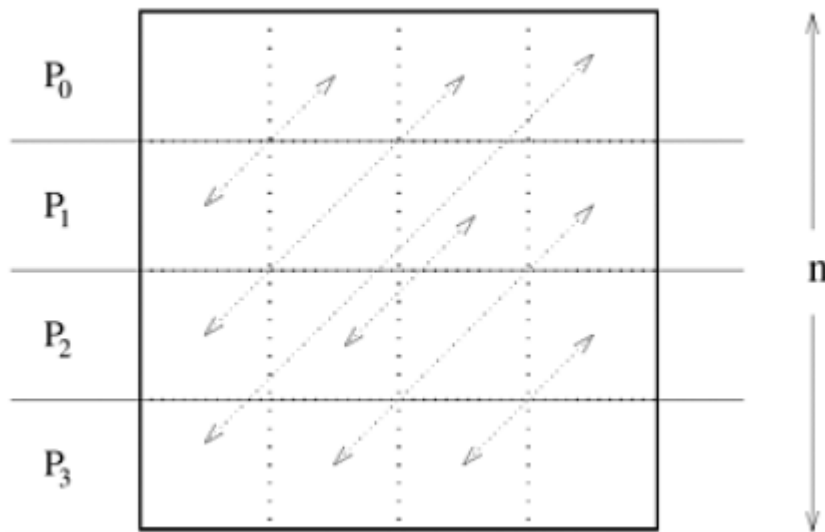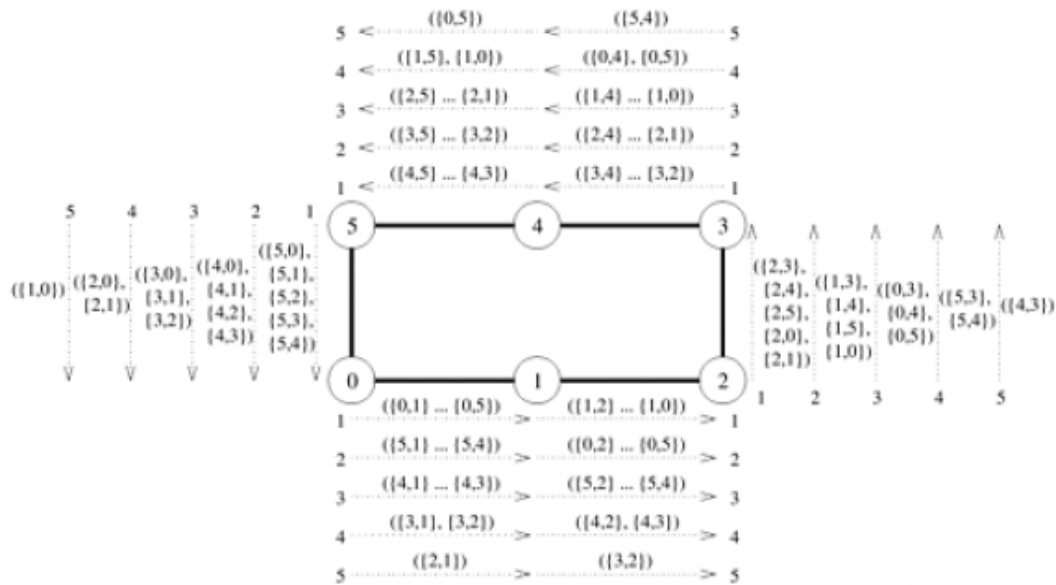# All-to-All Personalized Communication

All-to-all personalized communication in transposing a 4 x 4 matrix using four processes

