MODULE-3- Superconductivity:

Zero resistance state, Persistent current, Meissner effect, Critical temperature, Critical current (Silsbee Effect) – Derivation for a cylindrical wire using ampere's law, Critical field, Formation of Cooper pairs - Mediation of phonons, Two-fluid model, BCS Theory - Phase coherent state, Limitations of BCS theory, examples of systems with low and high electron-phonon coupling, Type-I and Type-II superconductors, Formation of Vortices, Explanation for upper critical field, Josephson junction, Flux quantization, DC and AC SQUID, Charge Qubit, Numerical Problems.

Module - 3 Blow-up

Subtopics	Topics to be covered	Duration
Zero Resistance state, Persistent Current & Meissner Effect	Superconducting transition, persistent currents, Meissner effect (qualitative)	1 Hour
Critical temperature, Critical current (Silsbee Effect)-Derivation for a cylindrical wire using ampere's law	Critical temperature, critical current, Silsbee effect, Derivation for a cylindrical wire using ampere's law, critical field	1 Hour
BCS Theory & Cooper Pairs	Concepts of phonon, Electron-phonon interactions, formation of Cooper pairs, energy gap concept	1 Hour
Two-fluid Model, Examples of systems with low and high electron-phonon coupling	Division into normal and superconducting electron fractions, explanation of thermal conductivity, Examples of systems with Low and High electron-Phonon Coupling.	1 Hour
Type I & II Superconductors	M-H characteristics, Type I (complete flux expulsion), Type II (vortex formation, Explanation for Upper Critical field)	½ Hour
Andreev Reflection	Cooper pair tunneling process, retro-reflection at N-S interface	½ Hour
Josephson Junction, flux quantization, DC & AC Josephson Effect	Josephson Junction, flux quantization, DC & AC Josephson effect,	1 Hour
SQUIDs & Numerical Problems	DC and RF SQUIDs (Qualitative), Numerical problems on critical field & critical current.	2 Hour

Superconductivity: **Introduction to Superconductors**

Kamerlingh Onnes discovered the phenomenon of superconductivity in the year 1911. When he was studying the temperature dependence of resistance of Mercury at very low temperature, he found that resistance of Mercury decreases with the decrease in temperature up to a particular temperature 4.15K. At this temperature, the resistance of mercury abruptly drops to zero. Between 4.15K and 0K, Mercury offered no resistance for the flow of electric current. The phenomenon is reversible and material becomes normal once again when temperature was increased above 4.15K. He called this phenomenon as superconductivity and material, which exhibited this property as superconductor. In subsequent decades, superconductivity was observed in several other materials.

"The phenomenon in which electrical resistance of certain metals, alloys and compounds drops to zero abruptly, below a certain temperature is called superconductivity".

Zero Resistant state:

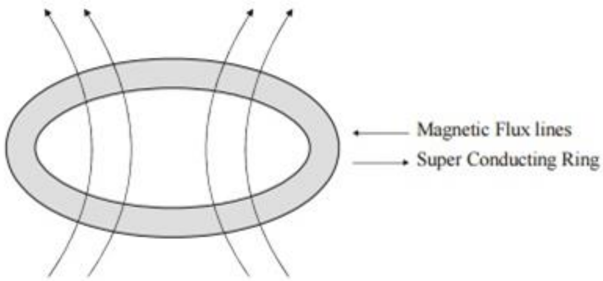
` A zero-resistance state (ZRS) refers to a condition where a material or system conducts electricity with no energy loss due to electrical resistance. The most common example is a superconductor, a material that exhibits zero electrical resistance below a certain critical temperature. However, ZRS also occurs in other systems, such as two-dimensional electron systems (2DES) subjected to strong magnetic fields and microwave irradiation, where the phenomenon is thought to be driven by radiation-induced effects that alter the electronic system's dynamics.

A superconductor, below its critical temperature, becomes a zero-resistance state, allowing electric current to flow indefinitely without any power dissipation.

Persistent current:

When a current of large magnitude is once induced in a superconducting ring then the current persists in the ring even after the removal of the field as shown in fig.

This is due to the diamagnetic property (i.e.,) the magnetic flux inside the ring will be trapped in it and hence the current persists.

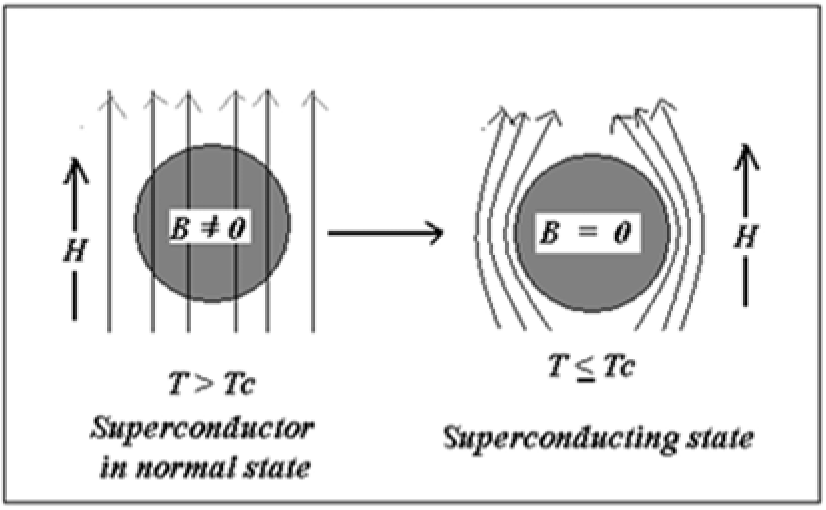


Persistant current

Meissner effect:

When a magnetic is applied to a superconducting specimen at a temperature below critical temperature (T_C), the magnetic flux lines are expelled from the interior of superconductor. This phenomenon is called Meissner effect.

When a superconductor is cooled below its critical temperature (T_c), it undergoes a phase transition and enters a superconducting state. In this state, the superconductor exhibits perfect diamagnetism, meaning it repels any external magnetic field from its interior as shown in the fig given below. This expulsion of magnetic flux is a characteristic feature of the Meissner effect.



The magnetic susceptibility of a superconductor is –ve and equal to 1. (i.e X= -1). Further, it is observed the Meissner effect is reversible. In a superconducting pure metal, the magnetic flux is expelled from the sample, irrespective of its geometry and sequence in which the magnetic field is applied.

The magnetization vector $\vec{M}$ ($\vec{M}$= magnetic dipole moment per unit volume) can be expressed in the following form,

$\vec{M} = X\vec{H}$

Where, *X is the magnetic susceptibility and* $\vec{H}$ the magnetic field intensity.

For a super conductor, X= -1. $\overline{\therefore \vec{M} = -\vec{H}}$

Therefore, magnetization in a superconductor is equal and opposite to that of magnetic field intensity.

Critical Temperature (Tc):

It is the temperature below which the material changes from normal conductor to superconductor and is denoted as Tc. The critical temperature is also called transition temperature.

Critical current (Silsbee effect):

The Silsbee effect explains how a sufficiently large current in a superconductor destroys its superconducting state.

When a current flows through a superconductor, it generates its own magnetic field (self-field). If this self-field, at the surface of the wire, reaches the material's critical magnetic field, the superconductivity will be destroyed, causing the material to become a normal conductor. The maximum current that can flow before this occurs is the critical current (Ic).

Critical Current (Ic):

Definition:

The critical current (Ic) is the maximum current a superconductor can carry without losing its superconductivity.

