MTH 430: Numerical Analysis & Scientific Computing II
Department of Mathematics and Statistics
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LAB 2

I. Consider the initial value problem

$$y' = -100y + 100t + 101$$
$$y(0) = y_0.$$

Given $y_0, h,$ and N, use Euler's method and Backward Euler's method to solve this IVP to obtain an approximations y_n^E and y_n^{BE} respectively for y at uniform time steps $t_n = nh, n = 0, 1, \ldots, N$ and plot the exact solution $y(t_n)$ and the approximate solution y_n^E on one graph and $y(t_n)$ and y_n^{BE} on a separate graph.

The first line of your Matlab implementation file `lab1_exercise1.m` should read

```
function [] = lab2_exercise1(y0, h, N)
```

where the input `y0` specifies the initial value y_0, h is the size of the uniform time stepping and N is the number of time steps for which the approximate solutions are to be computed.

Run this code for $y_0 = 1$, $y_0 = 0.99$ and $y_0 = 1.01$ with $h = 0.1$ and $N = 10$ and report your observations.

II. Recall the Lab Exercise II from Lab 1 – The populations of two species, a pray denoted by y_1 and predator denoted by y_2 can be modeled by the non-linear ODE

$$y' = \begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} y_1(\alpha_1 - \beta_1 y_2) \\ y_2(-\alpha_2 + \beta_2 y_1) \end{bmatrix} = f(y).$$

The parameters α_1 and α_2 are natural birth and death rates in isolation of prey and predators, respectively, and the parameters β_1 and β_2 determine the effect of interactions between the two populations, where the probability of interaction in proportional to product of the populations.

Run your Lab 1 Matlab function (in `lab1_exercise2.m` implementing the numerical solution of this problem using Euler's method)

```
function [ ] = lab1_exercise2(N, T, a1, a2, b1, b2, y10, y20)
```

for the final time $T = 250$ with parameter values $\alpha_1 = 1$, $\beta_1 = 0.1$, $\alpha_2 = 0.5$, $\beta_2 = 0.02$, and the initial population $y_1(0) = 100$ and $y_2(0) = 10$ while varying the value of N; take $N = 4000, 3500, 3000, 2500, 2000$ and 1500. Report and explain your observation.

Now, implement a Matlab function that uses the Backward Euler's method with uniform time steps $t_n = nT/N, n = 0, \ldots, N$, to solve the IVP between $t = 0$ and $t = T$. in the file `lab2_exercise2.m` with first line as

```
function [ ] = lab2_exercise2(N, T, a1, a2, b1, b2, y10, y20)
```

where the input `N` specifies the grid size N, the input `T` specifies the final time T, the inputs, `a1, a2, b1, b2` correspond to ODE parameters $\alpha_1, \alpha_2, \beta_1, \beta_2$ respectively and the inputs `y10, y20` specify the initial conditions $y_1(0)$, $y_2(0)$ respectively. Your function should again plot each of the two populations as a function of time (on the same plot) and, on a separate graph, plots the trajectory of the points $(y_1(t), y_2(t))$ in the plane as a function of time.

For the parameter values $\alpha_1 = 1$, $\beta_1 = 0.1$, $\alpha_2 = 0.5$, $\beta_2 = 0.02$, and the initial population $y_1(0) = 100$ and $y_2(0) = 10$, display the plots for $N = 4000, 3500, 3000, 2500, 2000, 1500, 1000$ and 500. Comment on your observations.