

**MODULE-2****Electrical Properties of Metals and Semiconductors:**

Failures of classical free electron theory, Mechanisms of electron scattering in solids, Matheissen's rule, Assumptions of quantum free electron theory(QFET), Density of states, Fermi Dirac statistics, Fermi energy, Variation of Fermi factor with temperature and energy, Success of QFET, Expression for carrier concentration, Derivation of electron concentration in an intrinsic semiconductor, Expression for electron and hole concentration in extrinsic semiconductor, Fermi level for intrinsic (with derivation) and extrinsic semiconductor (no derivation), Hall effect, Numerical Problems.

**Module - 2 Blow-up**

Subtopics	Topics to be covered	Duration
Classical Free Electron Theory(CFET) Mobility & Matthiessen's Rule	A Review (Self Study), Definition of drift velocity and mobility, Electron scattering mechanisms in Solids, Matthiessen's rule for resistivity.	1 Hour
Failures of Classical Theory	Specific heat of metals, temperature dependence inconsistencies, electron concentration dependence on conductivity (qualitative)	½ Hour
Quantum Free Electron Theory (QFET), Fermi Energy & Fermi Factor,	Assumptions, Density of States (Qualitative, Mention of DOS expression) Fermi-Dirac Statistics, Fermi Energy (Qualitative), Variation of Fermi factor $f(E)$ with energy & temperature, $T = 0$ K and finite T cases.	1 Hour
Expression for Carrier Concentration, Success of QFET	Derivation of Expression for 'n' using $N(E) dE = g(E) dE \cdot f(E)$ , Mention of Expression for electrical conductivity as per QFET, Success of QFET ( in line with the failures of CFET).	1 ½ Hours
Carrier Concentration and Fermi level in semiconductors	Intrinsic and Extrinsic Semiconductors (Self Study- Flipped Class), Derivation of electron concentration in an intrinsic semiconductor, Expression for electron and hole concentration in extrinsic semiconductor(Mention), Fermi level for intrinsic semiconductor (derivation) and extrinsic semiconductor(No derivation),	2 Hours
Hall Effect	Hall Effect , Explanation, Derivation of Expression for Hall Voltage. Applications	1 Hour
Numerical Problems	Numerical problems on Fermi factor and Hall coefficient	1 Hour

**Introduction:**

Electrical properties of a material refers to the response of a material to an applied electric field. One of the fundamental property of a material is its ability or non-ability to conduct electrical current. Based on this property, the materials are classified into conductors, semiconductors and insulators. In order to develop any electrical instruments, engineered devices, electronic circuits one has to understand the electrical properties of materials.

**Electrical conductivity ( $\sigma$ ):**

It is a measure of the material ability to allow charge carriers to flow through it. The SI unit of electrical conductivity is siemens per metre (S/m). A high conductivity indicates a material that readily allows the flow of electric current.

**Electrical resistivity ( $\rho$ ):**

It is a measure of material ability to oppose the flow of an electric current. The SI unit of electrical resistivity is ohm meter ( $\Omega\text{m}$ ). A high resistivity indicates a material that strongly opposes the flow of electric current.

**Drift velocity ( $v_d$ ):** The average velocity acquired by the free electron in a particular direction, due to the application of electric field is called as drift velocity.

$$v_d = \frac{I}{neA}$$

Where,  $v_d$  = drift velocity,  $I$  = current,  $n$  = free electron density  
 $e$  = charge of an electron,  $A$  = cross sectional area.

**Mobility ( $\mu$ ):** The ability of an electron to move through a metal in the presence of an applied electric field is called as electron mobility.

$$\mu = \frac{v_d}{E}$$

**Introduction to Electrical Conductivity: And Review of classical free electron theory:**

Valence electrons inside the atoms are loosely bound to the nucleus compare to other electrons in those atoms. These electrons (valence) are free to move throughout the volume of metal and constitute free electron gas. This free electron gas determine electrical, optical, thermal and magnetic properties of solids.

Explanation to electrical conductivity was developed in three stages,

- (i) Classical free electron theory: According to this theory, the free electrons obey laws of classical mechanics.
- (ii) Quantum free electron theory: According to this, free electrons obey the laws of quantum mechanics.
- (iii) Zone theory / Band theory: According to this theory, the free electrons move in a periodic field provided by lattice.

**Review of classical free electron theory:**

- The classical free electron theory was proposed by German physicist Paul Drude in 1900 and later improved by H.A. Lorentz, to explain the electrical conductivity in metals.

- According to this theory metals consists of free, or valence electrons move in a random direction similar to that of a gas molecule in a container.
- The free electrons obey the laws of classical mechanics and the theory as whole came to be known as Drude-Lorentz theory.
- The expression for electrical conductivity ( $\sigma$ ) based on this theory is given by,

$$\sigma = \frac{ne^2\tau}{m}$$

where,  $n \rightarrow$  number of free electrons per unit volume (electron concentration)

$e \rightarrow$  charge on the electron

$\tau \rightarrow$  relaxation time of the free electrons

$m \rightarrow$  mass of electron

### Assumptions of classical free electron theory

- The free electrons in the metal move through the positive ionic core and they are responsible for conduction when an electric potential is applied across the conductor.
- The free electrons are considered as gas molecules & assumed to obey the laws of kinetic theory of gases. The energy associated with each electron at a temperature  $T$  is given by  $(\frac{3}{2})kT$  and is related to kinetic energy through the relation,

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

- The motion of free electrons obeys the classical Maxwell-Boltzmann velocity distribution law.
- The free electrons travel under constant potential inside the metal and confine to the boundaries of metal.
- The attraction between the free electrons and the lattice ions, and the repulsion between the electrons themselves are ignored.
- The electric current in a metal due to an applied field is the effect of drift velocity in a direction opposite to the direction of the applied field.

### Mechanisms of Electron Scattering in Solids:

In solids, electrons do not move freely as in a vacuum. They interact with lattice vibrations, impurities, structural defects, other electrons, and boundaries of the material.

These interactions cause electron scattering, which determines the electrical resistivity, mobility, and conductivity of the solid. The scattering mechanisms define the mean free path  $\ell$  and the relaxation time  $\tau$  of the electrons.

**Major Scattering Mechanisms****1. Electron–Phonon (Lattice) Scattering:**

- Caused by thermal vibrations of the lattice atoms (phonons).
- As temperature increases, phonon population increases, enhancing scattering.
- Resistivity increases approximately linearly with  $T$  at high temperatures.
- Dominant mechanism above the Debye temperature.

**2. Electron–Impurity Scattering**

- Caused by foreign atoms or dopants, which disturb the periodic lattice potential.
- Scattering is elastic.
- Resistivity contribution is nearly temperature independent (residual resistivity).
- Dominant at low temperatures.

**3. Electron–Defect / Dislocation Scattering**

- Caused by vacancies, interstitials, grain boundaries, and dislocations.
- Defects break lattice periodicity, reducing mobility.
- Important in polycrystalline materials and alloys.

**4. Electron–Electron Scattering**

- Due to Coulomb interactions between conduction electrons.
- In pure metals at low temperature, it contributes significantly.
- Resistivity varies as  $T^2$  (Fermi liquid behaviour).

**5. Surface or Boundary Scattering**

- Caused by electron reflections at material surfaces.
- Significant in thin films, nanowires, and 2D materials, where dimensions are comparable to electron mean free path.
- Reduces electron mobility compared to bulk.

**Summary Outline:**

Mechanism	Cause	Temperature Dependence	Dominant Condition
Electron–Phonon	Lattice vibrations (phonons)	$\rho \propto T$ at high $T$	High temperature
Electron–Impurity	Foreign atoms / dopants	Nearly constant	Low temperature
Electron–Defect	Vacancies, dislocations, grain boundaries	Nearly constant	Polycrystals, alloys
Electron–Electron	Coulomb interactions	$\rho \propto T^2$	Very low $T$
Surface Scattering	Boundaries/interfaces	Size-dependent	Thin films, nanostructures

**Mattheissen's rule:**

Mattheissen's rule is a principle that describes the total electrical resistivity of a crystalline material as the sum of its individual resistivity's due to various scattering mechanisms.

In a crystalline material, the flow of electrical current is impeded by various scattering mechanisms that electrons encounter as they move through the lattice. These mechanisms can include impurities, defects, lattice vibrations (phonons), and electron-electron interactions.

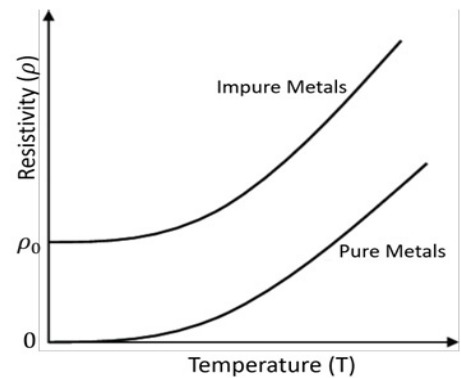


Figure : Mattheissen's rule

Mattheissen's rule states that the total electrical resistivity of the material is the sum of the resistivity's associated with each of these scattering processes. Mathematically, it can be expressed as:

$$\rho_{\text{total}} = \rho_{\text{impurities}} + \rho_{\text{defects}} + \rho_{\text{phonons}} + \rho_{\text{electron-electron interactions}}$$

Each term on the right side represents the contribution to the resistivity due to a specific scattering mechanism. By considering these individual contributions, Mattheissen's rule provides a way to estimate the total resistivity of a material.

