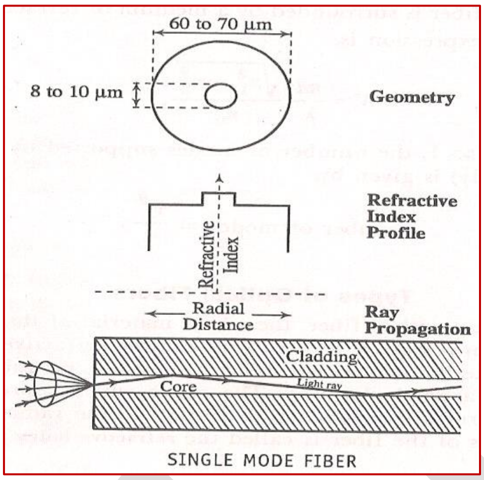
Based on the refractive index profile and mode of propagation, there are three types of optical fibers,

1. Single mode fiber
2. Step index multimode fiber
3. Graded index multimode fiber

(i) <u>Single mode fiber</u>

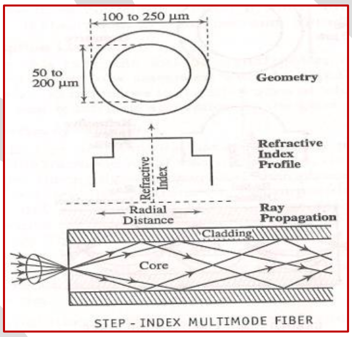
- Single mode fibers have a core material of uniform refractive index value.

- Cladding material also has a uniform refractive index but of lesser value than that of core.
- Thus its refractive index profile takes a shape of a step. The diameter of the core is about 8-10 μm and the diameter of the cladding is about 60-70 μm.



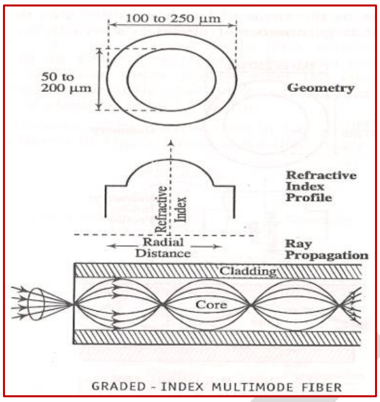
- Because of its narrow core, it can guide just a single mode as shown in above figure.
- Single mode fibers are the extensively used ones ant they are less expensive. They need LASERs as the source of light.

(ii) <u>Step index multimode fiber</u>



- A step index multimode fiber is very much similar to the single mode fiber except that its core is of large diameter. A typical fiber has a core diameter 50 to 200 μm and a cladding about 100 to 250μm outer diameter.
- Its refractive index profile is also similar to that of a single mode fiber but with a larger plane region for the core.
- Due to the large core diameter it can transmit a number of modes of wave propagation.
- The step index multimode fiber can accept either a LASER or an LED as source of light.
- It is the least expensive of all and its typical application is in data links which has lower bandwidth requirements.

(iii) <u>Graded index multimode fiber</u>



GRADED - INDEX MULTIMODE FIBER

- It is also called GRIN.
- The refractive index of core decreases in the radially outward direction from the axis of the fiber and becomes equal to that of cladding at the interface but the refractive index of the cladding remains uniform.
- Laser or LED is used as a source of light.
- It is the expensive of all. It is used in telephone trunk between central offices.

2c. Obtain the attenuation co-efficient of the given fiber of length 1500 m given the input and output power 100 mW and 70 mW.

$L = 1500\ m \Rightarrow L = 1.5\ Km$

$P_{in} = 100\,mW = 100 \times 10^{-3}\ W$

$P_{out} = 70\,mW = 70 \times 10^{-3}\ W$

$$\alpha = \frac{-10}{L} \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

$$= \frac{-10}{1.5} \times \log_{10}\left[\frac{70 \times 10^{-3}}{100 \times 10^{-3}}\right]$$

$$= \frac{-10}{1.5} \times \log(0.7)$$

$$= \frac{(-10 \times -0.154)}{1.5}$$

$$\boxed{\alpha = 1.03}$$

Module-2

Q3a. Setup SchrÖdinger time- independent wave equation in one dimension.

Consider a particle of mass 'm' moving with velocity 'v'. The de-Broglie wavelength 'λ' is

$$\lambda = \frac{h}{mv} = \frac{h}{P} \rightarrow (1)$$ Where 'mv' is the momentum of the particle.

The wave eqn is

$$\psi = Ae^{i(kx-\omega t)} \rightarrow (2)$$

Where 'A' is a constant and 'ω' is the angular frequency of the wave.

Differentiating equation (2) with respect to 't' twice

$$\frac{d^2\psi}{dt^2} = -A\omega^2 e^{i(kx-\omega t)} = -\omega^2\psi \quad \rightarrow (3)$$

The equation of a travelling wave is

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

Where 'y' is the displacement and 'v' is the velocity.

Similarly, for the de-Broglie wave associated with the particle

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2} \rightarrow (4)$$

where 'ψ' is the displacement at time 't'.

From eqns (3) & (4)

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2}\psi$$

But ω = 2πυ and v =υ λ where 'υ' is the frequency and 'λ' is the wavelength.

