

Decision Support

A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes

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Abstract

This work presents a new fuzzy multiple attributes decision-making (FMADM) approach, i.e., fuzzy simple additive weighting system (FSAWS), for solving facility location selection problems by using objective/subjective attributes under group decision-making (GDM) conditions. The proposed system integrates fuzzy set theory (FST), the factor rating system (FRS) and simple additive weighting (SAW) to evaluate facility locations alternatives. The FSAWS is applied to deal with both qualitative and quantitative dimensions. The FSAWS process considers the importance of each decision-maker, and the total scores for alternative locations are then derived by homo/heterogeneous group of decision-makers. Finally, a numerical example illustrates the procedure of the proposed FSAWS.

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1. Introduction

Effective supply chain management (SCM) is required for companies to meet continuously changing requirements in the marketplace. By reducing supply chain risk and uncertainty, companies can enhance customer service, optimize inventory levels, improve business processes and reduce cycle times, resulting in increased competitiveness and profitability. Facility location selection is one of the most critical decisions in supply chain design and management. To optimize logistical network configuration, facilities such as factories, warehouses, distribution centers (DCs) and retail outlets must be strategically located to maximize supply chain performance and profitability (Coyle et al., 2003; Simchi-Levi et al., 2003).

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Numerous attributes (factors) of potential facility locations must be considered during the location selection process, including labor costs, proximity to markets and customers, availability of suppliers, and even quality of life issues (Finch, 2003; Heizer and Render, 2004; Stevenson, 2005). These attributes can be classified into three categories (Liang and Wang, 1991; Heragu, 1997): (a) critical attributes, e.g., availability of utilities and community attitude; these attributes determine whether or not a location will be considered for further evaluation; (b) objective attributes, e.g., investment costs and labor costs, etc; these attributes are defined in monetary/quantitative terms; (c) subjective attributes, e.g., proximity to markets and customers, political stability and quality-of-life issues; these attributes are qualitative. The majority of these attributes can be assessed by human perception and human judgment. As such, facility location selection processes typically involve the imprecision and vagueness inherent in linguistic assessment and fuzzy multiple attributes decision-making (FMADM).

Recent studies have applied FST and its approaches to generate and solve facility location selection problems. Liang and Wang (1991) developed an algorithm for facility site selection based on FST concepts and hierarchical structure analysis. Kuo et al. (1999) proposed a decision support system (DSS) by integrating FST and the AHP in selecting a site for a new convenience store (CVS). Additionally, Kuo et al. (2002) developed a DSS for locating new CVSs by integrating fuzzy AHP and an artificial neural network (ANN). Liang (1999) created a fuzzy multiple attribute decision-making (FMADM) method to identify the optimal alternative based on ideal and anti-ideal point concepts. Chen (2001) developed a new FMADM approach for resolving the DC location selection problem under fuzzy environments based on a stepwise ranking procedure. Chu (2002) presented a fuzzy technique for order preference by similarity to ideal solution (TOPSIS) model to solve the facility location selection problem under GDM. Kahraman et al. (2003) used four fuzzy multi-attribute group decision-making (FMAGDM) approaches in evaluating facility locations.

Four primary conventional methods are frequently used for solving facility location selection problems: FRS, break-even analysis, center-of-gravity method and the transportation model (Finch, 2003; Kahraman et al., 2003; Heizer and Render, 2004; Stevenson, 2005). Among these four methods, only FRS is in the MADM class. The conventional factor rating system (FRS), also known as the multifactor rating system or scoring method, is a very popular and easily applied subjective decision-making method (Heragu, 1997). The simple additive weight (SAW) method, also known as the weighted sum method, is the most widely used MADM method (Hwang and Yoon, 1981; Chang and Yeh, 2001; Virvou and Kabassi, 2004). The basic principle of SAW is to obtain a weighted sum of the performance ratings of each alternative under all attributes (MacCrimmon, 1968; Chen and Hwang, 1992). The SAW consists of two basic steps (Hwang and Yoon, 1981; Kabassi and Virvou, 2004): (1) scale the values of all attributes to make them comparable; (2) sum up the values of the all attributes for each alternative.

The logic of the SAW method is used in FRS to derive total scores for individual alternatives which allows ranking by order of preference (Heragu, 1997; Finch, 2003; Heizer and Render, 2004; Stevenson, 2005). Although these conventional FRS approaches have been successfully applied to select facility locations, these approaches are less effective when dealing with the inherent imprecision of linguistic valuation in the decision-making process (Liang and Wang, 1991; Chen, 2001; Kahraman et al., 2003).

As indicated by literature review, many concepts and approaches have been integrated with the FST to enhance its capability to handle MADM problems with imprecise attributes. Variations of FST, FRS, and SAW can also be integrated to solve the facility selection problem under GDM. Integration of these methods has six key advantages.