But for our discussion, to make things simpler, we will say there are two kinds of contributions towards the total resistivity: a temperature-independent resistivity ( $\rho_0$ ) due to impurities and a temperature dependent resistivity ( $\rho_T$ ) which is the sum of all other resistivities. So we can write Mattheissen's rule simply as:

$$\rho_{\text{total}} = \rho_0 + \rho_T$$

The variation of resistivity with temperature for pure and impure metals is as shown in the above figure.

**Failure / Drawbacks of classical theory of free electrons:**

Though the classical free electron theory successfully explained the phenomena like electrical and thermal conductivity in metals, it failed to account for specific heat, temperature dependence of electrical conductivity and dependence of electrical conductivity on electron concentration.

- (1) **Specific heat:** According to classical free electron theory, the supplied heat energy is absorbed by the valence electrons. The energy associated with each electron is  $\frac{3}{2} kT$ .

Since one-kilo mole of metal consist  $N_A$  number of atoms, if each atom contribute one free electron then the total internal energy at temperature T is given by

$$U_T = \frac{3}{2} N_A kT$$

Where,  $N_A$  – Avogadro number.

Or 
$$U_T = \frac{3}{2} RT$$

The specific heat of metal at constant volume is given by

$$C_V = \left( \frac{\partial U_T}{\partial T} \right) = \frac{3}{2} N_A k \quad (\text{or}) \quad C_V = \frac{3}{2} R$$

But experimentally it was found that  $C_V = 10^{-4} RT$ . Therefore classical theory does not provide reason for experimental derivation.

## (2) Temperature dependence of electrical conductivity:

According to classical free electron theory

$$\frac{1}{2} m v_{th}^2 = \frac{3}{2} kT \quad \text{or} \quad v_{th} = \sqrt{\frac{3kT}{m}}$$

$$V_{th} \propto \sqrt{T} \quad \text{-----(1)}$$

But mean collision time ( $\tau$ ) is given by

$$\tau = \frac{\lambda}{v_{th}} \quad (\text{or})$$

$$\tau \propto \frac{\lambda}{v_{th}} \quad \text{-----(2)}$$

By comparing equations (1) and (2) we get

$$\tau \propto \frac{1}{\sqrt{T}} \quad \text{-----(3)}$$

But electrical conductivity is given by  $\sigma = \frac{ne^2\tau}{m}$

$$(\text{Or}) \quad \sigma \propto \tau \quad \text{-----(4)}$$

By comparing equations (3) and (4) we get

$$\sigma = \frac{1}{\sqrt{T}} \quad \text{-----(5)}$$

But experimentally the electrical conductivity is given as

$$\sigma_{exp} = \frac{1}{T} \quad \text{----- (6)}$$

From the above equations (5) and (6) it is clear that prediction of classical theory is not in agreement with the experimental observation.

## (3) Electron concentration:

According to classical free electron theory, the electrical conductivity is given by

$$\sigma = \frac{ne^2\tau}{m} \quad (\text{or}) \quad \sigma \propto n$$

Where  $n$  is electron concentration.

Consider copper and aluminum. Their electrical conductivities are  $5.88 \times 10^7 / \Omega\text{m}$  and  $3.65 \times 10^7 / \Omega\text{m}$ . The electron concentrations for copper and aluminum are  $8.45 \times 10^{28} / \text{m}^3$  and

$18.06 \times 10^{28}/\text{m}^3$ . Hence, the classical free electron theory fails to explain the dependence of  $\sigma$  on electron concentration.

Experimental results:

Metals	Electron Concentration (n)	Conductivity ( $\sigma$ )
Copper	$8.45 \times 10^{28}/\text{m}^3$	$5.88 \times 10^7/\Omega\text{m}$
Aluminum	$18.06 \times 10^{28}/\text{m}^3$	$3.65 \times 10^7/\Omega\text{m}$

However experimental results shows that electrical conductivity is not strictly (only) proportional to electron concentration.

#### (4) Mean free path :

According to classical free electron theory, the resistivity of metal is given by

$$\rho = \frac{m}{ne^2\tau} \quad \text{or}$$

$$\text{Relaxation time, } \tau = \frac{m}{ne^2\rho}$$

where  $m$  – mass of electron,  $n$  – number of electrons,  $e$  - charge of electrons

The mean free path  $\lambda = v\tau$

Where  $v$  – average velocity of electron

By substituting the values for typical metals, the mean free path can be calculated.

But these values do not match with experimental values. The classical free electron theory could not able to explain large variations in  $\lambda$ .

(5) The classical free electron model predicts only negative Hall constant. But some metals such as Be, Zn & Cd exhibit positive Hall constants.

#### Assumption of Quantum free electron theory:

- (1) The energy levels of conduction electrons are quantized.
- (2) The electrons travel in a constant potential inside the metal but stay within its boundaries
- (3) The attraction between electrons & lattice ions and repulsion among electrons are ignored.
- (4) The distribution of electrons in various energy levels follows Pauli's exclusion principle and Fermi Dirac quantum statistics.
- (5) Electrons are treated as wave like particle.

#### Fermi Dirac statistics:

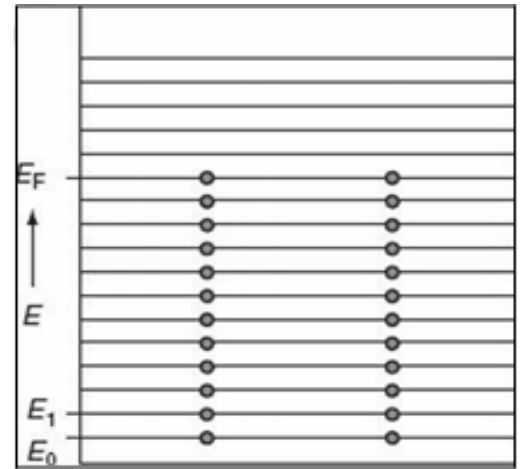
Fermi-Dirac statistics deals with the particles having half-integral spin like electron. Fermi distribution function gives information about the distribution of electrons among the various energy levels as a function of temperature.



Fermi-Dirac statistics describe the distribution of identical particles called fermions (like electrons) over available energy states, strictly adhering to the Pauli exclusion principle, which states that no two fermions can occupy the same quantum state simultaneously.

This statistical model is crucial for understanding the behavior of electrons in metals and semiconductors, explaining properties such as electrical conductivity and the band structure of solids.

The Fermi-Dirac distribution function provides the probability of a quantum state being occupied by a fermion at a given energy and temperature, forming a step-like distribution at absolute zero temperature that gradually "smears out" as temperature increases.



### Fermi Energy ( $E_F$ ), Fermi Factor $F(E)$ :

Fermi energy can be defined as "The highest energy occupied by the electrons at  $T = 0K$  is called Fermi energy" and is denoted as  $E_F$ . The probability of occupation of any energy level is given by a mathematical function given by

$$F(E) = \frac{1}{e^{\frac{E-E_F}{K_B T}} + 1} \quad \text{--- (1)}$$

Where,  $K_B$  is Boltzmann constant and  $T$  is Temperature.

The above equation is called Fermi factor.

**Fermi Factor:** Fermi factor is defined as the probability of finding the position of electron at different allowed energy levels.

### Dependence of Fermi Factor on Energy and Temperature:

The variation of Fermi factor with energy for various temperatures is discussed below.

#### CASE 1): Probability of occupation for $E < E_F$ , at $T = 0K$ :

i.e.,  $E < E_F$  at  $T = 0K$  (therefore neglecting term ' $E$ ' in denominator of eqn. (1) and  $T = 0$ )

Eqn (1) becomes 
$$F(E) = \frac{1}{e^{\frac{-E_F}{0}} + 1}$$

$$F(E) = \frac{1}{e^{-\infty} + 1} \quad \text{(Since } e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0)$$

$$F(E) = \frac{1}{0 + 1} = \frac{1}{1} = 1$$

$$\text{Therefore } F(E) = 1$$

Since  $F(E) = 1$  at  $T = 0K$ , Thus at  $T = 0K$ , there is 100 % chance for the electrons to occupy the energy levels below the Fermi level



**CASE 2): Probability of occupation for  $E > E_F$ , at  $T = 0K$ :**

i.e.,  $E > E_F$  at  $T = 0K$  (therefore neglecting term ' $E_F$ ' in denominator of eqn (1) and  $T = 0K$ )

Eqn (1) becomes 
$$F(E) = \frac{1}{e^{\frac{E}{0}} + 1}$$

$$F(E) = \frac{1}{e^{\infty} + 1}$$

$$F(E) = \frac{1}{\infty + 1} = \frac{1}{\infty} = 0$$

Therefore  $F(E) = 0$

Since  $F(E) = 0$  at  $T = 0K$

Thus, there is 0 % chance for the electrons to occupy energy levels above the Fermi energy level. It means that all energy states above the Fermi energy are empty at  $0K$ .

