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2005 J. Opt. B: Quantum Semiclass. Opt. 7 350

(http://iopscience.iop.org/1464-4266/7/11/006)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF OPTICS B: QUANTUM AND SEMICLASSICAL OPTICS

J. Opt. B: Quantum Semiclass. Opt. 7 (2005) 350–354 doi:10.1088/1464-4266/7/11/006

Quantitative measures of entanglement in pair-coherent states

G S Agarwal[1,3] **and Asoka Biswas**[2]

[1] Department of Physics, Oklahoma State University, Stillwater, OK-74078, USA
[2] Physical Research Laboratory, Navrangpura, Ahmedabad-380 009, India

E-mail: agirish@okstate.edu and asoka@prl.ernet.in

Received 17 July 2005, accepted for publication 6 October 2005
Published 21 October 2005
Online at stacks.iop.org/JOptB/7/350

Abstract
The pair-coherent states for a two-mode radiation field are known to belong to a family of states with non-Gaussian wavefunction. The nature of quantum entanglement between the two modes and some features of non-classicality are studied for such states. The existing criterion for inseparability are examined in the context of pair-coherent states.

Keywords: pair-coherent state, non-classicality, inseparability criterion

1. Introduction

Entanglement in continuous variables has been of great interest since the celebrated paper of Einstein, Podolsky and Rosen (EPR) [1] who constructed a two-particle state which was strongly entangled both in position and momentum spaces. The EPR state has been the subject of many discussions on the non-locality of quantum mechanics. It turns out that the EPR state can be physically realized in a high gain parametric amplifier. This opened up the possibility of a variety of new experiments [2–5] using entanglement in continuous variables. The Wigner function for such states is Gaussian in position and momentum variables [6]. The Gaussian states are very special in the sense that the information on the higher-order correlations can be extracted from second-order correlations. The criterion for entanglement for these states have been formulated in terms of second-order correlations between position and momentum variables [7, 8]. Mancini *et al* [9] derived an equivalent set of criterion. A great advantage of these inequalities is that the transpose criterion has been translated into something which is directly measurable. In this paper we focus our attention on the entangled character of a family of non-Gaussian states, namely, the pair-coherent states. These states are entangled since the expression for pair-coherent states is already in Schmidt form. We examine its entanglement character in terms of the Peres–Horodecki criterion. We calculate explicitly the eigenvalues of the partial transpose of the density matrix and show that some of these are negative. We also present results on the correlation entropy and linear entropy.

To set up an experiment to quantitatively measure entanglement, one can think of quasi-probability distribution functions, namely the Glauber–Sudarshan P-function, Q-function, and the Wigner function. We study the relationship between the non-classicality, the P-function of the state and entanglement. It is known that if the P-function of the state is well behaved, then the state is separable. We study the P-function for the pair-coherent state to show the entangled nature of the state. We also check the entanglement in the pair-coherent state using the inequalities for second-order moments.

The organization of this paper is as follows. In section 2, we introduce the family of bipartite non-Gaussian states of radiation field. We describe their properties briefly. In section 3, we investigate the inseparability of the pair-coherent states in light of the Peres–Horodecki criterion and von Neumann entropies. Later we discuss the relation between the entanglement and non-classicality of the P-function. Finally we study the existing separability inequality to detect entanglement in the pair-coherent state.

2. Pair-coherent state: an entangled non-Gaussian state

The simplest examples of non-Gaussian states of the field are, the single photon states. Other examples could be states generated by excitations on a Gaussian state [10, 11]. The state which has been extensively studied for its non-classical properties and violation of Bell inequalities [12, 13] is the pair-coherent state [14]. A pair-coherent state $|\zeta, q\rangle$ is the state of

[3] On leave of absence from: Physical Research Laboratory, Navrangpura, Ahmedabad, India.

a two-mode radiation field [14] with the following properties:

$$ab|\zeta, q\rangle = \zeta|\zeta, q\rangle, \tag{1a}$$

$$(a^\dagger a - b^\dagger b)|\zeta, q\rangle = q|\zeta, q\rangle, \tag{1b}$$

where a and b are the annihilation operators associated with two modes, ζ is a complex number, and q is the degeneracy parameter. The pair-coherent state for $q = 0$ (corresponding to equal photon number in both the modes) is given by

$$|\zeta, 0\rangle = N_0 \sum_{n=0}^{\infty} \frac{\zeta^n}{n!}|n, n\rangle, \tag{2}$$