First, from the perspective of a practical operating mechanism, decision-making or problem-solving procedures in the PDCA (plan, do, check, and action) management cycle should be easily understood and applied by any organization (Stevenson, 2005). A simulation by Zanakis et al. (1998) evaluated eight MADM methods: SAW; multiplicative exponential weighting (MEW); TOPSIS; elimination and (et) choice translating reality (ELECTRE); and four AHPs. The rank-reversal dimension indices in the simulation disclosed the following performance order for these eight methods: SAW and MEW performed the best, followed by TOPSIS and AHPs. The ELECTRE method performed the worst. In addition, Chang and Yeh (2001) confirmed the superiority of SAW in an empirical study of the three evaluation methods (SAW method, weighted product and TOPSIS). The findings of these studies suggest that simpler evaluation techniques are often superior. Second, the logic and principle of the SAW method is reflected in the FRS procedure; therefore both quali-

tative and quantitative dimensions can be considered in the FRS process by fusing FRS and SAW. Third, the most existing approaches such as AHP-based methods require complex computation. For example, in AHP-based approaches, exhaustive pair-wise comparison can be extremely time consuming if the MADM problem includes numerous attributes (Takeda, 1982). Fourth, although several fuzzy SAW approaches have been proposed previously by Baas and Kwakernaak (1977), Kwakernaak (1979), Cheng and McInnis (1980), Bonissone (1982), Dubois and Prade (1982), and Chen and Klein (1997) have been proposed, no study has yet integrated FST, FRS, and SAW to solve the facility location selection problem. Fifth, the availability of handy forms for combining subjective and objective attributes vaguely defined as well as precisely defined and the employment of linguistic terms facilitates human judgment for the assessment of preference. Sixth, in Kahraman et al. (2003) and the above-mentioned studies, the aggregating process of preferences of individual decision-makers in GDM problems did not explicitly consider the degree of importance of individual decision-makers.

This work therefore proposes a new fuzzy simple additive weighting system (FSAWS) for solving the facility location selection problem under fuzzy homo/heterogeneous GDM environments. The remainder of this paper is organized as follows. Section 2 presents a brief review of FST. Section 3 introduces and describes the FSAWS. Section 4 illustrates the procedures in the proposed system using a numerical example. Conclusions are drawn in Section 5.

2. Fuzzy set theory

Zadeh (1965) pioneered the use of FST to address problems involving fuzzy phenomena. In a universe of discourse X , a fuzzy subset \tilde{A} of X is defined with a membership function $\mu_{\tilde{A}}(x)$ that maps each element x in X to a real number in the interval $[0, 1]$. The function value of $\mu_{\tilde{A}}(x)$ signifies the grade of membership of x in \tilde{A} . When $\mu_{\tilde{A}}(x)$ is large, its grade of membership of x in \tilde{A} is strong (Keufmann and Gupta, 1991).

2.1. Preliminaries

In order to develop the proposed system, some definitions and properties relative to this study are needed. They are stated as follows.

Definition 1 (Dubois and Prade, 1978; Keufmann and Gupta, 1991). A fuzzy set $\tilde{A} = (a, b, c, d)$ on R , $a < b < c < d$, is called a trapezoidal fuzzy number if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ \frac{(x-d)}{(c-d)}, & c \leq x \leq d, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

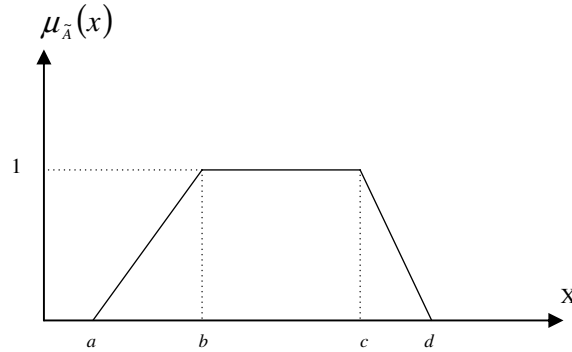
where a, b, c, d are real numbers. As Fig. 1 illustrates, the trapezoidal fuzzy number can be denoted by (a, b, c, d) . The x in interval $[b, c]$ gives the maximal grade of $\mu_{\tilde{A}}(x)$ i.e., $\mu_{\tilde{A}}(x)=1$; it is the most probable value of the evaluation data. Constants c and d are the lower and upper bounds of the available area for the evaluation data. These constants reflect the fuzziness of the evaluation data (Liang, 1999).

Trapezoidal fuzzy numbers are the most widely used forms of fuzzy numbers because they can be handled arithmetically and interpreted intuitively. Therefore, trapezoidal fuzzy numbers are used in this study.

Property 1 (Keufmann and Gupta, 1991; Liang and Wang, 1991; Chen and Hwang, 1992; Chiou et al., 2005). Given two trapezoidal fuzzy numbers $\tilde{A} = (a, b, c, d)$ and $\tilde{B} = (e, f, g, h)$, four main operations of these two fuzzy numbers can be expressed as follows:

(1) Addition of two trapezoidal fuzzy numbers \oplus

$$\tilde{A} \oplus \tilde{B} = (a + e, b + f, c + g, d + h), \quad a \geq 0, e \geq 0. \quad (2)$$

Fig. 1. A trapezoidal fuzzy number \tilde{A} .

(2) Multiplication of two trapezoidal fuzzy numbers \otimes

$$\tilde{A} \otimes \tilde{B} = (ae, bf, cg, dh), \quad a \geq 0, e \geq 0. \quad (3)$$

(3) Multiplication of any real number k and a trapezoidal fuzzy number \otimes

$$k \otimes \tilde{A} = (ka, kb, kc, kd), \quad a \geq 0, k \geq 0. \quad (4)$$

(4) Division of two trapezoidal fuzzy numbers $/$

$$\tilde{A} / \tilde{B} = \left(\frac{a}{h}, \frac{b}{g}, \frac{c}{f}, \frac{d}{e} \right), \quad a \geq 0, e \geq 0. \quad (5)$$

Property 2. Given any real number k and a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$, division operation ($/$) of the two numbers can be expressed as follows:

(1) Division of any real number k and a fuzzy number $/$

$$k / \tilde{A} = \left(\frac{k}{d}, \frac{k}{c}, \frac{k}{b}, \frac{k}{a} \right), \quad a \geq 0, k \geq 0. \quad (6)$$

(2) Division of a trapezoidal fuzzy number and any real number k $/$

$$\tilde{A} / k = \left(\frac{a}{k}, \frac{b}{k}, \frac{c}{k}, \frac{d}{k} \right) = \frac{1}{k} \otimes \tilde{A}, \quad a \geq 0, k \geq 0. \quad (7)$$

The proofs for the two operations are straightforward and thus omitted.