**CASE 3): Probability of occupation for  $E = E_F$ , at  $T > 0K$ :**

i.e.,  $E = E_F$  at  $T > 0K$

Eqn. (1) becomes 
$$F(E) = \frac{1}{e^{\frac{0}{k_B T}} + 1}$$

$$F(E) = \frac{1}{e^0 + 1}$$

$$F(E) = \frac{1}{1 + 1}$$

$$F(E) = \frac{1}{2} = 0.5$$

Since  $F(E) = \frac{1}{2}$  at  $T > 0K$

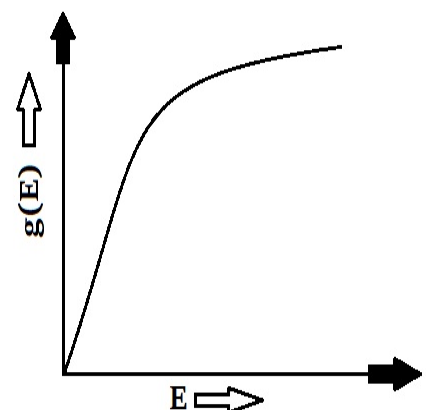
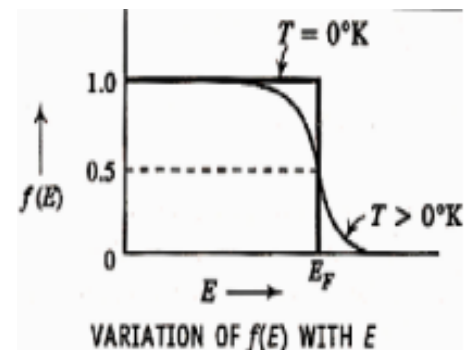
Hence, there is 50 % chance for the electrons to occupy Fermi level. Further, for  $E > E_F$  the probability value falls off rapidly to zero.

**Density of States  $g(E)dE$ :**

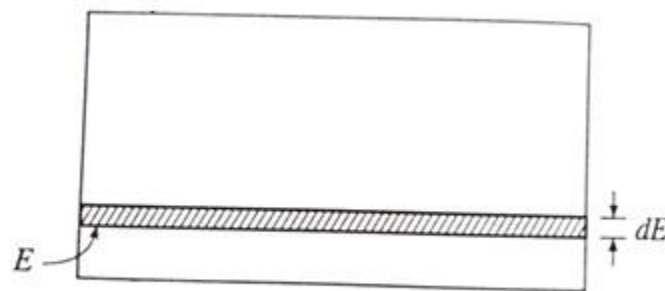
Density of states is defined as “the number of allowed energy levels / unit volume/ unit energy. It is denoted as  $g(E)dE$ . The product of  $g(E)dE$  gives the number of states per unit volume between the energy levels  $E$  and  $(E + dE)$  If  $E$  changes then  $g(E)$  also changes.

Density of states is given by

$$g(E)dE = \frac{\pi}{2} \left( \frac{8m}{h^2} \right)^{\frac{3}{2}} \sqrt{E} \cdot dE$$



As per the above equation it is clear that the number of energy levels in an energy interval  $dE$  is proportional to  $\sqrt{E}$ . A plot of  $g(E)$  versus  $E$  is shown in figure. The shape of the curve is a parabola.



**Density of states**

### Success / Merits of Quantum free electron theory:

- (1) Specific heat: According to classical free electron theory, all the electrons absorb the heat irrespective of energy levels occupied by free electrons, but according to quantum free electron theory, only few electrons near to Fermi level are capable of absorbing the heat energy. Hence the specific heat is given by quantum theory that

$$c_v = \left(\frac{2k}{E_f}\right) RT \quad (\text{or}) \quad 10^{-4}RT \text{ which agree well with experimental values.}$$

- (2) Temperature dependence of Electrical conductivity:

We know that electrical conductivity is

$$\sigma = \frac{ne^2\tau}{m}$$

But according to quantum free electron theory

$$\text{Relaxation time } \tau = \frac{\lambda}{v_f}$$

$$\sigma = \frac{ne^2\lambda}{mv_f} \text{-----(1)}$$

If the metal is heated, the ionic core start to vibrate, if  $r$  is the amplitude of vibration then the area of influence

(or) cross sectional area is  $\pi r^2$ . We say that

$$r^2 \propto T \text{-----(2)}$$

This cross sectional area scatter the electrons, hence

$$r^2 \propto \frac{1}{\pi r^2} \text{-----(3)}$$

Comparing equations (2) & (3) we get

$$\lambda \propto \frac{1}{T} \text{ -----(4)}$$

By comparing equation (1) & (4) we get

$\sigma \propto \frac{1}{T}$  thus dependence of  $\sigma$  on T is correctly explained on the basis of quantum free electron theory.

### (3) Electrical conductivity and electron concentration :

we know that electrical conductivity  $\sigma = \frac{ne^2}{m} \left( \frac{\lambda}{v_f} \right)$

So that electrical conductivity is depend on both concentration of electron (n) as well as  $\frac{\lambda}{v_f}$  also.

It explains the existence of long mean free path at low temperature from the following relation  $\lambda \propto \frac{1}{T}$

### (4) Quantum free electron theory explains successfully electrical conductivity & thermal conductivity.

It explains paramagnetic susceptibility and heat capacity successfully

### Carrier concentration in metals: Derivation:

Let  $N(E) dE$  represents the number of filled energy states between the interval of energy  $dE$ . Normally all the energy states will not be filled.

$$N(E)dE = Z(E)dE \cdot F(E) \dots \dots \dots (1)$$

$$Z(E)dE = \frac{\pi}{2} \left[ \frac{8m}{h^2} \right]^{\frac{3}{2}} E^{\frac{1}{2}} dE \dots \dots \dots (2)$$

We know that the probability of filled electrons in a given energy state is given by fermi function  $F(E)$ .

$$F(E) = \frac{1}{e^{\frac{E-E_F}{K_B T}} + 1} \dots \dots \dots (3)$$

We know that fermi energy at zero kelvin (0K) is maximum energy level that can be occupied by the electron is called fermi level at  $E_{F0}$

i. e., at Zero kelvin, for  $E < E_F$ , as per this condition the fermi energy  $F(E) = 1$

Therefore, equation (3) becomes  $F(E) = 1 \dots \dots \dots (4)$

Substitute equation (2) and (4) in equation (1)

$$N(E)dE = \frac{\pi}{2} \left[ \frac{8m}{h^2} \right]^{\frac{3}{2}} E^{\frac{1}{2}} dE \dots \dots \dots (5)$$

$\therefore$  integrating equation (5) within the limits 0 to  $E_{F0}$ ,

we can get the number of filled energy states electrons  $N(E)$  within fermi energy  $E_{F0}$

$$\therefore \int_0^N N(E) dE = \frac{\pi}{2} \left[ \frac{8m}{h^2} \right]^{\frac{3}{2}} \int_0^{E_{F0}} E^{\frac{1}{2}} dE$$

$$N = \frac{\pi}{2} \left[ \frac{8m}{h^2} \right]^{\frac{3}{2}} \frac{E_{F0}^{\frac{3}{2}}}{\frac{3}{2}}$$

$$N = \frac{\pi}{3} \left[ \frac{8m}{h^2} \right]^{\frac{3}{2}} E_{F0}^{\frac{3}{2}} \dots \dots \dots (6)$$

Equation (6) represents the expression for carrier concentration in metals and also for semiconductors too. **This equation is used to calculate the carrier concentration in metals and semiconductors.**

From equation (6) fermi energy at absolute zero temperature is given as

$$E_{F0} = \left[ \frac{3N}{\pi} \right]^{\frac{2}{3}} \cdot \frac{h^2}{8m} \dots \dots \dots (7)$$

#### Expression for electrical conductivity as per QFET:

Electrical conductivity  $\sigma$  of a conductor is given by

$$\sigma = \frac{ne^2\tau}{m}$$

Where ,  $n$  = number of electrons per unit volume

$e$  = charge of an electron

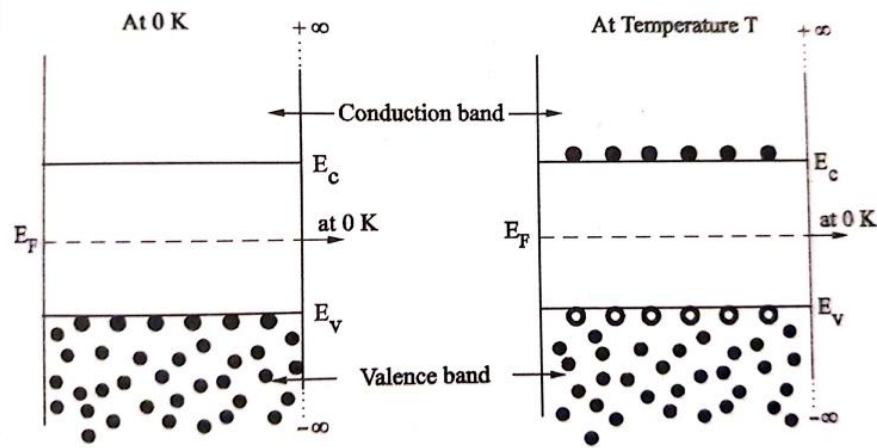
$\tau$  = mean collision time

$m$  = mass of an electron.

