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IEE/CSE 598 SPRING 24

COMPUTING FOR DATA DRIVEN OPTIMIZATION

Problem Statement: Consider a homogeneous fleet of vehicles, where all vehicles are identical. The set of customers is represented by numbers 1 through n, and there is a designated depot denoted as 0 to which all vehicles must start and return. For each customer $i = 1, \cdots, n$, the service time is denoted as s_i, representing the time a vehicle needs to stop upon arrival to service the customer. Travel times from customer i to customer j are denoted as t_{ij}, and t_{0j} represents the travel time from the depot to customer j. The assumption is that travel times are symmetric, i.e., $t_{ij} = t_{ji}$ for all $i, j = 0, 1, \cdots, n$. Each vehicle has a capacity q, and each customer i has a demand d_i. Assume $d_i \leq q$ for all $i = 1, \cdots, n$. Additionally, each customer i has a time window $[l_i, u_i]$, and a vehicle must arrive at the customer before u_i. If the vehicle arrives before the time window opens, it must wait until l_i to service the customer. The time window for the depot is $[l_0, u_0]$, representing the scheduling horizon. Vehicles may not leave the starting depot before l_0 and must return to the returning depot no later than time u_0. All parameters, including q, l_i, u_i, d_i, s_i, and t_{ij}, are assumed to be non-negative integers. The objective is to design a set of routes that minimizes the total number of vehicles utilized, while meeting the following criteria:

• Each customer is serviced exactly once.

• Every route originates and ends at vertex 0.

• The time windows of the customers and capacity constraints of the vehicles are fulfilled.

1. (Mixed-Integer Programming Formulation) You are going to formulate the illustrated
problem as a mixed-integer linear program with the following two sets of variables,
x and y:

• For each i, j = 0, · · · , n and each vehicle k
xijk = {1, if vehicle k travels directly from i to j
0, otherwise.
• For each customer i = 1, · · · , n and vehicle k, yik denotes the time vehicle k starts
service for customer i. In case vehicle k does not service customer i, yik has no meaning and consequently its value is considered irrelevant. We assume l0 = 0 and let 0 be the starting time of each vehicle k from the depot.

**(a) Find an upper bound V on the total number of vehicles that need to be utilized
to service all the customers.**

The upper bound V on the total number of vehicles needed can be calculated using the formula:

V = $\left\lceil \frac{D}{q} \right\rceil$

where,

V = total number of vehicles needed

D = Total demand

= $\Sigma_{i=1}^{n} d_i$, where di represents the demand of customer i.

Q = Capacity of each vehicle.

V = $\left\lceil \frac{\Sigma_{i=1}^{n} di}{q} \right\rceil$

(b) Formally write the mixed-integer linear programming problem and explain the

objective function and each constraint. (Hint: $\Sigma_{k=1}^{V}\Sigma_{i=1}^{n}\ x_{0ik}$ **will compute the total number of vehicles utilized; you would need to set the start time at the depot**
for each vehicle at zero; you may use Mij = max{ui + si + tij − lj , 0} for each i, j
for linearizing the time-window constraint).

Mixed – Integer Linear Programming Formulation:

Objective Function:

The objective is to minimize the total number of vehicles utilized.

Minimize $\Sigma_{k=1}^{V}\Sigma_{i=0}^{n}\Sigma_{j=0}^{n}\ x_{ijk}$

s.t.

1. Each customer must be serviced exactly once.

$\Sigma_{j=0}^{n}\ x_{ijk} = 1$ for all i = 1,...., n; and k=1,...., V

2. Each route originates and ends at vertex 0.

$\Sigma_{i=1}^{n}\ x_{0ik} = 1$ for all k=1,....,V

$\Sigma_{j=1}^{n}\ x_{j0k} = 1$ for all k=1,....,V

3.Time Window Constraints are satisfied.

$y_{ik} - y_{jk} + M_{ij}(1 - x_{ijk}) \geq s_i$ for all i,j = 1,...,n; and k = 1,.....,V.

where, $M_{ij} = \max\{u_i + s_i + t_{ij} - l_i, 0\}$.

4. Capacity constraints of the vehicles are fulfilled.

$\Sigma_{i=1}^{n}\ d_i\ x_{ijk} \leq q$ for all k=1,...,V

5. Binary Variables:

$x_{ijk} \in \{0,1\}$ For all i,k=0, ...,n; and k=1,....., V

$y_{ik} \geq 0$ for all i=1,....,n; and k=1,...., V

2. (Alternative Formulation with Huge Number of Variables) Suppose you have the set of all feasible routes P of vehicles that meet the capacity and the time window require-
ments. Each route p ∈ P is represented by a binary matrix [ˆxijp]i,j=0,··· ,n, where

ˆxijp = { 1, if vehicle k travels directly from i to j

0, otherwise

Objective Function:

The objective is to minimize the total number of vehicles utilized, which is represented as the sum of the z_p over all feasible routes p.