All-to-all personalized communication on a six-node ring

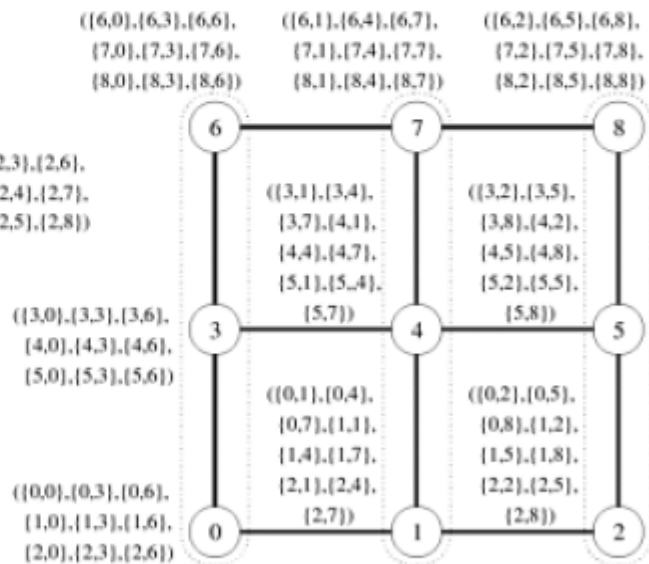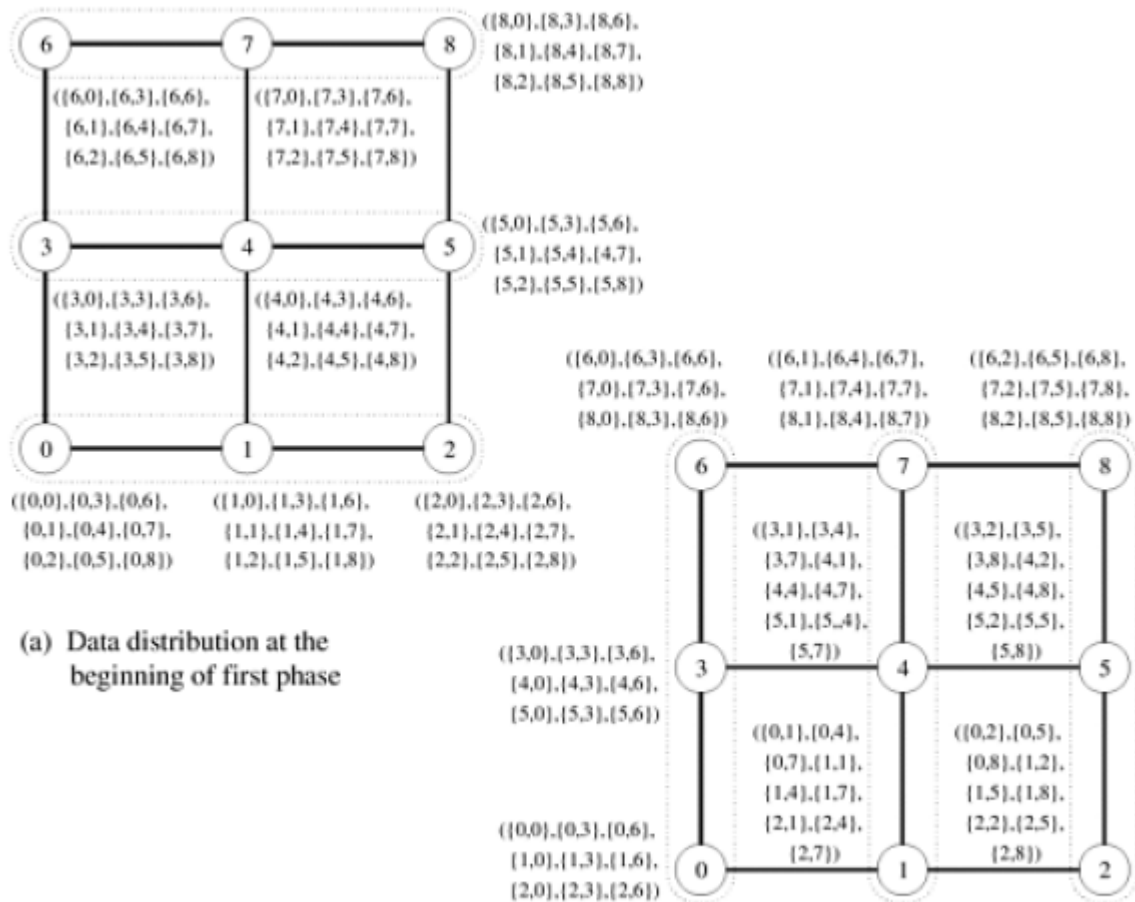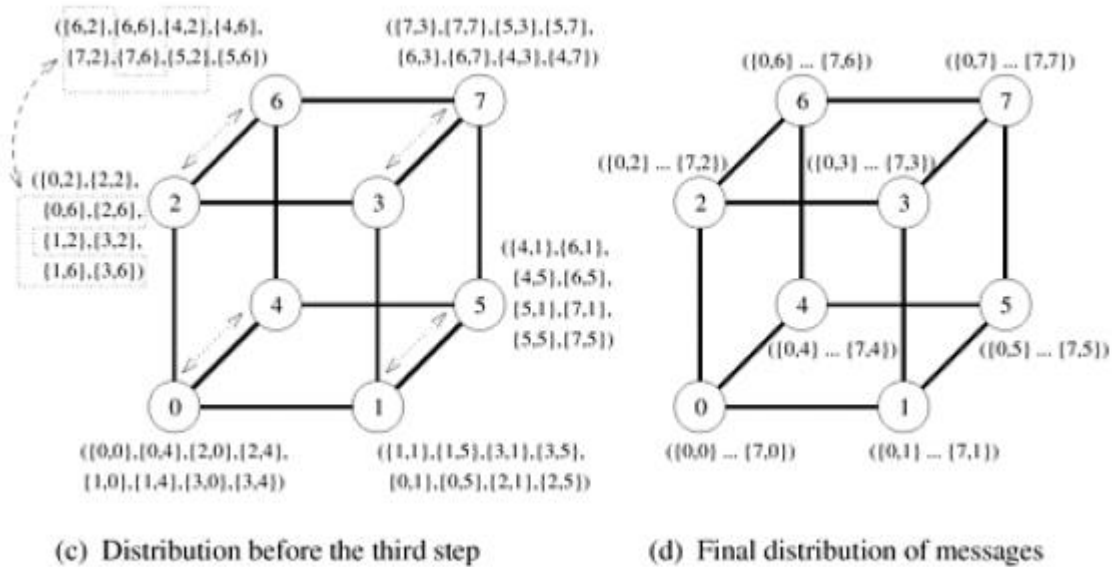