Relationship to H_c:

The value of Ic depends on the geometry of the superconductor and is related to the critical magnetic field. For a cylindrical wire of radius 'r', the critical current is given by the formula:

$$Ic = 2\pi rHc$$

Where:

Ic is the critical current

r is the radius of the wire

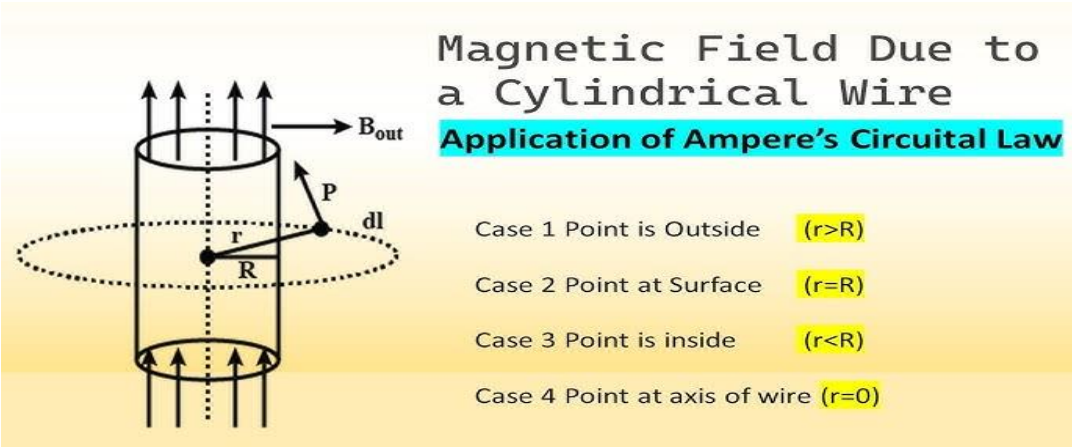
Hc is the critical magnetic field strength

Significance:

The Silsbee effect is a fundamental principle in understanding the limits of superconducting current carrying capacity. It helps explain why superconductors cannot carry infinite currents and how their superconducting state can be quenched by excessive current.

Ampere's law: Application to a infinite (long) straight cylindrical wire: (Derivation)

Consider an infinite long cylindrical wire. I is the current flows through the cylinder. To get magnetic field inside and outside the cylindrical wire, we consider 4 points and each point forms its amperian loop. Let's say P1 outside the wire, P2 outer surface of the wire,P3 inside the wire and P4 in the axis of the wire where there is no amperian loop can be formed.



We can take the point surface be at four different places.

Case 1. Point P (say P1) outside the cylindrical wire where r>R
Case 2. At a point P (say P2) at the surface (outer layer) of the cylindrical wire where r=R
Case 3. At point P (say P3) inside the cylindrical wire where r<R
Case 4. At point P (sayP4) at the axis of the wire where r=0

Case 1. Point P (say P1) outside the cylindrical wire where r>R

When the point P (Say P1) lying outside the magnetic field and amperian loop (r>R) is tangential to the each other. Therefore $\theta = 0, hence \cos 0 = 1$

By ampere's law, $$\oint \vec{B}.\vec{dl}\, cos\theta = \mu_0\, I$$

$$\oint \vec{B}.\vec{dl} = \mu_0\, I$$

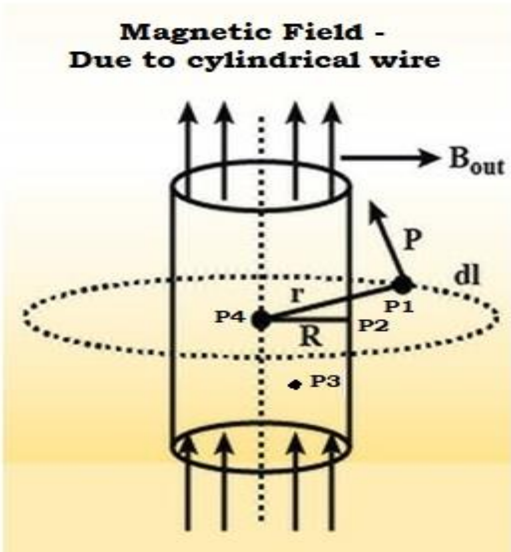
$$or\ \vec{B} \oint \vec{dl} = \mu_0\, I \qquad since\ \oint \vec{dl} = 2\pi r$$

$$\vec{B}.2\pi r = \mu_0\, I$$

$$\vec{B} = \frac{\mu_0\, I}{2\pi r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

So the magnetic field is directly proportional to radius(r),

$$\vec{B}\ \alpha\ \frac{1}{r} \ldots\ldots\ldots\ldots\ldots\ldots.\, at\ point\ P_1$$



Case 2. At a point P (say P2) at the surface (outer layer) of the cylindrical wire where r=R

Now taking point P (say P2) is on the surface of the wire where r=R,

Put r=R in equation (2), we get

$$\vec{B} = \frac{\mu_0 I}{2\pi R} = Constant \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

We can tell from equation (3) is constant because at every point of the surface, the magnetic field is constant. Hence magnetic field B is a Constant.

Case 3. At point P (say P3) inside the cylindrical wire where r<R

When point P (say P3) inside the wire, which is smaller than the present loop(P2) where r<R. Therefore, current through the third amperion loop (P3) is not same as the present loop (P2).

As current I flows uniformly, it flows across the cross-sectional area of πR^2. While the closed path encloses only part of the current which passes across the cross-sectional area of πr^2.

We know that the current flow I equal to Area of the wire, $\pi R^2 = I$

Hence current density enclosed by the path,

$$Current\ density\ per\ unit\ area\ of\ the\ cylinder\ of\ radius\ 'R'\ is,, J = \frac{I}{A} = \frac{I}{\pi R^2}$$

$$Current\ density\ per\ unit\ area\ of\ the\ amperian\ loop\ of\ radius\ 'r'\ is,, J = \frac{I}{A} = \frac{I}{\pi r^2}$$

The current density remains the same inside and outside the loop,

$$\therefore \quad J_{cylinder} = J_{amperian\ loop}$$

Total current density flowing through the πr^2 area at that point P3

$$\frac{I}{\pi R^2} = \frac{I_{amp}}{\pi r^2}$$

$$I_{amp}\ or\ \ I' = \frac{I.r^2}{R^2} \ldots\ldots\ldots\ldots (4)$$

By ampere's law at point P3,

$$\oint \vec{B}.\vec{dl} = \mu_0 I' \ldots\ldots\ldots\ldots.. (5)$$

Substitute equation (4) in (5) $\quad \vec{B} \oint \vec{dl} = \mu_0 \frac{I.r^2}{R^2}$

$$\vec{B}.2\pi r = \mu_0 \frac{I.r^2}{R^2} \qquad since\ \oint \vec{dl} = 2\pi r$$

$$\vec{B} = \frac{\mu_0 I}{2\pi R^2} r \ldots\ldots\ldots\ldots\ldots\ldots.. (6)$$

It is clear that magnetic field inside the cylindrical wire is directly proportional to “r”

$$i.e., \vec{B}\ \alpha\ r$$

At the surface r=R both the expressions coincide and the surface field is $\vec{B} = \frac{\mu_0 I}{2\pi R}$

A common practical criterion for critical current is that superconductivity is destroyed when the magnetic induction at the surface reaches a threshold value.

Using the surface-field $\vec{B}$=Bc condition and substitute the surface field from in the above equation and solve for the critical current I=Ic, we get

$$B_c = \frac{\mu_0\, I_c}{2\pi a}$$

We can get the critical current expression as ,

$$I_c = \frac{2\pi a\, B_c}{\mu_0}$$

Where Bc is the critical magnetic field.

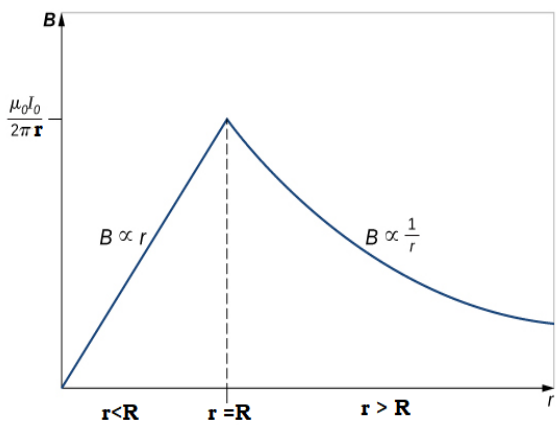
This is the required expression for the Derivation of a cylindrical wire.