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi \text{ or } \frac{1}{\lambda^2} = -\frac{1}{4\pi^2\psi}\frac{d^2\psi}{dx^2} \rightarrow (5)$$

$$K.E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{P^2}{2m} \rightarrow (6)$$

$$= \frac{h^2}{2m\lambda^2} \rightarrow (7)$$

Using eqn (5)

$$K.E = \frac{h^2}{2m}\left(-\frac{1}{4\pi^2\psi}\right)\frac{d^2\psi}{dx^2} = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2} \rightarrow (8)$$

Total Energy E = K.E + P.E

$$E = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2} + V$$

$$E - V = -\frac{h^2}{8\pi^2 m\psi}\frac{d^2\psi}{dx^2}$$

$$\frac{d^2\psi}{dx^2} = -\frac{8\pi^2 m}{h^2}(E - V)\psi$$

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0}$$

This is the time independent Schrodinger wave equation for one dimensional case.

For three dimensional case it can be eritten as follows.

$$\boxed{[\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2}] + \frac{8\pi^2 m}{h^2}(E - V)\psi(x,y,z)} = 0$$

3b. State and Explain Heisenberg's Uncertainty principle and Principle of Complementarity.

Heisenberg's Uncertainty Principle states that "It is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa".

If Δx and ΔP_x are the uncertainties in the measurement of position and momentum of the particle then the uncertainty can be written as

$$\Delta x \,.\, \Delta P_x \geq (h/4\pi)$$

In any simultaneous determination of the position and momentum of the particle, the product of the corresponding uncertainties inherently present in the measurement is equal to or greater than h/4π.

Similarly, 1) $\Delta E.\Delta t \geq h/4\pi$ 2) $\Delta L.\Delta\theta \geq h/4\pi$

Principle of complementarity as stated by Bohr "In a situation where the wave aspect of the system is revealed, its particle aspect is concealed (hidden) and in a situation where the particle aspect is revealed its wave aspect is concealed (hidden). Revealing both simultaneously is impossible; the wave and aspects are complementary."

Note: Meaning of complementary: things are different from each other but make a good combination.

Explanation: If an experiment is designed to measure the particle nature of matter, during this experiment errors of measurement of both position and time is zero and hence and hence momentum, energy and the wave nature of the matter are completely unknown. and vice versa.

3c. An electron is kinetic energy 500 keV is in vacuum. Calculate the group velocity and de Broglie wavelength assuming the mass of the moving electron is equal to the rest mass of electron

Given : $K.E = 500 \times 10^3 eV$

$V_g = ?$

$\lambda = ?$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 500 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$\boxed{\lambda = 1.737 \times 10^{-12} m}$$

$$\lambda = \frac{h}{mV}$$

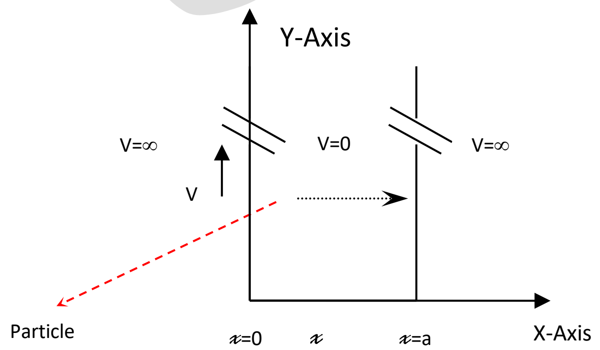
$$V_g = \frac{h}{m\lambda}$$

$$V_g = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.737 \times 10^{-12}}$$

$$\boxed{V_g = 4.19 \times 10^8 m/s}$$

OR

Q.04 a Discuss the motion of a quantum particle in a one-dimensional infinite potential well of width 'a' and also obtain the eigen functions and energy eigen states well (potential well of infinite depth) or of a particle in a box



Consider a particle of a mass 'm' free to move in one dimension along positive x-direction between x=0 to x=a. The potential energy outside this region is infinite and within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box. The Schrödinger equation outside the well is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-\infty)\psi = 0 \rightarrow (1) \qquad \because V = \infty$$

For outside, the equation holds good if ψ = 0 & |ψ|² = 0. That is particle cannot be found outside the well and also at the walls

The Schrodinger's equation inside the well is:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0 \rightarrow (2) \qquad \because V = 0$$

Let $\frac{8\pi^2 m}{h^2}E = k^2 \quad \rightarrow (3)$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The solution of above equation is:

ψ = C cos kx + D sin kx → (4)

at x = 0 → ψ = 0

0 = C cos 0 + D sin 0

∴ C = 0

Also x = a → ψ = 0

0 = C cos ka + D sin ka

But C = 0

∴D sin ka = 0 → (5

D≠0 (because the wave concept vanishes)

i.e. ka = $n\pi$ where n = 0, 1, 2, 3, 4… (Quantum number)

k = $\frac{n\pi}{a}$ → (6)

sub eqn (5) and (6) in (4)

$$\psi_n = D\sin\frac{n\pi}{a}x \rightarrow (7)$$

This gives permitted wave functions.

The Energy Eigen value given by

Substitute equation (6) in (3)

$$\frac{8\pi^2 m}{h^2}E = k^2 = \frac{n^2\pi^2}{a^2}$$

$$E = \frac{n^2 h^2}{8ma^2}$$

This is the expression for energy Eigen value.

