MTH 430: Numerical Analysis & Scientific Computing II
Department of Mathematics and Statistics
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LAB 7

Solve the following two-point boundary value problems

1.
$$\begin{aligned} u'' + u &= f, \quad -1 < t < 1, \\ u(-1) &= u(1) = 0, \end{aligned}$$

   with $f \in C([-1,1])$. Check your code on at least three problems with different choices of f (for which you know how to obtain the exact solution).

2.
$$\begin{aligned} u'' + u &= \frac{2(u')^2}{u}, \quad -1 < t < 1, \\ u(-1) &= u(1) = (e + e^{-1})^{-1}. \end{aligned}$$

using the collocation method with:

1. the grid $t_i = -1 + ih, i = 0, \ldots, n$, $h = 2/n$, and $\mathcal{P}_n$, the space of polynomials of degree n or less, as the approximation space. Implement this in two different ways, using the monomial basis $\{1, t, \ldots, t^n\}$ and using the Lagrange basis $\{\ell_0^{(n)}, \ell_1^{(n)}, \ldots, \ell_n^{(n)}\}$ where
$$\ell_i^{(n)} = \prod_{\substack{0 \le j \le n \\ j \ne i}} \frac{t - t_j}{t_i - t_j}.$$

2. the grid $t_i = -1 + ih, i = 0, \ldots, n$, $h = 2/n$, and, for the approximation space, use the space of piecewise linear polynomials where the elements restrict to linear polynomials when restricted to any interval $[t_i, t_{i+1}]$.

3. (bonus) the grid $t_j = \cos^{-1}(j\pi/n), j = 0, \ldots, n$, and $\mathcal{P}_n$ as the approximation space. Implement with the Chebyshev polynomials as basis, that is, use $\mathcal{P}_n = \text{span}\{T_0, T_1, \ldots, T_n\}$ where $T_k(t) = \cos(k \cos^{-1}(t))$.