Assignment - 1

1. Prove the following by induction by $n >= 0$

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

2. $\sqrt{2}$ is a irrational number

3. ~~Basic~~ construct DFA that accept all the string on (0,1) with substring of (0,1)

4. Construct DFA that accept all the strings on A, B consists any number of A's followed by B

5. Describe the language accepted by DFA

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$\delta =$

δ	0	1
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_2	q_1

$q_0 = \{q_0\}$

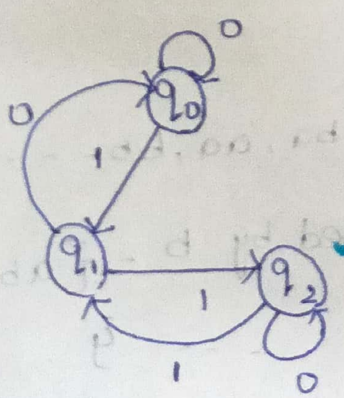
$F = \{q_1\}$

i) 10111

ii) 1011

Answers

5.



i) 10111

$\delta(q_0, 10111)$

$\delta(q_1, 0111)$

$\delta(q_0, 111)$

$\delta(q_1, 11)$

$\delta(q_2, 1)$

$\delta(q_1)$

$\therefore$ string is accepted

$\because$ final state is q_1

ii) 1011

$\delta(q_0, 1011)$

$\delta(q_1, 011)$

$\delta(q_0, 11)$

$\delta(q_1, 1)$

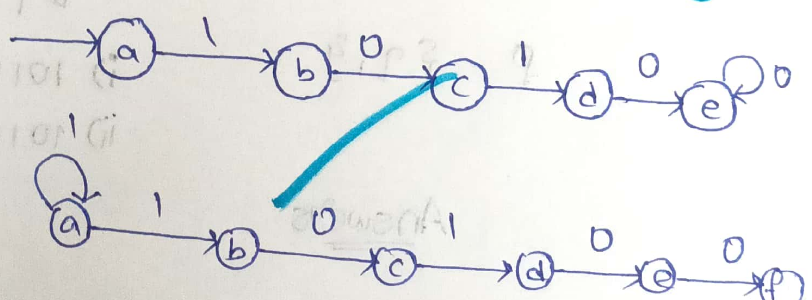
$\delta(q_2)$

∴ Here the string is not accepted as the final state is not q_1

(3) language = {01, 00, 10, 11, ...}

Sub string should be 01 = {10100, 110100... ..}

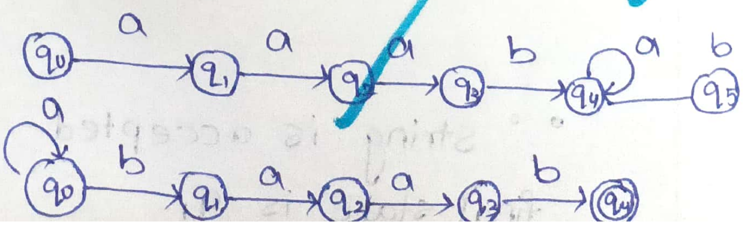
DFA :-



(4) language = {ab, ba, aa, bb, ...}

any no.of a's followed by b = {ab, aab, aaab, ...}

DFA :-



1. Mathematical induction

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} \rightarrow (1)$$

if $n=0$ in eq (1)

$$0^2 = \frac{0(0+1)(0+1)}{6}$$

$$0=0$$

if $n=k$ in eq (1)

$$1^2+2^2+3^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6} \rightarrow (2)$$

if $n=k+1$ in eq (1)

$$1^2+2^2+3^2+\cdots+k^2+(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\boxed{L.H.S \Rightarrow 1^2+2^2+3^2+\cdots+k^2+(k+1)^2}$$

by eq (2) $= \frac{k(k+1)(2k+1)}{6}+(k+1)^2$

$$= \frac{(k+1)}{6}\left[k(2k+1)+6(k+1)\right]$$

$$= \frac{(k+1)}{6}\left[2k^2+k+6k+6\right]$$

$$= \frac{(k+1)}{6}\left[2k^2+7k+6\right]$$

$$= \frac{(k+1)}{6}\left[2k^2+4k+3k+6\right]$$

$$= \frac{(k+1)}{6}\left[2k(k+2)+3(k+2)\right]$$

$$= \frac{(k+1)}{6}(k+2)(2k+3)$$

$$= R.H.S$$

$\therefore$ L.H.S = R.H.S

2. $\sqrt{2}$ is irrational !

lets assume $\sqrt{2}$ is rational

$\sqrt{2} = \frac{a}{b}$ a, b are coprimes

$b \neq 0$

$\sqrt{2}b = a$

squaring on b.s

$(\sqrt{2}b)^2 = a^2$

$2b^2 = a^2 \longrightarrow ①$

$\therefore$ [a^2 is divisible by 2]

[a is divisible by 2]

$a = 2c$ ($\because$ c is any integer)

put in eq① $2b^2 = (2c)^2$

$\cancel{2}b^2 = \cancel{4}c^2$

$b^2 = 2c^2$

$\therefore$ [b^2 is divisible by 2
b is divisible by 2]

a, b have common factors $\rightarrow$ 2

contradiction

$\therefore \sqrt{2}$ is an irrational number.