For n = 0 is not acceptable inside the well because $\psi_n = 0$. It means that the electron is not present inside the well which is not true. Thus the lowest energy value for *n* = 1 is called zero point energy value or ground state energy.

i.e. $E_{\text{zero-point}} = \frac{h^2}{8ma^2}$

The states for which n >1 are called exited states.

The energy of the particle at the ground state is

$$E_1 = \frac{h^2}{8ma^2} = E_0$$

The energy of the particle in the first excited state is $E_2 = 4E_0$.

The energy of the particle in the second excited state is $E_3 = 9\,E_0$.

i.e. $\int_0^a |\psi_n^2|\,dx = 1 \rightarrow (8)$

using the values of ψ_n from eqn (7)

$$\int_0^a D^2 \sin^2 \frac{n\pi}{a} x dx = 1$$

$$D^2 \int_0^a \left[\frac{1-\cos(2n\pi/a)x}{2}\right] dx = 1 \qquad \because \sin^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)$$

$$\frac{D^2}{2}\left[\int_0^a dx - \int_0^a \cos\frac{2n\pi}{a} x dx\right] = 1$$

$$\frac{D^2}{2}\left[x - \frac{a}{2n\pi}\sin\frac{2n\pi}{a}x\right]_0^a = 1$$

$$\frac{D^2}{2}[a-0] = 1$$

$$\frac{D^2}{2} a = 1$$

$$D = \sqrt{\frac{2}{a}}$$

Substitute D in equation (7)

the normalized wave functions of a particle in one dimensional infinite potential well is:

$$\psi_n = \sqrt{\frac{2}{a}} \sin\frac{n\pi}{a} x \rightarrow (9)$$

b Explain the physical significance of the Wave Function.

A physical situation in quantum mechanics is represented by a function called wave function. It is denoted by 'ψ'. It accounts for the wave like properties of particles. Wave function is obtained by solving Schrodinger equation.

Mathematically it is given by

$$\psi = Ae^{i(kx-\omega t)}$$

The wave function itself has no physical significance, the physical significance is given by a function called probability density or probability function.

According to max born interpretation, as square of the amplitude A2 for electromagnetic waves represent Intensity of the wave. In quantum mechanics square of the amplitude A2 represent the probability of finding the particle in certain position

Q4c. The speed of electron is measured to wit in an uncertainty of 2×10^4 ms^{-1} in one dimension. What is the minimum width required by the electron to be confined in an atom?

Given: $\Delta V = 2\times10^4\ ms^{-1}$

$$\Delta x . \Delta P \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4\pi m \Delta V} = \frac{6.63\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times2\times10^4}$$

$$\boxed{\Delta x = 2.9\times10^{-9}\ m}$$

Module-3

Q.05 Define a bit and qubit and explain the properties of qubit.

Bit is a basic unit in which information in a classical computer is stored in the form binary numbers 0 and 1.

Qubit is a basic unit in which information in a quantum computer is stored. Superposition, Entanglement, and Tunneling are all special properties that define a qubit.

Quantum bits, called qubits are similar to bits having two measurable states called 0 and 1 states. Qubits can also be in a superposition state of these 0 and 1 states as shown in the figure. A qubit $|\psi\rangle$ could be in $|0\rangle$ or $|1\rangle$ state which the superposition of both is $|0\rangle$ and $|1\rangle$ state.

This is written as, $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

Where α and β called the amplitude of the states which are a complex number.

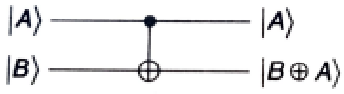
i) A qubit can be in a superposed state of the two states 0 and 1.Qubit is a superposition of both $|0\rangle$ and $|1\rangle$ state is given by
$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.

ii) If measurements are carried out with a qubit in superposed state then the results that we get will be probabilistic unlike how it's deterministic in a classical computer. The total probability of all the states of the quantum system must be 100%.
i.e. $|\alpha|^2 + |\beta|^2 = 1$ is called Normalization rule.

iii) Owing to the quantum nature, the qubit changes its state at once when subjected to measurement. This means, one cannot copy information fromqubits the way we do in the present computers and is known as "no cloning principle".

Q.05 b Discuss the CNOT gate and its operation on four different input states.

The CNOT gate is a typical multi-qubit logic gate and the circuit is as follows.



CNOT gate

The matrix representation of CNOT gate is given by

$$U_{CN} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The Transformation could be expressed as $|A, B\rangle \rightarrow |A, B \oplus A\rangle$

Consider the operations of CNOT gate on the four inputs $|00\rangle$, $|01\rangle$,$|10\rangle$ and $|11\rangle$.

Here in the inputs to the CNOT Gate the control qubit is $|0\rangle$. Hence no change in the state of Target qubit $|0\rangle$.$|00\rangle \rightarrow |00\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|0\rangle$. Hence no change in the state of Target qubit $|1\rangle$.$|01\rangle \rightarrow |01\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|1\rangle$. Hence the state of Target qubit flips from $|0\rangle$ to $|1\rangle$. $|10\rangle \rightarrow |11\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|1\rangle$. Hence the state of Target qubit flips from $|1\rangle$ to $|0\rangle$. $|11\rangle \rightarrow |10\rangle$

The Truth Table of operation of CNOT gate is as follows.