Property 3. Given two trapezoidal fuzzy numbers $\tilde{A} = (a, b, c, d)$, $\tilde{B} = (e, f, g, h)$ and any real number k , four commutative operations of these two numbers can be expressed as follows:

$$\tilde{A} \oplus \tilde{B} = \tilde{B} \oplus \tilde{A}, \quad (8)$$

$$k \oplus \tilde{A} = \tilde{A} \oplus k, \quad (9)$$

$$\tilde{A} \otimes \tilde{B} = \tilde{B} \otimes \tilde{A}, \quad (10)$$

$$k \otimes \tilde{A} = \tilde{A} \otimes k, \quad (11)$$

if $k \geq 0, a \geq 0, e \geq 0$.

The proofs for the four operations are straightforward and thus omitted.

Property 4. Yao and Wu, 2000. The signed distance of trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is defined as

$$d(\tilde{A}) = \frac{1}{4}(a + b + c + d). \quad (12)$$

Property 5. Yao and Chiang, 2003. From the perspective of membership grade, the signed distance method is superior to the centroid method for defuzzifying a fuzzy number.

In this study, fuzzy numbers represent aggregated fuzzy weights and total fuzzy scores. To identify the optimal alternative, fuzzy numbers are transformed into crisp real numbers to rank alternatives. Four well-known defuzzification methods are commonly employed: the centroid method (or center of area (COA)), mean of maximal (MOM), α -cut method and signed distance method (Zhao and Govind, 1991; Yager and Filev, 1994; Tsaur et al., 1997; Tang et al., 1999; Yao and Wu, 2000). Each method has advantages and disadvantages (Klic and Yan, 1995). Although the centroid method and the signed distance are the simplest and most popular in practice, the signed distance method is adopted to defuzzify the related fuzzy numbers based on the above property 5.

2.2. Linguistic variables and fuzzy numbers

In FST, conversion scales are applied to transform linguistic terms into fuzzy numbers. Determining the number of conversion scales is generally intuitive: while too few conversion scales reduce analytical discrimination capability, too many conversion scales make the system overly complex and impractical. Eight conversion scales are frequently used to convert linguistic terms to fuzzy numbers (Chen and Hwang, 1992). In the current study, a scale of 1–5 is used for importance weight and a scale of 1–9 is used for rating in the manner employed by Liang and Wang (1991) and Liang (1999).

Given the fuzzy nature of the facility location selection problem, importance weights of individual attributes and ratings of alternatives versus various subjective criteria are used as linguistic variables in this study. Table 1 lists importance weights of individual attributes and Table 2 presents ratings of alternatives versus various subjective criteria considered as linguistic variables. The trapezoidal fuzzy number is easily used and interpreted. For example, a very significant weight of a specific attribute can be measured by a trapezoidal fuzzy number denoted by (7, 10, 10, 10) (Table 1). Additionally, the trapezoidal fuzzy number can further represent monetary/quantitative terms (Liang, 1999; Zimmermann, 2001). For example, “approximately equal to

Table 1
Linguistic variables and fuzzy numbers for the importance weight

Linguistic variables	Fuzzy numbers
Very low (VL)	(0, 0, 0, 3)
Low (L)	(0, 3, 3, 5)
Medium (M)	(2, 5, 5, 8)
High (H)	(5, 7, 7, 10)
Very high (VH)	(7, 10, 10, 10)

Table 2
Linguistic variables and fuzzy numbers for the ratings

Linguistic variables	Fuzzy numbers
Very poor (VP)	(0, 0, 0, 20)
Between very poor and poor (B. VP & P)	(0, 0, 20, 40)
Poor (P)	(0, 20, 20, 40)
Between poor and fair (B. P & F)	(0, 20, 50, 70)
Fair (F)	(30, 50, 50, 70)
Between fair and good (B. F & G)	(30, 50, 80, 100)
Good (G)	(60, 80, 80, 100)
Between good and very good (B. G & VG)	(60, 80, 100, 100)
Very good (VG)	(80, 100, 100, 100)

\$700” can be represented by (690, 700, 700, 710); “between \$760 and \$800” can be represented by (750, 760, 800, 807); the crisp number \$300 can be represented by (300, 300, 300, 300) (Liang, 1999).

3. A new fuzzy simple additive weighting system

This section describes a new systematic FMADM approach, FSAWS, for location selection by integrating FST, FRS, and SAW. The FSAWS procedure is applicable to individual and group decision settings. In a group setting, fuzzy assessments by decision-makers can be aggregated by several methods. Among them, five are more popular, including mean, median, max, min, and mixed operators (Buckley, 1984). Although the mean (average) operator is the most widely used aggregation method (Liang and Wang, 1991), the importance (or reliability) of individual decision-makers (or experts) may be unequal in practice. Sometimes there are decision-makers, such as the executive manager of operations department, who are more important or reliable, or some experts who are more experienced than others in the decision group. In such a case, it is called heterogeneous (non-homogeneous) group decision-making problem. Another case would be considered homogeneous group decision-making problem (Ölçer and Odabaşı, 2005). The comparative importance of individual decision-makers must be considered to enhance the performance of evaluation in facility location decision-making problems. This study proposes a new FSAWS for solving the facility location selection problem for homo/heterogeneous GDM in a fuzzy environment.

