

On-chip generation of high-order single-photon W-states

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Quantum superposition is the quantum-mechanical property of a particle whereby it inhabits several of its possible quantum states simultaneously. Ideally, this permissible coexistence of quantum states, as defined on any degree of freedom, whether spin, frequency or spatial, can be used to fully exploit the information capacity of the associated physical system. In quantum optics, single photons are the quanta of light, and their coherence properties allow them to establish entangled superpositions between a large number of channels, making them favourable for realizations of quantum information processing schemes. In particular, single-photon W-states (that is, states exhibiting a uniform distribution of the photons across multiple modes) represent a class of multipartite maximally-entangled quantum states that are highly robust to dissipation. Here, we report on the generation and verification of single-photon W-states involving up to 16 spatial modes, and exploit their underlying multi-mode superposition for the on-chip generation of genuine random numbers.

Entanglement is one of the cornerstones of quantum mechanics. Its ability to inextricably link two or more seemingly separate entities lies at the heart of some of the most counter-intuitive quantum-inspired applications such as teleportation and quantum cryptography^{1–4}. Yet, despite its profound and multi-faceted implications, entanglement may often be a fragile quality when losses are involved. It is precisely for this reason that a certain class of maximally entangled multipartite configurations—the so-called W-states—has recently attracted considerable attention in the field of quantum computing and information science. These W-states are characterized by a coherent superposition of all the qubits involved, with equal probability amplitudes. As it turns out, such configurations exhibit the remarkable property of being intrinsically robust with respect to decoherence. Initially introduced for a three-qubit system⁵, this concept can be readily generalized to ensembles of N entangled qubits⁶. By virtue of their unique characteristics, quantum-optical W-states have since been suggested as a testbed for exploring fundamental aspects associated with the non-locality of single-particle quantum states^{7–15}. In their quantum optical representation, W-states describe configurations where single photons are coherently distributed among N spatially separated paths, or modes^{16,17}. In combination with the field coherence afforded by optics, such spatial quantum ‘parallelism’ holds great promise for a number of applications, ranging from complex quantum networks to robust quantum key distribution and communication schemes¹⁷. Along similar lines, it was recently demonstrated that single-photon W-states, when used as non-classical probes, can in principle facilitate the error-free readout of an arbitrary number of classical bits¹⁸. Up to now, the preparation of higher-order W-states of light has remained a challenging task, often involving complex bulk-optical set-ups^{16,18–20}. Clearly of interest will be the development of new

strategies through which quantum optical W-states can be reliably and efficiently synthesized on an integrated platform.

In the present work, we experimentally realize high-order W-states by judiciously manipulating the dynamic evolution of photons in systems of evanescently coupled waveguides^{21,22}. In doing so, we transform a single photon launched at the input of the device into a uniform coherent superposition of spatial modes. As a result, the initially localized photon emerges through any of the N output ports of the integrated device with exactly the same probability. To this end, two complementary approaches are presented that are capable of producing either even or odd single-photon W-states of, in principle, arbitrarily large order. Finally, we propose, implement and verify a W-state-based scheme for the direct generation of random bit sequences without the need for post-processing, and the capability for high bit rates due to the multiple output modes^{23–27}.

Generation of single-photon W-states

The uniform coherent distribution of a single photon over an ensemble of N channels corresponding to a quantum optical W-state $|W_N\rangle$ can be described in terms of the bosonic creation operators a_n^\dagger of the respective waveguide’s modes $n = 1 \dots N$ as $|W_N\rangle = N^{-1/2} \sum_{n=1}^N \exp(i\phi_n) a_n^\dagger |0\rangle$. In this context, ϕ_n are arbitrary relative phases and $|0\rangle$ denotes the vacuum state of the system. Let us first consider the case of an odd number $N = 2c - 1$ of channels. By virtue of the coupling taking place between adjacent channels, a single photon state $a_c^\dagger |0\rangle$ launched into the central waveguide c is gradually distributed across the lattice. This propagation along the longitudinal coordinate z is governed by the evolution operator, $U(z) = \exp(-iz\mathcal{M})$, where \mathcal{M} represents the z -invariant coupling matrix. This formalism is identical to the one employed in continuous-time quantum walks^{21,28}. The resulting state at a given z is characterized by the