Input	Output
$|00\rangle$	$|00\rangle$
$|01\rangle$	$|01\rangle$
$|10\rangle$	$|11\rangle$
$|11\rangle$	$|10\rangle$

Q.05c A Linear Operator 'X' operates such that X |0⟩ = |1⟩ and X |1⟩= |0⟩ Find the matrix representation of 'X'.

Let Given two basis states $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

X is Matrix assume 2x2 Matrix because the basis consists of two rows Hence to exists Matrix Multiplication and X is assumed as unitary operator.

$\boxed{\alpha = X}$

Hence $X = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$. $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$X|0\rangle = |1\rangle \Rightarrow \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\begin{bmatrix} x_{11} + 0 \\ x_{21} + 0 \end{bmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$x_{11} = 0$ and $x_{21} = 1$

$X|1\rangle = |0\rangle \Rightarrow \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} 0 + x_{12} \\ 0 + x_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\Rightarrow x_{12} = 1 \quad x_{22} = 0$

Hence the linear operator defined by it's matrix elements as

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$$

OR

Q.06

a State the Pauli matrices and apply Pauli matrices on the states |0⟩ and |1 ⟩ .

Pauli Matrices are set of 2 × 2 matrices. Which are very much useful in the study of quantum computation and quantum information. The pauli matrices are given by

$$\sigma_x = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \ , \ \sigma_y = Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \text{and} \ \sigma_z = Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\sigma_x|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$\sigma_x|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$\sigma_y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$$

$$\sigma_x|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle$$

$$\sigma_z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$\sigma_z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle$$

Q.06 b. Elucidate the differences between classical and quantum computing.

Classical computing	**Quantum computing**
It is large scale integrated multi-purpose computer.	It is high speed parallel computer based on quantum mechanics.
Information storage is bit based on voltage or charge etc.	Information storage is Quantum bit based on direction of an electron spin.
Information processing is carried out by logic gates e.g. NOT, AND, OR etc.	Information processing is carried out by Quantum logic gates.
Classical computers use binary codes i.e. bits 0 or 1 to represent information.	Quantum computers use Qubits i.e. 0, 1 and both of them simultaneously to run machines faster.
Operations are defined by Boolean Algebra.	Operations are defined by linear algebra over Hilbert Space and can be represented by unitary matrices with complex elements.
Circuit behaviour is governed by classical physics.	Circuit behavior is governed explicitly by quantum mechanics.

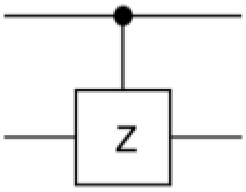
Q.06 c. Describe the working of controlled-Z gate mentioning its matrix representation and truth-table.

Controlled Z Gate, The operation of Z Gate is controlled by a Control Qubit. If the control Qubit is $|A\rangle$ is equal to $|1\rangle$ then only the Z gate transforms the Target Qubit $|B\rangle$ as per the Pauli-Z operation.

The action of Controlled Z-Gate could is specified by a matrix as follows.

$$U_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The schematic circuit of controlled Z gate and the truth table are as follows



Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|10\rangle$
$\|11\rangle$	$-\|11\rangle$

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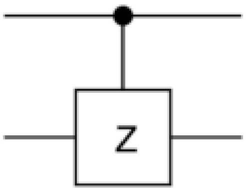


Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|10\rangle$
$\|11\rangle$	$-\|11\rangle$

MODULE-4

Q.07 a Define Fermi Factor and Discuss the variation of Fermi factor with temperature and energy

Fermi factor is the probability of occupation of a given energy state by the electrons in a material at thermal equilibrium.

The probability f(E) that a given energy state with energy E is occupied by the electrons at a steady temperature T is given by

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

$$f(E) = \frac{1}{e^{\frac{E-E_f}{kT}} + 1}$$

f(E) is called the Fermi factor.

The dependence of Fermi factor on temperature and energy is as shown in the figure.

i) Probability of occupation for $E<E_F$ at T=0K:

When T=0K and $E<E_F$

$$f(E) = \frac{1}{e^{-\infty}+1} = \frac{1}{0+1} = 1$$

The probability of occupation of energy state is 100%

f(E)=1 for E<E_F.

f(E)=1 means the energy level is certainly occupied and E<E_F applies to all energy levels below E_F. Therefore at T=0K all the energy levels below the Fermi level are occupied.

ii) Probability of occupation for E>E_F at T=0K:

When T=0K and E>E_F

$$f(E) = \frac{1}{e^{\infty}+1} = \frac{1}{\infty} = 0$$

The probability of occupation of energy state is 0%

f(E)=0 for E>E_F

∴ At T=0K, all the energy levels above Fermi levels are unoccupied. Hence at T=0K the variation of f(E) for different energy values, becomes a step function as shown in the above figure.

iii) The probability of occupation at ordinary temperature(for E≈E_F at T>0K)

At ordinary temperatures though the value of probability remains 1, for E<E_F it starts reducing from 1 for values of E close to but lesser than E_F as in the figure.