Case 4: At point P (sayP4) at the axis of the wire where r=0

When the point P (say P4) is at the axis of the cylindrical wire where r=0,

$$\text{Therefore}, \vec{B} = 0 \ldots\ldots\ldots (7)$$

At the axis of the wire, there is no amperion loop we can form. Hence, where there is no loop no current will be considered through the loop.



By the following cases, we can conclude that,

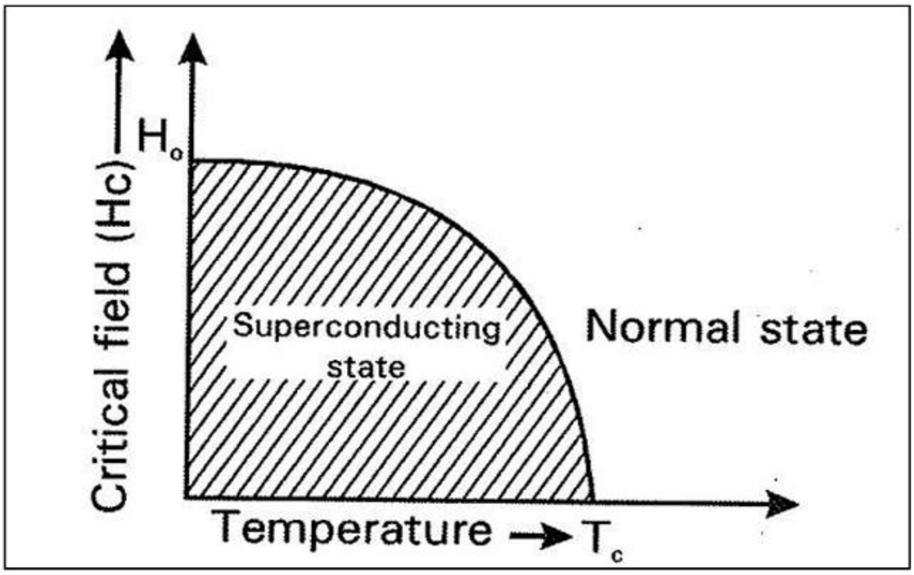
1) $\vec{B}\ \alpha\ \frac{1}{r}$ $\quad$ *outside the loop at point P1* $(r > R)$
2) $\vec{B} = Constant, at\ the\ surface\ of\ the\ point\ P2, (r = R)$
3) $B\ \alpha\ r, inside\ the\ loop\ of\ the\ point\ P3\ (r < R)$
4) $\vec{B} = 0$, t the axis *of the point P4* (r = 0)

Hence we can conclude that as radius (r) is increasing the magnetic field (B) also increase. The magnetic field becomes constant at r=R. As r>R, the magnetic field is inversely proportional to radius ‘r’ as shown in graphically.

<u>Critical field (Hc) and Temperature dependence of critical field:</u>

The magnetic field plays a significant role in the observation of the phenomenon of superconductivity. When superconductor is placed in a magnetic field, it expels magnetic lines of force completely out of the body and becomes a perfectly diamagnetic material. However, if the strength of the magnetic field is further increased, it is found that for a particular value of the magnetic field, material loses its superconducting property and becomes a normal conductor.

The value of the magnetic field at which superconductivity is destroyed is called the Critical magnetic field, denoted by Hc. It is different for different Superconductors. It was found that by reducing the temperature of the material further, superconducting property of the material could be restored. Thus, critical field does not destroy the superconducting property of the material but only reduces the critical temperature of the material. The variation of critical field with temperature for a given superconductor is shown below.



The minimum field required to destroy the superconducting property is given by

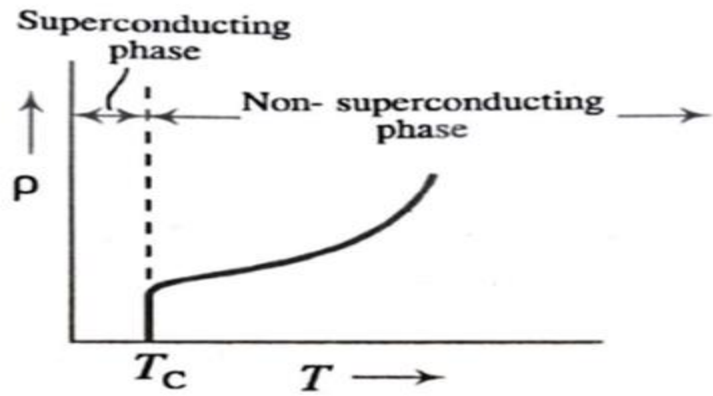
$$H_c = H_0\left(1 - \frac{T^2}{T_c^2}\right)$$

Where, H_0 is the field required to destroy the superconducting property at 0K, H_c the minimum field required to destroy the superconducting property at T in K and T_c the transition temperature of the material.

Under the influence of a magnetic field whose strength is greater than H_0, the material can never become superconductor however low the temperature may be.

Temperature dependence of resistivity of superconductors:

Temperature dependence of resistivity of superconductor is as given below.



The resistivity (ρ) decreases with decreasing temperature, similar to a normal metal, until it reaches a critical temperature Tc. At critical temperature Tc, resistivity drops abruptly to zero signifying the transition from normal state to the superconducting state of the material and it remains zero even up to 0 K.

Concept of Phonon:

Soviet physicist Igor Tamm introduced the concept of phonons in 1932. The name phonon is taken from the Greek word 'phone' which translates to sound or voice, because long wavelength phonons gives rise to sound.

"A phonon is the elementary excitation in the quantum mechanical treatment of vibrations in a crystal lattice or the quantum unit of a crystal lattice vibration."

A phonon is the quantum energy of the lattice vibration just like photons are the quantum energy of electromagnetic radiations. The study of phonon is very important in solid-state physics. They play a major role in many of the physical properties like thermal and electrical conductivity as well as in models of neutron scattering and related effects.

In the free electron theory, electrons in a crystal are treated as if they move independently, almost as if they were free particles in a metal. However, in reality, the crystal lattice, which consists of atoms or ions held in a regular repeating pattern, is not completely still. The atoms or ions in the lattice are in constant motion, vibrating about their equilibrium positions due to thermal energy. These vibrations give rise to what we call as phonons.

Phonons represent the vibrational modes of atoms or ions in a crystal lattice.

Phonons are quantized units of lattice vibrations, similar to how photons are quantized units of light. They carry both energy and momentum. When an electron moves through the crystal lattice, it can interact with phonons. These interactions can scatter electrons, leading to phenomena like electrical resistance and heat conduction. Phonons play a crucial role in understanding the behavior of electrons in a crystal, especially in explaining why some materials conduct electricity well while others do not.

Formation of Cooper pairs:

The formation of Cooper pairs is the fundamental concept of the Bardeen-Cooper-Schrieffer (BCS) theory, which explains the properties of superconductors, such as zero electrical resistance.

BCS theory (Qualitative)

Three scientists Barden, Cooper & Schrieffer in the year 1957, which is in short known as BCS theory, gave this theory of superconductivity. The BCS theory is based on the formation of Cooper pairs, which is purely a quantum mechanical concept. According to BCS theory, the basic interaction responsible for superconductivity is that two electrons attract each other via exchange of virtual phonons (they are called so because of their very short lifetime) which are emitted by lattice distortion. This is explained as.

When an electron approaches a positive ion core, it undergoes attractive coulomb interaction thus setting the ion in distortion which emits phonons. Smaller the mass of the positive ion core, the greater will be the distortion. This interaction called the electron-phonon interaction leads to scattering of the electron and causes electrical resistivity. The distortion causes an increase in the density of ions in the region of distortion. The higher density of ions in the distorted regions attracts, in its turn another electron. If a second electron comes near distorted ion, the energy of the second electron is lowered by exchange of energy between with lattice ion. Thus, a free electron exerts an attractive force on another electron through phonons.