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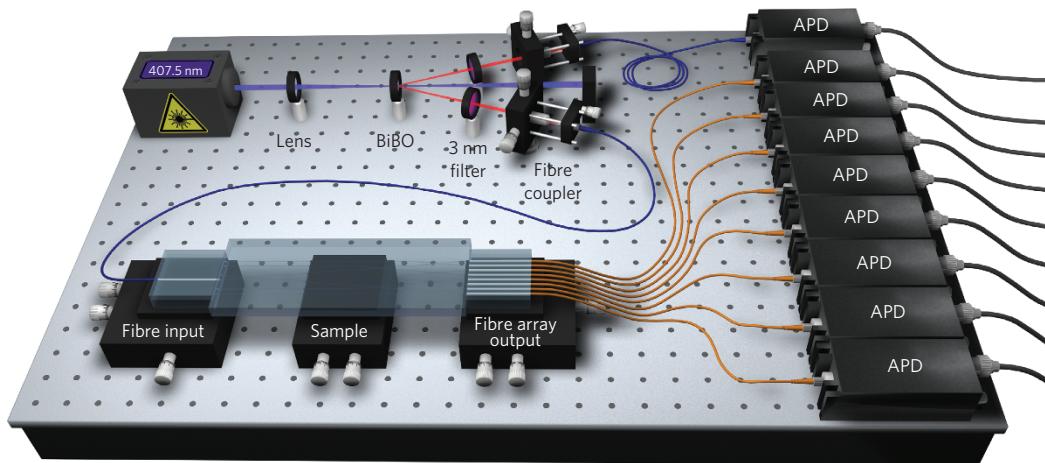


Figure 1 | Schematic of the experimental set-up. A heralded single-photon source at a wavelength of 815 nm was implemented by means of spontaneous parametric downconversion from a pump laser at 407.5 nm. Photons emerging at the output of the device were collected via a fibre array and subsequently fed into single-photon detectors (avalanche photodiodes, APDs) triggered by the herald.

complex probability amplitudes $U_{c,n}(z)$:

$$|\psi(z)\rangle = \left(\sum_{n=1}^N U_{c,n}(z) a_n^\dagger \right) |0\rangle \quad (1)$$

Evidently, $|\psi\rangle$ becomes a W-state when $|U_{c,1}(z_W)|^2 = \dots = |U_{c,N}(z_W)|^2$; that is, the probability of detecting this photon is the same for all sites at a specific propagation distance z_W . As recently shown, this condition can indeed be fulfilled for any arbitrary odd number of elements N by appropriately varying the coupling strengths in a symmetric fashion around site c (ref. 29).

Naturally, in systems with an even number N of channels, no such central site exists around which a symmetric evolution $U_{c,n}(z) = U_{c,N-n+1}(z)$ may be realized. To overcome this challenge, we break the z -invariance of the system by selectively enabling or suppressing the tunnelling of photons between specific lattice sites. In doing so, we make use of the fact that the coherent evolution in a pair of coupled waveguides periodically gives rise to entangled states of the type $|\psi(z_{BS}/2)\rangle = 2^{-1/2}(|10\rangle + i|01\rangle)$. We note that one can therefore obtain the state $|W_2\rangle$ by terminating the interaction between the channels at half a beating length z_{BS} . It readily follows that any even-numbered W-state $|W_N\rangle$ with $N = 2^m$ can be generated directly by cascading m stages of such integrated 50/50 couplers, or ‘beam splitters’³⁰, without the need for any post-selection process.