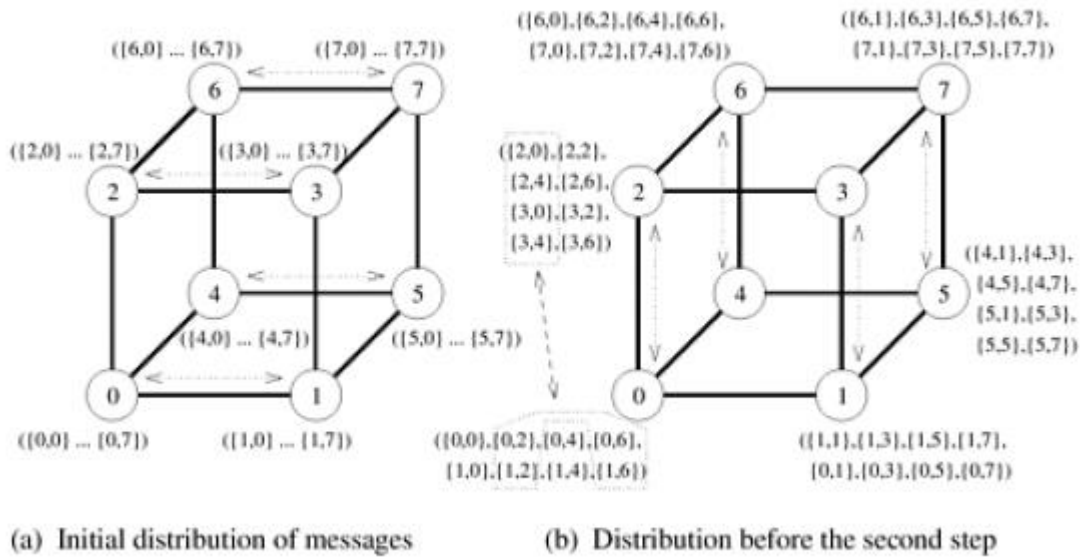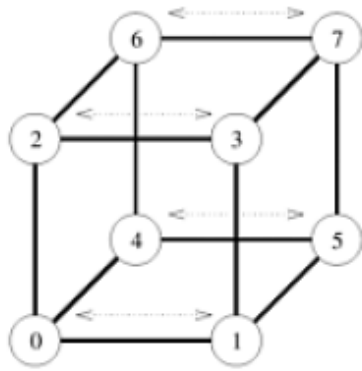# The distribution of messages at the beginning of each phase of all-to-all personalized communication on a 3 x 3 mesh

