2012-MA

EE24BTECH11042- srujana

1) The straight lines L_1 : x=0, L_2 : y=0; and L_3 :x+y=1 are mapped by the transformation $w = z^2$ into the curves C_1, C_2 and C_3 respectively. The angle of intersection between the curves at $w = 0$ is

   a) 0 b) $\pi/4$ c) $\pi/2$ d) π

2) In a tropical space, which of the following statements is NOT always true :
   a) Union of any finite family of compact sets is compact.
   b) Union of any family of closed sets is closed.
   c) Union of any family of connected sets having a non empty intersection is connected.
   d) Union of any family of dense subsets is dense.

3) Consider the following statements: P : The family of subsets $\left\{A_N = \left(\frac{-1}{n}, \frac{1}{n}\right), n = 1, 2, ...\right\}$ satisfies the finite intersection property.

   Q : On an infinite set X, a metric $d : X \text{ x } X \to R$ is defined as $d(x, y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$. The metric space (X, d) is compact.

   R : In a Frechet (T_1) topological space, every finite set is closed.

   S : If $f : R \to X$ is continuous, where R is given the usual topology and (X, T) is a Hausdorff (T_2) space, then f is one-one function.
   which of the above statements are correct?

   a) P and R b) P and S c) R and S d) Q and S

4) Let H be Hilbert space and $s^\perp$ denote the orthogonal complement of a set S ⊆ H. which of the following is INCORRECT?
   a) $For S_1, S_2 \subseteq H; S_1 \subseteq S_2 \implies S_1^\perp \subseteq S_2^\perp$.
   b) $S \subseteq (S^\perp)^\perp$
   c) $0^\perp = H$
   d) $S^\perp$ is always closed

5) Let H be a complex Hilbert space, $T : H \to H$ be a bounded linear operator and let T^* denote the adjoint of T . Which of the following statements are always TRUE?
   $P : \forall x, y \in H, \langle Tx, Y\rangle = \langle x, T^* y\rangle$ $Q : \forall x, y \in H, \langle x, Ty\rangle = \langle T^* x, y\rangle$
   $R : \forall x, y \in H, \langle x, Ty\rangle = \langle x, T^* y\rangle$ $S : \forall x, y \in H, \langle Tx, Ty\rangle = \langle T * x, T^* y\rangle$

a) P an Q b) P and R c) Q and S d) P and S

6) Let X={a,b,c} and let $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ be a topology defined on X . Then which of the following statements are TRUE?
$P : (X, \tau)$ is a Hausdroff space. $Q : (X, \tau)$ is a regular space.s
$R : (X, \tau)$ is a normal space. $S : (X, \tau)$ is a connected space.

a) P and Q b) Q and R c) R and S d) P and S

7) Consider the statements
P :If X is a normed linear space and M $\subseteq$ X is a subspace, then the closure $\overline{M}$ is also a subspace of X.

Q :If X is a banach space and $\sum X_n$ is an absolute convergent series in X, then $\sum X_n$ is convergent

$R : LetM_1$ and M_2 be subspaces of an inner product space such that $M_1 \cap M_2$=\{0\}.Then $\forall m_1 \in M_1, m_2 \in M_2; ||\mathbf{m_1} + \mathbf{m_2}||^2 = ||\mathbf{m_1}||^2 + |\mathbf{m_2}||^2$.

S :Let $f : X \rightarrow Y$ be a linear transformation from the banach space X into the Branch space Y. If f is continuous, then the graph of f is always compact.

The correct statements amongst the above are:

a) P and R only b) Q and R only c) P and Q only d) R and S only

8) A continuous random variable X has the probability density function

$$f(x) = \begin{cases} \frac{3}{5}e^{\frac{-3x}{5}}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

The probability density function Y=3X+2 is

a)
$$f(y) = \begin{cases} \frac{1}{5}e^{\frac{-1(y-2)}{5}}, & y > 2 \\ 0, & y \leq 2 \end{cases}$$

b)
$$f(y) = \begin{cases} \frac{2}{5}e^{\frac{3(y-2)}{5}}, & y > 2 \\ 0, & y \leq 2 \end{cases}$$

c)
$$f(y) = \begin{cases} \frac{3}{5}e^{\frac{-2(y-2)}{5}}, & y > 2 \\ 0, & y \leq 2 \end{cases}$$

d)
$$f(y) = \begin{cases} \frac{3}{5}e^{\frac{-4(y-2)}{5}}, & y > 2 \\ 0, & x \leq 2 \end{cases}$$

9) A simple random sample sample of size 10 from N$\left(\mu, \sigma^2\right)$ gives 98 % confidence interval $(20.49, 23.51)$.Then the null hypothesis $H_O : \mu = 20.5$ against $H_A : \mu \neq 20.5$
a) can be rejected at 2% level of significance
b) cannot be rejected at 5% level of significance

c) can be rejected at 10% level of significance
d) cannot be rejected at any level of significance

10) For the linear programming problem

Maximize $Z = X_1 + 2X_2 + 3X_3 - 4X4$

Subject to
$$2X_1 + 3_X2 - X_3 - X_4 = 15$$
$$6X_1 + X_2 + X_3 - 3X_4 = 21$$
$$8X_1 + 2X_2 + 3X_3 - 4X_4 = 30$$
$$X_1, X_2, X_3, X_4 \geq 0,$$

$X_1 = 4, X_2 = 3, X_3 = 0, X_4 = 2$ is

a) an optimal solution
b) a degenerate basic feasible solution
c) a non-degenerate basic feasible solution
d) a non-basic feasible solution

11) Which of the following statements is TRUE?

a) A convex set cannot have infinite many extreme points.
b) A linear programming problem can have infinite many extreme points.
c) A linear programming problem can have exactly two different optimal solutions.
d) A linear programming problem can have a non-basic optimal solution.

12) Let $\alpha = e^{2\pi i/5}$ and the matrix

$$M = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & 0 & \alpha^4 \end{bmatrix}$$

Then the trace of the matrix $I + M + M^2$ is

a) -5 b) 0 c) 3 d) 5

13) Let Let $V = \mathbb{C}^2$ be the vector space over the field of complex numbers and B=$\{(1, i), (i, 1)\}$ be a given ordered basis of V . Then for which of the following, $B^* = \{f_1, f_2\}$ is a dual basis of B over $\mathbb{C}$?

a) $f_1(z_1, z_2) = \frac{1}{2}(z_1 - iz_2), f_2(z_1, z_2) = \frac{1}{2}(z_1 + iz_2)$
b) $f_1(z_1, z_2) = \frac{1}{2}(z_1 + iz_2), f_2(z_1, z_2) = \frac{1}{2}(iz_1 + z_2)$
c) $f_1(z_1, z_2) = \frac{1}{2}(z_1 - iz_2), f_2(z_1, z_2) = \frac{1}{2}(-iz_1 + z_2)$
d) $f_1(z_1, z_2) = \frac{1}{2}(z_1 + iz_2), f_2(z_1, z_2) = \frac{1}{2}(-iz_1 - z_2)$