MTH 430: Numerical Analysis & Scientific Computing II
Department of Mathematics and Statistics
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LAB 8

Consider the problem

$$\begin{aligned}&\Delta u(x, y) = 0, \quad (x, y) \in \Omega\\ &u(x, y) = \log((x-3)^2 + (y-2)^2), \quad (x, y) \in \Gamma = \partial\Omega,\end{aligned}$$

where Ω is a domain that does not contain the origin $(3, 2)$. Verify that $u(x, y) = \log((x - 3)^2 + (y - 2)^2)$ solves this Dirichlet boundary value problem exactly. To solve this problem numerically using the finite difference method, utilize the h-grid $\Omega_h = \Omega \cap \mathbb{R}^2_h$ where $\mathbb{R}^2_h = \{(mh, nh) : m, n \in \mathbb{Z}\}$.

Write a functions `lab8_exercise1` and `lab8_exercise2` in the following formats

```
function [ x, y, u ] = lab8_exercise1(N)
function [ x, y, u ] = lab8_exercise2(N)
```

to implement its numerical solution using the five point finite difference approximation Δ_h of the Laplacian Δ, that is,

$$\Delta v(x, y) \approx \Delta_h v(x, y) = \frac{v(x + h, y) + v(x - h, y) + v(x, y + h) + v(x, y - h) - 4v(x, y)}{h^2},$$

where $h = 1/(2N)$. The input parameter `N` is used to specify the grid spacing h $(= 1/(2N))$. The output vectors `x, y` are one dimensional arrays containing the x and y components of the grid Ω_h arranged linearly in an order of your choice, and the output array `u` contains approximate solution u_h on the h-grid Ω_h in the same order. In `lab8_exercise1`, take

$$\Omega = (-1, 1) \times (-1, 1),$$

while for `lab8_exercise2`, take the rotated L-shaped domain

$$\Omega = ((0, 1) \times (0, 1)) \setminus ((0, 1/2] \times (0, 1/2]).$$