(a) Data distribution at the beginning of first phase

```
6 ——— 7 ——— 8   ({8,0},{8,3},{8,6},
                   {8,1},{8,4},{8,7},
({6,0},{6,3},{6,6},   {8,2},{8,5},{8,8})
{6,1},{6,4},{6,7},
{6,2},{6,5},{6,8})
({7,0},{7,3},{7,6},
{7,1},{7,4},{7,7},
{7,2},{7,5},{7,8})

3 ——— 4 ——— 5   ({5,0},{5,3},{5,6},
                   {5,1},{5,4},{4,7},
({3,0},{3,3},{3,6},   {5,2},{5,5},{5,8})
{3,1},{3,4},{3,7},
{3,2},{3,5},{3,8})
({4,0},{4,3},{4,6},
{4,1},{4,4},{4,7},
{4,2},{4,5},{4,8})

0 ——— 1 ——— 2

({0,0},{0,3},{0,6},   ({1,0},{1,3},{1,6},   ({2,0},{2,3},{2,6},
{0,1},{0,4},{0,7},    {1,1},{1,4},{1,7},    {2,1},{2,4},{2,7},
{0,2},{0,5},{0,8})    {1,2},{1,5},{1,8})    {2,2},{2,5},{2,8})
```

(b) Data distribution at the beginning of second phase

```
                   ({6,0},{6,3},{6,6},   ({6,1},{6,4},{6,7},   ({6,2},{6,5},{6,8},
                   {7,0},{7,3},{7,6},    {7,1},{7,4},{7,7},    {7,2},{7,5},{7,8},
                   {8,0},{8,3},{8,6})    {8,1},{8,4},{8,7})    {8,2},{8,5},{8,8})

6 ——— 7 ——— 8

        ({3,1},{3,4},        ({3,2},{3,5},
        {3,7},{4,1},         {3,8},{4,2},
        {4,4},{4,7},         {4,5},{4,8},
        {5,1},{5,4},         {5,2},{5,5},
        {5,7})               {5,8})

({3,0},{3,3},{3,6},
{4,0},{4,3},{4,6},   3 ——— 4 ——— 5
{5,0},{5,3},{5,6})

        ({0,1},{0,4},        ({0,2},{0,5},
        {0,7},{1,1},         {0,8},{1,2},
        {1,4},{1,7},         {1,5},{1,8},
        {2,1},{2,4},         {2,2},{2,5},
        {2,7})               {2,8})

({0,0},{0,3},{0,6},
{1,0},{1,3},{1,6},   0 ——— 1 ——— 2
{2,0},{2,3},{2,6})
```

An all-to-all personalized communication algorithm on a three-dimensional hypercube

([6,0] ... [6,7])   ([7,0] ... [7,7])

([2,0] ... [2,7])   ([3,0] ... [3,7])

([4,0] ... [4,7])   ([5,0] ... [5,7])

([0,0] ... [0,7])   ([1,0] ... [1,7])

(a) Initial distribution of messages

([6,0],[6,2],[6,4],[6,6],
[7,0],[7,2],[7,4],[7,6])

([6,1],[6,3],[6,5],[6,7],
[7,1],[7,3],[7,5],[7,7])

([2,0],[2,2],
[2,4],[2,6],
[3,0],[3,2],
[3,4],[3,6])

([4,1],[4,3],
[4,5],[4,7],
[5,1],[5,3],
[5,5],[5,7])

([0,0],[0,2],[0,4],[0,6],
[1,0],[1,2],[1,4],[1,6])

([1,1],[1,3],[1,5],[1,7],
[0,1],[0,3],[0,5],[0,7])

(b) Distribution before the second step

([6,2],[6,6],[4,2],[4,6],
[7,2],[7,6],[5,2],[5,6])

([7,3],[7,7],[5,3],[5,7],
[6,3],[6,7],[4,3],[4,7])

([0,2],[2,2],
[0,6],[2,6],
[1,2],[3,2],
[1,6],[3,6])

([4,1],[6,1],
[4,5],[6,5],
[5,1],[7,1],
[5,5],[7,5])