To demonstrate the feasibility of these approaches, we implemented systems for the generation of single-photon W-states spanning $N = 2, 4, 5, 8$ and 16 channels, respectively, by means of the femtosecond laser writing technique in fused silica glass^{22,31–34}. The devices were used in conjunction with heralded single photons obtained by spontaneous parametric downconversion³⁵ in a BiB_3O_6 crystal. A commercial V-groove fibre array was used to collect the photons from the individual output ports to be counted and then feed them to the respective avalanche photodiodes (Fig. 1). Subsequently, the photon probability distribution $P_n = \langle a_n^\dagger a_n \rangle$ was obtained from the count rates. We determined the ratio of two count events compared to single click events to be 10^{-4} at the output of the system used for $N = 2, 4$ and 8, which guarantees that multiphoton events have no significant influence on the measured statistics. As the ratio of single clicks to zero-photon events (that is, isolated detections of a herald photon) was measured to be 0.03, the possibility of weak coherent input states can be faithfully excluded. Similar ratios were obtained for the other devices. Moreover, by applying a herald as trigger, the ratio of dark counts to heralded counts is 10^{-9} , so the dark counts have a negligible influence on the

measurements. All arrangements were fabricated to match the standard fibre array spacing of 127 μm at the output facet. In the case of the $N = 5$ configuration, this was achieved by means of a three-dimensional fan-out interface structure designed to effectively suppress any additional crosstalk between the guides (Fig. 2a). Following the method outlined in ref. 29, the inter-channel coupling coefficients in the $N = 5$ arrangement were chosen in accordance with a length $z_W = 5$ cm of the functional region. Figure 2c shows the photon probability distribution measured at the output facet, illustrating the high fidelity of the device with $\bar{P}_{N=5} = (20.0 \pm 0.9)\%$.

Taking into account the hierarchical nature of the even- N configuration, we designed a discrete network of integrated 50/50 couplers that allows for the synthesis of two-, four- or eight-partite W-states by selecting the appropriate input channel of the device (Fig. 2b). For instance, if a single photon is coupled to input port 1, it encounters a single beam splitter, yielding $|W_2\rangle = 2^{-1/2}(a_1^\dagger + ia_2^\dagger)|0\rangle$. Input port 2, on the other hand, conveys the photon to two consecutive stages of couplers. The output state is thus given by $|W_4\rangle = 4^{-1/2}(ia_1^\dagger + a_2^\dagger + ia_3^\dagger - a_4^\dagger)|0\rangle$. Similarly, input port 3 yields the state

$$|W_8\rangle = 8^{-1/2}(-a_1^\dagger + ia_2^\dagger + a_3^\dagger + ia_4^\dagger - a_5^\dagger + ia_6^\dagger - a_7^\dagger - ia_8^\dagger)|0\rangle \quad (2)$$

Making use of the vertical degree of freedom, we realized a network facilitating the entanglement of 16 modes in two planes connected via a vertical coupler. Note that, in contrast to discrete-step quantum walks^{33,36}, no interference occurs between the individual instances during the build-up of the W-state. Figure 2c shows the experimentally measured photon number probability distributions for $|W_2\rangle, |W_4\rangle, |W_8\rangle$ and $|W_{16}\rangle$. As in the $|W_5\rangle$ device, the high uniformity in the output probabilities, that is, $\bar{P}_{N=2} = (50.0 \pm 0.5)\%$, $\bar{P}_{N=4} = (25.0 \pm 0.5)\%$, $\bar{P}_{N=8} = (12.5 \pm 0.7)\%$ and $\bar{P}_{N=16} = (6.25 \pm 0.8)\%$, is indicative of the fidelity of the device. The standard deviations given here represent the deviations from the ideal uniform probability distribution, which are mainly caused by the fabrication tolerances of the device. In contrast, the Poissonian count statistics would yield much smaller uncertainties, for example, $\pm 0.02\%$ in the case of $N = 8$.

Verification of multipartite entanglement

Conventionally, W-states are identified by means of state tomography. Given that the complexity of this approach dramatically increases with the number of entangled qubits, it is desirable to find an alternative characterization scheme with more favourable scaling behaviour.

