

# FUNCTIONS

SRUJANA-EE24BTECH11042

## SECTION-A(D)

- 1) Let  $g(x)$  be a function defined on  $[-1, 1]$ . If the area of the equilateral triangle with two of its vertices at  $(0, 0)$  and  $[x, g(x)]$  is  $\frac{\sqrt{3}}{4}$ , then the function  $g(x)$  is (1989-2 Marks)

- a)  $g(x) = \pm \sqrt{1-x^2}$  c)  $g(x) = -\sqrt{1-x^2}$   
 b)  $g(x) = \sqrt{1-x^2}$  d)  $g(x) = \sqrt{1+x^2}$

- 2) If  $f(x) = \cos \left[ \pi^2 \right] x + \cos \left[ -\pi^2 \right] x$ , where  $[x]$  stands for the greatest integer function, then (1991-2Marks)

- a)  $f\left(\frac{\pi}{2}\right) = -1$  c)  $f(-\pi) = 0$   
 b)  $f(\pi) = 1$  d)  $f\left(\frac{\pi}{4}\right) = 1$

- 3) If  $f(x) = 3x - 5$ , then  $f^{-1}(x)$  (1998-2Marks)

- a) is given by  $\frac{1}{3x-5}$   
 b) is given by  $\frac{x+5}{3}$   
 c) does not exist because  $f$  is not one-one  
 d) does not exist because  $f$  is not onto

- 4) If  $g(f(x)) = |\sin x|$  and  $f(g(x)) = (\sin \sqrt{x})^2$ , then (1998-2Marks)

- a)  $f(x) = \sin x^2, g(x) = \sqrt{x}$   
 b)  $f(x) = \sin x, g(x) = |x|$   
 c)  $f(x) = x^2, g(x) = \sin \sqrt{x}$   
 d)  $f$  and  $g$  cannot be determined

- 5) Let  $f : (0, 1) \rightarrow R$  be defined by  $f(x) = \frac{b-x}{1-bx}$ , where  $b$  is a constant such that  $0 < b < 1$ . Then

- a)  $f$  is not invertible on  $(0, 1)$   
 b)  $f \neq f^{-1}$  on  $(0, 1)$  and  $f^1(b) = \frac{1}{f^1(0)}$   
 c)  $(c)f = f^{-1}$  on  $(0, 1)$  and  $f^1(b) = \frac{1}{f^1(0)}$   
 d)  $f^{-1}$  is differentiable  $(0, 1)$

- 6) Let  $f : (-1, 1) \rightarrow IR$  be such that  $f(\cos 4\theta) = \frac{2}{2-\sec^2 \theta}$  for  $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ . Then the value(s) of  $f\left(\frac{1}{3}\right)$  is are

- a)  $1 - \sqrt{\frac{3}{2}}$  c)  $1 - \sqrt{\frac{2}{3}}$   
 b)  $1 + \sqrt{\frac{3}{2}}$  d)  $1 + \sqrt{\frac{2}{3}}$

- 7) The function  $f(x) = 2|x| + |x+2| - 2|x|$  has local minimum or local maximum at  $x =$  (JEE Adv.2013)

- a)  $-2$  b)  $\frac{-2}{3}$  c)  $2$  d)  $\frac{2}{3}$

- 8) Let  $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow R$  be given by  $f(x) = (\log(\sec x + \tan x))^3$ . Then (JEE Adv.2014)

- a)  $f(x)$  is an odd function

- b)  $f(x)$  is one-one function
- c)  $f(x)$  is an onto function
- d)  $f(x)$  is an even function

9) Let  $a \in \mathbb{R}$  and let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = x^5 - 5x + a$ . Then (JEE Adv.2014)

- a)  $f(x)$  has three real roots if  $a > 4$
- b)  $f(x)$  has only real root if  $a > 4$
- c)  $f(x)$  has three real roots if  $a < -4$
- d)  $f(x)$  has three real roots if  $-4 < a < 4$

10) Let  $f(x) = \sin\left(\frac{\pi}{6}\left(\frac{\pi}{2}\sin x\right)\right)$  for all  $x \in \mathbb{R}$  and  $g(x) = \frac{\pi}{2}\sin x$  for all  $x \in \mathbb{R}$ . Let  $(f \circ g)(x)$  denote  $f(g(x))$  and  $(g \circ f)(x)$  denote  $g(f(x))$ . Then which of the following are true ?

(JEE Adv 2015)

- a) Range of  $f$  is  $\left[\frac{-1}{2}, \frac{1}{2}\right]$
- b) Range of  $f \circ g$  is  $\left[\frac{-1}{2}, \frac{1}{2}\right]$
- c)  $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$
- d) There is an  $x \in \mathbb{R}$  such that  $(g \circ f)(x) = 1$

### SECTION-A(E)

1) Find the domain and the range of the function  $f(x) + \frac{x^2}{1+x^2}$ . Is the function one one? (1978)

2) Draw the graph of  $y = |x|^{\frac{1}{2}}$  for  $-1 \leq x \leq 1$ . (1978)

3) If  $f(x) = x^9 - 6x^8 - 2x^7 + 12x^6 + x^4 - 7x^3 + 6x^2 + x - 3$  find  $f(x)$  (1979)

4) Consider the following relations in the set of real numbers  $\mathbb{R}$ .

$$R = \{(x, y) : x \in \mathbb{R}, y \in \mathbb{R}, x^2 + y^2 \leq 25\} \quad R^1 = \{(x, y) : x \in \mathbb{R}, y \in \mathbb{R}, y \geq \frac{4}{9}x^2\}$$

Find the domain and the range of  $R \cap R^1$ . Is the relation  $R \cap R^1$  a function? (1979)

5) Let  $A$  and  $B$  be two sets each with a finite number of elements. Assume that there is an injective mapping from  $A$  to  $B$  and that there is an injective mapping from  $B$  to  $A$ . Prove that there is a bijective mapping from  $A$  to  $B$ .

(1981-2Marks)