The algorithm for the proposed approach will be developed in the following three major states: (1) rating state (2) aggregation state and (3) selection state. Fig. 2 illustrates a conceptual model of the proposed method. In the rating state, in order to establish the decision matrix for each decision-maker, decision-makers express their opinions (or performance ratings) of alternatives with respect to each attribute by questionnaires. These ratings are generally in fuzzy data form. The fuzzy data can be linguistic terms or verbal assessments. This state aims to convert fuzzy data into trapezoidal fuzzy numbers. In the aggregation state, a weight and attribute based aggregation method for a homo/heterogeneous group of decision-makers is employed. Aggregation is necessary for weights and subjective attributes. After the normalized weights are computed and the degree of importance of decision-makers is assessed, all performance ratings are aggregated under each subjective attribute for each alternative. In the selection state, the fuzzy weights of individual attributes and the total fuzzy scores of individual alternatives for homo/heterogeneous group of decision-makers are defuzzified in the defuzzification phase of the last state. These alternatives are then ranked by crisp value of total scores.

The FSAWS procedure based on above conceptual model is as follows:

Step 1: Form a committee of decision-makers. Choose the attributes and identify the prospective alternative facility locations.

A committee of decision-makers is formed to determine the most appropriate alternative location. The numerous attributes can be divided into three categories – critical, objective and subjective – in a location selection problem. An alternative is infeasible if it cannot meet the requirements of critical attributes. The subjective attributes are defined qualitatively and assessed in linguistic terms represented by fuzzy numbers. The objective attributes are defined in monetary/quantitative terms.

Step 2: Determine the degree of importance (or reliability) of the decision-makers. If the degrees of importance (or reliability) of decision-makers are equal, then the group of decision-makers is deemed a homogeneous group. Otherwise, the group is deemed a heterogeneous (non-homogeneous) group. Assume that there is a committee of k decision-makers (or experts) ($D_t, t = 1, 2, \dots, k$) who are responsible for assessing m alternatives ($A_i, i = 1, 2, \dots, m$) under each of the n attributes ($C_j, j = 1, 2, \dots, n$) as well as the importance of the attributes. The degrees of importance (or reliability) of decision-makers are $I_t, t = 1, 2, \dots, k$, where $I_t \in [0, 1]$ and $\sum_{t=1}^k I_t = 1$. If the importance (or reliability) and weight of each decision-maker is considered, then the fuzzy weights, $\tilde{\omega}_t, t = 1, 2, \dots, k$, of decision-makers are assigned according to the importance determined by interviewing the final decision-maker. Finally, the degree of importance I_t is defined as follows:

$$I_t = \frac{d(\tilde{\omega}_t)}{\sum_{t=1}^k d(\tilde{\omega}_t)}, \quad t = 1, 2, \dots, k, \quad (13)$$