Minimize: $\Sigma_{p \in P} z_p$

s.t.

1. Capacity Constraint: The total demand of customers on each route does not exceed the vehicle capacity.

$$\Sigma_{i=1}^{n} d_i \, a_{ip} \leq q z_{p,} \qquad \forall p \in P$$

where d_i is the demand of customer i
a_{ip} is a binary variable indicating whether customer i is on route p, and
q is the vehicle capacity.
2. Time window constraint: The time window constraints are met for each customer on each route.

$$y_{ik} + s_i + t_{ij} \leq u_i + M\left(1 - x_{ijk}\right), \forall i, j = 1, \ldots., n; \forall k; p \in P$$

$$y_{ik} + s_i + t_{ij} \leq l_i - M\left(1 - x_{ijk}\right), \forall i, j = 1, \ldots., n; \forall k; p \in P$$

where y_{ik} is the start time for servicing customer i by vehicle k, s_i is the service time for customer i, t_{ij} is the travel time from customer i to customer j, u_i and l_i are the upper and lower bounds of the time window for customer i respectively, and M is a sufficiently large constant.

3. Route membership constraint: Each customer is assigned to exactly one route:

$$\Sigma_{p \in P} a_{ip} = 1, \forall i = 1, \dots, n$$

4. Vehicle utilization constraint: The number of vehicles utilized on each route is a non- negative interger.

$$z_p \geq 0, \forall p \in P$$

3. (Column Generation) You are going to solve the reformulated problem using column generation.
(a) An initial set of trivial routes, denoted by P′, can be constructed as done for the cutting stock problem. Explain how.

To build an initial set of trivial routes, like the cutting stock problem, we can consider each customer demand separately. Each route will consist of only one customer. Therefore, for each customer i = 1, 2, ..., n, we create a route that starts from the depot, serves customer i, and returns to the depot. These initial routes cover the basic needs of each customer individually.

Mathematically, we denote the set of customers as {1, 2,....., n }. Each route p corresponds to a single customer $\hat{\imath}$ and is represented as p= {i}. Therefore, the initial set of trivial routes P' is defined as:

P' = {{1},{2} ...., {n}}

(b) Write the initial master problem with P′

Minimize $\Sigma_{p \in P'} z_p$

s.t. $\Sigma_{p \in P'} a_i^p z_p \geq d_i \quad \forall i = 1,2, \dots., n$

$z_p \geq 0, \forall p \in P'$

Here, z_p represents the number of vehicles utilized on route p, and a_{ip} is a binary indicator variable indicating whether customer i is included in route p. The objective is to minimize the total number of vehicles utilized, subject to meeting the demand constraints for each customer.

(c) Let πi denote the dual variable of the master problem, which is associated with the demand constraint for customer i. Formulate the pricing problem

that aims to find a route and its associated column (a1, · · · , an)T with the most negative reduced cost.

The pricing problem aims to minimize the reduced cost by finding the most negative reduced cost route. It can be formulated as:

Minimize $\Sigma_{i=1}^{n} \pi_i \, a_i \; - \; c$

s.t. $\Sigma_{i=1}^{n} a_i \; \leq 1$

$a_i \in \{0,1\} \, \forall i = 1,2, \dots , n$

where πi represents the dual variable associated with the demand constraint for customer i in the master problem.

The variable a_i denotes the inclusion of customer i in the route, while c signifies the reduced cost of the route.

4. (Implementation) You are going to implement the two approaches for solving the problem in the Final-Take-Home.ipynb file using Julia. The file includes data where the last customer serves as the depot, and travel times can be computed as the Euclidean distance based on the coordinate information. Following the instruction therein, encode the MILP formulation in Question 1 using Julia JuMP, and implement the column generation method in Question 3

Result of MILP implementation

```
Root node processing (before b&c):
  Real time             =    0.11 sec. (105.75 ticks)
Parallel b&c, 16 threads:
  Real time             =    0.00 sec. (0.00 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    0.11 sec. (105.75 ticks)
CPLEX Error  1217: No solution exists.
CPLEX Error  1217: No solution exists.
CPLEX Error  1217: No solution exists.
Version identifier: 22.1.1.0 | 2022-11-27 | 9160aff4d
Infeasibility row 'c1':  0  = -39.
Presolve time = 0.09 sec. (93.70 ticks)
```

Out[16]:

```
* Solver : CPLEX

* Status
  Result count       : 0
  Termination status : INFEASIBLE
  Message from the solver:
  "integer infeasible"

* Candidate solution (result #1)
  Primal status      : NO_SOLUTION
  Dual status        : NO_SOLUTION
  Objective bound    : 0.00000e+00

* Work counters
  Solve time (sec)   : 1.09000e-01
  Simplex iterations : 0
  Barrier iterations : 0
  Node count         : 0
```