This type of interaction, which is effective at very low temperature, is called **electron – lattice – electron interaction.** This interaction causes two electrons of opposite spin & equal and opposite momenta to attract each other in the presence of phonon field and forms a bound pair called Cooper pair.

Suppose, an electron of wave vector K emits a virtual phonon q which is absorbed by another electron having wave vector K^1. Then they are scattered as K-q and K^1+q as shown in figure. The resulting electron-electron interaction becomes attractive interaction (Vph). Two electrons interacting attractively in the phonon field to form a bonded pair is shown in figure.

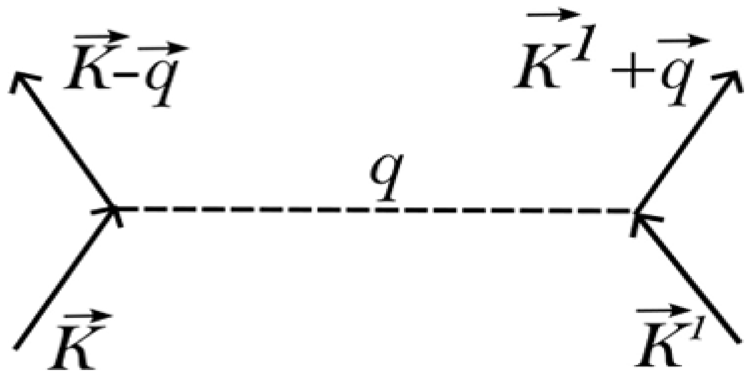


Figure: Cooper pair

At normal temperature, the attractive force is too small and pairing of electrons does not take place. At lower temperature i.e., below the critical temperature T c , the apparent force of attraction reaches a maximum value for two electrons of equal and opposite spins and opposite momentum. This force of attraction exceeds the Columbian force of repulsion between two electrons and the electron stick together and move as pairs. These pairs of electrons of opposite momenta are called **Cooper pairs.**

A pair of electrons formed by the interaction between the electrons with opposite spin and momenta in a phonon field is called a Cooper pair. The Cooper pair has a total spin of zero. As a result, the electron pairs in a superconductor are bosons. Each Cooper pair possesses a single wave function and wave functions associated with similar cooper pairs start overlapping and may extend over the entire super conductor. This dense cloud of Cooper pairs from a collective state where strong correlations arise among the motions of all pairs because of which they drift cooperatively through the material.

This leads to union of vast number of cooper pairs resulting in the entire union moving as a single unit. As we know that any number of Cooper pairs can occupy single energy state, all Cooper pairs occupy lower energy state called superconducting state. Since the density of Cooper pairs is quite high, even large currents require only a small velocity. The small velocity of Cooper pairs combined with their precise ordering minimizes collision process and hence they do not suffer any scattering. The extremely rare collisions of Cooper pairs with the lattice leads to zero resistivity.

Advantages: BCS THEORY

1. Provides an explanation of the superconductivity of Type I superconductors.
2. Explains superconductivity within the realms of classical mechanics.
3. Explains the relative difference in superconductivity between metals, better conductors at normal temperatures are terrible superconductors.
4. Provides an explanation of the superconductivity of Type I superconductors.

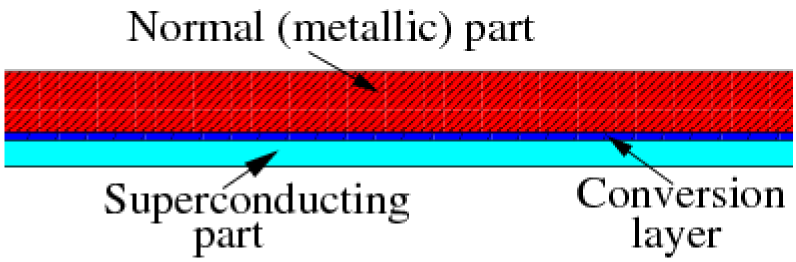
Limitations of BCS theory: Disadvantages:

1. BCS theory cannot explain high Tc superconductivity.
2. BCS assumes only weak electron-electron interaction. In some materials, motion of electron is strongly dependent on the position and spin of the neighbours' electrons. This strong interaction cannot be explained by BCS theory.
3. BCS theory is unable to predict new superconductors.
4. AC current affects the formation of cooper pairs. BCS theory cannot explain the cooper pairs are formed not only due to electron-phonon (or electron-lattice) interaction.
5. There are some other mechanisms such as d-wave pairing, p-wave pairing, BCS theory fails to explain the formation of cooper pairs by these mechanisms.

Mediations of Phonons:

We find that phonon-mediated superconductivity explains the main experimental findings, namely the displacement field and doping level dependence of the critical temperature, and the presence of two superconducting regions with different pairing symmetries that depend on the parent normal state.

Two fluid model:



When a material is cooled below its superconducting transition temperature Tc, it proceeds immediately into the superconducting phase. As the materials cools down below Tc, not all electrons in the materials jump into the superconducting phase at once. They become superconducting one by one (or rather pair –by–pair). As the material becomes colder, more electrons enters into the superconducting phase. There exists normal electrons and superconducting electrons simultaneously. The presence of two types of electrons, coexisting in the superconductor can be treated as two fluids of electrons. This is called two-fluid model.

The term fluid originates from Drude's classical fee electron theory where free electrons as treated as 'Fluid'.

So, the two fluid model of a superconductor treats each electron in the superconductor as participating in one of the two fluids: say the normal fluid or the superconducting fluid. These two fluids coexists, and they both flow simultaneously and in parallel through the superconducting fluid. The number density of electrons in normal fluid state Nn and that in superconducting fluid state Ns depends on the temperature. As temperature decreases Nn decreases and Ns increases.

According to this model, conduction electrons in a superconductor below its transition temperature divide into two types: super-electrons and normal electrons. Super electrons can move in a superconductor without any resistance, whereas normal electrons behave in the usual fashion suffering collisions and thereby experience resistance. The number of super-electrons is a function of temperature and can be expressed as

$$\boldsymbol{N_s = Nn\left[1 - \left(\frac{T}{T_C}\right)^4\right]}$$

Where, Nn is the total number of density of electrons.

Explanation:

Fig. shows the temperature variation of the fraction of super-electrons in a superconductor. Note that at absolute zero, all conduction electrons behave like super-electrons. As temperature increases, some of them start behaving like normal electrons I dependence of the and at the transition temperature, all the electrons become normal and superconductivity disappears.

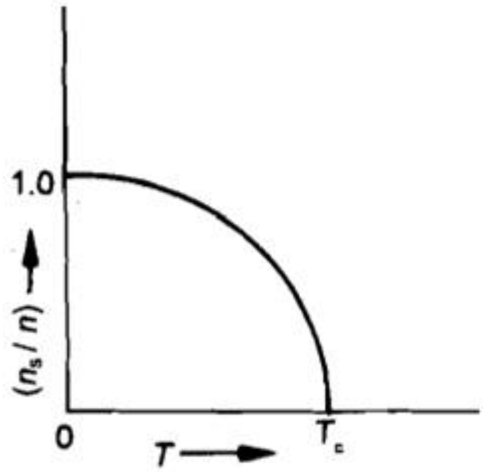


Fig. : Temperature dependence of the number of super-electrons relative to normal electrons

Number of super-electrons relative to normal electrons in superconducting state below the transition temperature, current can be carried by normal as well as super-electrons, You may ask: Why then we do not observe any resistance? In fact, the super-electrons short circuit normal electrons and we observe zero resistance. A superconducting specimen essentially behaves like two conductors connected in parallel; one having a normal resistance and the other zero resistance. This model could qualitatively explain zero resistance. But it failed to explain Meissner effect and other properties of superconductors. Moreover, this model is not based on basic principles.