Note: when  $\tau = \frac{\lambda}{v_f}$ , now conductivity becomes,  $\sigma = \frac{ne^2\lambda}{m.v_f}$

#### Carrier Concentration in Intrinsic Semiconductors:

We know, at 0K intrinsic pure semiconductor behaves as insulator. However, as temperature increases some electrons move from valence band to conduction band as shown in fig. Therefore, both electrons in conduction band and holes in valence band will contribute to electrical conductivity. Therefore, the carrier concentration (or) density of electrons ( $n_e$ ) and holes ( $n_h$ ) has to be calculated.



Assume that electron in the conduction band is a free electron of mass  $m_e^*$ , and the hole in the valence band behaves as a free particle of mass  $m_h^*$ . The electrons in the conduction band have energies lying from  $E_c$  to  $\infty$  and holes in the valence band have energies from  $-\infty$  to  $E_v$  as shown in fig. Here  $E_c$  represents the energy of the bottom (or) lowest level of conduction band and  $E_v$ , represents the energy of the top (or) the highest level of the valence band.

#### Derivation of electrons in conduction band in an intrinsic semiconductor (Derivation):

$$\text{Density of electrons in conduction band } n_e = \int_{E_c}^{\infty} Z(E) dE \cdot F(E) \dots \dots \dots (1)$$

$$\text{From Fermi-Dirac statistics we can write } Z(E)dE = 2 \times \frac{\pi}{4} \left[ \frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} E^{\frac{1}{2}} dE \dots \dots \dots (2)$$

Considering the minimum energy of conduction band as  $E_c$  and the maximum energy can go upto  $\infty$ , we can write equation(2) as

$$Z(E)dE = \frac{\pi}{2} \left[ \frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}} dE \dots \dots \dots (3)$$

We know that, Fermi Function, probability of finding an electron in a given energy state is,

$$F(E) = \frac{1}{e^{\frac{E-E_F}{K_B T}} + 1} \dots \dots \dots (4)$$

Substituting eqn.(4) and (3) in eqn(1) we have density of electron in conduction band within the limits  $E_c$  to  $\infty$  as

$$n_e = \int_{E_c}^{\infty} \frac{\pi}{2} \left[ \frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} \frac{(E - E_c)^{\frac{1}{2}}}{e^{\frac{E-E_F}{K_B T}} + 1} dE$$

$$\text{or } n_e = \frac{\pi}{2} \left[ \frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} \int_{E_c}^{\infty} \frac{(E - E_c)^{\frac{1}{2}}}{e^{\frac{E-E_F}{K_B T}} + 1} dE$$

Since to move an electron from valence bond to conduction band the energy required is greater than  $4K_B T$ . i.e.,  $E - E_F \gg K_B T$  or  $\frac{E - E_F}{K_B T} \gg 1$

$$\text{or } e^{\frac{E - E_F}{K_B T}} \gg 1$$

$$\therefore 1 + e^{\frac{E - E_F}{K_B T}} \approx e^{\frac{E - E_F}{K_B T}}$$

$$\therefore \text{equation (5) becomes } n_e = \frac{\pi}{2} \left[ \frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} \int_{E_c}^{\infty} \frac{(E - E_c)^{\frac{1}{2}}}{e^{\frac{E - E_F}{K_B T}}} dE$$

$$\text{or } n_e = \frac{\pi}{2} \left[ \frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} \int_{E_c}^{\infty} (E - E_c)^{\frac{1}{2}} \cdot e^{\frac{E_F - E}{K_B T}} dE \dots \dots \dots (6)$$

let us assume that,  $E - E_c = (x K_B T)$

$$\text{or } E = E_c + x K_B T$$

$$\text{differentiating w.r.t } x, \text{ we get } \frac{dE}{dx} = K_B T$$

$$\text{or } dE = K_B T \cdot dx$$

Applying limits: When  $E = E_c$ ;  $x = 0$

When  $E = \infty$ ;  $x = \infty$

$\therefore$  limits now becomes 0 to  $\infty$

$$\therefore \text{equation (6) can be written as } n_e = \frac{\pi}{2} \left[ \frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} \int_0^{\infty} (x K_B T)^{\frac{1}{2}} \cdot e^{\frac{(E_F - x K_B T - E_c)}{K_B T}} K_B T dx$$

$$n_e = \frac{\pi}{2} \left[ \frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} \int_0^{\infty} \left( x^{\frac{1}{2}} \right) (K_B T)^{\frac{3}{2}} \cdot e^{\frac{(E_F - E_c)}{K_B T}} e^{-x} K_B T dx$$

$$n_e = \frac{\pi}{2} \left[ \frac{8m_e^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_F - E_c)}{K_B T}} \int_0^{\infty} x^{\frac{1}{2}} \cdot e^{-x} dx$$

$$n_e = \frac{\pi}{2} \left[ \frac{8m_e^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_F - E_c)}{K_B T}} \frac{\sqrt{\pi}}{2} \quad \because \int_0^{\infty} x^{\frac{1}{2}} \cdot e^{-x} dx = \frac{\sqrt{\pi}}{2}$$

$$n_e = \frac{1}{4} \left[ \frac{8m_e^* \pi K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_F - E_c)}{K_B T}}$$

$\therefore$  Density of electrons in conduction band is

$$n_e = 2 \left[ \frac{2m_e^* \pi K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_F - E_c)}{K_B T}} \dots \dots \dots (7)$$

Similarly, Density of holes in valence band is,

$$n_h = 2 \left[ \frac{2m_h^* \pi K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_V - E_F)}{K_B T}} \dots \dots \dots (8)$$

Variation of fermi energy level in intrinsic semiconductor: Derivation

For an intrinsic semiconductor number of electron  $n_e$  will be same as that of number of holes  $n_h$

$$\text{i.e., } n_e = n_h$$

Substituting equation (7) and (8) in the above equation,

$$\begin{aligned} 2 \left[ \frac{2m_e^* \pi K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_F - E_C)}{K_B T}} &= 2 \left[ \frac{2m_h^* \pi K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_V - E_F)}{K_B T}} \\ (m_e^*)^{\frac{3}{2}} e^{\frac{(E_F - E_C)}{K_B T}} &= (m_h^*)^{\frac{3}{2}} e^{\frac{(E_V - E_F)}{K_B T}} \\ \left( \frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} &= \frac{e^{\frac{(E_F - E_C)}{K_B T}}}{e^{\frac{(E_V - E_F)}{K_B T}}} \\ \left( \frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} &= e^{\frac{(E_F - E_C - E_V + E_F)}{K_B T}} \\ \left( \frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} &= e^{\frac{(2E_F - (E_C + E_V))}{K_B T}} \end{aligned}$$

Taking log on both sides  $\frac{3}{2} \log \left( \frac{m_h^*}{m_e^*} \right) = \frac{(2E_F - (E_C + E_V))}{K_B T}$

$$\frac{3}{2} K_B T \log \left( \frac{m_h^*}{m_e^*} \right) = 2E_F - (E_C + E_V)$$

$$2E_F = E_V + E_C + \frac{3}{2} K_B T \log \left( \frac{m_h^*}{m_e^*} \right)$$

$$E_F = \frac{E_C + E_V}{2} + \frac{3}{4} K_B T \log \left( \frac{m_h^*}{m_e^*} \right) \dots \dots \dots (9)$$

If  $m_e^* = m_h^*$ , then  $\log \left( \frac{m_h^*}{m_e^*} \right) = 0$

( $\because \log 1 = 0$ )

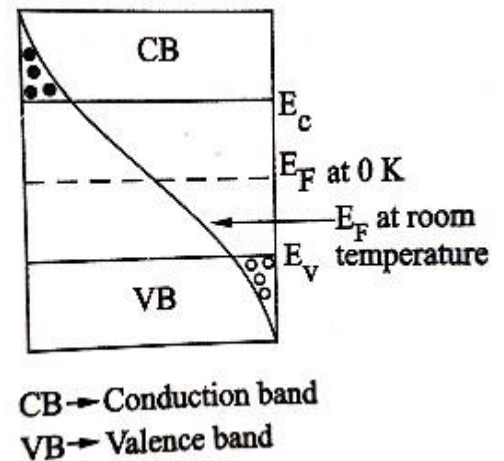
$$\therefore \text{equation (9) becomes } E_F = \frac{E_C + E_V}{2} \dots \dots \dots (10)$$

i.e., the fermi energy level lies in the midway between  $E_C$  and  $E_V$  as shown in fig 2.9 (since at 0K,  $T=0$ ). But in actual case  $m_e^* > m_h^*$  and the fermi energy level slightly increases with increase in temperature as shown in figure.

Density of electrons and holes in terms of energy gap or band gap ( $E_g$ ):

In terms of energy gap ( $E_g$ ), where  $E_g = E_C - E_V$  or  $-E_g = E_V - E_C$  we can get the expression of  $n_e$  and  $n_h$  by substituting the values of  $E_F$  in terms of  $E_C$  and  $E_V$ .