([0,0],[0,4],[2,0],[2,4],
[1,0],[1,4],[3,0],[3,4])

([1,1],[1,5],[3,1],[3,5],
[0,1],[0,5],[2,1],[2,5])

(c) Distribution before the third step

([0,6] ... [7,6])   ([0,7] ... [7,7])

([0,2] ... [7,2])   ([0,3] ... [7,3])

([0,4] ... [7,4])   ([0,5] ... [7,5])

([0,0] ... [7,0])   ([0,1] ... [7,1])
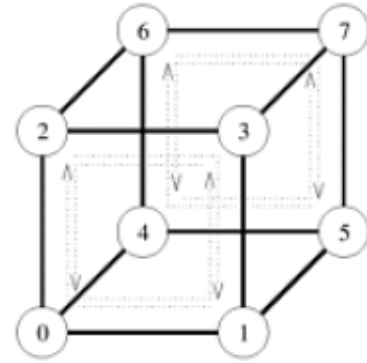
(d) Final distribution of messages

Seven steps in all-to-all personalized communication on an eight-node hypercube

(a)

(b)

(c)

(d)

(e)

(f)

(g)
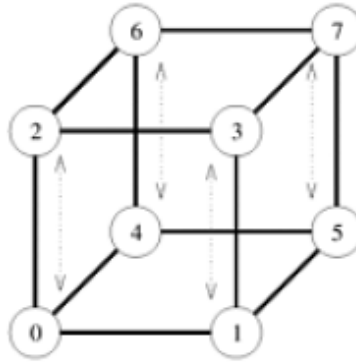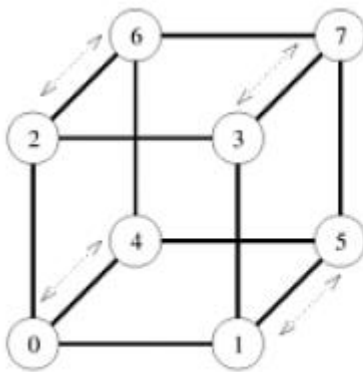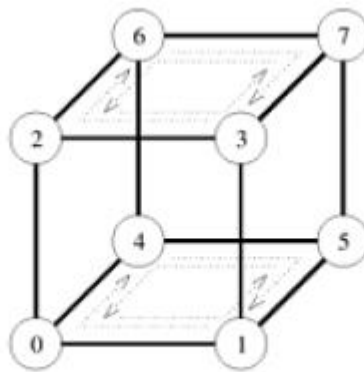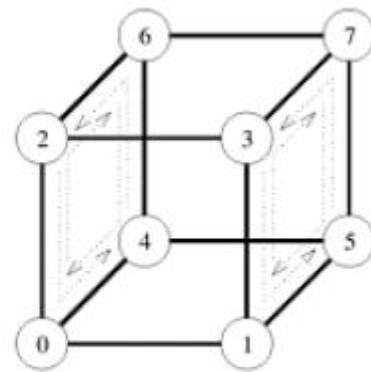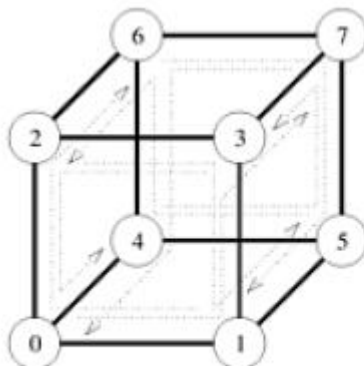
$$
\begin{array}{cccc}
0 & \longrightarrow 1 & \longrightarrow 3 & \longrightarrow 7 \\
1 & \longrightarrow 0 & \longrightarrow 2 & \longrightarrow 6 \\
2 & \longrightarrow 3 & \longrightarrow 1 & \longrightarrow 5 \\
3 & \longrightarrow 2 & \longrightarrow 0 & \longrightarrow 4 \\
4 & \longrightarrow 5 & \longrightarrow 7 & \longrightarrow 3 \\
5 & \longrightarrow 4 & \longrightarrow 6 & \longrightarrow 2 \\
6 & \longrightarrow 7 & \longrightarrow 5 & \longrightarrow 1 \\
7 & \longrightarrow 6 & \longrightarrow 4 & \longrightarrow 0 \\
\end{array}
$$