London theory of superconductivity deals with the discussion on electromagnetic principles of superconductors and could explain the Meissner effect. It also used the two-fluid model and is semi-phenomenological as it uses an equation which at that time could not be derived from the first principles. The more comprehensive theory of superconductivity was proposed by BCS, which is capable of explaining all the observed phenomena relating to conventional superconductors. It starts with first principles and using quantum concepts, explains the occurrence of zero resistance, the Meissner effect, the observed temperature dependence of heat capacity and so on. Since meaningful discussion of BCS theory is possible only using advanced quantum mechanical concepts and mathematical techniques, for simplicity, we discuss it here only qualitatively.

Phase coherent state in superconductors:

The electrons in a cooper pairs have opposite spins so that the total spin is zero and the pair behaves as bosons (Bosons means particle having zero spin). Any number of them can exist in the same quantum state at the same time. All the pairs are in the same ground state and can be explained by a single wave function. This is known as Phase coherence state.

In superconductors, the phase-coherent state refers to a macroscopic quantum state where the Cooper pairs; formed by two electrons, act as a single entity, behaving like bosons. In this state, the phase of the superconducting order parameter—the collective wave function of the Cooper pairs—is well defined and uniform throughout the material. This collective, phase-locked behavior is responsible for superconductivity's defining characteristics, such as dissipation less and super fluidity.

The ability of the superconductor to flow without resistance is a direct consequence of this phase-coherent state.

Formation of the Coherent State:

These Cooper pairs then condense into a single, macroscopic quantum state.

In this state, the quantum mechanical phase of the electronic wave function is not random but is "rigid," meaning the phase is uniform and consistent across the entire superconductor. This unified phase is known as the phase-coherent state.

Quantum Tunneling:

The phase coherence allows for quantum effects like the Josephson effect, where a supercurrent can tunnel through a thin insulating barrier between two superconductors. The cooper pairs can tunnel through the insulator (without being destroyed) giving a current from one superconductor into the other. This is known as **quantum tunneling or Cooper pair tunneling.** Also called as Josephson Effect.

Electron - phonon coupling: Definition:

Electron-phonon coupling (EPC) also provides in a fundamental way an attractive electron-electron interaction, which is always present and, in many metals, is the origin of the electron pairing underlying the macroscopic quantum phenomenon of superconductivity. The interaction between electron and quantized lattice vibrations (or phonons) is known as electron-phonon coupling. It can be weak or strong.

Examples of systems with low and high electron-phonon coupling:

Examples of systems with low electron-phonon coupling include gold and other simple noble metals, which exhibit weak interactions between electrons and lattice vibrations, while high coupling examples include superconductors like cuprates and complex materials such as MgB2 and aluminum, where the electron-phonon interaction is strong enough to significantly influence material properties like superconductivity and electron mobility.

Low Electron-Phonon Coupling Systems:

In weak interaction, there is no scattering of electrons, they move easily, and hence the conductivity is high. Example; Gold (Au), Silver (Ag), Copper (Cu) etc...

Gold is a well-known example of a material with low electron-phonon coupling. This is evident in its phonon dynamics, where phonon-phonon interactions are comparatively stronger than electron-phonon interactions.

High Electron-Phonon Coupling Systems:

In strong interactions (high electron-phonon coupling) , electrons strongly affected by the lattice vibrations and hence cooper pairs are formed thereby materials becomes superconductor, Examples: Lead (Pb), Aluminum (Al), some high pressure hydrides and etc…

Aluminum:

Aluminum exhibits a relatively strong electron-phonon coupling, particularly for longitudinal phonon modes, which couple more significantly to electrons than other phonon branches.

MgB_2:

This material shows strong electron-phonon coupling in specific phonon modes, such as the E2g in-plane phonons near the zone center, which strongly couple with sigma-bonding states.

Cuprates (High-Temperature Superconductors):

Electron-phonon coupling has been proposed as a significant component for high-temperature superconductivity in cuprates. While not the sole mechanism, strong electron-boson coupling is spectroscopically observed.

Semiconductors (e.g., $MoSe_2$):

In semiconductor photo catalysts like MoSe2, electron-phonon interactions slow electron mobility, increasing their effective mass, which influences device performance and efficiency.

Types of Super Conductors:

Superconductors are classified into two types;
1. Type I Superconductor or Soft Superconductor.
2. Type II Superconductor or Hard Superconductor.

Type-I Superconductor (Soft Superconductors)

Type I superconductors exhibit complete Meissner's Effect. The graph of magnetic moment V/s magnetic field is as shown in the Figure 1. As the field strength increases, the material becomes more diamagnetic until H becomes equal to Hc. Above Hc the material allows the magnetic flux to pass through and exhibits normal conductivity. The value of critical field Hc is very small for soft superconductors. Therefore, soft superconductors cannot withstand high magnetic fields. Therefore, they cannot be used for making superconducting magnets. Ex. Hg, Pb and Zn.

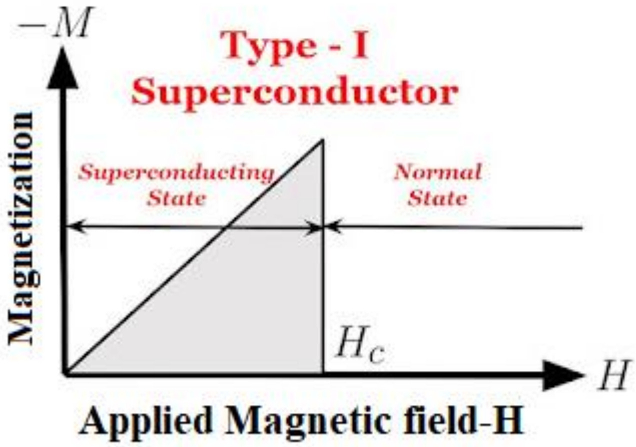


Figure 1

Type-II Superconductors (Hard Superconductors):

They do not exhibit complete Meissner effect. The superconductors are characterized by two critical fields H_{c1} and H_{c2} namely lower critical field and upper critical filed respectively. The graph of magnetic moment V/s magnetic field is as shown in the Figure 2. When $H<H_{c1}$ material exhibits perfect diamagnetism. Beyond Hc_1 partial flux penetrates and the material is

said to be Vortex State. Thus flux penetration occurs through small-channelized regions called filaments. As $H>H_{c1}$ more and more flux fills the body and thereby decreasing the diamagnetic property of the material. At H_{c2} the flux fills the body completely and material losses its diamagnetic property as well as superconducting property completely. The H_{c2} value is greater than Hc of soft superconductors. Therefore, they are used for making powerful superconducting magnets. Examples: NbTi, Nb_3Sn.