Substitute equation (9) in (7)





$$n_e = 2 \left[ \frac{2m_e^* \pi K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{\left( \frac{E_c + E_v}{2} + \frac{3}{4} K_B T \log \left( \frac{m_h^*}{m_e^*} \right) - E_c \right)}{K_B T}}$$

$$n_e = 2 \left[ \frac{2\pi K_B T}{h^2} \right]^{\frac{3}{2}} (m_e^*)^{\frac{3}{2}} e^{\left[ \frac{2E_c + 2E_v + 3K_B T \log \left( \frac{m_h^*}{m_e^*} \right) - 4E_c}{4K_B T} \right]}$$

$$n_e = 2 \left[ \frac{2\pi K_B T}{h^2} \right]^{\frac{3}{2}} (m_e^*)^{\frac{3}{2}} e^{\left[ \frac{(2(E_v - E_c) + 3 \log_e \left( \frac{m_h^*}{m_e^*} \right))}{2K_B T} \right]}$$

Since  $E_v - E_c = -E_g$ , we can write

$$n_e = 2 \left[ \frac{2\pi K_B T}{h^2} \right]^{\frac{3}{2}} (m_e^*)^{\frac{3}{2}} e^{\left[ \frac{(-E_g)}{2K_B T} + \log_e \left( \frac{m_h^*}{m_e^*} \right)^{\frac{3}{4}} \right]}$$

$$n_e = 2 \left[ \frac{2\pi K_B T}{h^2} \right]^{\frac{3}{2}} (m_e^*)^{\frac{3}{2}} \left( \frac{m_h^*}{m_e^*} \right)^{\frac{3}{4}} e^{\frac{(-E_g)}{2K_B T}}$$

$$n_e = 2 \left[ \frac{2\pi K_B T}{h^2} \right]^{\frac{3}{2}} (m_e^*)^{\frac{3}{4}} (m_h^*)^{\frac{3}{4}} e^{\frac{(-E_g)}{2K_B T}}$$

$$n_e = 2 \left[ \frac{2\pi K_B T}{h^2} \right]^{\frac{3}{2}} (m_e^* m_h^*)^{\frac{3}{4}} e^{\frac{(-E_g)}{2K_B T}} \dots \dots \dots (11)$$

Similarly by substituting equation (13) in equation (8) we get,

$$n_h = 2 \left[ \frac{2\pi K_B T}{h^2} \right]^{\frac{3}{2}} (m_h^* m_e^*)^{\frac{3}{4}} e^{\frac{(-E_g)}{2K_B T}} \dots \dots \dots (12)$$

Thus, it is found that  $n_e = n_h = n_i$

Where  $n_i$  is the intrinsic carrier concentration.

**Intrinsic carrier concentration in an intrinsic semiconductor:**

$$n_i^2 = n_e \times n_h$$

$$n_i^2 = 2^2 \left( \left[ \frac{2\pi K_B T}{h^2} \right]^{\frac{3}{2}} \right)^2 \left( (m_e^* m_h^*)^{\frac{3}{4}} e^{\frac{(-E_g)}{2K_B T}} \right)^2$$

$$n_i^2 = 4 \left[ \frac{2\pi K_B T}{h^2} \right]^3 (m_e^* m_h^*)^{\frac{3}{2}} e^{\frac{(-E_g)}{K_B T}} \dots \dots \dots (13)$$

Therefore, for intrinsic semiconductor even if impurity is added to increase  $n_e$  there will be decrease in  $n_h$  and hence the product of  $n_e n_h$  will remain constant. This is called the law of mass action.

Fermi level for extrinsic semiconductor:

Impure semiconductors in which the charge carriers are produced due to impurity atoms are called as extrinsic semiconductors. They are obtained by doping an intrinsic semiconductor with impurity atoms.

Based on the type of impurity added they are classified into two types

- (i) n – type semiconductor.
- (ii) p – type semiconductor.

**(i) n – type semiconductor**

n-type semiconductor is obtained by doping an intrinsic semiconductor with pentavalent impurity atoms (5 electrons in valence band) like Phosphorous, arsenic, antimony, etc.,

The 4 valence electrons of the impurity atoms bond with 4 valence electron in the semiconductor atom and the remaining one electron of the impurity atom is left free as shown in fig.

Therefore number of free electrons increases. As the electrons are produced in excess, they are the majority charge carriers in n-type semiconductor and holes are the minority charge carriers.

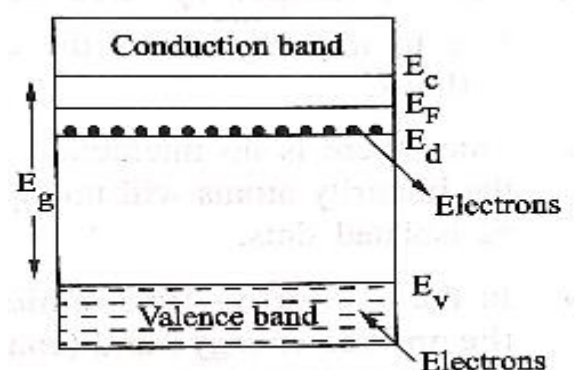
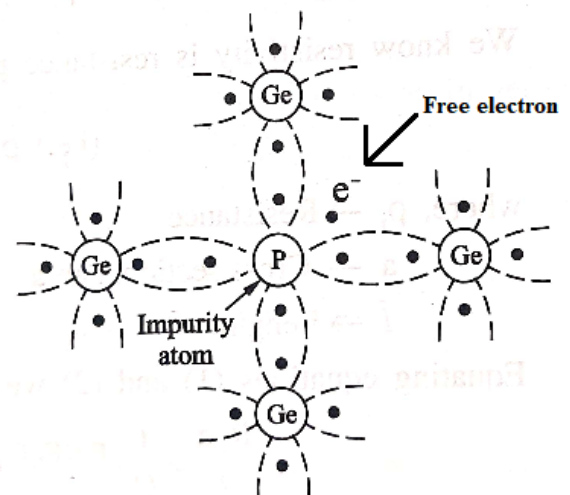
Since electrons are donated in this type of semiconductor the energy level of these donated electrons is called donor energy levels( $E_d$ ) as shown in figure.

$E_d$  is very close to conduction band and hence even at room temperature the electrons are easily excited to conduction band. The current flow in this type is due to electrons.

Expression for fermi level in n-type semiconductor:(Qualitative)

The energy band structure of n-type semiconductor is as shown in fig.

At OK,  $E_F$  will lie exactly between  $E_c$  and  $E_d$ , but even at low temperature, some electrons may go from  $E_d$  to  $E_c$ .



Expression for fermi energy in n-type semiconductor is

$$E_F = \frac{E_c + E_d}{2} + \frac{3}{2} K_B T \log \left( \frac{N_d}{2 \left[ \frac{2m_e^* \pi K_B T}{h^2} \right]^{\frac{3}{2}}} \right) \dots (1)$$

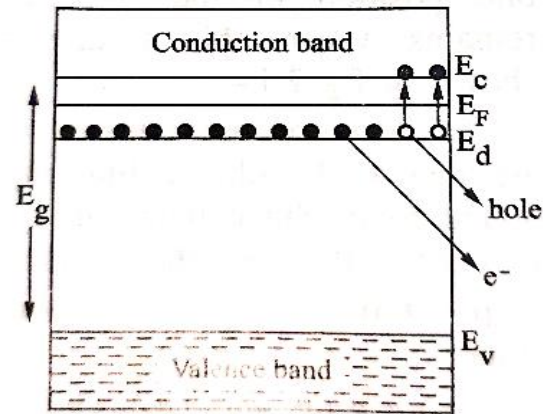
Where,  $E_F$  = Fermi energy

$E_c$  = conduction band

$E_d$  = donor energy level.

$N_d$  = number of donor energy level

$m_e^*$  = rest mass of an electron



At Zero Kelvin (OK), i.e., When  $T=0$ , we can write equation(1) as

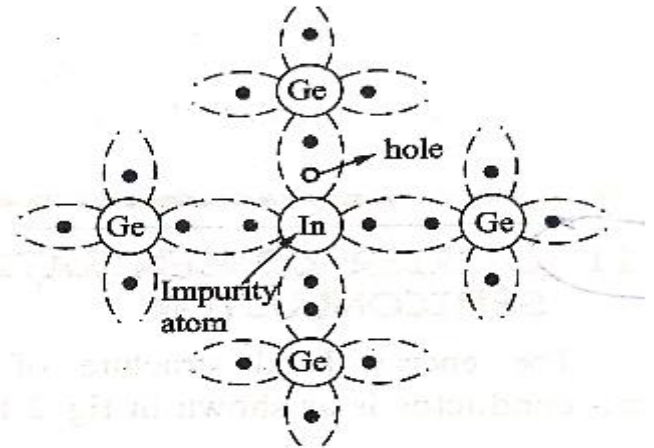
$$E_F = \frac{E_c + E_d}{2} \dots \dots \dots (2)$$

Equation (2) states that, at OK  $E_F$  will lie exactly in the midway between  $E_c$  and  $E_d$ .

