NCERT-10.4.2.4

EE24BTECH11042 - SRUJANA

QUESTION:
Find the two consecutive positive integers, the sum of whose square is 365
Theoretical Solution:

Simplify the Equation:

$$x^2 + (x+1)^2 = 365 \tag{0.1}$$
$$2x^2 + 2x + 1 = 365 \tag{0.2}$$
$$2x^2 + 2x = 364 \tag{0.3}$$
$$x^2 + x = 182 \tag{0.4}$$

Fixed Point Iteration:
Rearrange the equation as x=g(x) to create an iterative formula:

$$x = 182 - x^2 \tag{0.5}$$
$$x_{n+1} = g(x_n) \tag{0.6}$$
$$x_{n+1} = 182 - x_n^2 \tag{0.7}$$

Select an initial guess x0
However, it is observed that for any initial value x0, the iterative process diverges, with the values tending toward -∞. Therefore, this method is unsuitable.

Newton's Method
Newton's formula :

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.8}$$
$$f(x) = x^2 + x - 182 \tag{0.9}$$
$$f'(x) = 2x + 1 \tag{0.10}$$
$$x_{n+1} = x_n - \frac{x_n^2 + x_n - 182}{2x_n + 1} \tag{0.11}$$

Choose an initial guess, x0. Since the equation involves a square term, we estimate x0 around $\sqrt{182} \approx 13.5$
Compute $x_1, x_2, \ldots,$ until $|x_{n+1} - x_n| < \epsilon$, where ϵ is the tolerance (e.g., 10^{-6}).

Secant Method:

Unlike Newton's method, the derivative is approximated using two initial points:

$$x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n+1})} \tag{0.12}$$

$$\tag{0.13}$$

Start with two initial guesses, such as x0=13.5 and x1=14, and iterate until convergence.

Eigen values of companion matrix

In this method, we find the roots of any polynomial of the form $x^n + a_{n-1}x^{n-1} \dots ax + a_0 = 0$ by finding the eigenvalues of the Companion Matrix C given below:

$$C = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \vdots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{pmatrix} \tag{0.14}$$

For the Quadratic Equation $x^2 + x = 182$, we get the following companion Matrix

$$C = \begin{pmatrix} 0 & 1 \\ 182 & -1 \end{pmatrix} \tag{0.15}$$

The roots of the equation are the eigenvalues of the matrix C, which have been calculated using the QR Decomposition with shifts process:

QR algorithm:

In the QR algorithm, the matrix A_n is decomposed into matrices Q_n and R_n as:

$$A_n = Q_n R_n \tag{0.16}$$

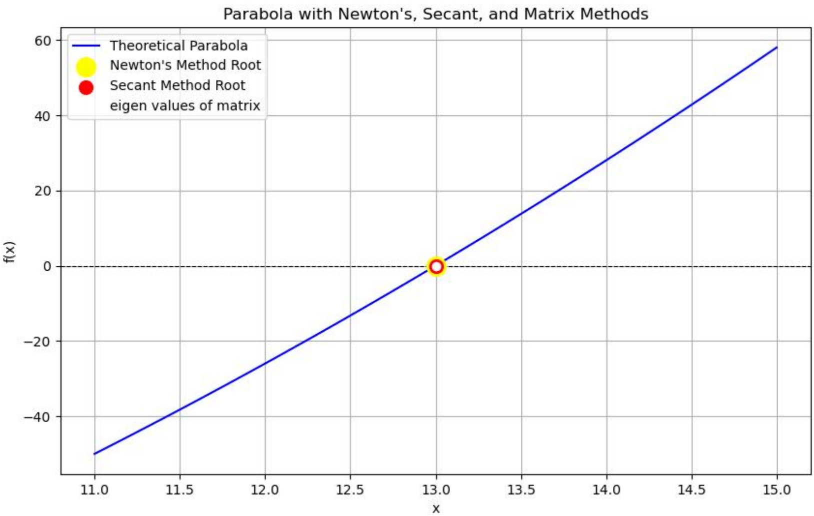
Then, the new matrix A_{n+1} is computed as:

$$A_{n+1} = R_n Q_n \tag{0.17}$$

This process is repeated until the off-diagonal elements of the matrix become negligibly small, at which point the diagonal elements approximate the eigenvalues of the original matrix.

Eigenvalues computed: [13.0 , -14.0]

we should consider only positive integer i.e .. 13


Parabola with Newton's, Secant, and Matrix Methods
Theoretical Parabola
Newton's Method Root
Secant Method Root
eigen values of matrix
f(x)
x
60
40
20
0
−20
−40
11.0
11.5
12.0
12.5
13.0
13.5
14.0
14.5
15.0