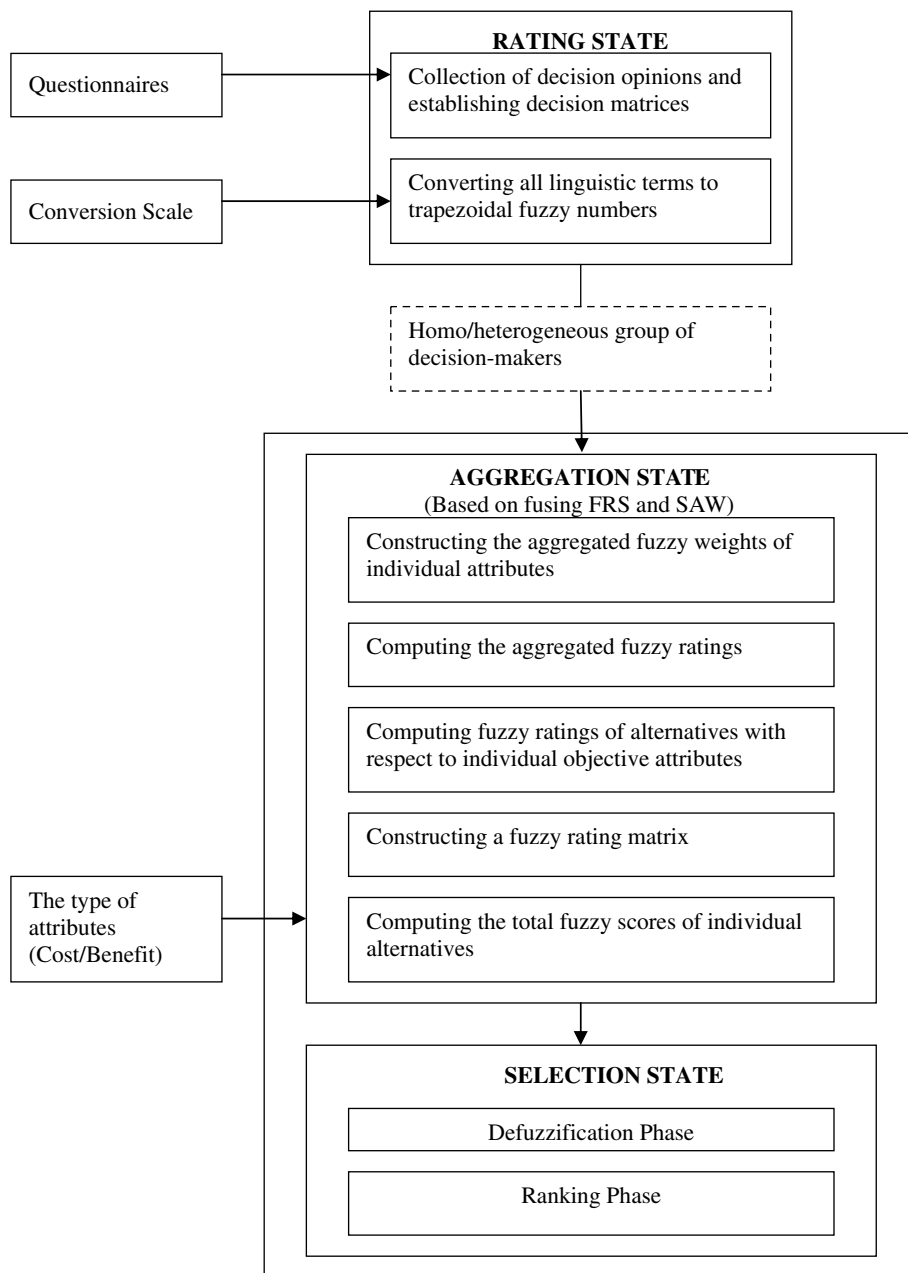


Fig. 2. The conceptual model of the proposed approach.

where $d(\tilde{\omega}_t)$ gives the defuzzified value of the fuzzy weight by using the signed distance.

If $I_1 = I_2 = \dots = I_k = \frac{1}{k}$, the group of decision-makers is called a homogeneous group; otherwise, the group of decision-makers is called a heterogeneous (non-homogeneous) group.

Step 3: Introduce linguistic weighting variables (Table 1) for decision-makers to assess attributes importance, and compute aggregated fuzzy weights of individual attributes. Let $\tilde{W}_{jt} = (a_{jt}, b_{jt}, c_{jt}, d_{jt})$, $j = 1, 2, \dots, n; t = 1, 2, \dots, k$, be the linguistic weight given to subjective attributes C_1, C_2, \dots, C_h , and objective attributes $C_{h+1}, C_{h+2}, \dots, C_n$ by decision-maker D_t . The aggregated fuzzy attribute weight, $\tilde{W}_j = (a_j, b_j, c_j, d_j)$, $j = 1, 2, \dots, n$, of attribute C_j assessed by the committee of k decision-makers is defined as

$$\tilde{W}_j = (I_1 \otimes \tilde{W}_{j1}) \oplus (I_2 \otimes \tilde{W}_{j2}) \oplus \cdots \oplus (I_k \otimes \tilde{W}_{jk}), \quad (14)$$

where $a_j = \sum_{t=1}^k I_t a_{jt}$, $b_j = \sum_{t=1}^k I_t b_{jt}$, $c_j = \sum_{t=1}^k I_t c_{jt}$, $d_j = \sum_{t=1}^k I_t d_{jt}$.

Step 4: Defuzzify the fuzzy weights of individual attributes; compute the normalized weights and construct the weight vector.

To defuzzify the weights of the fuzzy attributes, the signed distance is adopted. The defuzzification of \tilde{W}_j , denoted as $d(\tilde{W}_j)$, is therefore given by

$$d(\tilde{W}_j) = \frac{1}{4}(a_j + b_j + c_j + d_j), \quad j = 1, 2, \dots, n. \quad (15)$$

The crisp value of the normalized weight for attribute C_j , denoted as W_j , is given by

$$W_j = \frac{d(\tilde{W}_j)}{\sum_{j=1}^n d(\tilde{W}_j)}, \quad j = 1, 2, \dots, n, \quad (16)$$

where $\sum_{j=1}^n W_j = 1$. The weight vector $W = [W_1, W_2, \dots, W_n]$ is therefore formed.

Step 5: Use linguistic rating variables (Table 2) for decision-makers to assess fuzzy ratings of alternatives with respect to individual subjective attributes, and then pool them to obtain the aggregated fuzzy ratings. Let $\tilde{x}_{ijt} = (o_{ijt}, p_{ijt}, q_{ijt}, s_{ijt})$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, h$, $t = 1, 2, \dots, k$, be the linguistic suitability rating assigned to alternative location A_i for subjective attribute C_j by decision-maker D_t . Let us further define \tilde{x}_{ij} as the aggregated fuzzy rating of alternative A_i for subjective attribute C_j , such that

$$\tilde{x}_{ij} = (I_1 \otimes \tilde{x}_{ij1}) \oplus (I_2 \otimes \tilde{x}_{ij2}) \oplus \cdots \oplus (I_k \otimes \tilde{x}_{ijk}), \quad (17)$$

which can subsequently be represented and computed as

$$\tilde{x}_{ij} = (o_{ij}, p_{ij}, q_{ij}, s_{ij}), \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, h, \quad (18)$$

where $o_{ij} = \sum_{t=1}^k I_t o_{ijt}$, $p_{ij} = \sum_{t=1}^k I_t p_{ijt}$, $q_{ij} = \sum_{t=1}^k I_t q_{ijt}$, $s_{ij} = \sum_{t=1}^k I_t s_{ijt}$.

Step 6: The decision-makers in the committee assess the fuzzy (or crisp) costs or benefits associated with various alternatives versus the objective attributes and then compute fuzzy ratings of alternatives with respect to individual objective attributes.