where $N_0 = 1/\sqrt{I_0(2|\zeta|)}$ and $I_0(2|\zeta|)$ is the modified Bessel function of order zero. The coordinate space wavefunction is given by

$$\begin{aligned}\langle x_a, x_b|\zeta, 0\rangle &= N_0 \sum_{n=0}^{\infty} \frac{\zeta^n}{n!}\langle x_a|n\rangle\langle x_b|n\rangle \\ &= N_0 \sum_{n=0}^{\infty} \frac{\zeta^n}{n!}\frac{1}{\sqrt{\pi}}\frac{H_n(x_a)H_n(x_b)}{2^n n!}\exp\left[-\frac{x_a^2+x_b^2}{2}\right],\end{aligned} \tag{3}$$

where $\langle x_a|n\rangle$ is a harmonic oscillator wavefunction given in terms of the Hermite polynomial as

$$\langle x_a|n\rangle = \left(2^n n!\sqrt{\pi}\right)^{-1/2} H_n(x_a)\mathrm{e}^{-x_a^2/2}. \tag{4}$$

It is clear from the expression (3) that the wavefunction of the pair-coherent state is non-Gaussian. We have shown the quadrature distribution $P(x_a, x_b) = |\langle x_a, x_b|\zeta, 0\rangle|^2$ of the state (3) in figure 1. The distribution reflects the entanglement present in the state. Note that the pair-coherent state can be obtained by projecting the two-mode coherent state

$$|\alpha, \beta\rangle = \mathrm{e}^{-(|\alpha|^2+|\beta|^2)/2} \sum_{n,m=0}^{\infty} \frac{\alpha^n\beta^m}{\sqrt{n!m!}}|n, m\rangle \tag{5}$$

onto a space with a fixed difference of the number photons in two modes. The well-known squeezed vacuum state with Gaussian wavefunction is given by

$$|\zeta\rangle_{\mathrm{TP}} = \sqrt{1-|\zeta|^2}\sum_{n=0}^{\infty}\zeta^n|n, n\rangle. \tag{6}$$

Note that the expansion coefficients are different in (2) where the coefficients decrease quickly with increase in $n!$.

3. Inseparability of the pair-coherent state

In this section we quantitatively study the entanglement in the pair-coherent state. Note that the state (2) has an obvious form of Schmidt decomposition. This reflects the fact that this state is an entangled state. In the next subsections, we examine the other criterion to give an estimate of its entanglement.

3.1. Peres–Horodecki inseparability criterion

The Peres–Horodecki inseparability criterion [15] is known to be necessary and sufficient for the (2×2) and (2×3) dimensional states, but to be only sufficient for any higher dimensional states. This criterion states that if the partial transpose of a bipartite density matrix has at least one negative eigenvalue, then the state becomes inseparable. The density matrix ρ corresponding to the state $|\zeta, 0\rangle$ (which is a infinite dimensional state) can be written as

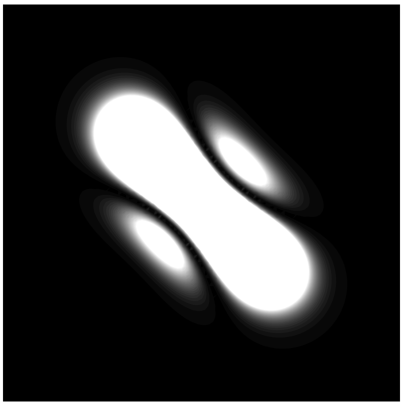

Figure 1. Contour plot of the quadrature distribution $P(x_a, x_b)$ for the pair-coherent state for $\zeta = -1$.

$$\rho = \left(\sum_{n=0}^{\infty} C_{nn}|n, n\rangle\right)\left(\sum_{m=0}^{\infty} C_{mm}^*\langle m, m|\right), \tag{7}$$

where $C_{mm} = N_0\frac{\zeta^m}{m!}$. Hence the partial transpose of ρ is given by

$$\rho_{\mathrm{PT}} = \sum_{n,m=0}^{\infty} C_{nn}C_{mm}^*|n, m\rangle\langle m, n|. \tag{8}$$

One can now calculate the eigenvalues and eigenfunctions of the matrix ρ_{PT} as follows. Let us start with the following set of two Hermitian conjugate terms for $n \neq m$ in the above equation:

$$C_{nn}C_{mm}^*|n, m\rangle\langle m, n| + C_{mm}C_{nn}^*|m, n\rangle\langle n, m|. \tag{9}$$