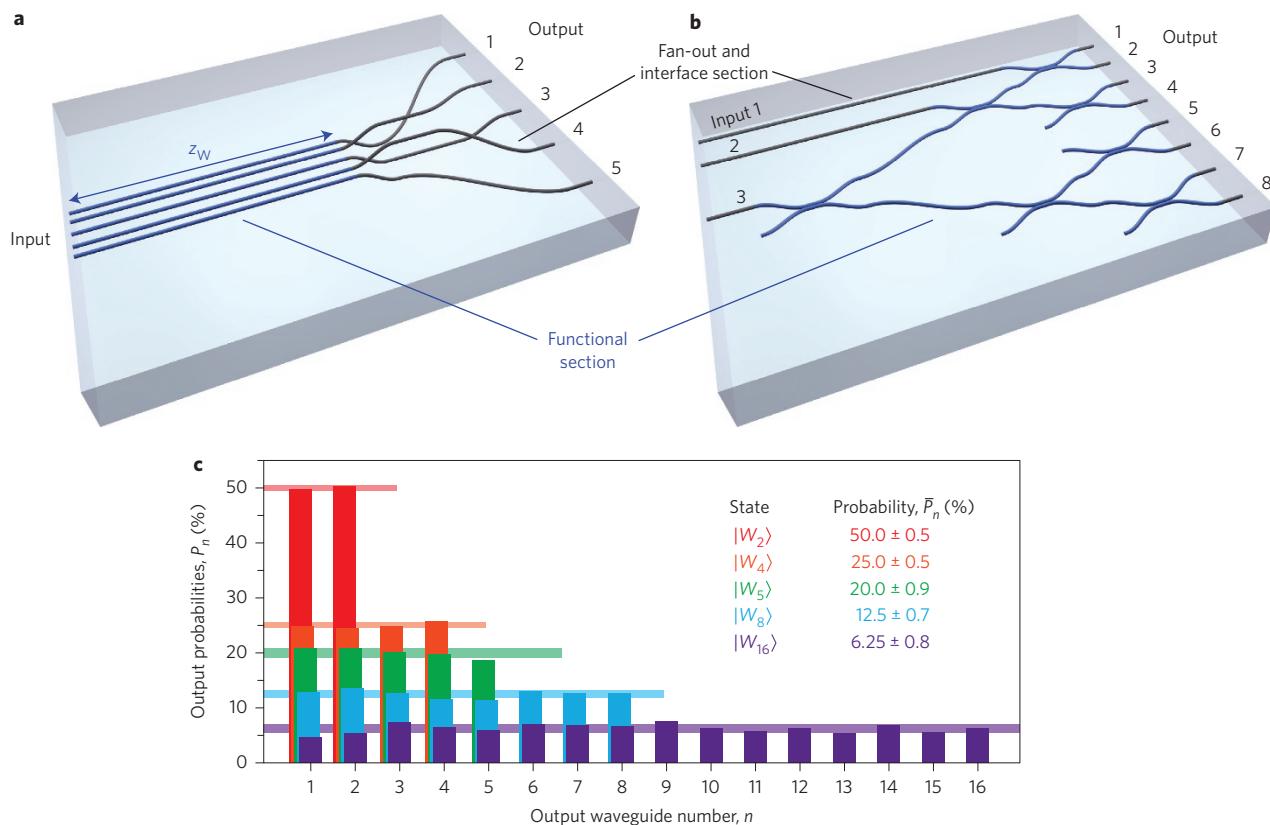


Figure 2 | Generation of high-order photonic W-states on an integrated platform. **a**, Odd number N of entangled channels. Here, the $|W_5\rangle$ state is obtained through the coherent evolution of a single photon injected into the central site of a lattice comprising N identical waveguides. Appropriately engineered couplings continuously transform the initially localized photon into the desired equal superposition of all N channels after distance z_W . Subsequently, a fan-out section designed to arrest the propagation dynamics serves as an interface to the output ports. **b**, Even number N of entangled channels. A hierarchical network of 50/50 couplers transforms the single-photon input state into W-states of order $N = 2^m$, where m is the number of sequential couplers. The specific structure shown here is capable of producing any of states $|W_2\rangle$, $|W_4\rangle$ and $|W_8\rangle$, depending on the choice of input port. **c**, Experimentally obtained probability distributions of states $|W_2\rangle$, $|W_4\rangle$, $|W_5\rangle$, $|W_8\rangle$ and $|W_{16}\rangle$. In the $N = 5$ case, the length of the functional section was $z_W = 5$ cm. The system used to synthesize the $N = 16$ state consisted of a pair of realizations of the structure in **b** arranged in parallel horizontal planes and connected to the input by a vertical 50/50 coupler. Standard deviations (from the uniform distribution) of the individual measurements are shown as horizontal bars.