The objective attributes are determined in various units and must be transformed into dimensionless indices (or ratings) to ensure compatibility with the linguistic ratings of the subjective attributes. The alternatives with the minimum cost (or maximum benefit) should have the highest rating. Based on the principle stated above, let $\tilde{r}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$, $i = 1, 2, \dots, m$, $j = q, q+1, \dots, n$, $q = h+1$, be the fuzzy (or crisp) cost or benefit associated with various alternatives to alternative location A_i for objective attributes C_j . Eqs. (19) and (20) shown in the following are applied transform the objective attributes:

$$\tilde{x}_{ij} = \left\{ \tilde{r}_{ij} / \max_i \{d_{ij}\} \right\} \otimes 100, \quad i = 1, 2, \dots, m, \quad j = q, q+1, \dots, n, \quad (19)$$

where $\max_i \{d_{ij}\} > 0$, \tilde{x}_{ij} denotes the transformed fuzzy rating of fuzzy (or crisp) benefit \tilde{r}_{ij} , \tilde{x}_{ij} can also be represented by fuzzy number $\tilde{x}_{ij} = (o_{ij}, p_{ij}, q_{ij}, s_{ij})$, $i = 1, 2, \dots, m$, $j = q, q+1, \dots, n$, $q = h+1$. In addition, \tilde{x}_{ij} becomes larger when \tilde{r}_{ij} is larger

$$\tilde{x}_{ij} = \left\{ \min_i \{a_{ij}\} / \tilde{r}_{ij} \right\} \otimes 100, \quad i = 1, 2, \dots, m, \quad j = q, q+1, \dots, n, \quad (20)$$

where $\min_i \{a_{ij}\} > 0$, \tilde{x}_{ij} denotes the transformed fuzzy rating of fuzzy (or crisp) cost \tilde{r}_{ij} , \tilde{x}_{ij} can also be represented by fuzzy number $\tilde{x}_{ij} = (o_{ij}, p_{ij}, q_{ij}, s_{ij})$, $i = 1, 2, \dots, m$, $j = q, q+1, \dots, n$, $q = h+1$, but \tilde{x}_{ij} becomes smaller when \tilde{r}_{ij} is larger.

Step 7: Construct a fuzzy rating matrix based on fuzzy ratings.

The fuzzy rating matrix \tilde{M} can be concisely expressed in matrix format

$$\tilde{M} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}, \quad (21)$$

where $\tilde{x}_{ij}, \forall i, j$ is the aggregated fuzzy rating of alternative $A_i, i = 1, 2, \dots, m$ with respect to attribute C_j .

Step 8: Derive total fuzzy scores for individual alternatives by multiplying the fuzzy rating matrix by their respective weight vectors.

Obtained total fuzzy score vector by multiplying the fuzzy rating matrix \tilde{M} by the corresponding weight vector W , i.e.,

$$\begin{aligned} \tilde{F} = \tilde{M} \otimes W^T &= \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \otimes \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix} = \begin{bmatrix} \tilde{x}_{11} \otimes W_1 \oplus \tilde{x}_{12} \otimes W_2 \oplus \cdots \oplus \tilde{x}_{1n} \otimes W_n \\ \tilde{x}_{21} \otimes W_1 \oplus \tilde{x}_{22} \otimes W_2 \oplus \cdots \oplus \tilde{x}_{2n} \otimes W_n \\ \vdots \\ \tilde{x}_{m1} \otimes W_1 \oplus \tilde{x}_{m2} \otimes W_2 \oplus \cdots \oplus \tilde{x}_{mn} \otimes W_n \end{bmatrix} \\ &= \begin{bmatrix} \tilde{f}_1 \\ \tilde{f}_2 \\ \vdots \\ \tilde{f}_m \end{bmatrix} = [\tilde{f}_i]_{m \times 1}, \end{aligned} \quad (22)$$

where $\tilde{f}_i = (r_i, s_i, t_i, u_i), i = 1, 2, \dots, m$.

Step 9: Compute a crisp value for each total score using a defuzzification method and select the alternative(s) with the maximum total score.

Rank total fuzzy scores $\tilde{f}_1, \tilde{f}_2, \dots, \tilde{f}_m$ by the signed distance to determine the best location. Determine crisp total scores of individual locations by the following defuzzification equation:

$$d(\tilde{f}_i) = \frac{1}{4}(r_i + s_i + t_i + u_i), \quad i = 1, 2, \dots, m, \quad (23)$$

where $d(\tilde{f}_i)$ gives the defuzzified value (crisp value) of the total fuzzy score of location A_i by using the signed distance.

The ranking of the locations can then be preceded with the above crisp value of the total scores for individual alternatives.

4. Illustrative example

In this section, the example shown in Liang and Wang's paper (Liang and Wang, 1991) and Liang's paper (Liang, 1999) is used to illustrate the facility location selection process employed in the proposed system.

Step 1: A high-tech company plans to identify a site to build a new plant. After initial screening and consideration of the critical attribute availability of skilled workers (classified as both critical and subjective), three alternatives A_1, A_2 and A_3 are selected for further evaluation. A committee of four decision-makers, D_1, D_2, D_3 and D_4 is formed to determine the best alternative site. Four selection attributes are considered: (a) transportation availability (C_1); (b) availability of skilled workers (C_2); (c) climatic conditions (C_3) and (d) investment cost (C_4).

These attributes are classified into two groups again (Table 3). The subjective attributes include transportation availability, availability of skilled workers, and climatic conditions. The objective attributes include only investment cost.

Step 2: After a committee meeting with all decision-makers, all decision-makers are considered equal in importance and reliability ($I_1 = I_2 = I_3 = I_4 = \frac{1}{4}$). Therefore, this committee is a homogeneous group.

Step 3: Use linguistic weighting variables and their respective fuzzy numbers (Table 1) to assess the importance weights for each attributes (Table 4). Based on the assessment values (Table 4), compute fuzzy weights of individual attributes by Eq. (14) (Table 5).

Step 4: Compute the defuzzified values of the aggregated fuzzy weights (Table 6) by Eq. (15) and the normalized weights of attributes using Eq. (16) (Table 6). These weights yield the values of the weight vector $W = [0.2116, 0.2455, 0.2635, 0.2794]$.