The values of f(E)becomes ½ at E=E_F

This is because for E=E_F

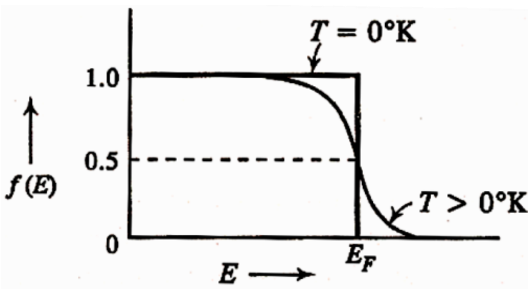
$$e^{\frac{(E-E_f)}{kT}} = e^0 = 1$$

$$\therefore f(E) = \frac{1}{e^{\frac{(E-E_f)}{kT}}+1} = \frac{1}{1+1} = \frac{1}{2}$$

The probability of occupation of energy state is 50%

Further for E>E_F the probability value falls off to zero rapidly.

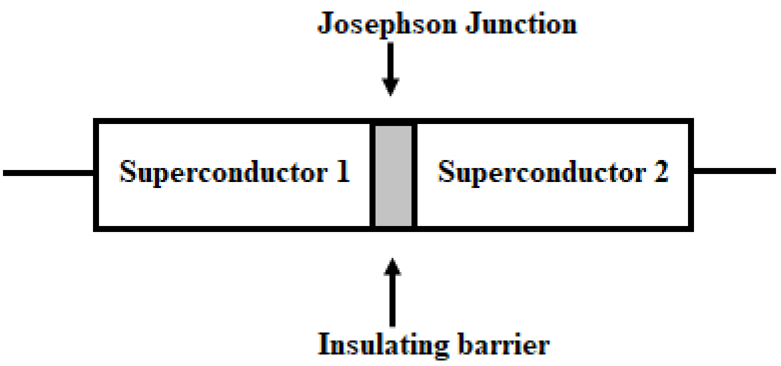
Hence, the Fermi energy is the most probable or the average energy of the electrons across which the energy transitions occur at temperature above zero degree absolute.



VARIATION OF f(E) WITH E

Q.07 b Explain DC and AC Josephson effects

Consider two superconductors separated by insulating barrier of thickness less than 10-20 Å, then the cooper pairs tunneling through the insulating barrier is known as **Josephson superconducting quantum tunneling**. The junction between the two superconductors with insulating barrier is known as **Josephson junction**.



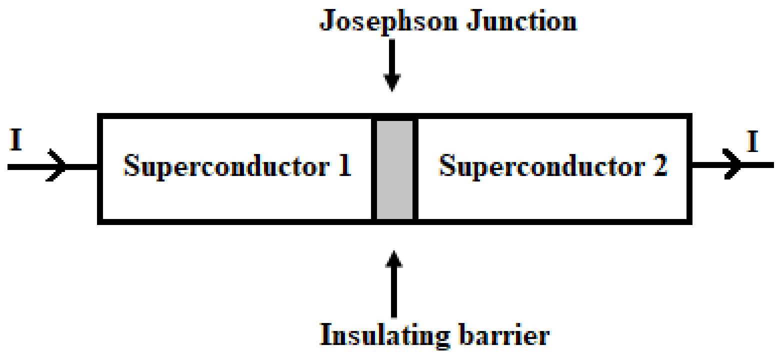
Josephson junction is an arrangement of two superconductors separated by an insulating barrier. When the barrier is thin enough, cooper pairs from one superconductor can tunnel through the barrier and reach the other superconductor.

Josephson proposed that this kind of tunneling leads to three kinds of effect, namely

1. dc Josephson effect
2. ac Josephson effect
3. quantum interference

1. <u>dc Josephson effect</u>

As per dc Josephson Effect, the tunneling of cooper pairs through the junction occurs without any resistance, which results in a steady dc current without any application of voltage between the two superconductors.



The super current through the junction is given by,

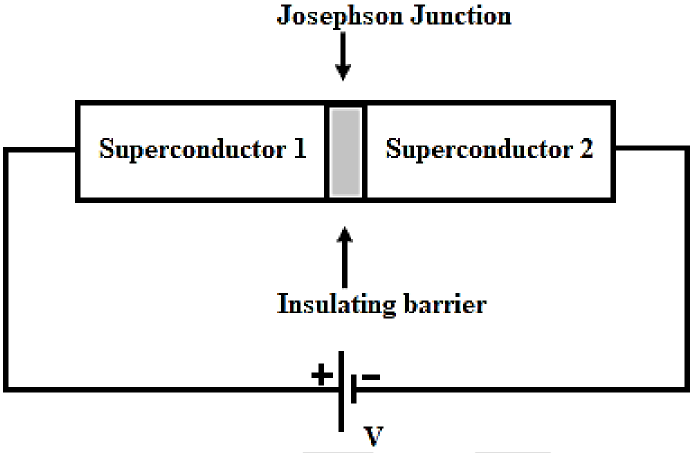
$$I_S = I_C \ sin \, sin \, \Phi_0$$

Where, Φ_0 = phase difference between the wave functions describing the cooper pairs on both the sides of the junction.

I_C = critical current at zero voltage which depends on the thickness and width of the insulating layer.