## (ii) p – type semiconductor

p-type semiconductor is obtained by doping an intrinsic semiconductor with trivalent impurity atoms (3 electrons in valence band) like Boron, Gallium, Indium, etc.,

The 3 valence electrons of the impurity atoms bond with 3 valence electron in the semiconductor atom and the remaining one electron of the impurity atom is left vacant, this is called hole as shown in fig.



Therefore number of holes increases. As the holes are produced in excess, they are the majority charge carriers in p-type semiconductor and electrons are the minority charge carriers. Since the impurity can accept the electrons in this energy level is called as acceptor energy level ( $E_a$ ) as shown in figure.

Here the current is mainly due to holes.

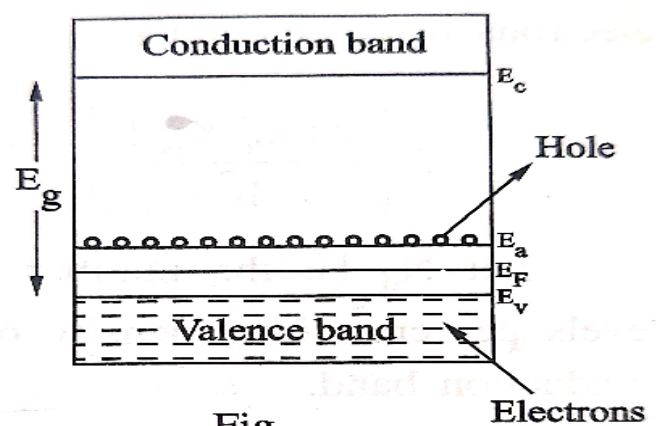


Fig.

**Expression for fermi level in p-type semiconductor: (Qualitative)**

For p-type semiconductor, at Zero kelvin (OK)  $E_F$  will be exactly in between  $E_a$  and  $E_v$ . At low temperatures, some electron from valence band fills the holes in the acceptor energy levels as shown in fig.

Expression for fermi energy in p-type semiconductor is

$$E_F = \frac{E_V + E_a}{2} - \frac{3}{2} K_B T \log \left( \frac{N_a}{2 \left[ \frac{2m_h^* \pi K_B T}{h^2} \right]^{\frac{3}{2}}} \right) \dots (1)$$

Where,

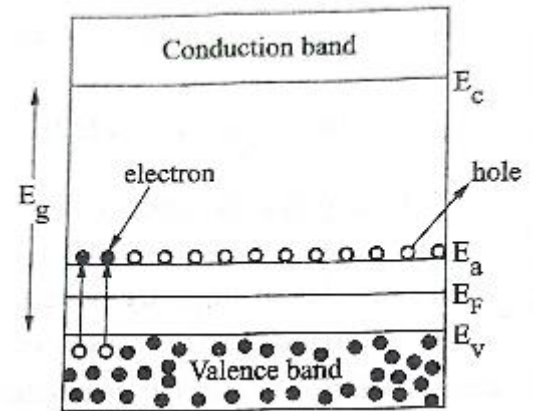
$E_F$  = Fermi energy

$E_V$  = Valence band

$E_a$  = Acceptor energy level.

$N_a$  = number of acceptor energy level

$m_h^*$  = rest mass of an holes.



At Zero Kelvin (OK), i.e., When  $T=0$ , we can write equation (1) as

$$E_F = \frac{E_V + E_a}{2} \dots \dots \dots (2)$$

Equation (2) states that, at OK  $E_F$  will lie exactly in the midway between  $E_c$  and  $E_d$ .

**Hall Effect: STATEMENT:**

When a magnetic field ( $B$ ) is applied perpendicular to a current carrying conductor or semiconductor a potential difference (electric field) is developed inside the conductor in a direction perpendicular to both current and magnetic field. This phenomenon is known as Hall Effect and the voltage thus generated is called Hall voltage.

**Hall Effect in n-type semiconductor:**

Let us consider a n-type semiconductor material in the form of rectangular slab. In such a material current flows in X-direction and magnetic field  $B$  applied in Z-direction. As a result, Hall voltage is developed along Y-direction as shown in figure.

Since the direction of current is from left to right the electrons moves from right to

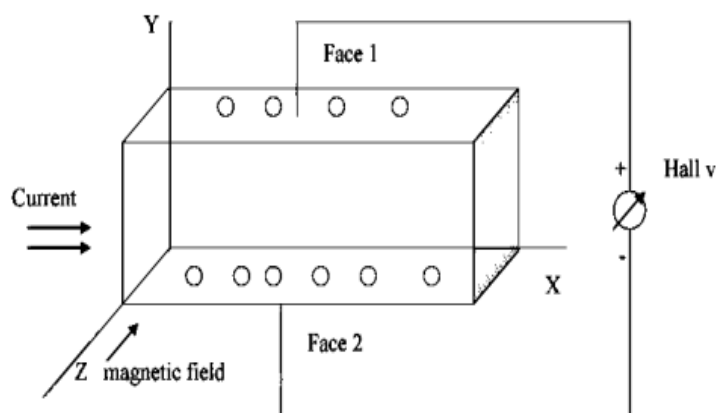


Fig -Hall Effect in N type semiconductor

left . When a magnetic field is applied the electrons are moving towards the bottom of the semiconductor.

$$\text{Force due to potential energy or Lorentz force} = -e E_H \text{ --- (1)}$$

$$\text{Force due to magnetic field} = -Bev \text{ --- (2)}$$

$$\therefore \text{At equilibrium, equation (1) = equation (2)}$$

$$-e E_H = -Bev$$

$$E_H = Bv \text{ --- (3)}$$

We know the current density  $J_x$  in the X – direction is

$$J_x = -n_e ev$$

$$\text{or } v = -\frac{J_x}{n_e e} \text{ --- (4)}$$

Substituting equation (4) in equation (3)

$$E_H = -\frac{B J_x}{n_e e} \text{ --- (5)}$$

$$E_H = R_H \cdot J_x \cdot B \text{ --- (6)}$$

where,  $R_H$  is known as the Hall coefficient and is give by

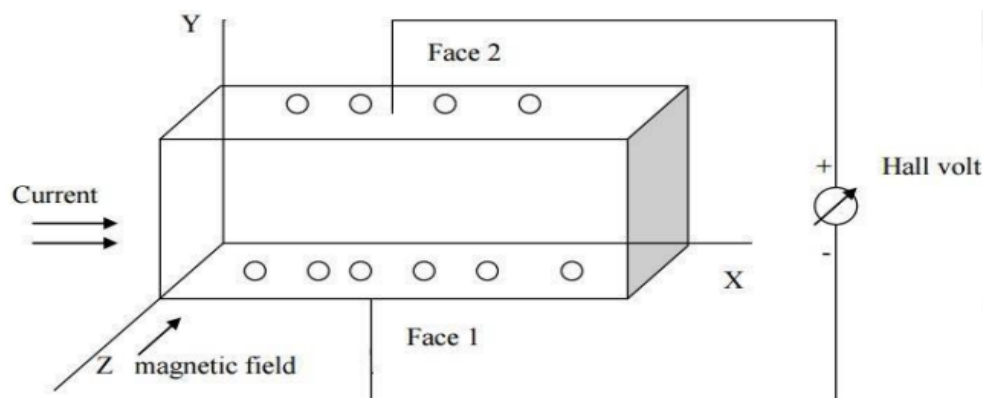
$$R_H = -\frac{1}{n_e e}$$

The negative sign indicates that the field is developed in the negative Y direction.

### Hall Effect in p –type semiconductor:

Let us consider a p –type-semiconducting material for which the current is passed along X – direction from left to right and magnetic field is applied along Z – direction as shown in fig. since the direction of current is from left to right, the holes will also move in the same direction as shown in fig.

Now due to magnetic field applied the holes moves towards downward direction with velocity  $v$  and accumulates at the face (1). A potential difference is established between face (1) and (2) in the positive Y - direction.



Force due to potential energy or Lorentz force =  $e E_H$  ————(7)

Force due to magnetic field =  $-Bev$  ————(8)

$\therefore$  At equilibrium, equation (7) = equation (8)

$$e E_H = Bev$$

$$E_H = Bv \text{ ———— (9)}$$

We know the current density  $J_x$  in the X – direction is

$$J_x = n_h e v$$

$$\text{or } v = -\frac{J_x}{n_h e} \text{ ———— (10)}$$

Substituting equation (4) in equation (3)

$$E_H = -\frac{B J_x}{n_h e}$$

$$E_H = R_H \cdot J_x \cdot B$$

where,  $R_H$  is known as the Hall coefficient and is give by

$$R_H = \frac{1}{n_h e} \text{ ———— (11)}$$

Equation (11) represents the hall Coefficient and the positive sign indicates that the hall field is developed in the positive Y direction.

The negative sign indicates that the field is developed in the negative Y direction.