Step 5: Assess the fuzzy ratings of the three alternatives by using the linguistic rating variables and their respective fuzzy numbers (Table 2) with respect to each subjective criterion, and then compute an aggregated fuzzy rating for each alternative-criterion combination by Eq. (17). Table 7 presents the computed results.

Step 6: Assess the fuzzy costs associated with various alternatives versus the objective criterion then compute fuzzy ratings of alternatives with respect to the objective criterion using Eq. (20). Table 8 presents the results.

Step 7: With the aggregated ratings (Tables 7 and 8), construct the fuzzy rating matrix (Table 9).

Step 8: Combine Tables 6 and 9 using Eq. (22) to obtain total fuzzy scores for each location. Table 10 shows the resulting scores.

Step 9: Use the defuzzification equation shown in Eq. (23), to obtain the crisp values of the total scores (Table 10).

Table 3
Categorized plant site selection attributes

Subjective attributes	Objective attributes
Transportation availability	Investment cost
Availability of skilled workers	
Climatic conditions	

Table 4
The importance weights of the attributes

Criteria	Decision-makers			
	D_1	D_2	D_3	D_4
C_1	M	VH	M	H
C_2	H	VH	VH	M
C_3	VH	H	H	VH
C_4	VH	H	VH	VH

Table 5
The fuzzy weights of the attributes and the aggregated fuzzy weights

Attributes	Decision-makers				AFW
	D_1	D_2	D_3	D_4	
C_1	(2, 5, 5, 8)	(7, 10, 10, 10)	(2, 5, 5, 8)	(5, 7, 7, 10)	(4, 6.75, 6.75, 9)
C_2	(5, 7, 7, 10)	(7, 10, 10, 10)	(7, 10, 10, 10)	(2, 5, 5, 8)	(5.25, 8, 8, 9.5)
C_3	(7, 10, 10, 10)	(5, 7, 7, 10)	(5, 7, 7, 10)	(7, 10, 10, 10)	(6, 8.5, 8.5, 10)
C_4	(7, 10, 10, 10)	(5, 7, 7, 10)	(7, 10, 10, 10)	(7, 10, 10, 10)	(6.5, 9.25, 9.25, 10)

Note: The AFW represents the aggregated fuzzy weights.

Table 6

The defuzzified values of the aggregated fuzzy weight and the normalized weights of attributes

Method	Attribute			
	C_1	C_2	C_3	C_4
Defuzzified values	6.6250	7.6875	8.2500	8.7500
Normalized weights	0.2116	0.2455	0.2653	0.2794

Table 7

The decision-makers' evaluation under subjective attributes and the aggregated ratings

Attribute	Alternatives	Decision-makers' assessment				Aggregated ratings
		D_1	D_2	D_3	D_4	
C_1	A_1	F	B. P & F	B. F & G	F	(22.5, 42.5, 57.5, 77.5)
	A_2	B. F & G	F	G	VG	(50, 70, 77.5, 92.5)
	A_3	B. P & F	G	B. P & F	B. F & G	(22.5, 42.5, 65, 85)
C_2	A_1	B. P & F	F	P	F	(15, 35, 42.5, 62.5)
	A_2	F	B. F & G	P	F	(22.5, 42.5, 50, 70)
	A_3	B. F & G	G	B. P & F	B. F & G	(30, 50, 72.5, 92.5)
C_3	A_1	G	G	G	B. F & G	(52.5, 72.5, 80, 100)
	A_2	B. F & G	P	B. F & G	VG	(35, 55, 70, 85)
	A_3	G	B. F & G	B. P & F	F	(30, 50, 65, 85)

Table 8

Fuzzy costs and ratings of the alternatives under objective attribute

A_i	Material costs (Million)	Labor costs (Million)	Total investment costs (Million)	Fuzzy objective ratings
A_1	(18, 20, 30, 32)	(15, 15, 15, 15)	(33, 35, 45, 47)	(46.8, 48.9, 62.9, 66.7)
A_2	(14, 15, 15, 16)	(8, 10, 15, 16)	(22, 25, 30, 32)	(68.8, 73.3, 88, 100)
A_3	(12, 16, 16, 18)	(10, 10, 10, 10)	(22, 26, 26, 28)	(78.6, 84.6, 84.6, 100)

Table 9

The fuzzy rating matrix

A_i	Attributes			
	C_1	C_2	C_3	C_4
A_1	(22.5, 42.5, 57.5, 77.5)	(15, 35, 42.5, 62.5)	(52.5, 72.5, 80, 100)	(46.8, 48.9, 62.9, 66.7)
A_2	(50, 70, 77.5, 92.5)	(22.5, 42.5, 50, 70)	(35, 55, 70, 85)	(68.8, 73.3, 88, 100)
A_3	(22.5, 42.5, 65, 85)	(30, 50, 72.5, 92.5)	(30, 50, 65, 85)	(78.6, 84.6, 84.6, 100)

The crisp total scores in Table 10 show that the ranking order in descending scores for the three alternatives is A_2 , A_3 , and A_1 . Therefore the committee should recommend alternative A_2 as the best facility site of the three alternatives for establishing a new plant. Table 12 illustrates this analytical result, which is consistent with similar studies by Liang and Wang (1991) and Liang (1999). Furthermore, the importance (or reliability) of decision-makers is assigned according to the importance determined by interviewing the final decision-maker. The fuzzy weights of decision-makers are $\tilde{\omega}_i$ ($\tilde{\omega}_1 = (5, 7, 7, 10)$, $\tilde{\omega}_2 = (7, 10, 10, 10)$, $\tilde{\omega}_3 = (0, 3, 3, 5)$, $\tilde{\omega}_4 = (2, 5, 5, 8)$). Through the procedure of computation in the proposed method, Table 11 presents the results. Based on the crisp total scores displayed in Tables 10 and 11 the ranking order (Table 12) of the three alternative sites for a homogeneous group is clearly A_2 , A_3 , A_1 and A_3 , A_2 , A_1 for a heterogeneous group. Before selecting A_3 or A_2 , the final decision-maker can conduct further investigations and sensitivity analyses with various $\tilde{\omega}_i$.