2. <u>ac Josephson effect</u>

When a dc voltage is applied across the junction, the tunneling of cooper pairs occur in such a way than an ac current would develop in the junction and this effect is called as ac Josephson Effect.
When a potential difference of ‘V’ is applied between the two sides of the junction then a radio frequency (RF) current oscillations across the junction is generated.



$$I = I_C \sin \sin \left(\Phi_0 + \Delta\Phi\right)$$

The energies of the cooper pairs on both sides of the barrier difference is E = hν = 2eV (Calculated using quantum mechanical concept).

Therefore it can be shown that, $\Delta\Phi = \omega t = 2\pi t \left(\frac{2\,eV}{h}\right)$

Hence, $I = I_C \sin \sin \left(\Phi_0 + 2\pi t \left(\frac{2\,eV}{h}\right)\right)$

I = Alternating current of frequency $\nu = \frac{2\,eV}{h}$

It shows that a photon of frequency ν is emitted or absorbed when a cooper pair crosses the junction. Thus when a voltage is applied across the junction, an ac current gets generated. This is known as ac Josephson Effect.

Q.07 c Calculate the probability of occupation of an energy level 0.2 eV above fermi level at temperature 27°C

Given : $T = 27°C$
$= 27 + 273$
$= 300K$

$E - E_f$
$\Rightarrow 0.2eV$
$= 0.2 \times 1.6 \times 10^{-19} J$

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{KT}\right)} + 1}$$

$k = 1.38 \times 10^{-23}$

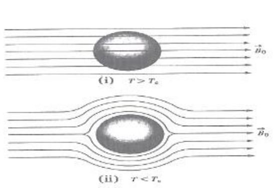
$$= \frac{1}{e^{\left(\frac{0.2 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}\right)} + 1}$$

$$\boxed{f(E) = 4.39 \times 10^{-4}}$$

OR

Q.08 a Describe Meissner's Effect and hence classifies superconductors into Soft and Hard superconductors using M-H graphs.

A superconducting material kept in a magnetic field expels the magnetic flux out of its body when it is cooled below the critical temperature and thus becomes perfect diamagnet. This effect is called Meissner effect.



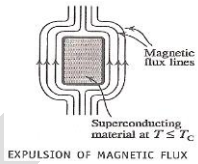


Fig: Superconductor sample subjected to an applied magnetic field with temperature (i) above and (ii) below T_C. The flux expulsion below Tc is called Meissner effect

When the temperature is lowered to T_c, the flux is suddenly and completely expelled, as the specimen becomes superconducting. The Meissner effect is reversible. When the temperature is raised the flux penetrates the material, after it reaches T_c. Then the substance will be in the normal state.

The magnetic induction inside the specimen

$$B = \mu_o (H + M)$$

Where 'H' is the intensity of the magnetizing field and 'M' is the magnetization produced within the material.

For $T < T_c$, $\quad B = 0$

$$\mu_0 (H + M) = 0$$

$$M = -H$$

$$M/H = -1 = \chi$$

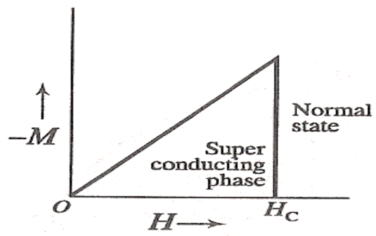
Susceptibility is -1 i.e. it is perfect diamagnetism.

Hence superconducting material do not allow the magnetic flux to exist inside the material.

There are two types of superconductors. They are type-I superconductors and type-II superconductors.

i)Type-I superconductors:

Type-I superconductors exhibit complete Meissner effect. Below the critical field it behaves as perfect diamagnetic. If the external magnetic field increases beyond H_c the superconducting specimen gets converted to normal state. The magnetic flux penetrates and resistance increase from zero to some value. As the critical field is very low for type-I superconductors, they are not used in construction of solenoids and superconducting magnets.



DEPENDENCE OF MAGNETIC MOMENT ON H

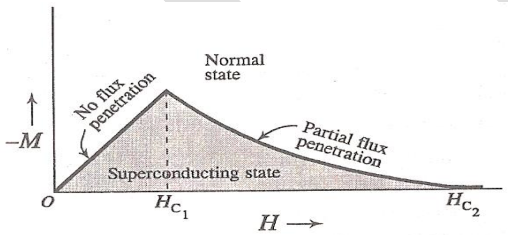
i) Type-II superconductors

Type-II superconductors are hard superconductors. They exist in three states

i) Superconducting state

ii) Mixed state

ii) Normal state



DEPENDENCE OF MAGNETIC MOMENT ON H

They are having two critical fields H_{c1} and H_{c2}. For the field less then Hc_1, it expels the magnetic field completely and becomes a perfect diamagnetic. Between Hc_1 and Hc_2 the flux starts penetrating throughout the specimen. This state is called vortex state. Hc_2 is 100 times higher than Hc_1. At Hc_2 the flux penetrates completely and becomes normal conductor. Type-II superconductors are used in the manufacturing of the superconducting magnets of high magnetic fields above 10 Tesla.

Q.08 b Enumerate the assumptions of Quantum free Electron Theory of Metals.

- The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
- The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.
- The electrons travel with a constant potential inside the metal but confined within its boundaries.
- The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

Q.08 c Lead has superconducting transition temperature of 7.26 K. If the initial field at 0K is 50 × 10^3 Am^{-1} Calculate the critical field at 6k.