### Hall coefficient in terms of hall voltage:

If the thickness of the sample is  $t$  and the voltage developed is  $V_H$ , then Hall voltage

$$V_H = E_H \cdot t \text{ ———— (12)}$$

Substituting equation (6 ) in equation (13), we have

$$V_H = R_H J_x B \cdot t \text{ ———— (13)}$$

$b$  is the width of the sample then

Area off the sample =  $b \cdot t$

$$\therefore \text{Current density} = J_x = \frac{I_x}{bt} \text{ ———— (14)}$$

Substitute equation (14) in equation (13) we get

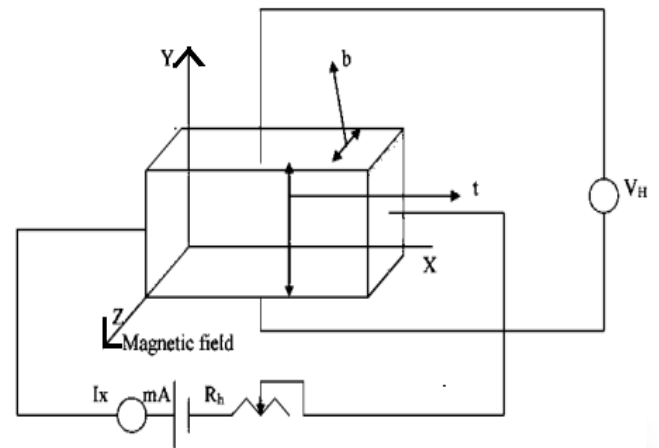
$$V_H = \frac{R_H I_x B \cdot t}{bt}$$

$$V_H = \frac{R_H I_x B}{b}$$

$$\therefore \text{Hall Coefficient, } R_H = \frac{V_H \cdot b}{I_x B}$$

**Experimental determination of Hall Effect:**

A semiconducting material is taken in the form of a rectangular slab of thickness 't' and breadth 'b'. A suitable current  $I_x$  ampere is passed through this sample along X- axis by connecting it to a battery. Now a semiconductor is placed in a magnetic field. A voltage is developed in the specimen, which can be measured by using the voltmeter connecting with the specimen. Then by using the formula.



$$\text{Hall Coefficient, } R_H = \frac{V_H \cdot b}{I_x B}$$

**Applications of Hall Effect:**

1. It is used to determine whether the material is p-type or n-type semiconductor. (ie) if  $R_H$  is negative then the material n-type. If the  $R_H$  is positive then the material p-type.
2. It is used to find the carrier concentration
3. It is used to find the mobility of charge carriers  $\mu_e$  and  $\mu_h$ . It is used to find the sign of the current carrying charges.
4. From the hall coefficient, carrier concentration and mobility can be determined.



## MODULE 2– Electrical Conductivity of Metals – NUMERICALS- FERMI ENERGY:

1. Calculate the Fermi velocity and the mean free path for the conduction electrons in silver, given that its Fermi energy is 5.5 eV and the relaxation time for electrons is  $3.83 \times 10^{-14}$  s.

*Data:* Fermi Energy,  $E_F = 5.5 \text{ eV} = 5.5 \times 1.602 \times 10^{-19} \text{ J}$

*Relaxation time for electrons,  $\tau = 3.83 \times 10^{-14}$  s*

*Mass of electron,  $m = 9.11 \times 10^{-31} \text{ kg}$*

*To find: Fermi velocity,  $v_f = ?$  and Mean free path,  $\lambda = ?$*

*Solution:* we have,  $v_f = \sqrt{\frac{2 E_F}{m}} = \sqrt{\frac{2 \times 5.5 \times 1.602 \times 10^{-19}}{9.11 \times 10^{-31}}} = 1.396 \times 10^6 \text{ m/s}$

*Mean free path,  $\lambda = v_f \times \tau = 1.396 \times 10^6 \times 3.83 \times 10^{-14} = 5.518 \times 10^{-8} \text{ m}$*

*Fermi velocity and mean free path for electrons in silver have the values*

**$1.396 \times 10^6 \text{ m/s}$  and  $5.518 \times 10^{-8} \text{ m}$  i.e  $518 \text{ \AA}$ .**

2. Find the temperature of which there is 1% probability that a state with an energy 0.5 eV above the Fermi energy is occupied. (Jan 2019 -18S | July 2022 – 21S)

*Data:* **Probability,  $f(E) = 1\% = 0.01$**

*Energy above  $E_F$  i.e.,  $(E - E_F) = 0.5 \text{ eV} = 0.5 \times 1.602 \times 10^{-19} \text{ J}$*

*To find :* *Temperature at which the above data holds good,  $T = ?$*

*Solution:* we have  $f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$

$$\frac{E-E_F}{kT} = \frac{0.5 \times 1.602 \times 10^{-19}}{1.38 \times 10^{-23} \times T} = \frac{5797}{T}$$

Substitute the above value, we have  $0.01 = \frac{1}{e^{\frac{5797}{T}} + 1}$

$$e^{\frac{5797}{T}} = \frac{1}{0.01} - 1 = 99$$

Taking natural log on both sides we have,  $\frac{5797}{T} = \ln 99$  ( $\therefore \ln 99 = 4.595$ )

$$\therefore T = \frac{5797}{4.595} = 1261.1 \text{ K}$$

**$\therefore$  Temperature at which the above data holds good,  $T = 1261.1 \text{ K}$**

3. Calculate the probability of an electron occupying an energy level 0.02 eV above the Fermi level at 200 K and 400 K in a material. (Jan 2020 | Jan 2021 -18S)

*Data:*  $E - E_F = 0.02 \text{ eV} = 0.02 \times 1.602 \times 10^{-19} \text{ J}$

*To find:* (i)  $f(E)$  at 200 K = ?

(ii)  $f(E)$  at  $400K = ?$ Solution: (i) Evaluation of  $f(E)$  at  $200K = ?$ 

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} = \frac{1}{e^{\frac{0.02 \times 1.602 \times 10^{-19}}{1.38 \times 10^{-23} \times 200}} + 1} = \frac{1}{e^{1.1594} + 1} = \frac{1}{3.188 + 1} = \frac{1}{4.188} = 0.24$$

$$\therefore f(E) \text{ at } 200K = 0.24$$

(ii) Evaluation of  $f(E)$  at  $400K = ?$ 

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} = \frac{1}{e^{\frac{0.02 \times 1.602 \times 10^{-19}}{1.38 \times 10^{-23} \times 400}} + 1} = \frac{1}{e^{0.5797} + 1} = \frac{1}{1.7855 + 1} = \frac{1}{2.7855} = 0.36$$

$$\therefore f(E) \text{ at } 400K = 0.36$$

$$\therefore \text{Evaluation of } f(E) \text{ at } 200K = 0.24$$

$$\therefore \text{Evaluation of } f(E) \text{ at } 400K = 0.36$$

4. Find the temperature of which there is 1% probability that a state with an energy 0.2eV above Fermi level is occupied. (Feb 2022 – 21S)

(Refer problem no.2 same type)

(Answer,  $T = 505K$ )

5. Calculate the probability of an electron occupying an energy level 0.02eV above the Fermi level and that in an energy level 0.02eV below the Fermi level at 200K.

Data: Energy level above the Fermi level, at 200K

(Jan 2020 | Jan 2021 -18S)

$$E - E_F = 0.02eV = 0.02 \times 1.602 \times 10^{-19}J$$

To find: (i)  $f(E)$  at  $200K = ?$ Solution: (i) Evaluation of  $f(E)$  at  $200K = ?$  (Energy level above the Fermi level)(ii) Evaluation of  $f(E)$  at  $200K =$ 

? (Energy level below the Fermi level)

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} = \frac{1}{e^{\frac{0.02 \times 1.602 \times 10^{-19}}{1.38 \times 10^{-23} \times 200}} + 1} = \frac{1}{e^{1.1594} + 1} = \frac{1}{3.188 + 1} = \frac{1}{4.188} = 0.24$$

$$\therefore f(E) \text{ at } 200K = 0.24$$

(i) Evaluation of  $f(E)$  at  $200K = ?$  (Energy level below the Fermi level)

Now, for the case of electron, occupying the energy level 0.02eV below the Fermi level  $E_F$ , at 200K, We have  $\Delta E = E_F - E = 0.02eV$

$$\therefore E - E_F = -0.02eV = -0.02 \times 1.602 \times 10^{-19}J$$

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} = \frac{1}{e^{\frac{0.02 \times 1.602 \times 10^{-19}}{1.38 \times 10^{-23} \times 200}} + 1} = \frac{1}{e^{-1.1594} + 1} = \frac{1}{0.3137 + 1} = \frac{1}{1.3137} = 0.76$$

$$\therefore f(E) = 0.24 \text{ for } E = 0.02eV \text{ above } E_F$$

$$\therefore f(E) = 0.76 \text{ for } E = 0.02eV \text{ below } E_F$$

6. The Hall coefficient of certain silicon specimen was found to be  $-7.35 \times 10^{-5} \text{ m}^3 \text{ C}^{-1}$  from 100K to 400 K. Determine the nature of the semiconductor. If the conductivity was found to be  $200 \text{ } \Omega^{-1} \text{ m}^{-1}$ . Calculate the density and mobility of the charge carrier.