Table 10
The fuzzy and crisp total scores under homogeneous group

Alternatives		Total scores
A_1	Fuzzy numbers	(35.44, 50.35, 61.51, 77.07)
	Defuzzified values (crisp)	56.10
A_2	Fuzzy numbers	(44.55, 60.22, 71.71, 87.10)
	Defuzzified values (crisp)	65.90
A_3	Fuzzy numbers	(41.99, 58.08, 72.32, 91.03)
	Defuzzified values (crisp)	65.86

Table 11
The fuzzy and crisp total scores under heterogeneous group

Alternatives		Total scores
A_1	Fuzzy numbers	(35.09, 50.30, 61.23, 76.90)
	Defuzzified values (crisp)	55.88
A_2	Fuzzy numbers	(43.32, 58.18, 70.21, 86.02)
	Defuzzified values (crisp)	64.18
A_3	Fuzzy numbers	(47.21, 63.47, 75.72, 94.49)
	Defuzzified values (crisp)	70.22

Table 12
The comparison of results for three papers

Papers	Basis	Ranking method	Decision group	Ranking order
Liang and Wang (1991)	The concepts of FST and the hierarchical structure analysis	Ranking fuzzy suitability indices	Homogeneous	A_2, A_3, A_1
Liang (1999)	The concepts of ideal and anti-ideal points, FST and the hierarchical structure analysis	TOPSIS	Homogeneous	A_2, A_3, A_1
This paper	The concepts of FST, SAW and the FRS	Ranking fuzzy total scores	Homogeneous	A_2, A_3, A_1
			Heterogeneous	A_3, A_2, A_1

Based on the above application, this work presents the following strong and weak points of the proposed approach.

The strong points of the model include the following:

- Objective and subjective attributes important to the specific site problem being addressed are classified as critical, objective, and subjective. The proposed approach incorporates subjective attributes as well as quantifiable costs and benefits by fusing the FRS method with the SAW method.
- The proposed model takes into consideration that there might be some real world situations in which decision-maker weights are also fuzzy in addition to attribute weights and performance ratings. The model can adequately handle the inherent uncertainty and imprecision of the human decision-making process.
- The proposed approach produces an overall desirability score for each alternative.
- The proposed model offers a simple and easily understood, flexible model in a wide range of such semi-structured decision-making problems.
- The proposed approach does not insist on consensus but rather synthesizes a representative outcome from decision-makers judgments.
- The proposed methodology is less complex than existing methods for ranking fuzzy numbers.

The weaknesses of the model include the following:

- The proposed model is applicable only to a multi-attribute, single-facility location.
- The proposed approach incorporates/requires rating and weighting factors based on subjective judgments.
- The proposed approach does not track the logical consistency of judgments used to determine ratings and weights.

5. Conclusions

This study proposed an effective FSAWS suitable for solving the facility location selection problem under a fuzzy homo/heterogeneous GDM environment.

For practical purposes, decision-making approaches and/or problem-solving methods employed during the planning stage of the PDCA management cycle should be easily understood and applied. Facility location selection involves subjective, vague and imprecise assessments which are by nature fuzzy. Fuzzy assessments expressed in linguistic terms are often intuitive and effective for decision-makers during the evaluation process. The proposed FSAWS, which fits such a profile, is established to solve the facility location selection problem, in which the importance weights of all attributes and the ratings of different alternative locations with respect to subjective attributes are assessed in linguistic variables represented by fuzzy numbers rather than crisp values. Additionally, the fuzzy (or crisp) costs or benefits of the objective attributes are transformed into fuzzy ratings to ensure compatibility with linguistic ratings of subjective attributes.

The main novel points and merits of the proposed method are threefold: First, the conventional FRS based on SAW combined with FST is extended to the domain location selection under homo/heterogeneous group decisions in fuzzy environments. In addition, the proposed system incorporates subjective and objective attributes.

Second, the proposed system allows MADM problems to accommodate linguistic terms represented as fuzzy numbers. This facilitates the creation of a decision procedure that is more realistic than existing systems. Consequently, the FSAWS procedure is intuitive, user friendly, effective. Furthermore, because the procedure is compliant with design rules for man–machine interface of human behavior and cause models in ergonomics, FSAWS is applicable to other management decision problems such as supplier evaluation, personnel selection and project management.

Third, because the proposed system uses simplified ranking fuzzy numbers rather than the complicated procedures mentioned in Liang's (1999) paper, computation is much faster. The proposed system gives the decision-maker the flexibility to select any FMADM methods.

The solution procedure for a FMADM problem is a non-linear recursive process evaluating related entities. Based on previous development and investigations of FMADM problems, the supporting method cannot be considered an effective means of discovering an “objective truth” or as a single-pass technique without a posteriori accommodations in practice. Useful FMADM models should function within a DSS context to aid managers in understanding problems and potential solutions so that optimal decisions can be made.

Future studies may extend the proposed FSAWS system to incorporate other factors in the analysis and computerization, e.g., the defuzzification method, the membership function for sensitivity (robustness), different conversion scales for fuzzy numbers and weights.

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