Given: $T_c = 7.26K$ $H_0 = 50 \times 10^3$ at $T = 0K$

$H_c = ?$ $T = 6K$

$$\therefore H_c = H_0 \left(1 - \left(\frac{T}{T_c}\right)^2\right)$$

$$H_c = 50 \times 10^3 \left(1 - \left(\frac{6}{7.26}\right)^2\right)$$

$$\boxed{H_c = 15.84 \times 10^3 Am^{-1}}$$

Module-5

Q.09 a Discuss timing in Linear motion, Uniform motion, slow in and slow out.

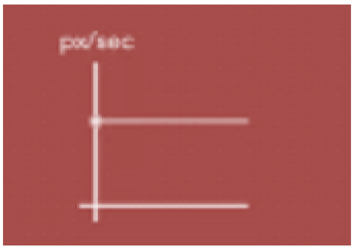
Timing animation refers to the duration of an action.

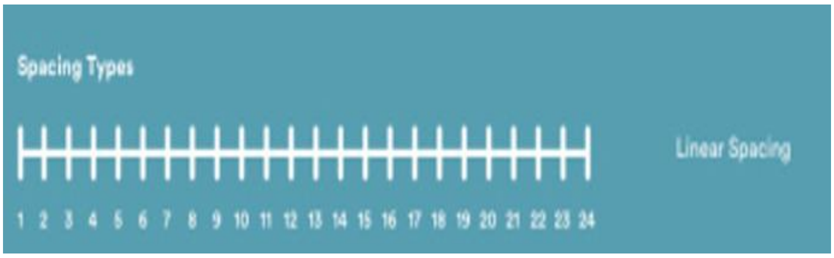
In animation, timing of action consists of placing objects or characters in particular locations at specific frames to give the illusion of motion.

Line of action: Individual drawings or poses have a **line of action**, which indicates the visual flow of action at that single image.

1) **Uniform motion**: It is the easiest to animate because the distance the object travels between frames is always the same.

The object moves the same distance between consecutive frames. The longer the distance between frames, the higher the speed.

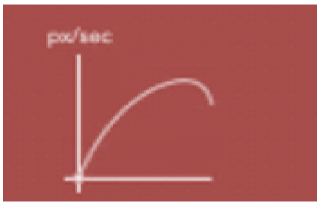




2) Ease out / Speed up

The object is speeding up i.e it's speed increases gradually, often from a still position.

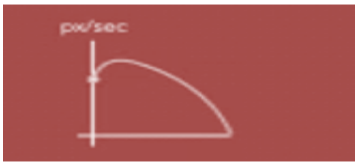
The frames are located such that, initially the frames are closely spaced with gradual increase in the spacings.



3) Ease in/ Slowed down.

The object is slowing down, it's speed decreases gradually often in preparation for stopping.

The frames are located such that, initially the frames are widely spaced with gradual decrease in the spacings of the frames.

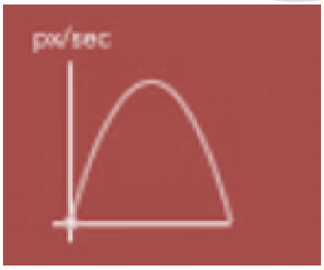


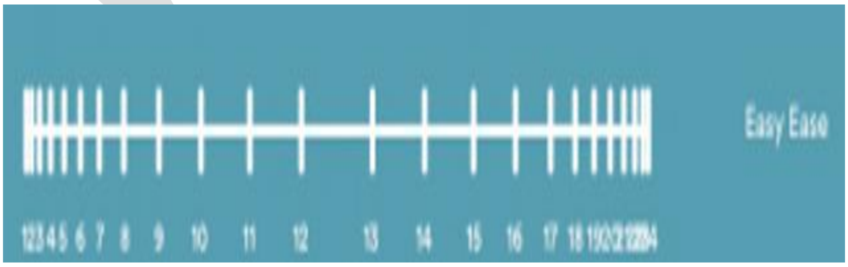
4) Ease out- Ease in or Ease-Ease.

It is the combination of speed up and slowed down. That is the object initially gets speed up initially and finally comes to still position with slowing down.

In the beginning the frames are located such that, initially the frames are closely spaced with gradual increase in the spacings up to middle position.

From the middle position onwards, the frames are widely spaced with gradual decrease in the spacings of the frames towards the still position.





Q.09 b Distinguish between descriptive and inferential statistics

Statistical physics is a branch of physics that evolved from a foundation of statistical mechanics, which uses methods of probability theory and statistics, particularly the mathematical tools for dealing with large populations and approximations, in solving physical problems.

Descriptive statistics: The term "descriptive statistics" refers to summarizing and organizing the characteristics of a data set. A data set is a collection of responses or observations from a sample or entire population.

In quantitative research, after collecting data, the first step of statistical analysis is to describe characteristics of the responses, such as the average of one variable (e.g., age), or the relation between two variables (e.g., age and creativity).

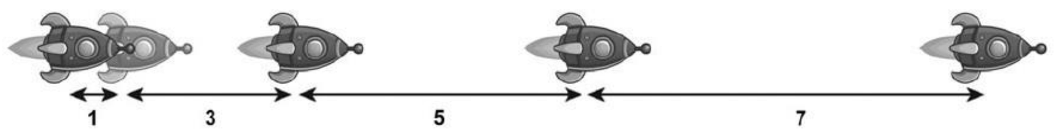
Descriptive statistics comprises three main categories – Frequency Distribution, Measures of Central Tendency, and Measures of Variability.