To this end, we make use of our *a priori* knowledge that the output state is the result of the evolution of a single-photon state. To provide unequivocal evidence that the produced states are indeed W-states (that is, coherent superpositions of single-photon states), we therefore verify that coherence is preserved during propagation. In other words, if W-states are considered as an input for a linear optical system (for example, an optical interferometer), the response in terms of the average photon number distribution will be given by the modulus square of the coherent sum of that system's Green's functions. This impulse response describes the individual output distributions corresponding to each single-mode state of which the W-state is composed^{16,37}. In contrast, when the same optical system is excited by an incoherent mixture of single-waveguide Fock states, its response will be described by the incoherent sum of the average photon number distributions arising from the individual Fock state excitations of the system.

Along these lines, we make use of interferometric arrangements based on coupled waveguides. Interestingly, the very unitary $U(z_W)$ used to synthesize the $N = 5$ state displays a high sensitivity with respect to the coherence of an input state (Fig. 3a). The analytical model shows that an incoherent excitation would result in a uniform distribution over all channels at the interferometer output (Fig. 3b, top right). Due to the symmetry of $U(z_W)$, this condition can be experimentally mimicked by separately injecting (at $z = 0$) single photons into all input channels of the interferometer and

summing all individually measured average photon number distributions at $z = 2z_W$ (Fig. 3b, bottom right). In contrast, after propagating a distance z_W , the coherent W-state should yield the characteristic interference pattern shown in Fig. 3b (top left). Indeed, the measured average photon number distribution (Fig. 3b, bottom left) closely matches the theoretical predictions, strongly indicating that the device in fact produces the desired W-state $|W_5\rangle$.

Following a different approach in the even- N arrangement, we directly verify multipartite entanglement for W-state $|W_8\rangle$. For this purpose we follow a scheme similar to the one described in ref. 38, where one applies a projection into a basis of N orthogonal W-states (all varying in their phase distribution), including the desired state (equation (2)). This is achieved by a unitary transformation U , which maps each of the W-states to a specific output channel, in our case the desired state to the eighth mode $U|W_8\rangle = |0_1, 0_2, \dots, 1_8\rangle = a_8^\dagger|0\rangle$. The density matrix of an unknown state ρ will evolve as $\tilde{\rho} = U\rho U^\dagger$, such that its fidelity with $|W_8\rangle$ is given by the count rate in the eighth channel:

$$\begin{aligned} \mathcal{F} &= \langle W_8 | \rho | W_8 \rangle = \langle W_8 | U^\dagger U \rho U U^\dagger | W_8 \rangle \\ &= \langle 0_1, 0_2, \dots, 1_8 | \tilde{\rho} | 0_1, 0_2, \dots, 1_8 \rangle = \tilde{\rho}_{88} \end{aligned} \quad (3)$$

It can be shown that all biseparable states have to satisfy the condition $\mathcal{F} \leq (N - 1)/N$ (refs 39,40). Hence, a violation of this