**Solution:**

$$\text{Conductivity } \sigma = 200 \text{ } \Omega^{-1} \text{ m}^{-1}$$

$$\text{Hall co-efficient } R_H = -7.35 \times 10^{-5} \text{ m}^3 \text{ C}^{-1} \quad \dots (1)$$

**a) Density of electrons**

$$n = \frac{-1}{R_H e} \text{ (from equation (1))}$$

$$n = \frac{1}{(7.35 \times 10^{-5} \times 1.609 \times 10^{-19})}$$

$$\text{(i.e.)} = 8.455 \times 10^{22} \text{ m}^{-3}$$

We know Conductivity

$$\sigma = n e \mu_e$$

**b) Mobility**

$$\mu = \frac{\sigma}{n e} = \frac{200}{8.455 \times 10^{22} \times 1.6 \times 10^{-19}} = 0.0147$$

$$\text{Mobility } \mu = 0.0147 \text{ m}^2 \text{ v}^{-1} \text{ s}^{-1}$$

$$\text{Density of electrons (n)} = 8.053 \times 10^{22} \text{ m}^{-3}$$

$$\text{Mobility } (\mu) = 0.0147 \text{ m}^2 \text{ v}^{-1} \text{ s}^{-1}$$

7. In a P-type germanium,  $n_i = 2.1 \times 10^{19} \text{ m}^{-3}$  density of boron  $4.5 \times 10^{23} \text{ atoms /m}^3$ . The electron and hole mobility are 0.4 and  $0.2 \text{ m}^2 \text{ v}^{-1} \text{ s}^{-1}$  respectively. What is its conductivity before and after addition of boron atoms?

**Solution:**

**Given:**

$$\text{Intrinsic carrier concentration } n_i = 2.1 \times 10^{19} \text{ m}^{-3}$$

$$\text{Mobility of electrons } \mu_e = 0.4 \text{ m}^2 \text{ v}^{-1} \text{ s}^{-1}$$

$$\text{Mobility of holes } \mu_h = 0.2 \text{ m}^2 \text{ v}^{-1} \text{ s}^{-1}$$

**a) Conductivity before the addition of boron atoms**

$$\begin{aligned} \sigma &= n_i e (\mu_e + \mu_h) \\ &= 2.1 \times 10^{19} \times 1.6 \times 10^{-19} (0.4 + 0.2) \\ &= 2.016 \text{ } \Omega^{-1} \text{ m}^{-1} \end{aligned}$$

**b) Conductivity after the addition of boron atoms, Boron is a P-type impurity atom**

$$\begin{aligned}\sigma &= p e \mu_h \\ &= 4.5 \times 10^{23} \times 1.6 \times 10^{-19} \times 0.2 \\ \sigma &= 14400 \Omega^{-1} \text{ m}^{-1}\end{aligned}$$

8. An N-type semiconductor has hall coefficient =  $4.16 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}$ . The conductivity is  $108 \Omega^{-1} \text{ m}^{-1}$ . Calculate its charge carrier density ' $n_e$ ' and electron mobility at room temperature.

**Given:**

$$\text{Hall Co-efficient } R_H = 4.16 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}$$

$$\text{Conductivity } \sigma = 108 \Omega \text{ m}^{-1}$$

1. For 'n' type the charge carriers density  $n_e = \frac{-1}{R_H e}$  Here the negative sign indicates the field direction alone.

$$\begin{aligned}n_e &= \frac{3\pi}{8} \frac{-1}{R_H e} \\ n_e &= \left[ \frac{3 \times 3.14}{8} \right] \left[ \frac{1}{1.6 \times 10^{-19} \times 4.6 \times 10^{-4}} \right] \\ n_e &= 1.7690 \times 10^{22} \text{ m}^{-3}\end{aligned}$$

2. Electron mobility

$$\begin{aligned}\mu_e &= \frac{\sigma_e}{n_e e} \\ &= \frac{108}{(1.7690 \times 10^{22} \times 1.6 \times 10^{-19})} \\ \mu_e &= 0.0381 \text{ m}^2 \text{ v}^{-1} \text{ s}^{-1}\end{aligned}$$

9. In an N-type semiconductor, the concentration of electron is  $2 \times 10^{22} \text{ m}^{-3}$ . Its electrical conductivity is  $112 \Omega^{-1} \text{ m}^{-1}$ . Calculate the mobility of electrons.

**Given:**

$$\text{Conductivity } \sigma = 112 \Omega^{-1} \text{ m}^{-1}$$

Carrier concentration of electron

$$n_i = 2 \times 10^{22} \text{ m}^{-3}$$

$$\text{Hall coefficient } R_H = \frac{1}{ne}$$

$$\begin{aligned}&= \frac{1}{2 \times 10^{22} \times 1.6 \times 10^{-19}} \\ &= 3.125 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}\end{aligned}$$

$$\begin{aligned}\text{Mobility } \mu &= \sigma R_H = 112 \times 3.125 \times 10^{-4} \\ \mu &= 0.035 \text{ m}^2 \text{ v}^{-1} \text{ s}^{-1}\end{aligned}$$

10. A semiconducting crystal with 12 mm long, 5 mm wide and 1 mm thick has a magnetic density of  $0.5 \text{ Wbm}^{-2}$  applied from front to back perpendicular to largest faces. When a current of 20 mA flows length wise through the specimen, the voltage measured across its width is found to be  $37 \mu\text{V}$ . What is the Hall coefficient of this semiconductor?

**Solution:**

**Given:**

**Solution:**

**Given:**

$$\text{Hall voltage } V_H = 37 \mu\text{V} = 37 \times 10^{-6} \text{ V}$$

$$\text{Breath of the material } t = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

$$\text{Current } I_H = 20 \text{ mA} = 20 \times 10^{-3} \text{ A}$$

**Magnetic flow density**

$$B = 0.5 \text{ Wbm}^{-2}$$

$$\begin{aligned} \text{Hall coefficient } R_H &= \frac{V_H t}{I_H B} \\ &= \frac{37 \times 10^{-6} \times 1 \times 10^{-3}}{20 \times 10^{-3} \times 0.5} \end{aligned}$$

$$R_H = 3.7 \times 10^{-6} \text{ C}^{-1} \text{ m}^3$$

$$\therefore \text{Hall coefficient } R_H = 3.7 \times 10^{-6} \text{ C}^{-1} \text{ m}^3$$

11. Hall coefficient of a specimen of depend silicon found to be  $3.66 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}$ . The resistivity of the specimen is  $8.93 \times 10^{-3} \text{ m}$ . Find the mobility and density of the charge carriers.

**Solution:**

$$\text{Hall coefficient } R_H = 3.66 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}$$

$$\text{Resistivity } \rho = 8.93 \times 10^{-3} \Omega\text{m}$$

**i) Density of holes**

$$\begin{aligned} n_h &= \frac{1}{R_H e} \\ &= \frac{1}{3.66 \times 10^{-4} \times 1.6 \times 10^{-19}} \\ n_h &= 1.7076 \times 10^{22} \text{ m}^{-3} \end{aligned}$$

**ii) Mobility of holes  $\mu_n = \frac{1}{\rho n e}$**

$$\begin{aligned} &= \frac{1}{8.93 \times 10^{-3} \times 1.7076 \times 10^{22} \times 1.6 \times 10^{-19}} \\ \mu_n &= 0.041 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1} \end{aligned}$$

12. The intrinsic carrier density of a semiconductor is  $2.1 \times 10^{19} \text{ m}^{-3}$ . The electron and hole mobilities are  $0.4$  and  $0.2 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$  respectively. Calculate the conductivity.

**Solution:**

**Given data:**

Intrinsic carrier concentration  $n_i = 2.1 \times 10^{19} \text{ m}^{-3}$

Mobility of electron  $\mu_e = 0.4 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$

Mobility of hole  $\mu_h = 0.2 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$

$$\begin{aligned} \text{Conductivity } \sigma &= n_i e (\mu_e + \mu_h) \\ &= 2.1 \times 10^{19} \times 1.6 \times 10^{-19} \times (0.4 + 0.2) \end{aligned}$$

Conductivity  $\sigma = 2.016 \Omega^{-1} \text{ m}^{-1}$

13. The electron mobility and hole mobility in Si are  $0.135 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$  and  $0.048 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$  respectively at room temperature. If the carrier concentration is  $1.5 \times 10^{16} \text{ m}^{-3}$ . Calculate the resistivity of Si at room temperature.

**Solution:**

**Given data:**

Carrier concentration  $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$

Mobility of electron  $\mu_e = 0.135 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$

Mobility of hole  $\mu_h = 0.048 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$

**i) Electrical Conductivity  $\sigma$**

$$\begin{aligned} \sigma &= n_i e (\mu_e + \mu_h) \\ &= 1.5 \times 10^{16} \times 1.6 \times 10^{-19} \times (0.135 + 0.048) \end{aligned}$$

$$\sigma = 0.4392 \times 10^{-3} \Omega^{-1} \text{ m}^{-1}$$

**ii) Resistivity  $\rho$  of silicon**

$$\rho = \frac{1}{\sigma}$$

$$\rho = \frac{1}{0.4392 \times 10^{-3}}$$

$$\rho = 2.2768 \Omega \text{ m}$$