Inferential Statistics:

Inferential Statistics is a method that allows us to use information collected from a sample to make decisions, predictions, or inferences from a population. The major inferential statistics are based on statistical models such as Analysis of Variance, chi-square test, student's t distribution, regression analysis, etc.

Q09.c Illustrate the odd rule and odd rule multipliers with a suitable example.

- When acceleration is constant, The Odd Rule is used (Simple Pattern of Odd Numbers) to time the frames.
- Between consecutive frames, the distance moved by the object is a multiple of an odd number.
- For acceleration, the distance between frames increases by multiples of 1, 3, 5, 7, etc.
- For deceleration, the multiples start at a higher odd number and decrease, for example 7, 5, 3, 1.
- The Odd Rule is a multiplying system based on the smallest distance (base distance) travelled between two frames in the sequence



Rocket speeding up using the Odd Rule.

OR

Q.10 a Describe Jumping and parts of jump.

A jump action includes a take-off, free movement through the air, and a landing.

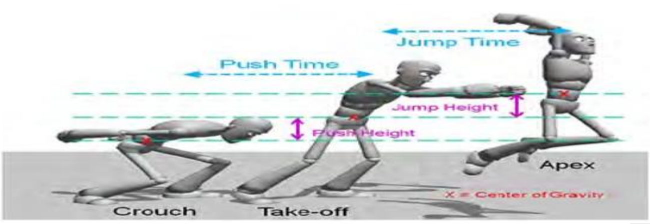
Parts of Jump :

Crouch—A squatting pose taken as preparation for jumping.

Take off—Character pushes up fast and straightens legs with feet still on the ground. The amount of time (or number of frames) needed for the push is called the push time.

In the air— Both the character's feet are off the ground, and the character's CG moves in a parabolic arc as any free-falling body would.

Landing—Character touches the ground and bends knees to return to a crouch. The distance from the character's CG when her feet hit to the ground to the point where the character stops crouching is called the stop height. The stop height is not always exactly the same as the push height.



Push height: The distance between Center of gravity (CG) in crouch position to CG of Take off position

Jump Height: The distance between CG in takeoff position to CG of position at air.

Stop Height: The distance between CG in Landing position to CG of Crouch position during landing

$$\text{Jump Height} = 1.2\text{m}$$

$$\text{Jump Time} = t = \sqrt{\frac{2h}{g}} = 0.5s$$

$$\text{Jump Time at 30 fps} = 0.5 \times 30 = 15 \text{ frames.}$$

Q.10 b Discuss the salient features of Normal distribution using bell curves.

The bell curve is a normal probability distribution of variables plotted on the graph and is like a bell shape where the highest or top point of the curve represents the most probable event out of all the series data.

CHARACTERISTICS

1. The Normal Curve is Symmetrical: The normal probability curve is symmetrical around its vertical axis called ordinate which represents the mean of distribution. The symmetry about the ordinate at the central point of the curve implies that the size, shape, and slope of the curve on one side of the curve is identical to that of the other. In other words, the left and right halves of the middle central point are mirror images, as shown in the figure given here.

2. The Normal Curve is Unimodel: Since there is only one maximum point in the curve, thus the normal probability curve is unimodal, i.e. it has only one mode.

3. The Normal Curve is Bilateral: The total area under the curve is 1, the 50% area of the curve lies to the left side of the maximum central ordinate and 50% of the area lies to the right side. Hence the curve is bilateral.

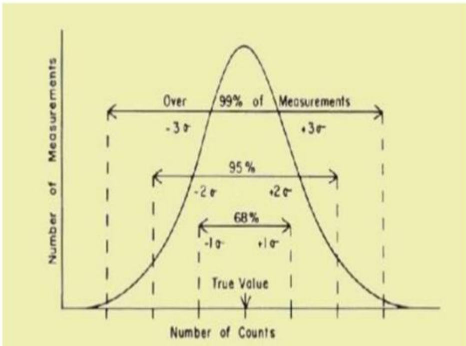
4. The Normal Curve is a mathematical model in behavioral Sciences: This curve is used as a measurement scale. The measurement un it of this scale is ± 1σ (the unit standard deviation).

Standard Deviations: The standard normal distribution is a normal probability distribution that has a mean of 0 and a standard deviation of 1. The Standard Deviation is a measure of how spread-out numbers are. As per 3 sigma rule of normal distribution,

68% of values are within 1 standard deviation of the mean.

95% of values are within 2 standard deviations of the mean.

99.7%of values are within 3 standard deviations of the mean



Q.10 c The number of particles emitted per second by a random radioactive source has a Poisson's distribution with λ = 4. Calculate the probability of P(X = 0) and P(X = 1) .

Given: $\lambda = 4$ $P(x=0) = ?$ $P(x=1) = ?$

$$f(k:\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$P(x=0) = f(0:4) = \frac{4^0 e^{-4}}{0!}$$

$$= 0.0183$$

$$P(x=1) = f(1:4) = \frac{4^1 e^{-4}}{1!}$$

$$= 0.0732$$

****

ALL THE BEST