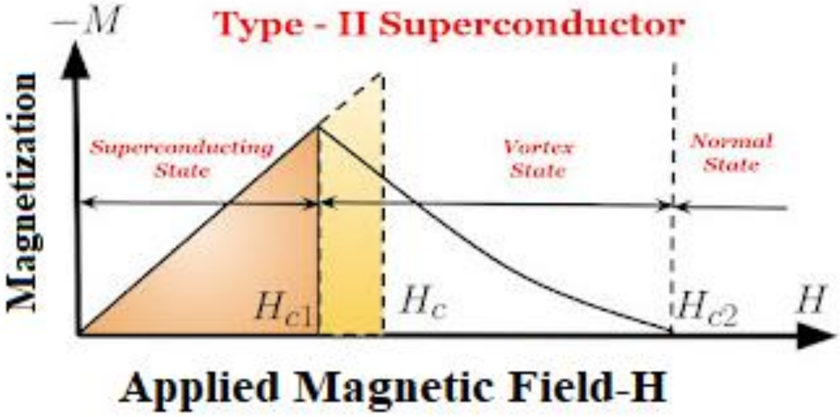
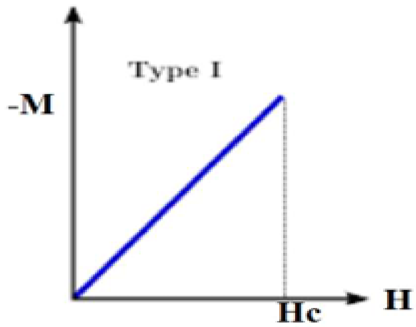
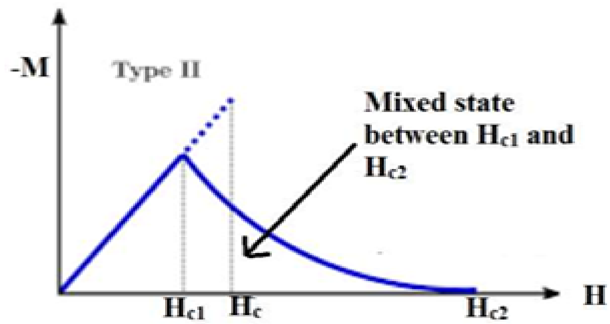
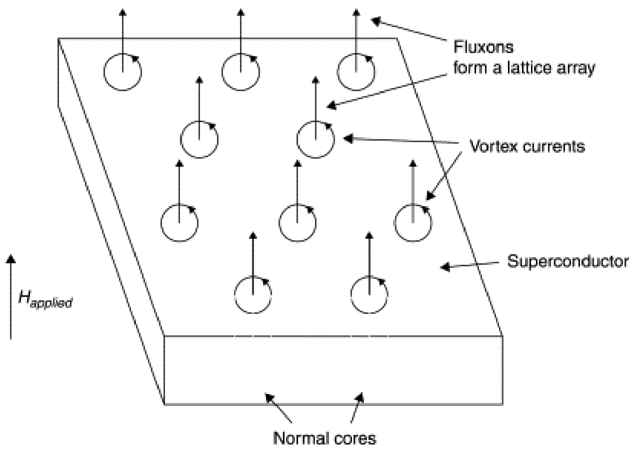


Figure-2

Difference between Type-I and Type-II superconductors.

Type-I superconductors	**Type-II superconductors**
	
➢ The material becomes normal conductor, if we increase the magnetic field beyond H_c ➢ Below T_c, they are perfect diamagnetic and exhibit complete Meissner effect ➢ They have only one critical field (H_c). At the critical field the magnetization drops to zero. ➢ Also known as soft superconductors ➢ Eg. Aluminium, lead, indium etc.	➢ They have two critical fields, H_{c1} and H_{c2} ➢ The material is perfect diamagnet below H_{c1} ➢ At H_{c1} the magnetic field lines begin penetrating the material. ➢ Mixed state is present in these materials ➢ The material becomes a normal conductor, if we increase the magnetic field beyond H_{c2} ➢ They are known as hard superconductors ➢ Eg. Nb-Sn, Nb-Ti, Va-Ga etc are examples

<u>Formation of Vortices:</u>



Vortices form in Type-II superconductors above a critical magnetic field strength (Hc1) when an external magnetic field penetrates the material in discrete flux tubes, called Abrikosov vortices, which contain a non-superconducting core surrounded by circulating supercurrents. This mixed state allows for magnetic flux to be partially expelled from the bulk of the superconductor, while the remaining flux is confined to these vortices arranged in a lattice to minimize system energy. This process enables the superconductor to remain superconducting to a higher applied magnetic field (Hc2) than Type-I superconductors, which exhibit complete magnetic expulsion.

Upper critical field (H_{c2}):

The upper critical field (Hc2) is the maximum magnetic field strength at which a material remains superconducting at a given temperature and pressure. Above Hc2, superconductivity is destroyed, and the material returns to its normal (non-superconducting) state. .

<u>Explanation for upper critical field:</u>

The upper critical field (Hc2) is the maximum magnetic field strength at which a type-II superconductor can still maintain its superconducting state at a given temperature; above this field, superconductivity is completely destroyed.

This value is a crucial characteristic for both understanding the fundamental properties of a superconductor, such as the pairing mechanism of electrons, and for practical applications like superconducting magnets.

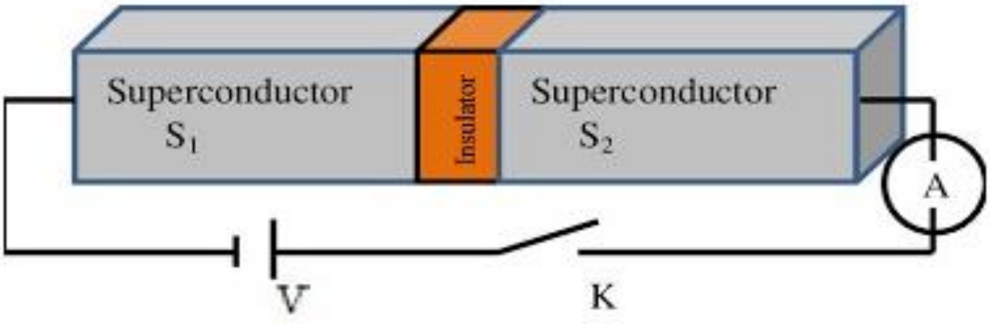
It is temperature-dependent, decreasing as temperature increases towards the critical temperature (Tc), and is influenced by factors such as the material's coherence length, Pauli paramagnetism (spin-related effects), and orbital effects (magnetic field interactions).

This property is crucial for the practical applications of superconductors, defining their operational limits in devices like superconducting magnets. It is a temperature-dependent function, and the value at absolute zero (0 K) is often considered for theoretical analysis.

<u>Josephson Junctions (Qualitative)</u>

In 1962, B.D. Josephson predicted a number of remarkable phenomena about superconductivity, which are used to understand the superconducting properties. **Persistent current in dc voltage is the principle used in Josephson device.**

An insulating material of thickness nearly 1 to 2 nm sandwiched between two different superconducting materials (S_1 and S_2) is known as Josephson device or Josephson junction. The Josephson device is shown in figure, where S_1 and S_2 are the two different superconductors.



DC Josephson Effect:

The pair of electrons present in a superconducting material is in the same phase. Whenever a Josephson junction is formed by sandwiching an insulator in between two different superconductors, the Cooper pairs present in two different superconducting materials need not be in the same phase and their density may also be different. There will be a tunnelling of superconducting electrons from one side with higher electron density to another side with lower electron density across the junction. Due to this tunnelling of electrons, a dc voltage appears across the Josephson device, even though no field is applied there. This phenomenon is said to be DC Josephson effect.

The insulating layer introduces a phase difference (ϕ) between wave function of cooper pairs on either side and hence super current appears even though the applied voltage is zero. Super current (or junction current) is given by

$$I_J = I_c sin\phi$$

Where, Ic is the maximum junction current that depends on the thickness of insulating layer. It is quite small (between 1μA to 1mA).

AC Josephson Effect:

If we apply DC voltage across the Josephson junction, it introduces an additional phase difference between the Cooper pairs and an alternating current is generated. This is known as AC Josephson Effect. The frequency of alternating current is directly proportional to applied voltage V and is given by

$$\nu = \frac{2eV}{h}$$

The photon energy of emission or absorption at the junction is hν = 2eV

$$i.e., \nu = \frac{2eV}{h} = \frac{2 \times 1.602 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$\nu = 483.5 \times 10^{12} \text{V Hz}$$

For an applied voltage of 1μV, the frequency of the ac signal is

$$\nu = 483.5 \times 10^{12}\ \text{V} = 483.5 \times 10^{12} \times 10^{-6} =$$

$$\nu = 483.5\ \text{MHz}$$

By measuring the frequency of the AC signal accurately, the value of e/h and the photon energy, 2eV are accurately measured.

SQUIDS: DC and RF SQUIDs (Qualitative)

SQUID is an acronym for ***Superconducting Quantum Interference Device***. It is based on the principle of Josephson effect. SQUID is basically a sensitive magnetometer and it is used to measure extremely weak magnetic fields as small as 10^{-21}T.