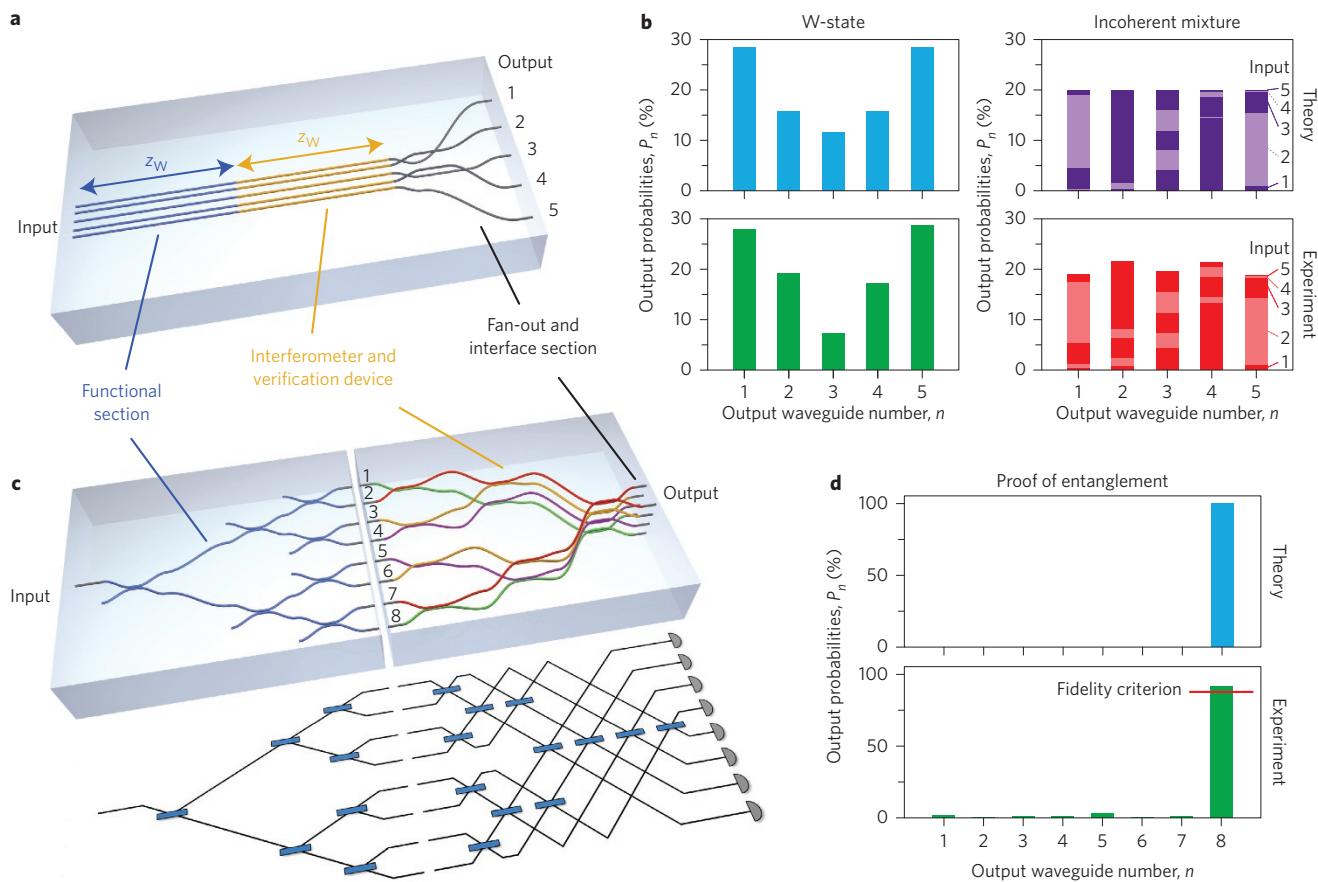


Figure 3 | Interferometric validation of W-states, given *a priori* knowledge of a single-photon input state. **a**, To demonstrate coherence between the outputs of the $N = 5$ device, we allowed the created state to undergo interference while propagating through a second section of length z_W before separating the channels in the fan-out arrangement. **b**, Theoretical probability distributions (top row) assuming coherent (left) or fully incoherent (right) conditions at the end of the functional section. The measured interferogram (bottom, left) proves a coherent superposition of the channels, that is, state $|W_5\rangle$. As a comparison, the incoherent case, with its characteristic absence of interference, was observed by subsequently launching single photons into each channel of the array (alternating colour) and adding the probability distributions, incoherently (bottom right). **c**, In the $N = 8$ case, an additional sequence of integrated 50/50 couplers were attached to verify entanglement of the $|W_8\rangle$ output. **d**, The theoretical output probability distribution of this arrangement under perfect conditions exhibits a projection to only one output (top); the experimental results clearly exceed the required fidelity condition of 7/8 (bottom).

fidelity criterion is a sufficient condition for the presence of genuine N -mode entanglement in state ρ .

