

# NCERT-10.3.4.1.2

EE24BTECH11042 - SRUJANA

## QUESTION:

Solve the following pair of linear equations:

$$3x + 4y = 10$$

$$2x - 2y = 2$$

## Theoretical Solution:

$$x = 1 + y \quad (1)$$

$$3(1 + y) + 4y = 10 \quad (2)$$

$$7y = 7 \quad (3)$$

$$y = 1, \quad x = 2 \quad (4)$$

## LU Decomposition Method

LU decomposition is a mathematical method used to solve systems of linear equations  $Ax = b$ , where  $A$  is a square matrix,  $x$  is an unknown vector, and  $b$  is a constant vector. It is especially useful for efficiently solving multiple systems of equations with the same coefficient matrix.

In this method, we decompose  $A$  into the product of two matrices  $L$  and  $U$ , where  $L$  is a lower triangular matrix and  $U$  is an upper triangular matrix.

This decomposition allows us to solve the system in two steps:

1) Solve  $Ly = b$

2) Solve  $Ux = y$

Given the system of equations:

$$3x + 4y = 10 \quad (5)$$

$$2x - 2y = 2 \quad (6)$$

We represent the system in matrix form:

$$A = \begin{bmatrix} 3 & 4 \\ 2 & -2 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ 2 \end{bmatrix} \quad (7)$$

## Step 1: LU Decomposition

$$A = LU \quad (8)$$

$$L = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix} \quad (9)$$

From  $LU = A$ :

$$u_{11} = 3, \quad u_{12} = 4 \quad (10)$$

$$l_{21} = \frac{2}{3}, \quad u_{22} = -2 - \frac{2}{3} \times 4 = -\frac{14}{3} \quad (11)$$

Thus:

$$L = \begin{bmatrix} 1 & 0 \\ \frac{2}{3} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 3 & 4 \\ 0 & -\frac{14}{3} \end{bmatrix} \quad (12)$$

**Step 2: Solve  $Ly = b$**

$$\begin{bmatrix} 1 & 0 \\ \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix} \quad (13)$$

$$y_1 = 10 \quad (14)$$

$$\frac{2}{3}y_1 + y_2 = 2 \quad \Rightarrow \quad y_2 = 2 - \frac{2}{3} \times 10 = -\frac{14}{3} \quad (15)$$

**Step 3: Solve  $Ux = y$**

$$\begin{bmatrix} 3 & 4 \\ 0 & -\frac{14}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -\frac{14}{3} \end{bmatrix} \quad (16)$$

$$3x_1 + 4x_2 = 10 \quad (17)$$

$$-\frac{14}{3}x_2 = -\frac{14}{3} \quad \Rightarrow \quad x_2 = 1 \quad (18)$$

$$3x_1 = 6 \quad \Rightarrow \quad x_1 = 2 \quad (19)$$

**Final Solution**

$$x = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad (20)$$