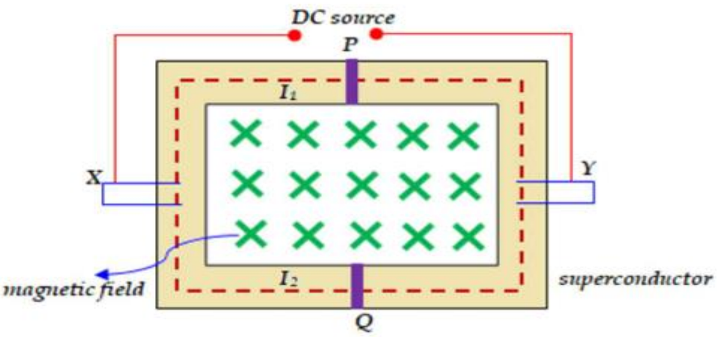
There are two types of SQUID namely

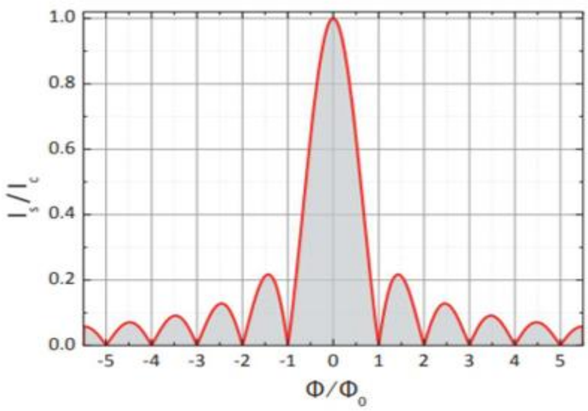
- ✓ DC SQUID and
- ✓ RF SQUID (or AC SQUID).

DC-SQUID:

DC-SQUID consists of two Josephson junctions (P and Q) arranged in parallel to form a loop and a DC source is connected across X and Y as shown in the figure. As a result, biasing current enters at X and leaves at Y. Let I_1 and I_2 be the tunneling currents across P and Q. When a magnetic field is applied perpendicular to the arrangement, a phase difference is introduced between I_1 and I_2 and they overlap to produce interference effect (very much similar to Young's double slit experiment). In superconductors the current is caused by the Cooper pairs. Cooper pairs are associated with de-Broglie waves. Applied magnetic field introduces phase shift between these waves and hence they interfere when arrive at Y. The resultant current is

$$I_r = \frac{I_c sin\frac{\pi\phi}{\phi_0}}{\frac{\pi\phi}{\phi_0}}$$



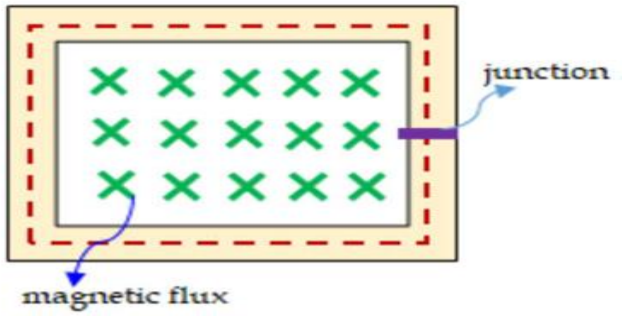


Where, ϕ is the flux linked with the SQUID and $\phi_0 = (\frac{h}{2e} = 2.06 \times 10^{-15} wb/m^2) called\ fluxoid.$

A graph of $\frac{I_r}{I_C}$ V/S $\frac{\phi}{\phi_0}$ is as shown above. The graph is exactly similar to intensity distribution 0 in single slit diffraction of light. By measuring the resultant current in the SQUID, one can determine the applied magnetic field. It is possible to measure very small magnetic fields using this arrangement.

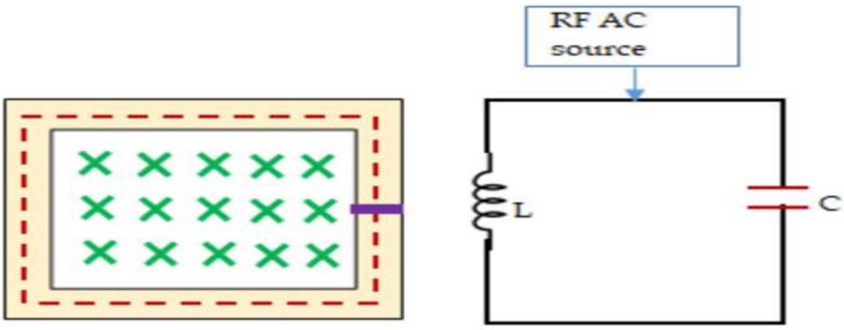
RF-SQUID (or) AC-SQUID:

The RF (Radio Frequency) SQUID is a one-junction SQUID loop and is used as a magnetic field detector. Although it is less sensitive than the DC SQUID, it is cheaper and easier to manufacture and is therefore more commonly used. It is as shown in fig (i).



RF SQUID loop is placed near a LC circuit which is connected to RF AC source as shown in Fig (ii).

The loop is immersed in a magnetic field whose flux is Φ (to be measured).
Now pass an oscillating current (I) through LC circuit from the RF source. It induces magnetic flux Φ_{RF}. This flux is coupled with the loop. The total external flux is



(ii)

$$\Phi_{ex} = \Phi + \Phi_{RF}$$

The loop is linked to circuit through mutual inductance. By chance if the flux in the loop (Φ) changes, there will be corresponding changes in Φ_{ex}. Any changes in the total flux Φ ex will induce emf (according to Faraday's law) in the LC circuit and hence voltage (V) across LC changes. By measuring the change in voltage V, one can measure the magnetic flux (Φ) and its variation w.r.t. time.

SQUIDS are used in

- Measurement of magnetic fields as small as 10^{-21}T produced by biological currents such as in human heart and brain.
- Geophysical measurements (connected with rocks, seismic waves etc)
- Nondestructive testing (testing the material without damaging it)
- Fabrication of qubits (in quantum computing)

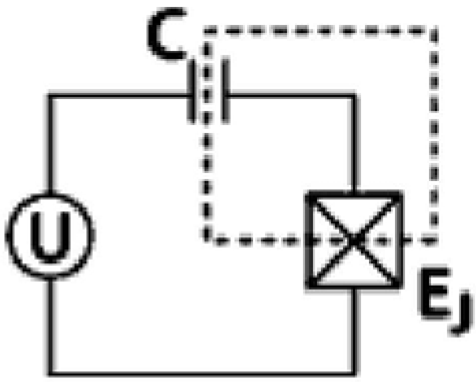
Applications in Quantum Computing:

Charge Qubit:

In quantum computing, a charge qubit ia also known as cooper pairs box. It is a qubit whose basis states are charge states. The states represent the presence of absence of excess cooper pairs in the island (dotted region in the figure). In superconducting quantum computing, a charge qubit is formed by a tiny superconducting is an coupled by Josephson junction to a superconducting reservoir.

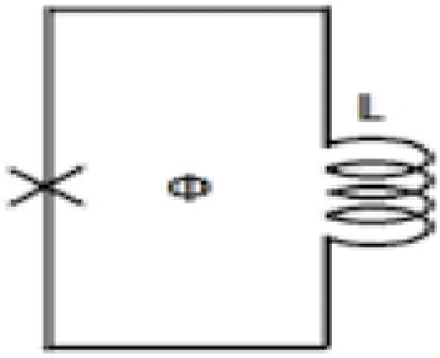
Flux Qubit:

Flux qubits (also called as persistent current qubits) are micrometer sized loops of superconducting metal that is interrupted by a number of Josephson junctions. These devices function as quantum bits. The Josephson junctions are designed so that the persistent current will flow continuously when external magnetic field is applied. Only integer number of flux quanta are allowed to penetrate the superconducting ring.