Diagonalizing the above block of the matrix ρ_{PT} we find the following eigenvalues:

$$\begin{aligned}\lambda_{nn} &= \frac{1}{I_0(2|\zeta|)}\frac{|\zeta|^{2n}}{(n!)^2}, && \forall n \\ \lambda_{nm}^{\pm} &= \pm\frac{1}{I_0(2|\zeta|)}\frac{|\zeta|^{n+m}}{n!m!}, && \forall n \neq m\end{aligned} \tag{10}$$

and the corresponding eigenfunctions $|n, n\rangle$ and $(|n, m\rangle \pm \mathrm{e}^{-\mathrm{i}\theta}|m, n\rangle)/\sqrt{2}$, where θ is the relative phase of the amplitudes C_{nn} and C_{mm} and is defined by $\mathrm{e}^{\mathrm{i}\theta} = C_{nn}C_{mm}^*/|C_{nn}||C_{mm}|$. Clearly the matrix ρ_{PT} has several negative eigenvalues. Hence according to the Peres–Horodecki criterion, the pair-coherent state is an inseparable state[4]. Note that if the phase of the parameter ζ is random, then the state becomes separable, as then terms corresponding to different values of n and m drop out of the double summation in (8).

[4] The eigenvalues of the partial transpose of the density matrix for the squeezed vacuum state are also given by (10) with $n!$ and $m!$ replaced by unity.

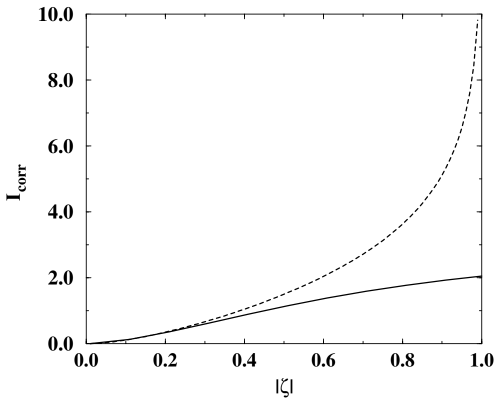

Figure 2. Variation of the correlation entropy of the pair-coherent state (solid line) and the squeezed state (dashed line) with $|\zeta|$. For large values of $|\zeta|$ the correlation entropy varies linearly with $|\zeta|$.

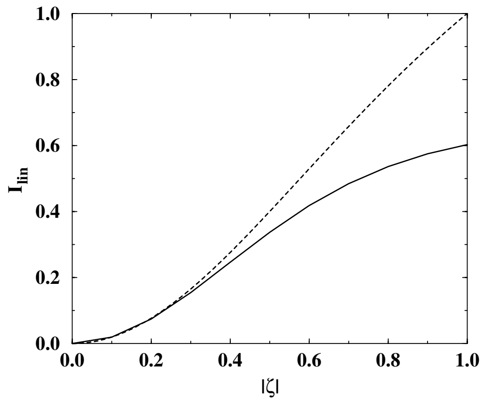

Figure 3. Variation of the linear entropy I_{lin} for the pair-coherent state (solid line) and squeezed vacuum state (dashed line) with $|\zeta|$.

3.2. Correlation entropy

The correlation entropy of a bipartite system consisting of subsystems a and b is given by [16]

$$I_{\text{corr}} = S_a + S_b - S_{ab}, \tag{11}$$

where S_k is the von Neumann entropy of the system k. If a and b are uncorrelated (separable), then I_{corr} vanishes. Now for any bipartite pure state, S_{ab} is zero. We have calculated $S_{a,b}$ for the pair-coherent state as

$$S_a = S_b = -\sum_{n=0}^{\infty} \frac{|\zeta|^{2n}}{I_0(2|\zeta|)n!^2} \log_2 \left(\frac{|\zeta|^{2n}}{I_0(2|\zeta|)n!^2} \right). \tag{12}$$