In this vein we fabricated an interferometric set-up to provide unitary U (Fig. 3c). The fidelity is then simply given by the detection probability in channel eight. By attaching the interferometer to the chip used for the generation of $|W_8\rangle$, we experimentally obtained $\mathcal{F} = 0.918 \pm 0.004 > 7/8$, which clearly violates the criterion and thus proves the eight-mode entanglement of the $|W_8\rangle$ state. Note that, although the deviation from unity is mainly caused by deviations from the uniform probability distribution, the uncertainty of the fidelity is determined by the Poissonian count statistics in the output channels of the verification device.

Generation of genuine random numbers

Finally, we demonstrate a direct application of the W-state generation scheme by exploiting the intrinsic uncertainty in the uniform photon number distribution of the synthesized state $|W_8\rangle$ for the generation of genuine random numbers. To this end, we associate a number $m = n - 1$, with $n \in [1, 8]$, to every successful detection of a photon at output port n , which corresponds to a three-digit binary number. It follows that random numbers with more than three digits can be readily obtained by concatenating the results of multiple measurements. We note that, for a given

number of output channels N , the largest number presentable by M measurements scales as N^M . In our $|W_8\rangle$ arrangement, a measurement of merely three photons can yield a random number between 0 and $8^3 - 1 = 511$, whereas by using a $|W_{16}\rangle$ state one could span the range 0 ... 4,095. In comparison to existing two-mode schemes²⁵, this provides the advantage of an increased bit rate for a given photon flux. Moreover, as the latter cannot be increased arbitrarily due to detector saturation, the multimodal scheme offers a larger maximally attainable bit rate for a given detector technology. Importantly, this method for W-state-based random bit generation only requires the conversion from photon counting to the digital format, and no post-processing. The maximum deviation from the ideal 50/50 ratio between ‘zeros’ and ‘ones’ in our experiment was found to be 3.444×10^{-4} for a sample sequence of 1 Mbit at an average rate of 60 kbit s^{-1} . The true statistical randomness of this implementation was verified by using the standard statistical test suite for random number generators provided by NIST⁴¹ to characterize an arbitrarily chosen bit sequence (see Supplementary Information). We emphasize that the bit-generation rate in our setting is limited only by the fluence of the single-photon source and the speed of the photon-counting circuitry, and not by the photonic system utilized in generating the W-states. Hence, the bit-generation rate can be readily improved

by several orders of magnitude, as brighter single-photon sources and gigahertz single-photon sources and photon-counting circuitry become more available⁴².

Conclusions

We have successfully demonstrated the generation of high-order optical single-photon W-states for odd ($N=5$) as well as even ($N=2, 4, 8, 16$) numbers of entangled channels. Our approach provides a robust, monolithically integrated solution to this task, and the number of modes can be readily tuned. For the case of $N=8$, we measured a fidelity of 91.8% between the output and target state $|W_8\rangle$. This exceeds the upper bound for biseparable states, thus verifying entanglement among all modes. Subsequently, we utilized the intrinsic properties of multipartite W-states to generate quantum-based random numbers. Our results may pave the way towards the robust and versatile sources needed for the wide-scale deployment of quantum-cryptographic communication protocols.

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Author contributions

M.G., R.H., A.P.-L. and D.N.C. conceived the idea. M.G., R.H. and R.K. designed the samples and performed the measurements. A.P.-L., M.G., R.H. and R.K. analysed the data. A.S. supervised the project. All authors discussed the results and co-wrote the manuscript.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to A.S.

Competing financial interests

The authors declare no competing financial interests.

CORRIGENDUM

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In the version of this Article originally published, the contribution of Demetrios N. Christodoulides to conceiving the idea behind the work was not acknowledged in the Author Contributions section. This error has now been corrected in the HTML and PDF versions of the Article.