Phase Qubit:

A phase qubit is a current-biased Josephson junction, operated in the zero voltage state with a non-zero current bias. This employs a single Josephson junction and the two levels are defined by quantum oscillations of the phase different between the electrodes of the junction. DC squid is a type of phase qubit.



1. Low temperature superconductors:

When the transition temperature is low (<20 K), such superconductors are called as low temperature superconductors and they have no practical use.

2. High temperature superconductors:

When the transition temperature is high, (>20 K) they are called as high temperature superconductors.

<u>High Temperature superconductivity:</u>

Superconducting materials which exhibit superconductivity at relatively higher temperatures are called high temperature superconductors. Thus high temperature superconductors posses higher value of critical temperature compared to conventional superconductors. Most of the high temperature superconductors are found to fall into the category of ceramics. All high temperature superconductors are oxides of copper and bear Perovskite crystal structure characterized by large number of copper-oxygen layers. It was found that addition of extra copper-oxygen layer pushes the critical temperature Tc to higher values. The super currents are strong in the copper-oxygen layer and weak in the direction perpendicular to the planes. Ex: in 1986, George Bednorz and Alex Muller discovered a compound containing Lanthanum, Barium, Copper and Oxygen having Tc =30K was developed. In 1987 scientists developed a compound which is an oxide of the form YBa2Cu3O7 which is referred to as 1-2-3 compound with Tc > 90K was discovered. Following is the list of High Temperature Superconductors.

<u>High temperature superconductors:</u>

The low temperature superconductors usually have transition temperatures below 30K and must be cooled using liquid helium (4K) in order to achieve superconductivity. High temperature superconductors have been observed with transition temperatures as high as 134 K and can be cooled to superconductivity using liquid nitrogen (77K) which is much cheaper and easily accessible as compared to liquid helium.

• 1986, George Bednorz and Karl Müller at IBM, Zurich confirmed superconductivity in Perovskite structured lanthanum based cuprate oxide material Tc= 35 K).

• 1987 Nobel Prize in Physics for the discovery of "high temperature" superconductivity in ceramic materials.

• **Examples of High temperature superconductors.**

• $YBa_2Cu_3O_7$ (92K), $Bi_2Sr_2Ca_2Cu_3O_6$ (110K), $Tl_2Ba_2Ca_2Cu_3O_{10}$ (125K) and $HgBa_2Ca_2Cu_3O_8$ (134K).

• The chemical substitution in perovskite cuprates push the transition temperatures well beyond 35 Kelvin. The reason of superconductivity in high temperature superconductors cannot be explained using BCS theory. Several models emerged for explaining high temperature superconductivity but none of them fully explained the phenomenon.

Superconductor	Year	T_c (K)
K_yWO_3	1967	6.0
$LiTi_{2+y}O_4$	1973	1.2
$BaPb_{1-y}Bi_yO_3$	1975	13
$La_{2-y}Ba_yCuO_4$	1986	30
$YBa_2Cu_3O_{7-y}$	1987	90
$Ba_{1-y}K_yBiO_3$	1988	20
$BiSrCaCu_2O_{6+y}$	1988	105
$TlBa_2Ca_2Cu_3O_{9+y}$	1989	110
$HgBa_2CaCu_2O_{6+y}$	1993	120
$GdFeAsO_{1-y}$	2008	53.5

MODULE 3 – SUPERCONDUCTIVITY - NUMERICALS

1. The critical temperature of Nb is 9.15K. At zero kelvin, the critical field is 0.196T. Calculate the critical field at 8K?

$$H_C = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$
$$H_C = 0.196\left[1-\left(\frac{8}{9.15}\right)^2\right]$$
$$H_C = 0.046\,T$$

2. The superconducting transition temperature of Lead is 7.26 K. The initial field at 0K is $50\times\ 10^3\ Amp\ m{-}1$. Calculate the critical field at 6 K.

(CSE Stream, JAN/FEB – 2024 | 22S)

$$Given\ \ T_c = 7.26K, T = 6K\ and\ H_0 = 50\times10^3 Am^{-1}$$
$$H_C = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$
$$H_C = 50\times10^3\left[1-\left(\frac{6}{7.26}\right)^2\right]$$
$$H_C = 15849.32\,T\ Or\ 15.849\times10^3 Amp\ m^{-1}$$

3. Lead in the superconducting state has a critical temperature of 6.2K at zero magnetic field and a critical field of $0.064Mam^{-1}$ at 0K. Determine the critical filed at 4K.

$$H_C = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$
$$H_C = 0.064\left[1-\left(\frac{4}{6.2}\right)^2\right]$$
$$H_C = 0.0373\,T$$

4. The superconducting transition temperature of lead of 7.26 K. The initial field at 0 K is 64×10^3 Amp m^{-1}. Calculate the critical field at 5 K.

Solution:

Given data:

Critical temperature $T_C = 7.26$ K

Critical field $H_0 = 64\times10^3$ Amp m^{-1}

Temperature $T = 5K$

The critical field $H_C = H_0[1-(T/T_C)^2]$

$= 64\times10^3[1-(5/7.26)^2]$

$= 64\times10^3\times0.5257$

$H_C = 33.644\times10^3$ Amp m^{-1}

5. Calculate the critical current and current density for a wire of a lead having a diameter of 1 mm at 4.2 K. The critical temperature for lead is 7.18 K and H = 6.5 × 10^4 Am–1.

Solution:

Given data:

Critical temperature T_C = 7.18 K 7.18 K

Critical field H_o = 6.5×10^4 A m^{-1}

Temperature T = 4.2 K

Radius of the wire r = 0.5×10^{-3} m

The critical magnetic field $H_c = H_0\left[1-\left[\frac{T^2}{T_c^2}\right]\right]$

$$= 6.5 \times 10^4\left[1-\left[\frac{4.2}{7.18}\right]^2\right]$$

$$= 4.276 \times 10^4 \text{ A m}^{-1}$$

i) Critical current $I_c = 2\pi r H_c$

$$= 2 \times 3.14 \times 0.5 \times 10^{-3} \times 4.276 \times 10^4$$

$$= 134.39 \text{ A}$$

ii) Critical density $J_c = \frac{I_C}{\pi r^2}$

$$= \frac{134.39}{3.15 \times (0.5 \times 10^{-3})^2}$$

$$= 1.71 \times 10^8 \text{ A m}^{-2}$$

Critical current I_c = 134. 39 A

Critical density $J_c = 1.71 \times 10^8$ A m^{-2}

6. Find the critical current which can pass through a long thin superconducting wire of aluminum of diameter 2 mm, the critical magnetic field for aluminum is 7.9 × 10^3 A m^{-1}.

Solution:

Given data:

The critical magnetic field $H_c = 7.9 \times 10^3$ A m^{-1}

Radius r $= \frac{\text{Diameter}}{2}$

$$= \frac{2}{2} = 1 \times 10^{-3}$$

$$\text{Critical current} \quad I_C = 2\pi r H_C$$
$$= 2 \times 3.14 \times 1 \times 10^{-3} \times 7.9 \times 10^3 \text{ A m}^{-1}$$
$$I_C = 49.65 \text{ A}$$
$$\text{Critical current} \quad I_C = 49.65 \text{ A}$$

7. Calculate the critical current, which can flow through a long thin super conducting wire of diameter 1 mm. The critical magnetic field is 7.9×10^3 Amp m.

Soution:

Given data:

$$\text{Diameter of the wire } d = 1 \text{ mm} = 1 \times 10^{-3} \text{m}$$

$$\text{radius of the wire } r = \frac{d}{2} = \frac{1 \times 10^{-3}}{2} \text{ m}$$

$$\text{The critical magnetic field } H_c = 7.9 \times 10^3 \text{ Amp m}^{-1}$$

Critical current flowing through the wire

$$I_c = 2\pi r H_c$$

$$= 2 \times 3.14 \left(\frac{1 \times 10^{-3}}{2}\right) (7.9 \times 10^3)$$

$$\boxed{I_c = 24.81 \text{ Amp}}$$