We plot the correlation entropy (11) for the pair-coherent state with $|\zeta|$ in figure 2. The correlation entropy for the pair-coherent state remains non-zero for all values of $|\zeta|$, which signifies that the state is inseparable (entangled) for all $|\zeta|$. For smaller values of $|\zeta|$ the entropy increases slowly; but at larger values of $|\zeta|$ it saturates. Note that the squeezed vacuum state (6) is a very special kind of Gaussian non-classical state, the entanglement properties of which have been much studied in the literature [3]. For a better understanding of the inseparability of the pair-coherent state, we have compared the correlation entropies of the pair-coherent state and of the squeezed vacuum state in figure 2. Clearly the correlation entropy of the squeezed vacuum state grows much faster than that of the pair-coherent state. This is because, as $|\zeta|$ approaches unity, the squeezed vacuum state becomes much more incoherent than the pair-coherent state. At $|\zeta| = 1$ the squeezed vacuum state becomes completely random, as all the possible states of the either mode then become equally probable. It is well known that the correlation entropy of a system with all basis states equally probable is $2\log_2 N$, where N is the number of possible basis states of the system. For squeezed vacuum state, as $N \to \infty$, the correlation entropy diverges for $|\zeta| = 1$. On the other hand, in the case of a pair-coherent state, the picture is different at $|\zeta| = 1$. In this case the states with lower occupation number become more probable than the states with higher occupation number. Thus the correlation entropy for the pair-coherent state remains less than that of the squeezed vacuum state for $|\zeta|$ approaching unity, suggesting that the pair-coherent state remains much more coherent than the squeezed vacuum.

3.3. Linear entropy

We further calculate the linear entropy of the pair-coherent state, which is given by $I_{\text{lin}} = 1 - \text{Tr}(\rho_k^2)$, where ρ_k is the reduced density matrix of the subsystem k. For a pure state density matrix ρ, I_{lin} vanishes as $\text{Tr}(\rho^2) = 1$. But for an entangled state, ρ_k does not have the form of a pure state density matrix. Thus, any non-zero I_{lin} provides a signature of entanglement present in the state. Note further that the linear entropy is closely related to the entanglement measure in terms of Schmidt number [17]. For a pair-coherent state, the linear entropy is given by

$$I_{\text{lin}} = 1 - \frac{1}{I_0(2|\zeta|)^2} \sum_{n=0}^{\infty} \frac{|\zeta|^{4n}}{n!^4}. \tag{13}$$

We show the variation of the quantity I_{lin} with $|\zeta|$ in figure 3. Clearly for any non-zero $|\zeta|$, I_{lin} becomes non-zero and positive, implying entanglement in the pair-coherent state. For $|\zeta| \sim 0.4$, $I_{\text{lin}} \sim 0.25$, i.e., $\text{Tr}(\rho_k^2)$ is close to unity. This implies that the state represented by ρ_k behaves more like a pure state than a mixed state. On the other hand, for $|\zeta| = 1$, $I_{\text{lin}} \sim 0.6$, which means that ρ_k represents more a mixed state than a pure state. Thus the state (2) is more entangled. Further, we have plotted the linear entropy of the squeezed vacuum state (6) in figure 3. Clearly, for small $|\zeta|$ the linear entropy of both the states show similar behaviour. But at larger values of $|\zeta|$, the linear entropy of the squeezed vacuum state is greater than for the pair-coherent state, i.e., the degree of mixedness of the pair-coherent state remains lower than the squeezed vacuum state for larger values of $|\zeta|$.

3.4. Entanglement and non-classicality of the P-function

So far we have investigated the entanglement in the pair-coherent state in terms of the Peres–Horodecki inseparability criterion and various entropies. However all these criterion are not possible to verify in experiments. We will now study the non-classicality of the P-function for the pair-coherent state as the P-function of a density matrix can be measured in experiments. For bipartite systems, the two-mode density matrix can be written in terms of the diagonal coherent state representation as

$$\rho = \int\int d^2\alpha\, d^2\beta\, P(\alpha, \alpha^*; \beta, \beta^*)|\alpha, \beta\rangle\langle\alpha, \beta|. \qquad (14)$$

It is known that if the P-function has a non-classical character, then the state is entangled. We examine the inseparability of the pair-coherent state from the point of view of the non-classicality of the P-function. The Glauber–Sudarshan P-distribution function gives a quasi-probability distribution in phase space, which can assume negative and singular values for non-classical fields. There is another distribution function called the Q-function which is related to the P-function by

$$Q(\alpha, \alpha^*; \beta, \beta^*) = \frac{1}{\pi^2}\langle\alpha, \beta|\rho|\alpha, \beta\rangle$$
$$= \frac{1}{\pi^2}\int P(\gamma, \gamma^*; \delta, \delta^*)e^{-|\gamma-\alpha|^2}e^{-|\delta-\beta|^2}\, d^2\gamma\, d^2\delta, \qquad (15)$$

which is always positive. Note that if the function $P(\gamma; \delta)$ were like a classical probability distribution, then $Q(\alpha; \beta) > 0\ \forall\alpha, \beta$. However if Q is zero, then P must become at least negative (referring to the non-classicality of the state) in some parts. Hence the exact zeros of the Q-function are also a signature for the non-classicality of the field. In order to see the non-classicality of the pair-coherent state, we examine the structure of the Q-function. The Q-function for the pair-coherent state can be calculated as

$$Q(\alpha, \alpha^*; \beta, \beta^*) = \frac{1}{\pi^2 I_0(2|\zeta|)}e^{-(|\alpha|^2+|\beta|^2)}\left|I_0\left(2\sqrt{\zeta\alpha^*\beta^*}\right)\right|^2. \qquad (16)$$

