LAB 3

1. Consider the initial value problem

$$\begin{aligned} y' &= -100y + 100t + 101, \\ y(0) &= y_0. \end{aligned}$$

Given y_0, h, and N, where the input y_0 specifies the initial value, h is the size of the uniform time stepping and N is the number of time steps for which the approximate solutions are to be computed, use the 2-step Adams Moulton method started with y_1 chosen as (i) the exact value $y(h)$ and (ii) the value obtained using the Euler's method, to solve this IVP to obtain approximations y_n^{EX} and y_n^{EM} respectively for y at uniform time steps $t_n = nh$, $n = 0, 1, \cdots, N$. The first line of your Matlab implementation should read

```
function [y,yEX,yEM] = lab3_ex1(y0,h,N)
```

2. Suppose that a body of mass m is orbiting a second body of much larger mass M. From Newton's law of motion and gravitation, the orbital trajectory $(x(t), y(t))$ is described by the system of second-order ODEs

$$\begin{aligned} x'' &= -GMx/r^3, \\ y'' &= -GMy/r^3, \end{aligned}$$

where G is the gravitational constant and $r = (x^2 + y^2)^{1/2}$ is the distance between the center of mass of the two bodies. For this exercise, we assume that the units are such that $GM = 1$ and we solve the corresponding system of differential equations with the initial conditions

$$x(0) = 1 - e, \quad y(0) = 0, \quad x'(0) = 0, \quad y'(0) = \left(\frac{1+e}{1-e}\right)^{1/2},$$

where e is the eccentricity of the resulting elliptical orbit which has period 2π. Try values $e = 0$ (which should give a circular orbit), $e = 0.5$ and $e = 0.9$. For each case, solve the problem for at least one period and obtain output through enough intermediate points (constant time steps) to draw a smooth plot of orbital trajectory. Make separate plots of x versus t, y versus t, and y versus x. If you trace the trajectory through several periods, does the orbit tend to wander or remain steady. Check how well your numerical solutions conserve the energy

$$E(t) = \frac{(x'(t))^2 + (y'(t))^2}{2} - \frac{1}{r(t)}$$

and the angular momentum

$$A(t) = x(t)y'(t) - y(t)x'(t).$$

For the numerical solution, implement two set of schemes, namely, k step Adams-Bashford methods in the file named `lab3_ex2a.m` and k step Adams-Moulton methods in the file `lab3_ex2b.m`; each of them for $k = 1, 2$ and 3.

(a) For Adams-Bashford (AB) methods, the first line of your Matlab implementation should read

```
function [E, A] = lab3_ex2a(P, N, k, e)
```

where the input P is the number of periods for which the solution is to be computed, N is the number of steps used for one period (that is, the constant time step size is $h = 2\pi/N$), k is the order of the Adam-Bashford methods (implement for $k = 1, 2$ and 3 using `if-elseif-else`) and e is the eccentricity used in the initial condition. The output E and A are vectors of size $(PN+1)$ that respectively contain the energy and angular momentum at each time step. To start the k step Adam-Bashford method, appropriately use lower order Adam-Bashford methods (that is, to obtain the solution at $t_1 = h$, use AB1, to obtain the solution at $t_2 = 2h$, use AB2, and so on).

(b) For Adams-Moulton (AM) methods, the first line of your Matlab implementation should read

```
function [E, A] = lab3_ex2b(P, N, M, k, e)
```

where the input P is the number of periods for which the solution is to be computed, N is the number of steps used for one period (that is, the constant time step size is $h = 2\pi/N$), M is the number of fixed point iterations used for the non-linear solve at each time step, k is the order of the Adam-Moulton methods (implement for $k = 1, 2$ and 3 using `if-elseif-else`) and e is the eccentricity used in the initial condition. The output E and A are vectors of size $(PN+1)$ that respectively contain the energy and angular momentum at each time step. To start the k step Adam-Moulton method, appropriately use lower order Adam-Moulton methods (that is, to obtain the solution at $t_1 = h$, use AM1, to obtain the solution at $t_2 = 2h$, use AM2, and so on).