This function has zeros only if α and β are out of phase for real positive ζ, and if $2\sqrt{\zeta|\alpha||\beta|} = z_0$, where z_0 are the exact zeros of the Bessel function $J_0(z)$. The smallest few values of z_0 are 2.4048, 5.52, 8.6537, 11.7915, 14.9309, etc. The existence of these zeros proves that the pair-coherent state is a non-classical state.

However, it is worth mentioning that the notion that, if the P-function of a state is not well-behaved, then the state inseparable, is not always true. For example, for a two-mode separable Fock state $|n, m\rangle$, the P-function is not at all well-behaved. This motivates us to investigate some alternative inseparability criterion, which can be verified in experiments. We will show that existing inseparability inequalities are quite useful in detecting entanglement in the non-Gaussian state like (2) in experiments, albeit under certain conditions.

3.5. Separability inequalities

Duan *et al* [7] and Simon [8] independently have derived the separability criterion of a bipartite continuous-variable system in terms of the second-order correlations. This criterion states that if a state is separable, then the uncertainties in a pair of EPR-like operators u and v satisfy

$$M = \langle(\Delta u)^2\rangle + \langle(\Delta v)^2\rangle \geqslant m^2 + \frac{1}{m^2}, \qquad (17)$$

where

$$u = |m|x_a + \frac{1}{m}x_b, \qquad (18a)$$

$$v = |m|p_a - \frac{1}{m}p_b, \qquad (18b)$$

for any arbitrary non-zero real number m. Here $x_k = (k + k^\dagger)/\sqrt{2}$ and $p_k = (k - k^\dagger)/i\sqrt{2}$ ($k = a, b$) are the position and momentum operators for the subsystem k satisfying the commutation relation $[x_k, p_{k'}] = i\delta_{kk'}$. Conversely, violation of this criterion provides a sufficient condition for inseparability of states, albeit with a lower bound

$$\left|m^2 - \frac{1}{m^2}\right| \leqslant M < m^2 + \frac{1}{m^2}, \qquad (19)$$

which, for $m = \pm 1$, reads as

$$0 \leqslant M < 2. \qquad (20)$$

For a bipartite Gaussian state, the criterion (17) is also sufficient for separability.

Equivalent necessary and sufficient conditions for separability of Gaussian states have been derived by Englert and Wódkiewicz [18] using density operator formalism. They have shown that the positivity of the partial transposition and P-representability of the separable Gaussian states are closely related. These criterion have been experimentally verified via the interaction of a linearly polarized field with cold atoms [19], in atomic ensembles [20], and with squeezed light fields [21, 22]. Mancini *et al* [9] have shown that the separability of a state leads to the following uncertainties in a pair of EPR-like variables:

$$M_x = \langle(\Delta u)^2\rangle\langle(\Delta v)^2\rangle \geqslant 1, \qquad m = 1, \qquad (21)$$

where $u = x_a + x_b$ and $v = p_a - p_b$. Violation of this inequality provides a sufficient criterion of inseparability in Gaussian states.

We will now discuss the validity of the criterion (21) in the case of a pair-coherent state which is a non-Gaussian state. We calculate the uncertainties $\langle(\Delta u)^2\rangle$ and $\langle(\Delta v)^2\rangle$ (for $m = 1$) for the pair-coherent state $|\zeta, 0\rangle$. We find the averages $\langle x_a + x_b\rangle = 0$ and

$$\langle(x_a + x_b)^2\rangle = \langle\zeta|(1 + a^\dagger a + b^\dagger b + ab + a^\dagger b^\dagger)|\zeta\rangle$$
$$= 1 + 2|\zeta|\frac{I_1(2|\zeta|)}{I_0(2|\zeta|)} + 2|\zeta|\cos\phi. \qquad (22)$$

Here $I_1(2|\zeta|)$ is the modified Bessel function of order one and $\zeta = |\zeta|e^{i\phi}$. Thus $\langle(\Delta u)^2\rangle = \langle u^2\rangle - \langle u\rangle^2 = \langle(x_a + x_b)^2\rangle$. In a similar way, one can calculate the variance $\langle(\Delta v)^2\rangle$, which is found to be equal to $\langle(\Delta u)^2\rangle$. In figure 4, we have shown the variation of product $\langle(\Delta u)^2\rangle\langle(\Delta v)^2\rangle$ with $|\zeta|$. Clearly, the inequality (21) is violated for all $|\zeta|$. We can thus infer that the pair-coherent state is an inseparable state.

Now we will discuss whether the criterion (equation (19)) is applicable for the pair-coherent state. The total variance $M = \langle(\Delta u)^2\rangle + \langle(\Delta v)^2\rangle$ can be calculated as

$$M = \left(|m|^2 + \frac{1}{m^2}\right) + 2\left(|m|^2 + \frac{1}{m^2}\right)|\zeta|\frac{I_1(2|\zeta|)}{I_0(2|\zeta|)}$$
$$+ 4\frac{|m|}{m}|\zeta|\cos\phi. \qquad (23)$$

We show the variation of the above quantity with $|\zeta|$ in figure 5 for $\phi = \pi$. The figure shows that the total variance remains less than $|m|^2 + \frac{1}{m^2}$ for all $|\zeta|$. Thus the inequality (19) is

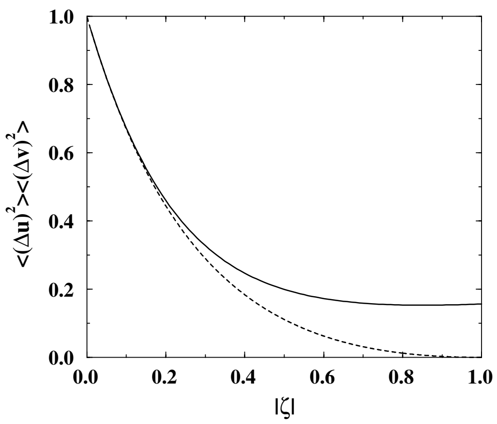

Figure 4. Variation of product of the variances of the joint position and momentum of two subsystems in a pair-coherent state (solid line) and in a two-mode squeezed vacuum state (dashed line) with the squeezing parameter $|\zeta|$, for $\phi = \pi$ and $m = 1$. The product remains less than unity for all non-zero $|\zeta|$.

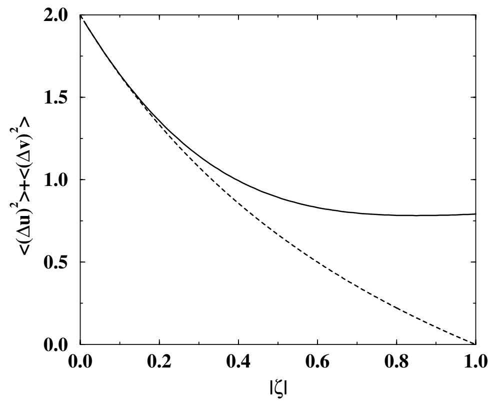

Figure 5. Variation of total variance in the EPR-like variables u and v with $|\zeta|$ for the pair-coherent state (solid line) and squeezed vacuum state (dashed line) for $m = 1$. We have chosen $\phi = \pi$.

satisfied for the pair-coherent state under the condition

$$(\text{sign of } m)(\text{sign of } \cos\phi) < 0. \tag{24}$$

Thus the criterion (19) is sufficient for the inseparability of the pair-coherent state only if the above condition is satisfied.

We have also compared the degree of violation of (21) and (19) of the pair-coherent state with that of the squeezed vacuum state (6) in figures 4 and 5. It shows that the criterion (21) and (19) are violated more by the squeezed vacuum state for larger $|\zeta|$. Thus the degree of inseparability is more for the squeezed vacuum state at larger $|\zeta|$. This is due to the fact that the expansion coefficients in the pair-coherent state (see equation (2)) decrease quickly with the increase of $n!$.

4. Conclusions

In conclusion, we have studied the inseparability of a special family of non-classical states, called the pair-coherent states which are non-Gaussian in nature. We confirmed the inseparability of pair-coherent states in the light of the Peres–Horodecki criterion and various entropies. We then demonstrated that the existing inseparability criterion (19) based on second-order correlation is applicable to these kind of non-Gaussian states only under certain constraints.

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